6.004 Worksheet Questions
L06 – Boolean Algebra and Logic Synthesis

Note: A small subset of essential problems are marked with a red star (★). We especially encourage you to try these out before recitation.

Problem 1. ★

Consider the truth table on the right, which defines two functions F and G of three input variables (A, B, and C).

For each function, write it in **normal form**, then find a **minimal sum of products** (minimal SOP) expression.

A	B	C	F	G
0	0	0	1	1
0	0	1	1	1
0	1	0	0	1
0	1	1	1	0
1	0	0	1	1
1	0	1	0	0
1	1	0	0	1
1	1	1	1	0

Normal form for F(A,B,C) = _____

Minimal sum of products for F(A,B,C) = _____

Normal form for G(A,B,C) = _____

Minimal sum of products for G(A,B,C) = _____

Problem 2. ★

Consider the 3-input Boolean function G(A,B,C) = $\overline{A} \cdot \overline{C} + A \cdot \overline{B} + \overline{B} \cdot \overline{C}$

1. How many 1's are there in the output column of G's 8-row truth table?

2. Give a minimal sum-of-products expression for G.

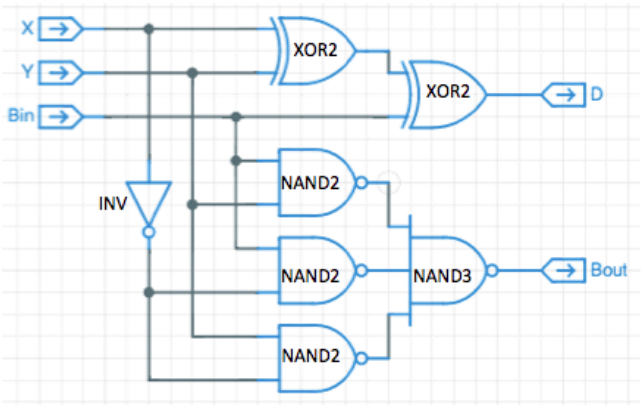
3. There's good news and bad news: the bad news is that the stockroom only has G gates. The good news is that it has as many as you need. Using only combinational circuits built from G gates, one can implement (choose the best response):

   (A) Any Boolean function (G is functionally complete)
   (B) Only functions with 3 inputs or less
   (C) Only functions with the same truth table as G

4. Can a sum-of-products expression involving 3 input variables with greater than 4 product terms *always* be simplified to a sum-of-products expression using fewer product terms?

Problem 3. ★

Consider the logic diagram shown below, which includes XOR2, NAND2, NAND3, and INV (inverter) gates.



Gate	t_{PD}
INV	1.0ns
NAND2	1.5ns
NAND3	1.8ns
XOR2	2.5ns

1. Using the t_{PD} information for the gate components shown in the table above, compute the t_{PD} for the circuit.

t_{PD} = ___

2. Find minimal sum-of-products expressions for both outputs, **D** and **Bout**.

   NOTE: The gates implement the following functions:

   - $NAND2(a,b) = \overline{a \cdot b}$
   - $NAND3(a,b,c) = \overline{a \cdot b \cdot c}$
   - $XOR2(a,b) = a \cdot \overline{b} + \overline{a} \cdot b$

Minimal sum of products for D(X,Y,Bin) = _____

Minimal sum of products for Bout(X,Y,Bin) = _____

Problem 4.

Simplify the following Boolean expressions by finding a *minimal sum-of-products expression* for each one:

1. $\overline{ac + b + c}$
2. $(a + b)c + \overline{c}a + b(\overline{a} + c)$
3. $a\overline{(b + c)}(b + a(b + c))$
4. $a(b + c(d + ef))$

Problem 5.

There are some Boolean expressions for which no assignment of values to variables can produce True (e.g., $a\overline{a}$). Those Boolean expressions are said to be *non-satisfiable*. Are the following Boolean expressions satisfiable? If the expression is satisfiable, give an assignment to variables that makes the expression evaluate to True. If the expression is non-satisfiable, prove it.

1. $(a + b)c + \overline{c}a + b(\overline{a} + c)$
2. $(x + y)(x + \overline{y})(z + \overline{y})(y + \overline{x})$
3. $(x + y + z)(x + y + \overline{z})(x + \overline{y} + z)(\overline{x} + y + z) \cdot (x + \overline{y} + \overline{z})(\overline{x} + y + \overline{z})(\overline{x} + \overline{y} + z)(\overline{x} + \overline{y} + \overline{z})$
4. $\overline{xyz + xy\bar{z} + x\bar{y}z + x\bar{y}\bar{z} + \bar{x}yz + \bar{x}y\bar{z} + \bar{x}\bar{y}\bar{z}}$

Problem 6.

