Stats 506, F18, Problem Set 3

Chen Xie, chenxie@umich.edu November 17, 2018

Question 1

Part a

The percent of homes have stucco construction as the major outside wall material within each division is shown in the table 1 and figure 1 below.

Table 1: Proportion of homes with stucco construction within each census division in 2015. Estimates are based on the residential energy consumption survey.

| Census Division | % Stucco Homes (95% CI) |
|--------------------|--------------------------|
| Mountain South | 64.2% (55.4, 73.0) |
| Pacific | 44.6% (41.3, 47.9) |
| Mountain North | $16.6\% \ (10.2,\ 23.0)$ |
| South Atlantic | 10.6% (7.8 , 13.4) |
| West North Central | 4.9% (0.9, 8.8) |
| West South Central | 3.0% (1.6, 4.3) |
| Middle Atlantic | 2.1% (0.6, 3.5) |
| New England | 1.2% (0.0, 2.8) |
| East North Central | $0.7\% \ (\ 0.1,\ 1.2)$ |
| East South Central | $0.4\% \ (\ 0.0,\ 1.2)$ |
| | |

From this table, we can know that the division 'Mountain South' has the highest proportion, and 'East South Central' has the lowest proportion.

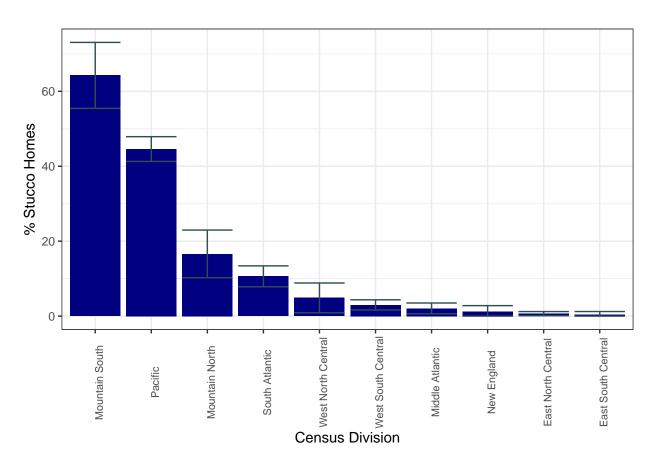


Figure 1: Estimated percent of homes within each census division with major wall type of stucco.

Part b

The average total electricity usage in kilowatt hours in each division is shown in the table 2 and figure 2 below.

 $\label{thm:constraint} \mbox{Table 2: } \mbox{$Average$ annual electricity utilization by $Census$ $Division$ in $kwh/home.}$

| Census Division | Average Electricity Usage, kwh/home (95% CI) |
|--------------------|--|
| East South Central | 14,536, (13,320 - 15,752) |
| West South Central | 14,324, (13,495 - 15,153) |
| South Atlantic | 13,447, (12,904 - 13,989) |
| West North Central | 10,524, (9,635 - 11,413) |
| Mountain South | 10,442, (7,950 - 12,934) |
| East North Central | 9,129, (8,730 - 9,528) |
| Middle Atlantic | 8,465, (8,071 - 8,860) |
| Mountain North | 8,384, (7,121 - 9,648) |
| Pacific | 8,100, (7,750 - 8,450) |
| New England | 7,515, (6,472 - 8,557) |

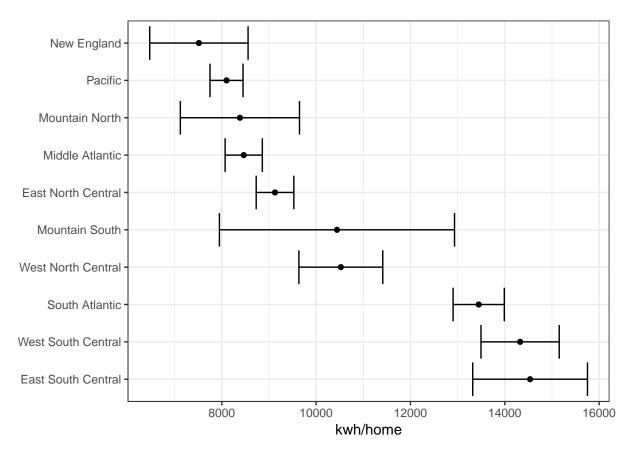


Figure 2: Estimated average annual electricity usage in khw/home for each of 10 census divisions.

