6.004 Worksheet Questions L16 – Caches

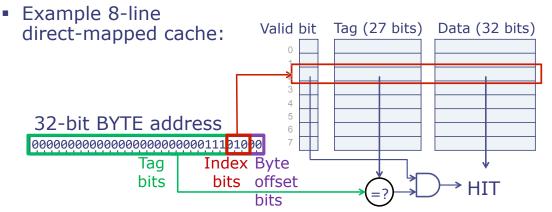
Keep the most often-used data in a small, fast SRAM (often local to CPU chip). The reason this strategy works: LOCALITY.

- Temporal locality: If a location has been accessed recently, it is likely to be accessed (reused) soon
- Spatial locality: If a location has been accessed recently, it is likely that nearby locations will be accessed soon

AMAT(Average Memory Access Time) = HitTime + MissRatio * MissPenalty

Direct-Mapped Caches

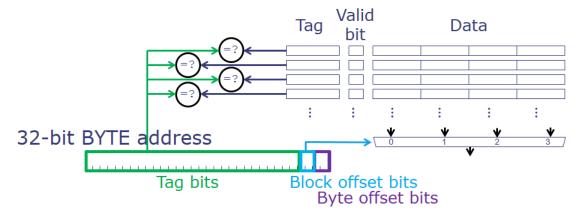
- Each word in memory maps into a single cache line
- Access (for cache with 2^w lines):
 - Index into cache with W address bits (the index bits)
 - Read out valid bit, tag, and data
 - If valid bit == 1 and tag matches upper address bits, HIT



Fully-Associative Cache

Opposite extreme: Any address can be in any location

- No cache index!
- Flexible (no conflict misses)
- Expensive: Must compare tags of all entries in parallel to find matching one

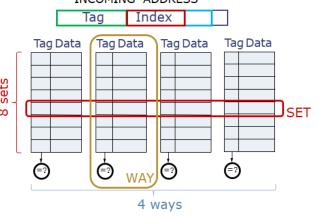


N-way Set-Associative Cache

 Use multiple direct-mapped caches in parallel to reduce conflict misses
 INCOMING ADDRESS



- # Rows = # Sets
- # Columns = # Ways
- Set size = #ways
 = "set associativity"
 (e.g., 4-way → 4 lines/set)
- Each address maps to only one set, but can be in any way within the set
- Tags from all ways are checked in parallel



 Fully-associative cache: Extreme case with a single set and as many ways as cache lines

Cache summary

How to break up address:

- Bottom 2 bits of memory address are always 00 (for word alignment) these 2 bits are called the byte offset.
- Number of block offset bits = log2 (words per block)
- Number of index bits = log2 (number of sets (rows))

Cache structure:

- In a direct mapped cache, number of sets = number of rows = number of cache lines
- In a set-associative cache, number of sets = number of rows = number of cache lines / # of ways (e.g., 2-way set associative cache has 2 cache lines in a set. A cache line = a block so it may contain multiple words).
- In a fully-associative cache, number of sets = 1 so there are no index bits.

Problem 1 (parts A and B are from worksheet 15).

The RISC-V Engineering Team is working on the design of a cache. They've decided that the cache will have a total of $2^{10} = 1024$ data words, but are still thinking about the other aspects of the cache architecture.

First assume the team chooses to build a direct-mapped cache with a block size of 4 words.

|) Please answer the following questions: |
|---|
| Number of lines in the cache: |
| Number of bits in the tag field for each cache entry: |
| This cache takes 2 clock cycles to determine if a memory access is a hit or a miss and, if it's a |

(B) This cache takes 2 *clock cycles* to determine if a memory access is a hit or a miss and, if it's a hit, return data to the processor. If the access is a miss, the cache takes 20 *additional clock cycles* to fill the cache line and return the requested word to the processor. If the hit rate is 90%, what is the processor's average memory access time in clock cycles?

Average memory access time assuming 90% hit rate (clock cycles): _____

Now assume the team chooses to build a 2-way set-associative write-back cache with a block size of 4 words. *The total number of data words in the entire cache is still 1024*. The cache uses a LRU replacement strategy.

