CS-E4850 Computer Vision
Exercise Round #12
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Exercise 1. Course feedback.
Exercise 2. Epipolar geometry.

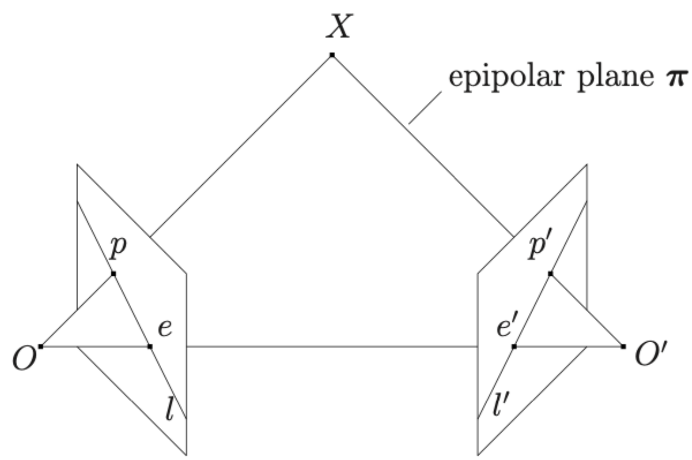

Figure 1: Epipolar geometry

Proof.

$$\begin{aligned} \overrightarrow{O'p'} \cdot (\overrightarrow{O'O} \times \overrightarrow{Op}) &= 0 \qquad (1) \\ \Rightarrow \mathbf{x}' \cdot (\mathbf{t} \times (\mathbf{R}\mathbf{x} + \mathbf{t})) &= 0 \\ \mathbf{x}' \cdot (\mathbf{t} \times (\mathbf{R}\mathbf{x}) + \mathbf{t} \times \mathbf{t})) &= 0 \\ \mathbf{x}' \cdot (\mathbf{t} \times (\mathbf{R}\mathbf{x})) &= 0 \\ \mathbf{x}'^{\top}[\mathbf{t}]_{\times}\mathbf{R}\mathbf{x} &= 0 \\ \Rightarrow \mathbf{x}'^{\top}\mathbf{E}\mathbf{x} &= 0 \end{aligned}$$

where $\mathbf{E} = [\mathbf{t}]_{\times}\mathbf{R}$ □

Exercise 3. Stereo vision.

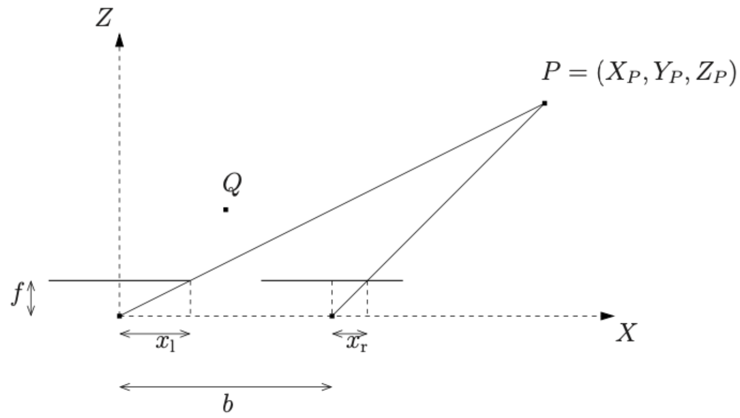

Figure 2: Stereo vision

a)

$$Z_p = \frac{b}{d} f = 6\ cm$$

b)

$$\Delta Z = \frac{b}{\Delta d} f = 60\ m$$

c)

$$\mathbf{x}_l^Q = \mathbf{P}_l \mathbf{X}^Q$$

$$\mathbf{x}_l^Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix}$$

$$= 3 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

So the in-homogeneous coordinate of Q on the image plane of the camera on the left is

$$\mathbf{x}_l^Q = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

From the Epipolar constraint:

$$\mathbf{x}'^{\top} \mathbf{E} \mathbf{x} = 0,\ \mathbf{E} = [\mathbf{t}]_{\times} \mathbf{R}$$

$$\mathbf{R} = \mathbf{I}$$

$$\begin{aligned}
\mathbf{t} &= (T, 0, 0) = (-6, 0, 0) \\
\therefore \mathbf{E} &= [\mathbf{t}]_{\times}\mathbf{R} \\
&= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 6 \\ 0 & -6 & 0 \end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
\begin{pmatrix} u' & v' & 1 \end{pmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 6 \\ 0 & -6 & 0 \end{bmatrix} \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} &= 0 \\
\begin{pmatrix} u' & v' & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 6 \\ -6v \end{pmatrix} &= 0 \\
6v' - 6v &= 0 \\
v' = v &= 0
\end{aligned}$$

The corresponding epipolar line on the image plane of the camera on the right is

$$v' = 0$$

Exercise 4. Fundamental matrix estimation.