Problem Set 2

Analytical Politics II

Question 1:

a.

To check Nash equilibrium, we should check whether π_A choose policy $\frac{1}{2}$ is best response of π_B choose policy $\frac{1}{2}$, vice versa.

① if π_A choose $\frac{1}{2}$,

$$U_B = \begin{cases} -(1-\frac{1}{2})^2 = -\frac{1}{4} & \text{if B choose } 0 \\ \frac{1}{2}\cdot-(1-\frac{1}{2})^2 + \frac{1}{2}-(0-\frac{1}{2})^2 = -\frac{1}{4} & \text{if B choose } \frac{1}{2} \\ -(1-\frac{1}{2})^2 = -\frac{1}{4} & \text{if B choose } 1 \end{cases}$$
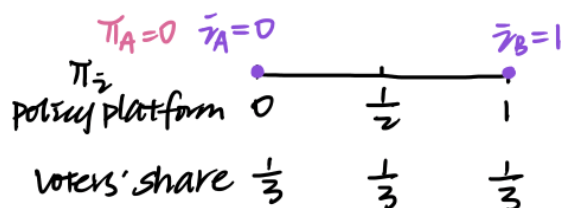
② if π_B choose $\frac{1}{2}$,

$$U_A = \begin{cases} -(0-\frac{1}{2})^2 = -\frac{1}{4} & \text{if A choose } 0 \\ \frac{1}{2}\cdot-(1-\frac{1}{2})^2 + \frac{1}{2}-(0-\frac{1}{2})^2 = -\frac{1}{4} & \text{if A choose } \frac{1}{2} \\ -(0-\frac{1}{2})^2 = -\frac{1}{4} & \text{if A choose } 1 \end{cases}$$

→ Thus, if one candidate chose policy $\frac{1}{2}$, no matter another candidate choose what policy, the final policy that implemented is policy $\frac{1}{2}$.

→ If we consider winning probability, if A choose $\frac{1}{2}$ policy, the best response for B is also choose policy $\frac{1}{2}$, because B choose 0 or 1 will lose the election under $\frac{2}{3}$ fraction voter vote for A with policy $\frac{1}{2}$.

→ To sum up, if one of candidate choose policy $\frac{1}{2}$, the best respons of another candidate is chose $\frac{1}{2}$ policy as well with probability of winning $=\frac{1}{2}$.

b.

$$\pi_A = 0 \quad \bar{\gamma}_A = 0 \qquad\qquad \bar{\gamma}_B = 1$$

$\pi_{\frac{1}{2}}$

policy platform $\quad 0 \qquad \frac{1}{2} \qquad 1$

voters' share $\quad \frac{1}{3} \qquad \frac{1}{3} \qquad \frac{1}{3}$

if party A commits to policy $\pi_A = 0$:

1) if party B commits to policy $\pi_B = 1$
there's a tie as $\frac{1}{2}$ voters vote for $\pi_A = 0$
while the rest $\frac{1}{2}$ voters vote for $\pi_B = 1$
in this case,

$$u_B(x_B, \pi) = -(x_B - \pi)^2$$
$$= (\tfrac{1}{2})\left[-(1-0)^2\right] + (\tfrac{1}{2})\left[-(1-1)^2\right]_0$$
$$= -\tfrac{1}{2}$$

2) if party B commits to policy $\pi_B = \frac{1}{2}$
$\frac{2}{3}$ voters would vote for party B, while $\frac{1}{3}$ voters
vote for party A.
$\Rightarrow$ Party B wins and the policy $\pi_B = \frac{1}{2}$
would be implemented.
in this case,

$$u_B(x_B - \pi) = -(1 - \tfrac{1}{2})^2 = -\tfrac{1}{4}$$

3) if party B commits to policy $\pi_B = 0$
Both party A and party B commits to policy $\pi = 0$.
$\rightarrow$ There's a tie between two parties.
$\rightarrow$ Finally, policy $\pi = 0$ will be implemented.
in this case,

$$u_B(x_B - \pi) = (\tfrac{1}{2})\left[-(1-0)^2\right] + (\tfrac{1}{2})\left[-(1-0)^2\right]$$
$$= -\tfrac{1}{2} - \tfrac{1}{2}$$
$$= -1$$

Overall, when party A commits to policy $\pi_A = 0$,
committing to policy $\pi_B = \frac{1}{2}$ can bring party B higher utility
in contrast with committing to policy $\pi_B = 1$ (i.e., $-\tfrac{1}{4} > -\tfrac{1}{2} > -1$).
That's to say, party B's best response to $\pi_A = 0$ is $\pi_B = \frac{1}{2}$
instead of $\pi_B = 1$. Thus, $(\pi_A, \pi_B) = (0, 1)$ is not a NE.

c.