(C) Please answer the following questions

Address bits used as block offset: A[:]

Address bits used as cache line index: A[:]

Address bits used for tag comparison: A[_:_]

(D) To implement the LRU replacement strategy this cache requires some additional state for each set. How many state bits are required for each set?

Number of state bits needed for each set for LRU: _____

To test this set-associative cache, the team runs the benchmark code shown on the right. The code sums the elements of a 16-element array. The first instruction of the code is at location 0x0 and the first element of the array is at location 0x10000. Assume that the cache is empty when execution starts and remember *the cache has a block size of 4 words*.

(E) How many instruction misses will occur when running the benchmark?

| Number of instruction | misses when | running the | benchmark: | |
|-----------------------------|-------------------|---------------|--------------|--|
| i tuilibei oi ilibei uetioi | i iiiigges wiieii | i uniming the | benemina ix. | |

(F) How many data misses (i.e., misses caused by the memory access from the LD instruction) will occur when running the benchmark?

| Number | of da | ta misses | when | running | the | benchmark: | |
|--------|-------|--------------|---|--------------|------|--------------|--|
| TIGHT | UI UI | ttt IIIIBBCB | , ,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,, | 1 W111111115 | LIIL | Denemina it. | |

(G) What's the exact hit rate when the complete benchmark is executed?

Benchmark hit rate: ____

- . = 0x0
 mv x3, x0 // index
 mv x1, x0 // sum
 // x4 = 0x10000
 lui x4, 0x10
- L: add x5, x4, x3 lw x2, 0(x5) add x1, x1, x2 addi x3, x3, 4 slti x2, x3, 64 bnez x2, L unimp // halt

 $. = 0 \times 10000$

A: .word 0x1 .word 0x2

.word 0xF .word 0x10

Problem 2

Assume, the program shown on the right is being run on a RISC-V processor with a cache with the following parameters:

| 2-way set-associative block size of 2, i.e., 2 data words are stored in | . = 0x240 test: | // start of program |
|--|---|--|
| each cache line | addi x4, x0, 16 | <pre>// initialize loop index J // to size of array</pre> |
| total number of data words in the cache is 32 LRU replacement strategy | mv x1, x0 | // x1: sum |
| (A) The cache will divide the 32-bit address supplied by the processor into four fields: 2 bits of byte offset, B bits of block offset, L bits of cache line index, and T bits of tag field. Based on the cache parameters given above, what are the appropriate values for B, L, and T? | subi x4, x4, 1 slli x2, x4, 2 lw x3, 0x420(x2 add x1, x1, x3 | <pre>// add up elements in array // decrement index // convert to byte offset)// load value from A[J] // add to sum // loop N times</pre> |
| | j test | // perform test again! |
| value for B: value for L: value for T: (B) If the SLLI instruction is resident in a cache line, what will be its cache line index? the value of the tag field for the cache? Cache line i | // allocate space . = 0x420 A: .word A[0] .word A[1] ndex for SLLI when res | |
| Tag | field for SLLI when res | sident in cache: |
| (C) Given that the code begins at address 0x240 at there are 16 elements in the array as shown in where the location holding the value A[j] will instruction in the program. | the code above, list all th | e values $j (0 \le j \le 16)$ |
| List all j where A[j] h | ave the same cache line | index as SLLI: |
| (D) If the outer loop is run many times, give the st that the number of compulsory misses as the c the number of hits and misses during execution | ache is first filled are insi | |
| | Steady-stat | e hit ratio (%): |
| | | |

Problem 3 *

Consider a 2-way set-associative cache where each way has 4 cache lines with a **block size of 2 words**. Each cache line includes a valid bit (V) and a dirty bit (D), which is used to implement a write-back strategy. The replacement policy is least-recently-used (LRU). The cache is used for both instruction fetch and data (LD,ST) accesses. Please use this cache when answering questions (A) through (D).

(A) Using this cache, a particular benchmark program experiences an average memory access time (AMAT) of 1.3 cycles. The access time on a cache hit is 1 cycle; the miss penalty (i.e., additional access time) is 10 cycles. What is the hit ratio when running the benchmark program? You can express your answer as a formula if you wish:

| Hit | ratio | for | benchmark | program: | |
|-----|-------|-----|-----------|----------|--|
| | | | ~ | D - 0 5 | |

(B) The circuitry for this cache uses various address bits as the block offset, cache line index and tag field. Please indicate which address bits A[31:0] are used for each purpose by placing a "B" in each address bit used for the block offset, "L" in each address bit used for the cache line index, and "T" in each address bit used for the tag field.

