Chapter 13

Game Theory

Game Theory

- In the previous three chapters, we've talked about how monopolies and competitive firms make choices
- Our models were too simple — determining equilibrium requires more than just equating supply with demand
- Firms choose their best strategies (price or quantity) given the choices of all the other firms in a market

Introduction

- **Game theory** is the study of strategic interactions between two or more actors
- We will examine behavior when players are making **strategic decisions** — actions based on anticipation of others' actions
- These concepts can be applied to market contexts as well as any number of human (and non-human) interactions

Three Categories of Games

1. Simultaneous Games
   - The participants choose their actions simultaneously without knowing their opponents' strategies
2. Repeated Games
   - A series of simultaneous games long the same set of economic actors
   - Successful collusion (e.g. cartels) is made possible by the cyclic repetition of output decisions
3. Sequential Games
   - Players take turns making decisions

Introduction

- Some things to remember:
    - Game theory is about seeing the world through the eyes of your opponent
    - As with consumer theory, we assume players are self-interested
    - Rules often determine the outcome of a game — important to understand the timing of moves, allowable allocations, etc.

Three Elements of a Game

1. Players
   - A player is a participant in an economic game that must decide her actions based on the actions of others
2. Strategy
   - The action taken by a player — may be simple or complex, and depend on the actions (anticipated or actual) of the other players
3. Payoffs
   - The outcome a player receives from playing the game
   - The payoff to one player depends on the actions of other players, otherwise the former would have no incentive to act strategically

Static Games

- In a **static game** each player acts simultaneously, only once, and has complete information about the payoff functions but incomplete information about rivals' moves
    - Examples: employer negotiations with a potential new employee, street vendors' choice of locations and prices, a penalty kick in soccer
- Consider a **normal form** static game of complete information which specifies the players, their strategies, and the payoffs for each combination of strategies
    - Competition between United and American Airlines on the OKC-Dallas route

The Payoff Matrix

- Quantities, q, are in thousands of passengers per quarter; profits are in millions of dollars per quarter

		American Airlines	
		$q_A = 64$	$q_A = 48$
United Airlines	$q_U = 64$	4.1 (upper right); 4.1 (lower left)	3.8 (upper right); 5.1 (lower left)
	$q_U = 48$	5.1 (upper right); 3.8 (lower left)	4.6 (upper right); 4.6 (lower left)

Note: Quantities are in thousands of passengers per quarter; (rounded) profits are in millions of dollars per quarter.

Dominant and Dominated Strategies

- Predicting behavior in games relies on finding the **optimal strategy** for each player — the strategy that results in highest expect payoff

- **Dominant Strategy**: a winning strategy for a player, regardless of her opponents' strategies

  - If a player has a dominant strategy, that strategy is always chosen because it will always make the player win

Dominant and Dominated Strategies

- **Dominanted Strategy**: a losing strategy for a player, regardless of her opponents' strategies
  - Dominated strategies are never chosen, and once identified, can be ignored
- If there is a dominant strategy, all other strategies are dominated
- If there is a dominated strategy, there is not necessarily a dominant strategy

Predicting a Game's Outcome

- Rational players will avoid strategies that are **dominated** by other strategies

- We can precisely predict the outcome of any game in which every player has a **dominant strategy**

- Airline game:

  - If United chooses *high-output*, American's *high-output* strategy maximizes its profits

  - If United chooses *low-output*, American's *high-output* strategy still maximizes its profits

  - For American, *high-output* is a dominant strategy

Quantity Setting Game

- The *high-output* strategy is dominant for American and for United — dominant strategy equilibrium

		American Airlines	
		$q_A = 64$	$q_A = 48$
United Airlines	$q_U = 64$	4.1 4.1	3.8 5.1
	$q_U = 48$	5.1 3.8	4.6 4.6

Note: Quantities are in thousands of passengers per quarter; (rounded) profits are in millions of dollars per quarter.

