CS-E4850 Computer Vision
Exercise Round #5
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Exercise 1. Total least squares line fitting.
1)

Proof. Given a line $ax + by - d = 0$, a parallel line which goes through point (x_i, y_i) is

$$ax_i + by_i - d' = 0$$

where $d' = ax_i + by_i$. The distance between the point and the line is actually the distance between these two parallel lines, which is

$$distance = \frac{|d - d'|}{\sqrt{a^2 + b^2}}$$

Since $a^2 + b^2 = 1$, this becomes

$$distance = \frac{|d - d'|}{\sqrt{a^2 + b^2}} = |d - d'| = |ax_i + by_i - d|$$

$\square$

2)
Solution

$$\begin{aligned}\frac{\partial E}{\partial d} &= \sum_{i=1}^{n} -2(ax_i + by_i - d) \\ &= 2nd + \sum_{i=1}^{n} -2(ax_i + by_i) \\ &= 0\end{aligned}$$

$$\begin{aligned}\Rightarrow d &= \frac{1}{n}\sum_{i=1}^{n}(ax_i + by_i) \\ &= a\frac{1}{n}\sum_{i=1}^{n}(x_i) + b\frac{1}{n}\sum_{i=1}^{n}(y_i) \\ &= a\bar{x} + b\bar{y}\end{aligned}$$

3)

Proof.

$$
\begin{aligned}
E &= \sum_{i=1}^{n} (a x_i + b y_i - d)^2 \\
&= \sum_{i=1}^{n} (a(x_i - \bar{x}) + b(y_i - \bar{y}))^2 \\
&= \left\| \begin{pmatrix} x_1 - \bar{x} & y_1 - \bar{y} \\ \vdots & \vdots \\ x_n - \bar{x} & y_n - \bar{y} \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \right\|^2 \\
&= \begin{pmatrix} a & b \end{pmatrix} \begin{pmatrix} x_1 - \bar{x} & y_1 - \bar{y} \\ \vdots & \vdots \\ x_n - \bar{x} & y_n - \bar{y} \end{pmatrix}^\top \begin{pmatrix} x_1 - \bar{x} & y_1 - \bar{y} \\ \vdots & \vdots \\ x_n - \bar{x} & y_n - \bar{y} \end{pmatrix} \begin{pmatrix} a & b \end{pmatrix}^\top \\
&= \begin{pmatrix} a & b \end{pmatrix} U^\top U \begin{pmatrix} a & b \end{pmatrix}^\top
\end{aligned}
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