Fill in each box with "B", "L", or "T"

| ; | 31 | 30 | 29 | 28 | 27 | 26 | 25 | 24 | 23 | 22 | 21 | 20 | 19 | 18 | 17 | 16 | 15 | 14 | 13 | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |
|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 0 | 0 |

(C) This cache needs room to store new data and based on the LRU replacement policy has chosen the cache line whose information is shown to the right for replacement. Since the current contents of that line are marked as dirty (D = 1), the cache must write some information back to main memory. What is the address of each memory location to be written? Please give each address in hex.

Way: 0

Cache line index: 3 Valid bit (V): 1 Dirty bit (D): 1 Tag field: 0x123

Addresses of each location to be written (in hex):

(D) This cache is used to run the following benchmark program. The code starts at memory address 0; the array referenced by the code has its first element at memory address 0x200. First determine the number of memory accesses (both instruction and data) made during each iteration through the loop. Then estimate the steady-state average hit ratio for the program, i.e., the average hit ratio after many iterations through the loop.

```
addi x1, x1, 1 // increment checksum
add x1, x1, x2 // include data value in checksum
addi x3, x3, 4 // byte index of next array element
slti x2, x3, 1000 // process 250 entries
bnez x2, loop
unimp // halt

. = 0x200
array:
... array contents here ...

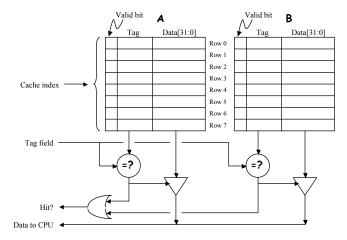
Number of memory accesses made during each iteration of the loop:

Estimated steady-state average hit ratio:
```

Problem 4

Consider the diagram to the right for a 2-way set associative cache to be used with our RISC-V processor. Each cache line holds a single 32-bit word of data along with its associated tag and valid bit (0 when the cache line is invalid, 1 when the cache line is valid).

(A) The RISC-V produces 32-bit byte addresses, A[31:0]. To ensure the best cache performance, which address bits should be used for the cache index? For the tag field?



address bits used for cache index: A[:]

address bits used for tag field: A[_:_]

(B) Suppose the processor does a read of location 0x5678. Identify which cache location(s) would be checked to see if that location is in the cache. For each location specify the cache section (A or B) and row number (0 through 7). E.g., 3A for row 3, section A. If there is a cache hit on this access what would be the contents of the tag data for the cache line that holds the data for this location?

cache location(s) checked on access to 0x5678: ______
cache tag data on hit for location 0x5678 (hex): _____

(C) Assume that checking the cache on each read takes 1 cycle and that refilling the cache on a miss takes an *additional* 8 cycles. If we wanted the *average* access time over many reads to be 1.1 cycles, what is the minimum hit ratio the cache must achieve during that period of time? You needn't simplify your answer.

minimum hit ratio for 1.1 cycle average access time:

(D) Estimate the approximate cache hit ratio for the following program. Assume the cache is empty before execution begins (all the valid bits are 0) and that an LRU replacement strategy is used. Remember the cache is used for both instruction and data (LD) accesses.

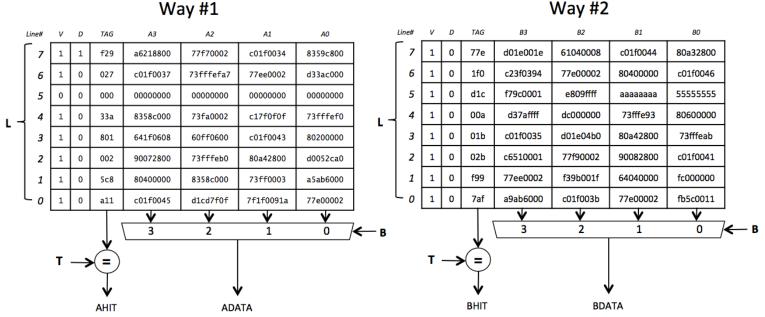
```
. = 0
      addi x4, x0, 0x100
      mv x1, x0
                           // x2 = 0x1000
      lui x2, 1
loop: lw x3, 0(x4)
      addi x4, x4, 4
      add x1, x1, x3
      addi x2, x2, -1
      bnez x2, loop
      sw x1, 0x100(x0)
                            // halt
      unimp
       = 0x100 
source:
      . = . + 0x4000 // Set source to 0x100, reserve 0x1000 words
                                                approximate hit ratio: _____
```