- Players choose strategies that don't maximize joint profits (i.e. they're competitive, not cooperative)

- **Prisoner's Dilemma**: all players have dominant strategies that lead to a profit that is less than if they cooperated

Nash Equilibrium

- When iterative elimination fails to predict a unique outcome, we can use a related approach
- The **best response** is a strategy that maximizes a player's payoff given its beliefs about a rival's strategies
- A set of strategies is a **Nash equilibrium** if, when all other players use these strategies, no player can obtain a higher payoff by choosing a different strategy
    - No player has an incentive to deviate from a Nash equilibrium

Nash Equilibrium

- Every game has at least one Nash equilibrium and every dominant strategy equilibrium is a Nash equilibrium

		American Airlines	
		$q_A = 64$	$q_A = 48$
United Airlines	$q_U = 64$	4.1 4.1	3.8 5.1
	$q_U = 48$	5.1 3.8	4.6 4.6

Note: Quantities are in thousands of passengers per quarter; (rounded) profits are in millions of dollars per quarter.

Best Response and Nash equilibrium

- In a game without dominant strategies, calculate best responses to determine the Nash equilibrium

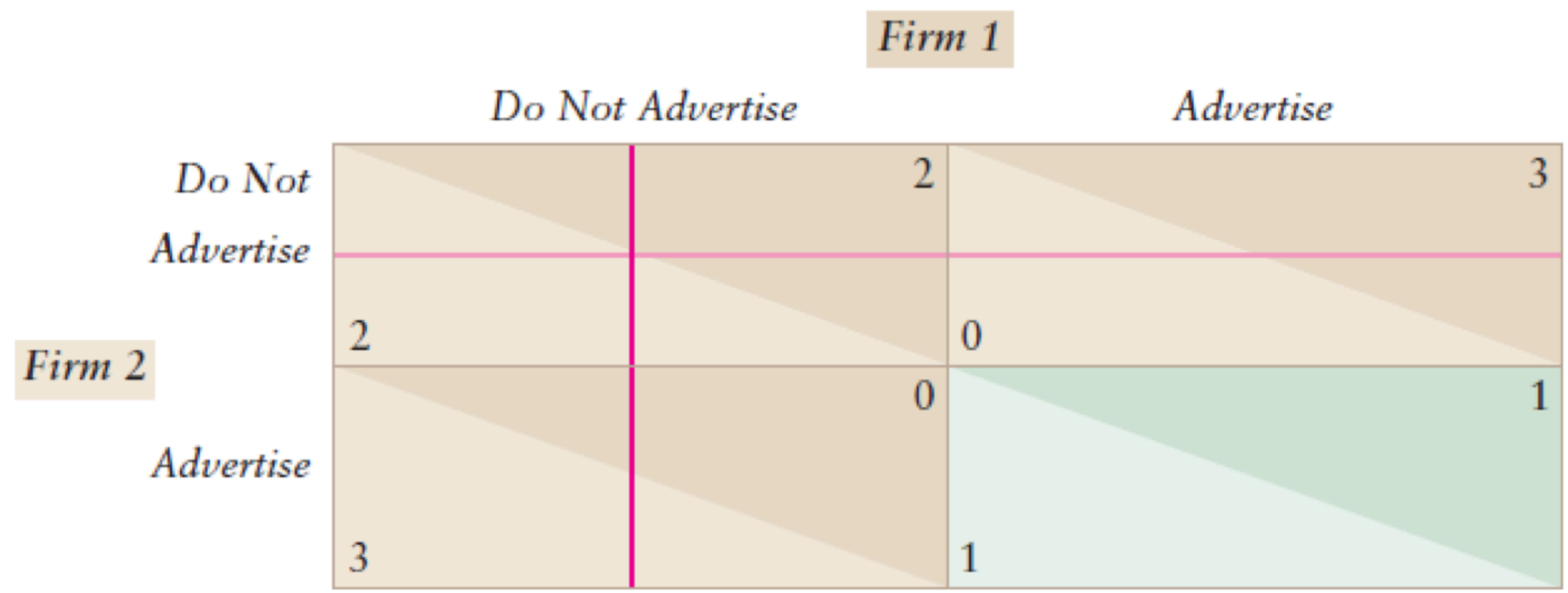
		American Airlines		
		$q_A = 96$	$q_A = 64$	$q_A = 48$
United Airlines	$q_U = 96$	0 (United), 0 (American)	3.1 (United), 2.0 (American)	4.6 (United), 2.3 (American)
	$q_U = 64$	2.0 (United), 3.1 (American)	4.1 (United), 4.1 (American)	5.1 (United), 3.8 (American)
	$q_U = 48$	2.3 (United), 4.6 (American)	3.8 (United), 5.1 (American)	4.6 (United), 4.6 (American)

Note: Quantities are in thousands of passengers per quarter; (rounded) profits are in millions of dollars per quarter.