Suppose $(\Pi_A, \Pi_B) = (0, 1)$
Theoretically, if all voters vote, there would be a tie.

1) For voters prefer $\Pi = 0$:

    i. if $\Pi_A = 0$ is implemented:
      $U_{0 \text{ voters}}(\text{preferred}) = -(0-0)^2 = 0$

    ii. if $\Pi_B = 1$ is implemented:
      $U_{0 \text{ voters}}(\text{not preferred}) = -(0-1)^2 = -1$

    Expressive utility $= U_{0 \text{ voters}}(\text{preferred}) - U_{0 \text{ voters}}(\text{not})$
    (for 0 voters) $= |0-(-1)| = 1$

    Thus, these voters will vote if $c < 1$.

2) For voters prefer $\Pi = \frac{1}{2}$:
    They're indifferent between $\Pi_A = 0$ and $\Pi_B = 1$.

    i. if $\Pi_A = 0$ is implemented:
      $U_{\frac{1}{2} \text{ voters}} = -(\frac{1}{2} - 0)^2 = -\frac{1}{4}$

    ii. if $\Pi_B = 1$ is implemented:
      $U_{\frac{1}{2} \text{ voters}} = -(\frac{1}{2} - 1)^2 = -\frac{1}{4}$

    Expressive Utility (for $\frac{1}{2}$ voters) $= |(-\frac{1}{4}) - (-\frac{1}{4})| = 0$

    Thus, those voters would not vote as $c > 0$.

3) For voters prefer $\Pi = 1$:

    i. if $\Pi_B = 1$ is implemented:
      $U_{1 \text{ voters}}(\text{preferred}) = -(1-1)^2 = 0$

    ii. if $\Pi_A = 0$ is implemented:
      $U_{1 \text{ voters}}(\text{not preferred}) = -(1-0)^2 = -1$

    Expressive utility (for 1 voters) $= |0-(-1)| = 1$

    Those voters would always like to vote as $c < 1$.

d.

0 voter's net value from voting is $1 - 0_1^2 > 0$

$\Rightarrow \frac{1}{3}$ voter with ideal point $= 0$ will participate

$\Rightarrow \frac{1}{3}$ fraction of voter vote for A with policy 0

$\frac{1}{2}$ voter's net value from voting is $0 - 0_1^2 < 0$ (Not participate in vote)

1 voter's net value from voting is $1 - 0_1^2 > 0$

$\Rightarrow \frac{1}{3}$ voter with ideal point $= 1$ will participate

$\Rightarrow \frac{1}{3}$ fraction of voter vote for B with policy 1

$\frac{1}{2}$ probability A win, $\frac{1}{2}$ probability B win

$E(U_A) = \frac{1}{2} \cdot -(0-0)^2 + \frac{1}{2} \cdot -(0-1)^2 = \frac{1}{2} \cdot -1 = -\frac{1}{2}$

e.

For ideal point = 0 voter.

$$U_V(0, \tfrac{1}{2}) = -(0-\tfrac{1}{2})^2 = -\tfrac{1}{4} \text{ if A win at policy } \tfrac{1}{2}$$

$$U_V(0, 1) = -(0-1)^2 = -1 \text{ if B win at policy } 1$$

Expressive utility for voter ideal point = 0 is $|-1 + \tfrac{1}{4}| = \tfrac{3}{4}$

For ideal point = $\tfrac{1}{2}$ voter.

$$U_V(\tfrac{1}{2}, \tfrac{1}{2}) = -(\tfrac{1}{2}-\tfrac{1}{2})^2 = 0 \text{ if A win at policy } \tfrac{1}{2}$$

$$U_V(\tfrac{1}{2}, 1) = -(\tfrac{1}{2}-1)^2 = -\tfrac{1}{4} \text{ if B win at policy } 1$$

Expressive utility for voter ideal point = $\tfrac{1}{2}$ is $|-\tfrac{1}{4} - 0| = \tfrac{1}{4}$

For ideal point = 1 voter.

$$U_V(1, \tfrac{1}{2}) = -(1-\tfrac{1}{2})^2 = -\tfrac{1}{4} \text{ if A win at policy } \tfrac{1}{2}$$

$$U_V(1, 1) = -(1-1)^2 = 0 \text{ if B win at policy } 1$$

Expressive utility for voter ideal point = 1 is $|-\tfrac{1}{4} - 0) = \tfrac{1}{4}$

f.