(E) After the program of part (D) has finished execution what information is stored in row 4 of the cache? Give the addresses for the two locations that are cached (one in each of the sections) or briefly explain why that information can't be determined.

Addresses whose data is cached in "Row 4": _____ and _____

Problem 5★

A standard unpipelined RISC-V is connected to a 2-way set-associative cache containing 8 sets, with a block size of 4 32-bit words. The cache uses a LRU replacement strategy. At a particular point during execution, a snapshot is taken of the cache contents, which are shown below. All values are in hex; assume that any hex digits not shown are 0.



(A) The cache uses bits from the 32-bit byte address produced by the processor to select the appropriate set (L), as input to the tag comparisons (T) and to select the appropriate word from the data block (B). For correct and optimal performance what are the appropriate portions of the address to use for L, T and B? Express your answer in the form "A[N:M]" for N and M in the range 0 to 31, or write "CAN'T TELL".

Address bits to use for L: A[:] Address bits to use for T: A[:] Address bits to use for B: A[:]

(B) For the following addresses, if the contents of the specified location appear in the cache, give the location's 32-bit contents in hex (determined by using the appropriate value from the cache). If the contents of the specified location are NOT in the cache, write "MISS".

Contents of location 0xA1100 (in hex) or "MISS": _____ Contents of location 0x548 (in hex) or "MISS": _____

(C) Ignoring the current contents of the cache, is it possible for the contents of locations 0x0 and 0x1000 to both be present in the cache simultaneously?

Locations 0x0 and 0x1000 present simultaneously (circle one): YES ... NO

(D) Give a one-sentence explanation of how the D bit got set to 1 for Line #7 of Way #1. At what point should the D bit be reset to 0?

One sentence explanation

(E) The following code snippet sums the elements of the 32-element integer array X. Assume this code is executing on a RISC-V processor with a cache architecture as described above and that, initially, the cache is empty, i.e., all the V bits have been set to 0. Compute the hit ratio as this program runs until it executes the *unimp* instruction, a total of 2 + (6*32) + 1 = 195 instruction fetches and 32 data accesses.

Hit ratio: . = 0mv x4, x0 // loop counter mv x1, x0 // accumulated sum loop: slli x2, x4, 2 // convert loop counter to byte offset 1w x3, 0x100(x2)// load next value from array // add value to sum add x1, x1, x3 // increment loop counter
// finished with all 32 elements? addi x4, x4, 1 slti x2, x4, 32 bnez x2, loop // nope, keep going unimp // all done, sum in x1 . = 0x100X: .word 1 // the 32-element integer array X .word 2 .word 32

Problem 6 *

After his geek hit single *I Hit the Line*, renegade singer Johnny Cache has decided he'd better actually learn how a cache works. He bought three RISC-V processors, identical except for their cache architectures:

- **Proc1** has a 64-line direct-mapped cache
- *Proc2* has a 2-way set associative cache, LRU, with a total of 64 lines
- *Proc3* has a 4-way set associative cache, LRU, with a total of 64 lines

Note that each cache has the same total capacity: 64 lines, each holding a single 32-bit **word** of data or instruction. All three machines use the same cache for data and instructions fetched from main memory.

Johnny has written a simple test progr

```
// Try a little cache benchmark
// Assume x7 = 0x2000 (data region A)
// Assume x8 = 0x3000 (data region B)
// Assume x9 = 16 (size of data regions in BYTES!)
. = 0 \times 1000
                           // start program here
     addi x6, x0, 1000
P:
                          // outer loop count
     // Loop index i (array offset)
Q:
R:
                        // read B[i]
     1w \times 2, 0(\times 10)
     bnez x3, R
     addi x6, x6, -1
                          // repeat many times
     bnez x6, Q
                           // halt
     unimp
```

Johnny runs his program on each processor, and finds that one processor model outperforms the other two.

