MANCHESTER 1824
The University of Manchester

MATH10202 Linear Algebra A 2021-22
2^{nd} Semester - Week 1

Tutorial Class

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Content

- Contraposition
- Injective, Surjective, Bijective
- Invertible

Domain, Codomain and Range

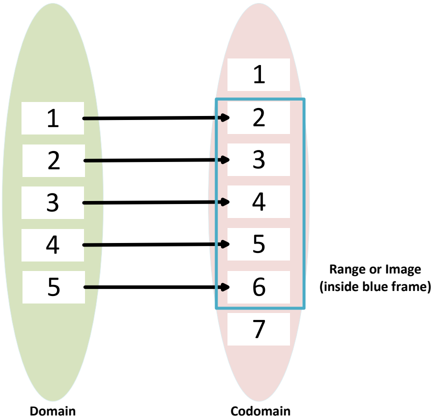

Figure: Domain, Codomain and Range.

Contraposition

"A conditional statement is true if, and only if, its contrapositive is true."

The condition statement is denoted by $P \to Q$, then

- Contraposition: $\neg Q \to \neg P$
- Converse: $Q \to P$
- Negation: $\neg(P \to Q)$

The converse is the contrapositive of the inverse, so they share the same truth value but are not always dependent on the truth of the original proposition. However, the negation always reaches the opposite conclusions to the original proposition, that is, if the negation is true the original proposition is false and vice versa.
Question: Is it correct?: "All flying pigs carry umbrellas."

Contraposition

We can write the statement "All flying pigs carry umbrellas." as "$P \to Q$" where

- P: "Some Pigs can fly."
- Q: "Those pigs carry umbrellas."

So "$\neg Q \to \neg P$" is "Those pigs that do not carry umbrellas cannot fly". Obviously, the statement of "$\neg Q \to \neg P$" is correct, so "$P \to Q$" is correct, then the statement of "All flying pigs carry umbrellas." is correct.

Injective, Surjective, Bijective

Denote by a function $f : A \to B$,

- Injective: f is injectiuve if, for all $x, y \in A$, $f(x) = f(y) \Rightarrow x = y$.
- Surjective: f is surjective if, for all $y \in B$, there is some $x \in A$ such that $f(x) = y$.
- Bijective: f is bijective if it is both injective and surjective.

Injective, Surjective, Bijective

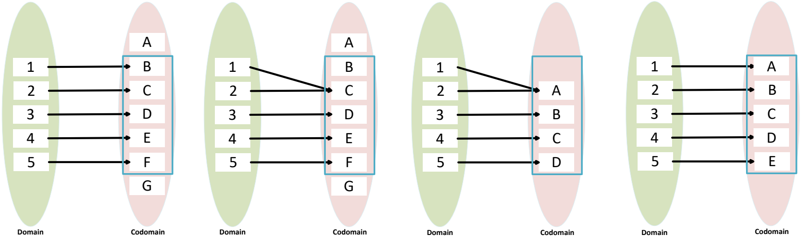

Figure: Question: Injective, Surjective or Bijective?

Injective, Surjective, Bijective

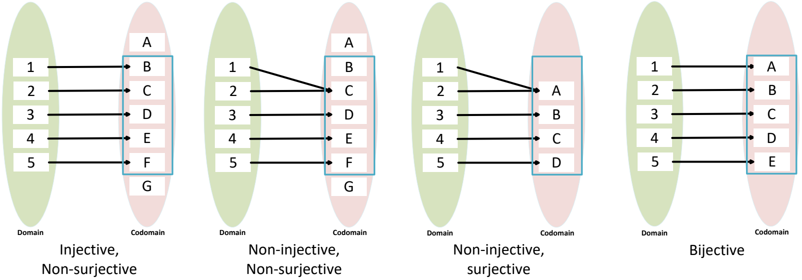

Figure: Answer.

Invertible

A function $f : A \to B$ is invertible if there is a function $g : B \to A$ such that $fg = \mathbb{1}_B$ and $gf = \mathbb{1}_A$ where $\mathbb{1}_A$ and $\mathbb{1}_B$ denote the identity functions on A and B respectively.

Hint: Recall that a function map any element of domain to only one output in the codomain.
Note: A function is bijective if and only if it is invertible.

End

Thanks for your attention!