

HYLU User Guide

(Version 20251203)

Xiaoming Chen (chenxiaoming@ict.ac.cn)

HYLU (*Hybrid Parallel Sparse LU Factorization*) is a general-purpose parallel solver for efficiently solving sparse linear systems ($\mathbf{Ax} = \mathbf{b}$) on multi-core shared-memory architectures. It employs an innovative parallel up-looking LU factorization algorithm, which dynamically adapts to varying matrix sparsity patterns by leveraging hybrid numerical kernels. The numerical stability is maintained by combining static pivoting, dynamic diagonal supernode pivoting, dynamic scaling, and iterative refinement.

HYLU is implemented in C and offers seamless integration with C/C++ applications. The multi-threading of **HYLU** is implemented based on operating system's native threading interface (OpenMP is NOT needed).

1. Functions

HYLU offers 6 groups of functions for solving sparse linear systems. Each group provides both 32-bit and 64-bit integer versions for both real number and complex number systems. The following table lists the format of the **HYLU** function names and their corresponding versions. In the following text, the function names of the 32-bit integer/real number version will be used to explain the functionality of **HYLU**. In the 32-bit integer version, only the input matrix indices use 32-bit integers, while internal LU factor data structures maintain 64-bit integers for scalability. For detailed function prototypes and argument descriptions of the **HYLU** functions, please refer to `<hylu.h>`.

<code>HYLU_XXXX</code>	32-bit integer/real number
<code>HYLU_L_XXXX</code>	64-bit integer/real number
<code>HYLU_C_XXXX</code>	32-bit integer/complex number
<code>HYLU_CL_XXXX</code>	64-bit integer/complex number

(1) `HYLU_CreateSolver`

This function creates a solver instance, retrieves a pointer to the parameter array, and spawns working threads for parallel execution. Input parameters are initialized to their default values during this process. The number of working threads should not exceed the number of idle cores.

(2) `HYLU_DestroySolver`

This function frees all allocated memory, terminates the created threads, and destroys the solver instance.

(3) `HYLU_Analyze`

This function performs preprocessing steps including static pivoting, matrix

reordering, and symbolic factorization. The matrix format is **compressed sparse row (CSR)**, where column indexes in each row are **NOT required to be sorted**.

The second argument, 'repeat', specifies whether the linear system will be solved repeatedly with an identical matrix structure. This is common in applications such as circuit simulation. In such a scenario, **preprocessing is needed only once**. When enabled, this argument impacts the default values of certain parameters. Specifically, in the repeated solving scenario, *HYLU* prioritizes minimizing nonzeros in the LU factors, which may increase preprocessing time but improves subsequent factorization efficiency.

It is optional to provide the matrix values (the argument 'ax') to preprocessing, but providing values is strongly recommended. Without matrix values, static pivoting and static scaling cannot be carried out.

(4) `HYLU_Analyze2`

This function provides similar functionality to `HYLU_Analyze` with user-provided ordering. The user-provided ordering must **ensure a zero-free diagonal**. Static pivoting and static scaling are disabled.

(5) `HYLU_Factorize`

This function computes the numerical LU factorization of the preprocessed matrix, decomposing it into a lower triangular matrix (**L**) and an upper triangular matrix (**U**).

Dynamic diagonal supernode pivoting is performed during this process. If no suitable pivot is found, a pivot perturbation strategy is applied.

(6) `HYLU_Solve`

This function computes the solution vector by performing forward (**Ly = b**) and backward (**Ux=y**) substitutions using the computed LU factors.

If pivot perturbation has occurred in the last numerical LU factorization, iterative refinement is automatically performed to correct the solution.

2. Parameters

This table describes all individual components of the parameter array. Input parameters are initialized to their default values (marked with an asterisk *) upon calling `HYLU_CreateSolver`. All modifications (if needed) to input parameters should be completed prior to invoking `HYLU_Analyze`.

Parameter	Description	
<code>parm[0]: output</code>	Version.	
<code>parm[1]: input</code>	Timer control. When enabled, <code>parm[7]</code> returns the wall time of the last function call.	
	0*	Disabled.
	>0	High-precision timer (microsecond precision).
	<0	Low-precision timer (millisecond precision).
<code>parm[2]: input</code>	Ordering method for fill-in reduction.	
	0*	The ordering method is automatically decided according to