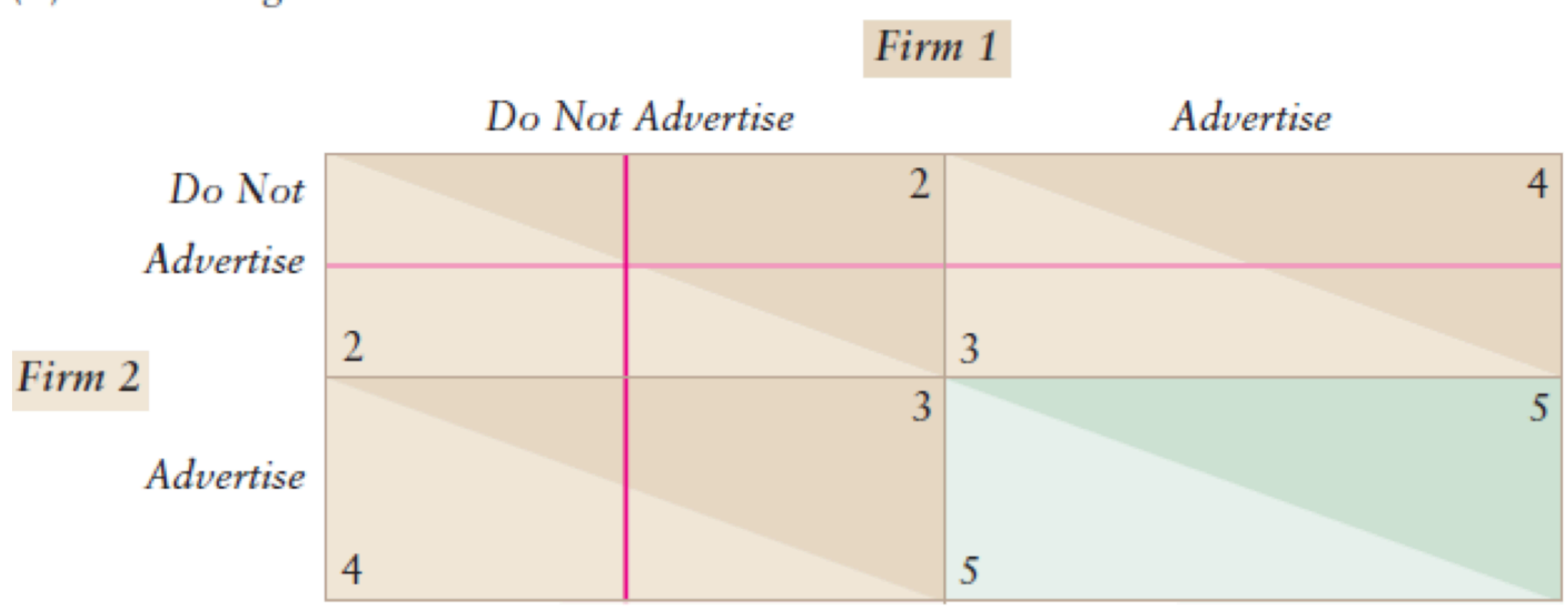
Failure to Maximize Joint Payoffs

- In the Nash equilibrium of the first advertising game, firms maximize joint profits — here, they do not