The average total electricity usage in kilowatt hours in each division for urban and rural is shown in the table 3 and figure 3 below.

Table 3: Average electricity utilization in kwh per home for urban and rural areas within each census division.

| Census Division | Rural, kwh/home (95% CI) | Urban, kwh/home (95% CI) |
|--------------------|---------------------------|---------------------------|
| East South Central | 16,333, (14,088 - 18,578) | 13,747, (12,197 - 15,298) |
| West South Central | 16,317, (14,067 - 18,567) | 13,629, (12,852 - 14,405) |
| South Atlantic | 15,942, (14,839 - 17,045) | 12,725, (12,134 - 13,316) |
| West North Central | 14,174, (12,608 - 15,740) | 9,467, (8,722 - 10,211) |
| Pacific | 14,115, (12,001 - 16,229) | 7,349, (6,905 - 7,793) |
| East North Central | 13,500, (12,022 - 14,978) | 7,980, (7,552 - 8,408) |
| Middle Atlantic | 12,223, (10,633 - 13,814) | 7,987, (7,659 - 8,316) |
| Mountain North | 9,356, (5,698 - 13,014) | 8,099, (7,396 - 8,803) |
| New England | 9,001, (6,766 - 11,236) | 6,964, (5,918 - 8,010) |
| Mountain South | 8,610, (6,536 - 10,685) | 10,743, (8,178 - 13,308) |

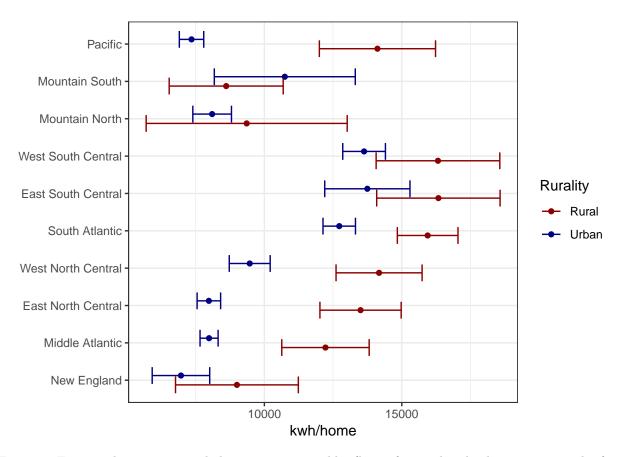


Figure 3: Estimated average annual electricity usage in khw/home for rural and urban areas in each of 10 census divisions.

Part c

The disparity between urban and rural in internet access within each division is shown in the table 4 and figure 4 below.

The division 'Mountain South' has the largest disparity between urban and rural areas in terms of the proportion of homes with internet access.

Table 4: Urban and rural disparity in internet access for the ten US Census Division in 2015.

| Census Division | Urban Internet Access, % (95% CI) | Rural Internet Access, % (95% CI) | Difference, $\%$ (95% CI) |
|--------------------|-----------------------------------|-----------------------------------|---------------------------|
| Mountain South | 85.3% (81.3, 89.2) | $66.7\% \ (58.3, 75.2)$ | 18.5% (7.2, 29.8) |
| East South Central | $78.4\% \ (70.5, 86.2)$ | $69.0\% \ (63.5, 74.6)$ | $9.3\% \ (-1.4, \ 20.1)$ |
| West North Central | 88.0% (84.6, 91.4) | 80.3% (71.5, 89.2) | 7.7% (-2.5, 17.8) |
| Mountain North | 87.4% (82.0, 92.9) | 81.9% (73.8, 90.0) | 5.5% (-6.2, 17.2) |
| West South Central | 81.6% (76.4, 86.8) | $76.5\% \ (72.1,\ 80.9)$ | $5.1\% \ (-2.3,\ 12.5)$ |
| Pacific | 88.7% (86.2, 91.2) | 85.3% (77.4, 93.1) | 3.4% (-4.5, 11.4) |
| South Atlantic | 85.3% (82.6, 88.0) | 82.0% (76.3, 87.8) | 3.3% (-3.5, 10.1) |
| New England | 87.6% (82.5, 92.6) | 85.8% (82.4, 89.2) | $1.8\% \ (-2.5, \ 6.0)$ |
| East North Central | 86.3% (83.8, 88.7) | 86.2% (81.6, 90.8) | $0.0\% \ (-5.3, \ 5.4)$ |
| Middle Atlantic | 89.3% (83.9, 94.8) | 91.3% (85.3, 97.3) | -1.9% (-9.1, 5.2) |