	$(T_{UA}=0, T_{UB}=1)$	$(T_{UA}=\frac{1}{2}, T_{UB}=1)$	
0 voter	$1-C$	$\frac{3}{4}-C$	less incentive
$\frac{1}{2}$ voter	$0-C$	$\frac{1}{4}-C$	higher incentive
1 voter	$1-C$	$\frac{1}{4}-C$	less incentive

1

When party A moderates while keeping $\pi B = 1$ fixed:

- Voters whose ideal policy is 0 (i.e., party A's core supporters) would experience a decrease in their expressive utility from 1 to 3/4, resulting in relatively lower turnout. Intuitively, as those voters feel less satisfied and less motivated by the moderate policy proposed by party A in contrast with their ideal point, those voters would have lower incentives to turn out.

- Voters whose ideal policy is 1 (i.e., party B's core supporters) would also experience a decrease in their expressive utility, but with larger magnitude from 1 to 1/4, resulting in even lower turnout. Intuitively, as those voters perceive reduced threat as party A's moderation, those voters would have much lower incentives to turn out.

- Meanwhile, this decision would also increase the participation and turnout of moderate voters (i.e., voters whose ideal policy is 1/2) by increasing their expressive utility from 0 to 1/4. Intuitively, as the moderate policy proposed by party A closely aligns with their ideal point, those voters would have higher incentives and increased motivation to vote for the moderate policy.

g.

0 voter's net value from voting is $(\frac{3}{4} - 0.1) < 0$ (Not participate in vote)

$\frac{1}{2}$ voter's net value from voting is $\{\frac{1}{4} - 0.1\} < 0$ (Not participate in vote)

1 voter's net value from voting is $\{\frac{1}{4} - 0.1\} < 0$ (Not participate in vote)

→ No voter will participate in Vote, cost too high.

→ $\frac{1}{2}$ prob that A win at policy $\frac{1}{2}$ and $\frac{1}{2}$ prob b win at policy 1

$E(U_A) = \frac{1}{2} \cdot -(0 - \frac{1}{2})^2 + \frac{1}{2} \cdot -(0 - 1)^2$

$= \frac{1}{2} \cdot -\frac{1}{4} + \frac{1}{2} \cdot -1$

$= -\frac{1}{8} - \frac{1}{2} = -0.625$

h. When c = 0.9 and party B chooses $\pi_B = 1$, party A's best response is committing to the policy that well aligns with its ideal point, that's choosing $\pi_A = 0$, as this polarization can brings party A higher expected utility (i.e., -1/2 > -5/8).

This answer does differ from the answer in part (a), because part (a) assumes that all voters will participate and turn out to vote as long as their preferred policy can bring them higher expected utility. However, in this case, we take into account voters' costs of voting and they will vote if and only if their expressive utility is greater than the costs. As a result, changing from polarization to moderation will also affect the turnout of voters, which in turn, would further change party A's best response.

i.   The model developed in the previous parts does benefit us to understand Biden's current situation:

- Currently, Biden is facing a challenge in motivating black voters to support him in re-election. There's an obvious decline in the turnout of his supporters, specifically among black voters: the percentage of black voters supporting him has decreased from 87 percent in 2020 to 71 percent (Barbaro, 2024).

- The decrease in black voter turnout can be explained by different factors in relation to the model:

  ○   Unfulfilled promises, such as those related to student loan debt, and controversial political decisions, especially Biden's support for Israel in the war against Gaza, do deviate from the ideal point of his supporters, particularly those polarized black voters. The unexpected outcome that no longer aligns with the preferences of those black voters would inevitably diminish their enthusiasm to turn out to vote for Biden. Theoretically, this issue can be explained by the model as Biden's proposed policy moving away from the ideal point of his supporters. As we discussed in part (f), as the policy proposed and truly implemented by Biden fails to meet the expectations and needs of faithful supporters, those black voters are more likely to experience a decrease in their expected utility of implementing so-called preferred policy as well as their expressive utility/value (i.e., U(preferred) - U(not preferred)). As a result, even if their perceived costs of voting remain unchanged, a decreasing number of voters would be willing to turn out to vote due to the divergence between the proposed (and even implemented) policy and the preferences of those black voters.