(A) Simplify the following Boolean expressions by finding a minimal sum-of-products expression for each one. (*Note:* These expressions can be reduced into a minimal SOP by repeatedly applying the Boolean algebra properties we saw in lecture.)

$\overline{(a + b \cdot \overline{c})} \cdot d + c$

$a \cdot \overline{(b + c)}(c + a)$

(B) There are Boolean expressions for which no assignment of values to variables can produce True (e.g., $a \cdot \overline{a}$). These Boolean expressions are said to be *non-satisfiable*.

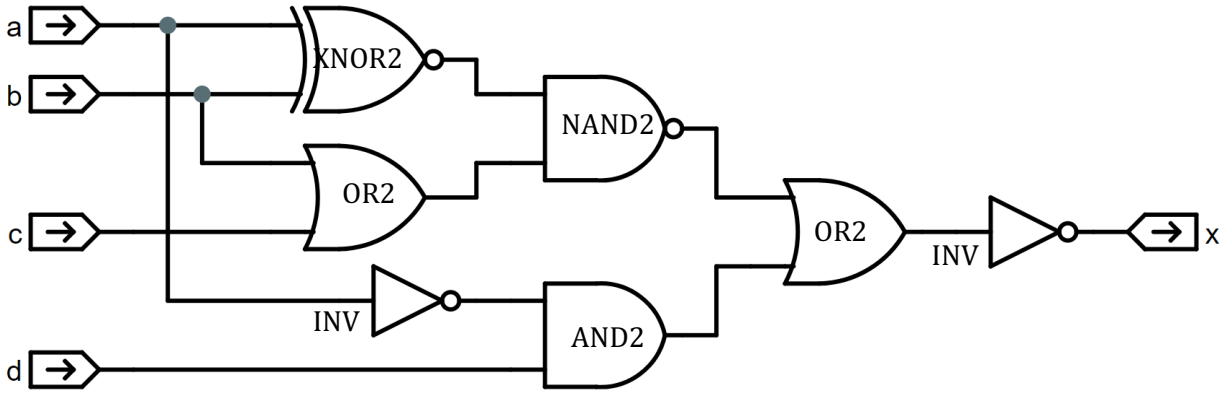
Are the following Boolean expressions satisfiable? If the expression is satisfiable, give an assignment to variables that makes the expression evaluate to True. If the expression is non-satisfiable, explain why.

1. $(\overline{x} + y\overline{z}) \cdot (\overline{y}x + z) \cdot (\overline{z}y + x)$

2. $(\overline{x} + y\overline{z}) \cdot (\overline{y}x + z) \cdot (\overline{z}y + x) + \ (\overline{x} + yz) \cdot (\overline{y}x + z) \cdot (\overline{z}y + x)$

Problem 7. From Past Quizzes (Spring 2020 Quiz 1) ★

(A) Consider the logic diagram below, which includes XNOR2, OR2, NAND2, AND2, and INV. Using the t_{PD} information for the gate components shown in the table below, compute the t_{PD} for the circuit.

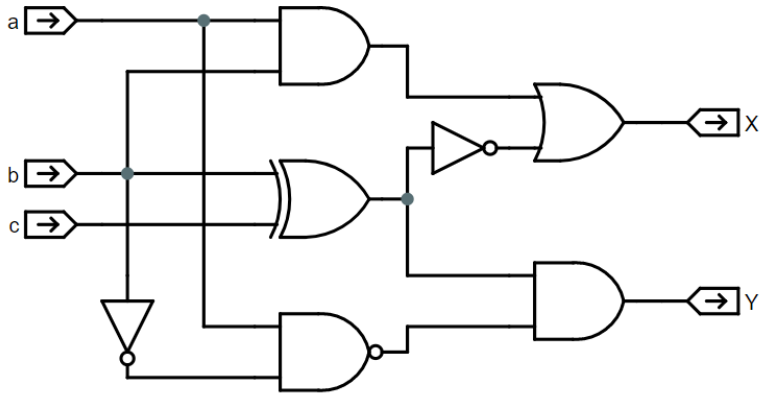


Gate	t_{PD}
XNOR2	7.0ns
OR2	5.5ns
NAND2	3.0ns
AND2	5.0ns
INV	2.0ns

t_{PD} (ns) = _____

(B) Given the circuit shown below, construct the truth table for outputs **X** and **Y**

a	**b**	**c**	**X**	**Y**
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		



(C) Find a minimal sum-of-products expression for output **X** of the circuit described by the truth table shown below.

a	b	c	d	X
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

Minimal sum of products for X = _____

(D) For each of the following expressions determine if it is satisfiable. If satisfiable, provide a minimal sum-of-products. Otherwise, show why it is not satisfiable.

1. $\overline{\overline{c}(a+b)(a+d)}(ab\overline{c})$

2. $(x+y)(x\overline{y}z+y\overline{z}+\overline{y})$