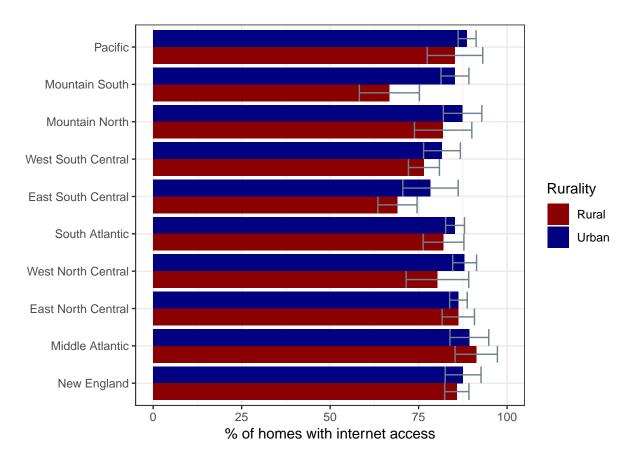


Figure 4: Urban and rural disparity in internet access for the ten US Census Division in 2015

Part d

The average number of bedrooms for different type of housing units is shown in the table 5 and figure 5. We can find that the type 'Single-family detached house' will have most bedrooms.

 $\begin{tabular}{ll} Table 5: Average numbers of bedrooms for different type of housing units \end{tabular}$

| Type of housing units | Average numbers of bedrooms, (95% CI) |
|--|---------------------------------------|
| Mobile home | 2.609, (2.59 - 2.627) |
| Single-family detached house | 3.235, (3.235 - 3.235) |
| Single-family attached house | 2.567, (2.558 - 2.575) |
| Apartment in a building with 2 to 4 units | 1.755, (1.739 - 1.772) |
| Apartment in a building with 5 or more units | 1.546, (1.536 - 1.556) |

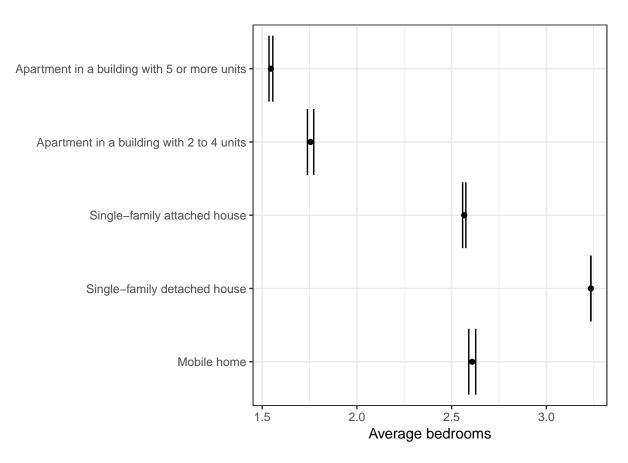


Figure 5: Average numbers of bedrooms for different type of housing units

Question 2

Part a & b

The function in Part a is called 'mc_pval'.

I choose that $\Sigma = I_{p \times p}$, Σ is a $p \times p$ Identity matrix, and $\sigma = 10$.

The estimates of parameters of between the function in Part a is the same as the result of R function lm().

The estimates of parameters of X1 to X6 is shown in the talbe 6, to relance = 1e-3,

Table 6: Compare estimates of parameters of X1 to X6 between the function in Part a and the R function lm(), tolerance=1e-3

| | Pval of Part_a | Pval of lm() | Difference |
|----|----------------|--------------|------------|
| X1 | 0.0042 | 0.0043 | 0 |
| X2 | 0.0002 | 0.0002 | 0 |
| X3 | 0.0926 | 0.0929 | 0 |
| X4 | 0.0948 | 0.0952 | 0 |
| X5 | 0.0203 | 0.0205 | 0 |
| X6 | 0.1762 | 0.1765 | 0 |

Part c & d & e

The Monte Carlo estimates of the family wise error rate, the false discovery rate, the sensitivity, and the specificity of different types of adjusted p-values in multiple comparison are shown in the table 7 and figure 6.

When p-value is not adjusted, the family wise error rate is extremely large, and the false discovery rate is large also. But the adjusted p-values can fix this problem in some levels, and also improve the specificity.