  ○   Considerations of age and charisma may also affect the black voter turnout, as a new generation of black voters become more like moderate voters rather than faithful polarized supporters for Biden. In fact, the podcast points out that the emerging new generation of black voters see Biden as "representative of a bygone era of politics" and struggle to form a

personal connection with him (Barbaro, 2024). Consequently, those new generations of black voters exhibit indifference between Democrat Biden and Republican Trump. Intuitively, as more of those new generations actively engage in elections, the overall proportion of faithful/polarized black voters who support Biden would decrease, as only older black voters remain unwavering in their support. Meanwhile, the overall proportion of centrist, encompassing the new generations of black voters, would increase. And as centrists usually experience lower expressive utility/value in contrast with polarized supporters, they would be less willing to turn out to vote for the person/party that they're indifferent to.

However, this model overlooks the crucial aspects of reality: that's, the potential for voters to undergo a complete shift in their ideal point, moving to the extreme opposite. In other words, for those voters who currently have an ideal point at 0, it's also possible that under certain circumstances, they would totally change their idea point to 1 and vote for the opposing side. In relation to Biden's situation, those black voters who are initially classified as faithful and polarized supporters for Democrats and Joe Biden may also undergo a substantial change in their opinions and change to vote for the Republican candidate, like Trump, due to reasons like unfulfilled promises, lack of personal connection, etc. In fact, as the podcast points out, former President Trump did increase his support among black voters from 8 percent in 2020 to 22 percent last year and "if former President Trump were to really have more than 20 percent support from Black voters, that would be a historic shift in Black voting behavior" (Barbaro, 2024). However, in our model, we ignore this possibility.

Question 2:

a. Code:

```
##tell R not to use scientific notation
options(scipen = 999)
options(digits = 10)

# Load necessary libraries
library(dplyr)
```

```
library(ggplot2)

##set directory
setwd("~/Desktop/PPHA 31610/problem set")

##read data
data = read.csv("Turnout_Chile.csv")

##Q2.a
#Combine data from both rounds
combined_data = data %>%
  group_by(age) %>%
  summarize(avg_turnout = mean(voted))
#Create a bar plot
ggplot(combined_data, aes(x = age, y = avg_turnout)) +
  geom_bar(stat = "identity", fill = "blue", color = "black") +
  labs(title = "Average Voter Turnout by Age",
       x = "Age",
       y = "Average Voter Turnout") +
  theme_minimal()
```

Output:

Average Voter Turnout by Age
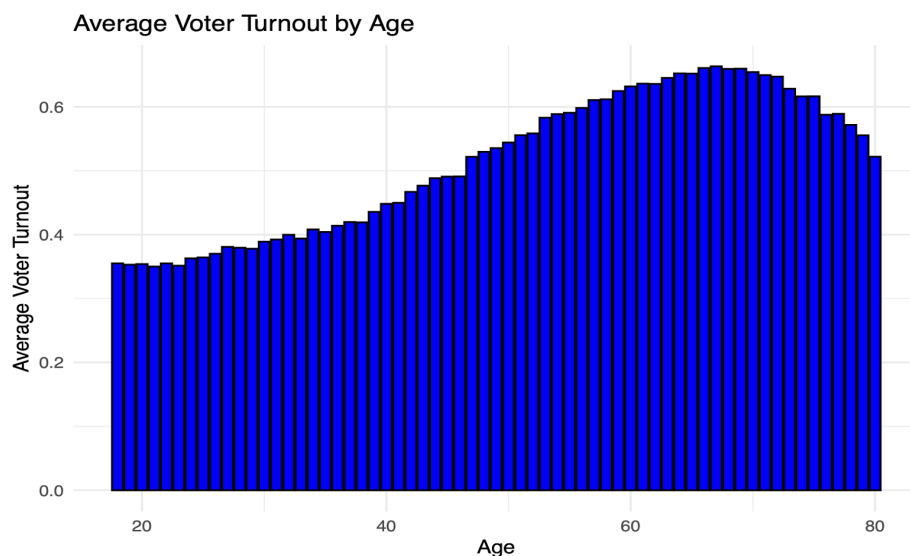
Describe your findings:

When the age gets older, the average voter turnout increases, and it increases to the max when age is around 70, and then the average voter turnout starts to decrease. The possible explanation for this is that younger individuals, particularly those in their late teens and early twenties, may be less experienced or motivated to participate in the electoral

process. As individuals age, they may develop a stronger sense of civic responsibility and engagement, leading to an increase in voter turnout. Voter turnout might decrease among the oldest age groups due to health-related issues or decreased mobility. Older individuals may face challenges in physically reaching polling stations, particularly if they have mobility issues or health concerns that limit their ability to participate in the electoral process.

b. There are basically two implications of the results in part (a) for political representation. The first one is representation of older individuals. With voter turnout reaching its highest point around age 70, policymakers may show increased responsiveness to the concerns and preferences of this age group. Policies aligned with the interests of older individuals are likely to garner greater attention and consideration. But there should be a potential risk for the oldest demographics which could lead to their perspectives being overlooked in the political decision making process. The second one is potential for intergenerational policy gaps. The observed pattern may contribute to intergenerational policy gaps which preferences and needs of the younger generation being underrepresented.

For effect on environmental regulations, if older individuals are more engaged in the political process, there could be an impact on environmental regulation. Policies related to environmental issues may be influenced by the preferences of the more active older demographic. And if older individuals are more engaged in the political process, there could be an impact on environmental regulation. Policies related to environmental issues may be influenced by the preferences of the more active older demographic.

For effect on pension reform, given that voter turnout tends to be higher among older individuals, policies related to pension reform may be shaped by the preferences of this demographic. There may be a greater focus on issues such as retirement benefits and social security. A decrease in voter turnout among older age groups after the peak might lead to challenges in addressing evolving needs related to pension reform, potentially resulting in policies that are less reflective of the broader aging population's interests.

c. For expectations of higher turnout rate in the presidential run-off, a presidential run-off usually marks an important phase in the electoral process by offering voters a direct choice between the top two candidates. The increased significance of this decision is likely to serve as a motivating factor, driving higher voter turnout for the run-off election. For expectations of lower turnout rate in the presidential run-off, if the initial election process was highly involved an extended campaign period, voters may experience fatigue. This could result in a lower turnout for the run-off, while some individuals may feel less motivated to participate in a next round of voting.

d. <u>Code:</u>

```
##Q2.d
# Calculate average voter turnout by age for each round
combined_data_rounds = data %>%
  group_by(age, runoff) %>%
  summarize(avg_turnout = mean(voted))
```

```
# Create a bar plot
ggplot(combined_data_rounds, aes(x = age, y = avg_turnout, fill = factor(runoff))) +
  geom_bar(stat = "identity", position = "dodge", color = "black") +
  labs(title = "Average Voter Turnout by Age and Round",
       x = "Age",
       y = "Average Voter Turnout") +
  scale_fill_manual(values = c("blue", "green"), name = "Round",
                    labels = c("First Round", "Runoff")) +
  theme_minimal()
```

<u>Output:</u>

Average Voter Turnout by Age and Round
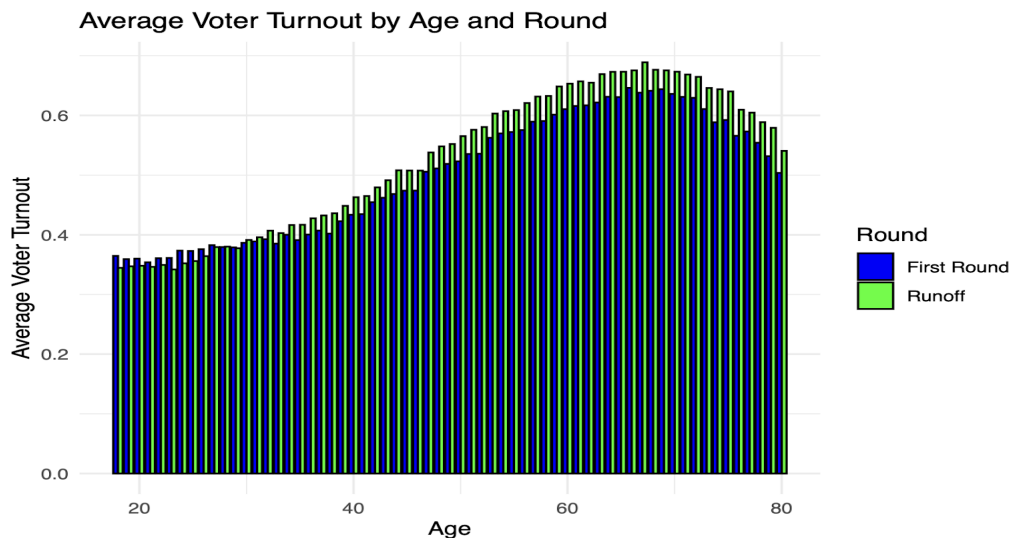
<u>Describe your findings:</u>