(A) Which processor model gets the highest hit ratio on the above benchmark?

Circle one: Proc1 Proc2 Proc3

(B) Johnny changes the value of **B** in his program to **0x2000** (same as **A**), and finds a substantial improvement in the hit rate attained by one of the processor models (approaching 100%). Which model shows this marked improvement?

Circle one: Proc1 Proc2 Proc3

(C) Finally, Johnny moves the code region to 0x0 and the two data regions **A**, and **B** each to 0x0, and sets x9 to 64. What is the TOTAL number of cache misses that will occur executing this version of the program on each of the processor models?

TOTAL cache misses running on Proc1: ____; Proc2: ____; Proc3: ____

Problem 7 ★ (A) We would like to design a cache with an AMAT (average memory access time) of 1.5 cycles. Accessing our cache takes 1 cycle, and on a miss, it takes an *additional* 10 cycles to retrieve the data from main memory and update the cache. What does our hit ratio need to be in order to achieve the target AMAT?

| Hit | ratio | = | | | | |
|-----|-------|---|--|--|--|--|
| | | | | | | |

We choose to implement a 2-way set-associative cache with a block size of 4 (i.e. 4 words per line). The number of sets in the cache is 4. Assume that addresses and data words are 32 bits.

(B) To ensure the best cache performance, which address bits should be used for the block offset, the cache index, and the tag field?

Address bits used for byte offset: A[1 : 0]

Address bits used for block offset: A[... : ...]

Address bits used for cache index: A[... : ...]

Address bits used for tag field: A[... : ...]

(C) Assuming the cache uses a writeback policy, what is the total number of bits per cache line? Please show your work for partial credit.

bits

We want to analyze the performance of this cache on the following assembly program, which iterates through a 1000-word array A and sets each element to A[i] = -A[i]. The base address of array A is 0x3000.

```
. = 0x100 // The following code starts at address 0x100
// Assume the following registers are initialized:
// x1=0 (loop index)
// x2=1000 (number of array elements)
// x3=0x3000 (base address of array A)
loop:
   slli x5, x1, 2 // x5 = byte offset of the ith element
   add x6, x5, x3 // x6 = address of A[i]
  1w x7, 0(x6)
                   // x7 = A[i]
  sub x7, x0, x7
                   // x7 = -A[i]
   sw x7, 0(x6)
                   // store A[i] = -A[i]
   addi x1, x1, 1 // increment i
  blt x1, x2, loop // continue looping
```

(D) Below is the cache state the first time the program is about to enter the loop at loop. Assume that the cache uses a least-recently used (LRU) replacement policy, and that all cache lines in Way 1 are currently the least-recently used. Mark up the cache below to indicate the state of the cache immediately after one loop iteration (after executing the blt instruction for the first time). You do not need to specify the value of data words, but do specify the values of D (dirty bit), V (valid bit), and Tag.

Way 0

| D | V | Tag | Word 0 | Word 1 | Word 2 | Word 3 |
|---|---|-----|--------|--------|--------|--------|
| 0 | 1 | 0x0 | | | | |
| 0 | 1 | 0x0 | | | | |
| 0 | 1 | 0x0 | | | | |
| - | 0 | - | | | | |

Way 1

| D | V | Tag | Word 0 | Word 1 | Word 2 | Word 3 |
|---|---|-----|--------|--------|--------|--------|
| - | 0 | - | | | | |
| - | 0 | - | | | | |
| - | 0 | 1 | | | | |
| - | 0 | - | | | | |

| (E) | How many instruction fetches and data accesses occur per iteration of the loop? |
|-----|--|
| | |
| | |
| | |
| | Number of instruction fetches: |
| | Number of data accesses: |
| (F) | After the program has been running for many loop iterations, what is the steady-state hit ratio for instruction fetches and data accesses? |
| | |
| | |
| | |
| | |
| | Steady-state hit ratio for instruction fetches: |
| | Steady-state hit ratio for data accesses: |
| | |
| | |
| | |
| | |
| | |
| | |
| | |