(a) Advertising Only Takes Customers from Rivals



(b) Advertising Attracts New Customers to the Market



Multiple Equilibria

- Many oligopoly games have more than one Nash equilibrium

		Firm 1	
		Do Not Enter	Enter
Firm 2	Do Not Enter	0 0	1 0
	Enter	0 1	–1 –1

Mixed Strategies

- So far, the firms have used **pure strategies**, meaning each player chooses a single action
- A **mixed strategy** is when a player chooses among possible actions according to probabilities that the player assigns
    - A pure strategy assigns a probability of 1 to an action
    - A mixed strategy is a probability distribution over actions
- When a game has multiple pure-strategy Nash equilibria, a mixed-strategy Nash equilibrium can help predict the outcome of the game

Simultaneous Entry Game

- This game has two Nash equilibria in pure strategies and one mixed-strategy Nash equilibrium

		Firm 1	
		Do Not Enter	Enter
Firm 2	Do Not Enter	0 0	1 0
	Enter	0 1	–1 –1

Dynamic Games

- In a **dynamic game**:
  - Players move either sequentially or repeatedly
  - Players have complete information about payoff functions
  - At each move, players have perfect information about the previous moves of all players

Dynamic Games

- Dynamic games are analyzed in their extensive form, which specifies:

  1. The number of players n
  2. The sequence of their moves
  3. The actions they can take at each move
  4. The information each player has about all players' previous moves
  5. The payoff function over all possible strategies

Actions and Strategies

- In games where players move sequentially, we distinguish between an action and a strategy
  - An **action** is a move that a player makes at a specified point
  - A **strategy** is a battle plan that specifies the action a player will make based on information available at each move

Repeating Games

- In a repeating game, a firm can influence its rival's behavior by **signaling** and **threatening to punish**
    - One airline could use a low-quantity strategy for a few periods to **signal** to the other firm its desire that the two firms cooperate and produce that low quantity in the future
    - The airline can threaten to **punish** a rival for not restricting output

Repeating Airline Game

- American will produce the smaller quantity at each period as long as United does the same
- If United produces the larger quantity in period t, American will produce the larger quantity in period $t + 1$ and all subsequent period

		American Airlines	
		$q_A = 64$	$q_A = 48$
United Airlines	$q_U = 64$	4.1 / 4.1	3.8 / 5.1
	$q_U = 48$	5.1 / 3.8	4.6 / 4.6