The voter turnout is higher in the run-off election than in the first round. And this may mainly be attributed to mainly two reasons. The first one is that in the first round of the election, voters might face a larger field of candidates, making it challenging to choose a clear favorite. The run-off election simplifies the decision-making process by narrowing the options to the top two candidates. This clarity can motivate more voters to participate, knowing that their choice in the run-off holds greater weight. The second one is that in the run-off, voters may adopt a more strategic approach, especially if their preferred candidate did not advance from the first round. The elimination of other candidates allows voters to consolidate their support behind one of the remaining contenders, potentially boosting turnout as voters see a more direct impact of their choice.

e. <u>For expectations of higher turnout rate among women,</u> women's participation in civic and political activities has been on the rise globally. In recent years, there has been a growing awareness of gender issues, and women may feel a heightened sense of responsibility to have their voices heard. This increased gender engagement could lead to higher turnout among women. <u>For expectations of lower turnout rate among women,</u> some regions or demographics may still face barriers that disproportionately affect women's ability to participate in elections. These barriers could include logistical challenges, cultural norms, or discriminatory practices. In such cases, women may experience lower turnout due to systemic obstacles that hinder their access to the electoral process.

f.

<u>Code:</u>

```
##Q2.f
# Calculate average voter turnout by age and gender for each round
combined_data_gender = data %>%
  group_by(age, female, runoff) %>%
  summarize(avg_turnout = mean(voted))
# Create a bar plot
ggplot(combined_data_gender, aes(x = age, y = avg_turnout, fill = factor(female))) +
  geom_bar(stat = "identity", position = "dodge") +
  labs(title = "Average Voter Turnout by Age and Gender",
       x = "Age",
       y = "Average Voter Turnout") +
  scale_fill_manual(values = c("skyblue", "pink"), name = "Gender",
                    labels = c("Male", "Female")) +
  theme_minimal()
```

<u>Output:</u>
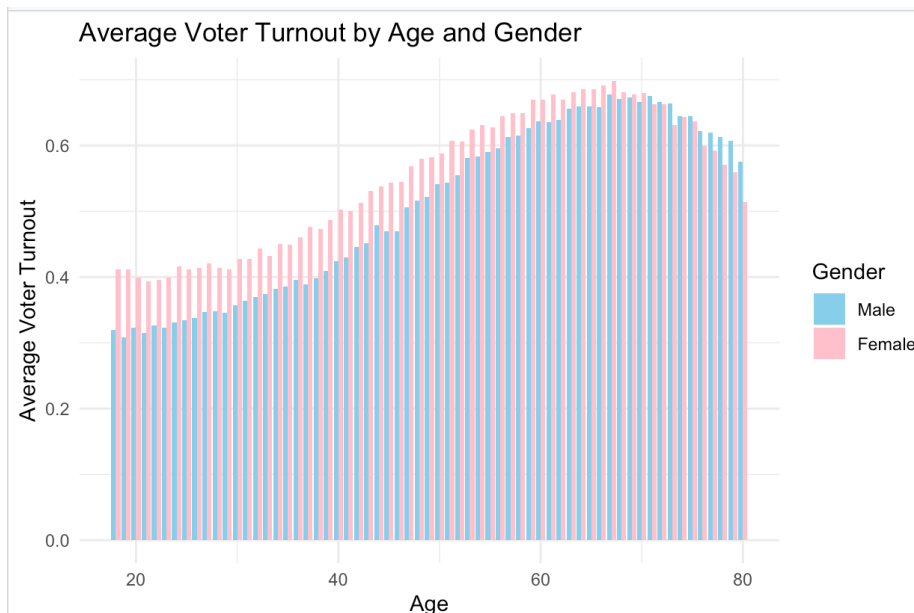
<u>Describe your findings:</u>

The graph illustrates that females have higher average voter turnout. There are two possible reasons for this. The first one is that in some societies, there has been a growing

emphasis on empowering women politically. Efforts to encourage gender equality and female participation in civic activities may have resulted in higher voter turnout among women. The second one is that increased representation of women in political leadership roles can serve as a positive motivator for female voter turnout. Seeing women in key positions may inspire others to actively participate in the political process.