Table 7: Monte Carlo estimates of the family wise error rate, the false discovery rate, the sensitivity, and the specificity of different types of adjusted p-values in multiple comparison

| | Family wise error rate | False discovery rate | Sensitivity | Specificity |
|------------------------|------------------------|----------------------|-------------|-------------|
| Unadjusted | 0.989 | 0.3344 | 0.8465 | 0.9497 |
| Bonferroni | 0.001 | 0.0051 | 0.0212 | 1 |
| Holm | 0.001 | 0.0051 | 0.0212 | 1 |
| BH(Benjamini-Hochberg) | 0.203 | 0.0405 | 0.4805 | 0.9975 |
| BY(Benjamini-Yekuteli) | 0.007 | 0.0034 | 0.152 | 0.9999 |

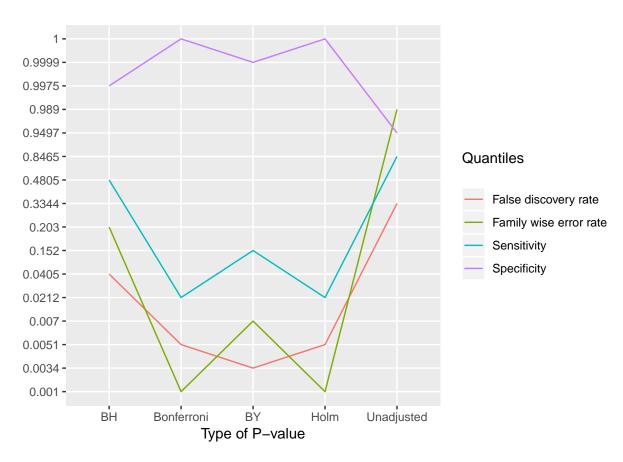


Figure 6: Monte Carlo estimates of the family wise error rate, the false discovery rate, the sensitivity, and the specificity of different types of adjusted p-values in multiple comparison

Question 3

Part a

Computed by data.table, the table of univariate regression coefficients with groups is shown below.

 $\label{thm:coefficients} \begin{tabular}{ll} Table 8: The univariate regression coefficients with groups computed by data. table \end{tabular}$

| Number of cylinders(Group) | Beta of Displacement | Beta of Gross horsepower | Beta of Weight |
|----------------------------|----------------------|--------------------------|----------------|
| 4 | -0.1351 | -0.1128 | -5.6470 |
| 6 | 0.0036 | -0.0076 | -2.7801 |
| 8 | -0.0196 | -0.0142 | -2.1924 |

Part b

 ${\bf beta_dt_fn}$ is the function for an arbitrary dependent, independent, and grouping variables. The results computed by ${\bf beta_dt_fn}$ is shown below. It matches the Part a.

Table 9: The univariate regression coefficients with groups computed by a function computing by data.table

| Number of cylinders(Group) | Beta of Displacement | Beta of Gross horsepower | Beta of Weight |
|----------------------------|----------------------|--------------------------|----------------|
| 4 | -0.1351 | -0.1128 | -5.6470 |
| 6 | 0.0036 | -0.0076 | -2.7801 |
| 8 | -0.0196 | -0.0142 | -2.1924 |

Part c

Computed by dplyr summarize_at, the table of univariate regression coefficients with groups is shown below.

Table 10: The univariate regression coefficients with groups computed by dplyr

| Number of cylinders(Group) | Beta of Displacement | Beta of Gross horsepower | Beta of Weight |
|----------------------------|----------------------|--------------------------|----------------|
| 4 | -0.1351 | -0.1128 | -5.6470 |
| 6 | 0.0036 | -0.0076 | -2.7801 |
| 8 | -0.0196 | -0.0142 | -2.1924 |

Part d

beta_dplyr_fn is the function for an arbitrary dependent, independent, and grouping variables. The results computed by **beta_dplyr_fn** is shown below. It matches the Part c.

Table 11: The univariate regression coefficients with groups computed by a function computing by data. table

| Number of cylinders(Group) | Beta of Displacement | Beta of Gross horsepower | Beta of Weight |
|----------------------------|----------------------|--------------------------|----------------|
| 4 | -0.1351 | -0.1128 | -5.6470 |
| 6 | 0.0036 | -0.0076 | -2.7801 |
| 8 | -0.0196 | -0.0142 | -2.1924 |