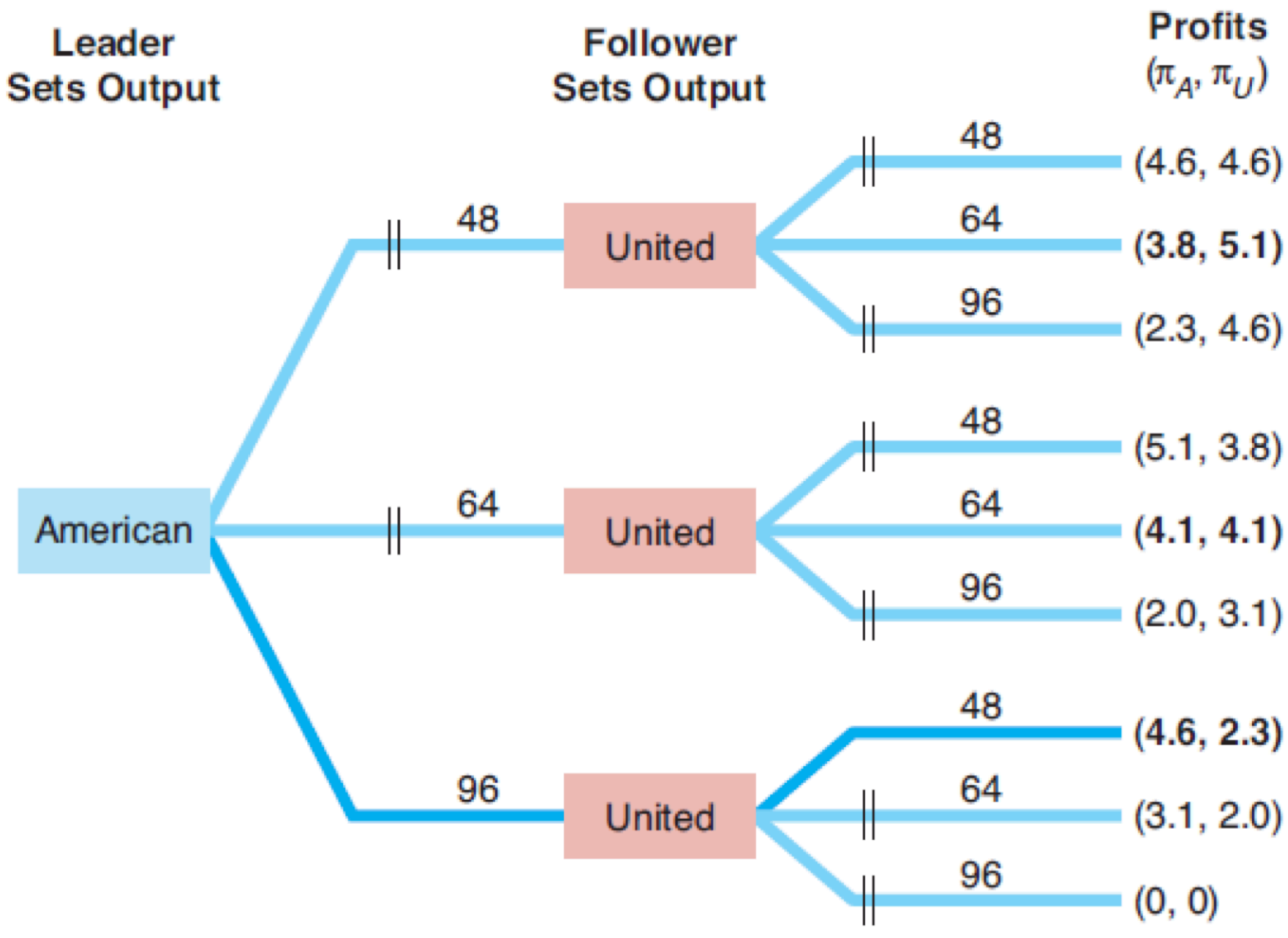
Note: Quantities are in thousands of passengers per quarter; (rounded) profits are in millions of dollars per quarter.

Sequential Games

- A **Stackelberg game tree** shows:

    1. **Decision nodes** indicating which player's turn it is
    2. **Branches** indicating all possible actions available
    3. **Subgames** — subsequent decisions available given previous actions

Sequential Games



Subgame Perfect Nash Equilibrium

- To predict the outcome of the Stackelberg game tree, we use a strong version of Nash equilibrium
- A set of strategies forms a **subgame perfect Nash equilibrium** if the players' strategies are a Nash equilibrium in every subgame
    - The previous game has four subgames; three at the second stage where United makes a decision and an additional subgame at the time of the first decision
    - We can solve for the subgame perfect Nash equilibrium using **backwards induction**

Backwards Induction in the Airline Game

- Backwards induction is where we:

  1. Find the best response by the last player to move

  2. Find the best response for the player who made the next-to-last move

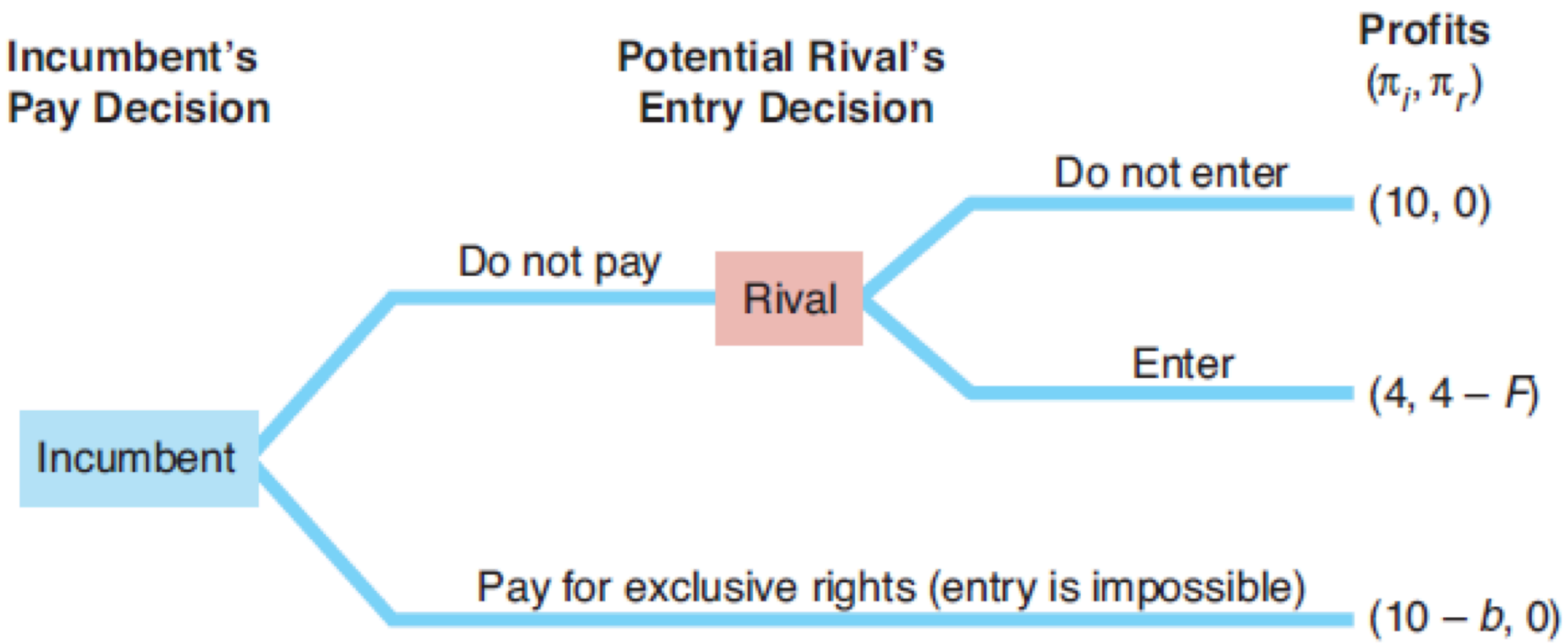
  3. Repeat the process until we reach the beginning of the game

Backwards Induction in the Airline Game

- Airline game:
  - If American chooses 48, United selects 64 — American's profit = 3.8
  - If American chooses 64, United selects 64 — American's profit = 4.1
  - If American chooses 96, United selects 48 — American's profit = 4.6
- Therefore, American chooses 96 in the first stage

Dynamic Entry Game

- Entry occurs unless the incumbent acts to deter entry by paying for exclusive rights to be the only firm in the market



Behavioral Game Theory

- Behavioral Economics seeks to augment the rational economic model so as to better understand and predict economic decision making
- Example: **The Ultimatum Game**
    - Proposer makes a take-it-or-leave-it offer to responder

The Ultimatum Game

- In the subgame perfect equilibrium, proposer makes the lowest possible offer and responder accepts
- But such rational behavior isn’t a good predictor of actual outcomes
- Experimentally, the lowest-possible offer is rarely made, and low offers are frequently rejected
- Responders reject low offers due to notions of fairness and reciprocity

Practice Problems

1. The following payoff matrix represents a single-period, simultaneous move game to be played by two firms. Does Apple have a dominant strategy and if so, what is it? Does Samsung have a dominant strategy and if so, what is it? Solve for the pure strategy Nash equilibrium (note: there could be none, one, or multiple pure strategy NE)

Samsung

Apple		Advertise	Don't advertise
	Advertise	15, 30	90, 25
	Don't advertise	12, 100	100, 70

Practice Problems

2. Assume this game is a single period, simultaneous move game. Identify all the best response actions. Is there a unique, stable Nash equilibrium outcome that can be predicted? If yes, what is that outcome and why is it a NE? If no, why is there no NE?

3. Now assume instead that the game, while still only played once, becomes a sequential move game. Draw a game tree that depicts the game if Apple moves first, and a second game tree that depicts if Samsung moves first. Is the equilibrium outcome the same?

		Samsung	
		Enter	**Stay Out**
Apple	**Enter**	-5, -5	25, 0
	Stay out	0, 25	0, 0