		the matrix dimension and the 'repeat' argument of HYLU_Analyze.
	1	Approximate minimum degree.
	2	Approximate minimum degree variant.
	3	Nested dissection method 1.
	4	Nested dissection method 2.
	5	Best of 1 and 2.
	6	Best of 3 and 4.
	7	Best of 1-4.
parm[3]: input		Ordering method switch point. For nested dissection, once a partition size is smaller than parm[3], it switches to a constrained approximate minimum degree method. Smaller parm[3] values generally yield better ordering results of nested dissection at the cost of increased ordering time.
	0*	Automatic control. The value is determined according to the 'repeat' argument of HYLU_Analyze.
	>=64	Allowable range.
parm[4]: output		Selected ordering method (1-4), reported by HYLU_Analyze.
parm[5]: input		Minimum number of columns of a supernode. A supernode must contain at least parm[5] columns.
	32*	Default value.
	>=8	Allowable range.
parm[6]: input		Maximum number of rows of a supernode. A supernode may contain at most parm[6] rows. A larger supernode is split into multiple supernodes.
	0*	Automatic control. The value is determined by whether working threads have been spawned.
	>=8	Allowable range.
parm[7]: output		Wall time (in microseconds) of last function call.
parm[8]: output		Number of off-diagonal pivots, reported by HYLU_Factorize.
parm[9]: output		Number of supernodes, reported by HYLU_Analyze or HYLU_Analyze2.
parm[10]: input		Pivot perturbation coefficient. Zero or small pivots will be replaced by $\text{sign}(\text{pivot}) \times 10^{\text{parm}[10]} \times \ \mathbf{A}\ _\infty$.
	-15*	Default value.
	<0	Allowable range.
parm[11]: output		Number of perturbed pivots, reported by HYLU_Factorize.
parm[12]: output		Current memory usage (in bytes), or required memory size (in bytes) when -4 is returned, reported by HYLU_Analyze or HYLU_Analyze2.
parm[13]: output		Maximum memory usage (in bytes), reported by HYLU_Analyze or HYLU_Analyze2.
parm[14]: output		Thread numbers stored in 3 shorts. Use the following method to get them: <code>const short *threads = (short *) &parm[14];</code>

	threads [0]: number of physical cores (may be incorrect). threads [1]: number of logical cores. threads [2]: number of created working threads.
parm[15]: input	Maximum number of refinement iterations.
	0* Whether to perform iterative refinement and the iteration count are determined automatically.
	>0 If <i>HYLU</i> decides to perform iterative refinement, execute parm[15] iterations.
	<0 Execute -parm[15] iterations.
parm[16]: output	Number of refinement iterations performed, reported by <i>HYLU_Solve</i> .
parm[17]: output	Number of nonzeros in L (including diagonal), reported by <i>HYLU_Analyze</i> or <i>HYLU_Analyze2</i> .
parm[18]: output	Number of nonzeros in U (excluding diagonal), reported by <i>HYLU_Analyze</i> or <i>HYLU_Analyze2</i> .
parm[19]: output	Number of floating-point operations of factorization (excluding scaling), reported by <i>HYLU_Analyze</i> or <i>HYLU_Analyze2</i> .
parm[20]: output	Number of floating-point operations of solving (excluding scaling), reported by <i>HYLU_Analyze</i> or <i>HYLU_Analyze2</i> .
parm[21]: input	Matrix scaling method.
	>0* Dynamic matrix value scaling.
	0 Disabled.
	<0 Static matrix value scaling.
parm[22]: input	Symbolic factorization method. Symmetric symbolic factorization helps reduce preprocessing time, but increases fill-ins for structurally unsymmetric matrices.
	>0 Unsymmetric symbolic factorization.
	<0 Symmetric symbolic factorization.
	0* Automatic control.
parm[23]: input	Parallel nested dissection.
	1* Enabled.
	0 Disabled.

3. Return Values

Every function of *HYLU* returns an integer to indicate the error code. This table describes the meanings of the return codes.

Return value	Description
0	Successful.
-1	Invalid instance handle.
-2	Function argument error (e.g., negative matrix dimension, NULL pointer).
-3	Invalid input matrix (e.g., matrix indices error).
-4	Out of memory. parm[12] will return the needed memory size.
-5	Structurally singular.

-6	Numerically singular.
-7	Threads error.
-8	Calling procedure error.
-9	Integer overflow, please use the <code>HYLU_L_*</code> functions.
-10	Internal error.

4. Notes

- (1) **User must not free the memory of the parameter array**, which is managed by *HYLU*.
- (2) By default, *HYLU* operates in **row-major order**. For matrices stored in column-major order, set the argument 'transpose' of *HYLU_Solve* to true to solve transposed systems ($\mathbf{A}^T \mathbf{x} = \mathbf{b}$). However, the parallel scalability of column-wise solve is not as good as that of row-wise solve.
- (3) Please ensure that the number of threads created by *HYLU* is less than or equal to the number of idle cores, otherwise the performance of *HYLU* will degrade severely.
- (4) Due to the limited pivoting range, for some linear systems *HYLU* may produce inaccurate solutions. In such cases, consider adjusting `parm[6]` to a larger value and utilizing sequential factorization. `parm[10]` and `parm[21]` may also affect the accuracy of results.
- (5) Memory leaks have been observed in *HYLU* libraries linked with Intel MKL BLAS. These leaks are attributed to the MKL BLAS functions themselves.
- (6) The results of sequential and parallel computations can be slightly different.