Problem 8

| Anne and Ben just learned about caches in 6.004 and decided to design their own. |
|---|
| (A) They would like to design a cache with an AMAT (average memory access time) of 3 cycles. Accessing the cache should take 1 cycle, and on a miss, it should take an <i>additional</i> 16 cycles to retrieve the data from main memory, update the cache, and return the requested word to the processor. What should their hit ratio be in order to achieve the target AMAT? |
| |
| Hit ratio: |
| Ben suggests implementing a 2-way set-associative cache with a block size of 4 (i.e. 4 words per line). The number of sets in the cache is 8. Assume that addresses and data words are 32 bits wide. |
| (B) To ensure the best cache performance, which address bits should be used for the block offset, the cache index, and the tag field? |
| |
| Address bits used for byte offset: A[_1_: _0_] |
| Address bits used for block offset: A[:] |
| Address bits used for cache index: A[:] |
| Address bits used for tag field: A[:] |
| (C) Anne agrees with the appropriateness of a 2-way set-associative implementation, but she suspects that a larger block size might result in a higher hit rate. Suppose the block size of the cache is doubled to 8. If the total number of data words in the cache remains unchanged, how would the number of cache lines change? |
| |
| |
| Change in # of cache lines (select one of the choices below): |

UNCHANGED ... +1 ... -1 ... 2x ... 0.5x ... CAN'T TELL

Ultimately, they choose to implement Ben's 2-way, 4-block cache. Below is a snapshot of the cache during the execution of some unknown code. The column labeled $Word\ x$ corresponds to the x^{th} word of the block. The V bit specifies whether or not the line is valid, and the D bit specifies whether or not the line is dirty.

| V | D | Tag | Word | Word | Word | Word |
|---|----------------------------|--|--|---|--|---|
| | | | 0 | 1 | 2 | 3 |
| 1 | 0 | 0x32 | 0x0A | 0x1A | 0x2A | 0x3A |
| 1 | 0 | 0x32 | 0x4B | 0x5B | 0x6B | 0x7B |
| 1 | 0 | 0x32 | 0x3C | 0x2C | 0x1C | 0x0C |
| 1 | 0 | 0x32 | 0x7D | 0x6D | 0x5D | 0x4D |
| 1 | 1 | 0x50 | 0x33 | 0x23 | 0x13 | 0x03 |
| 1 | 1 | 0x50 | 0x44 | 0x34 | 0x24 | 0x14 |
| 0 | 0 | 0x43 | 0x55 | 0x65 | 0x75 | 0x85 |
| 1 | 0 | 0x66 | 0x66 | 0x76 | 0x86 | 0x96 |
| | 1 1 1 1 1 1 | 1 0 1 0 1 0 1 0 1 0 1 1 1 1 0 0 | 1 0 0x32 1 0 0x32 1 0 0x32 1 0 0x32 1 0 0x32 1 1 0x50 1 1 0x50 0 0 0x43 | 0 1 0 0x32 0x0A 1 0 0x32 0x4B 1 0 0x32 0x3C 1 0 0x32 0x7D 1 1 0x50 0x33 1 1 0x50 0x44 0 0 0x43 0x55 | 0 1 1 0 0x32 0x0A 0x1A 1 0 0x32 0x4B 0x5B 1 0 0x32 0x3C 0x2C 1 0 0x32 0x7D 0x6D 1 1 0x50 0x33 0x23 1 1 0x50 0x44 0x34 0 0 0x43 0x55 0x65 | 0 1 2 1 0 0x32 0x0A 0x1A 0x2A 1 0 0x32 0x4B 0x5B 0x6B 1 0 0x32 0x3C 0x2C 0x1C 1 0 0x32 0x7D 0x6D 0x5D 1 1 0x50 0x33 0x23 0x13 1 1 0x50 0x44 0x34 0x24 0 0 0x43 0x55 0x65 0x75 |

| V | D | Tag | Word | Word | Word | Word |
|---|---|------|------|------|------|------|
| | | | 0 | 1 | 2 | 3 |
| 1 | 0 | 0x33 | 0x80 | 0x81 | 0x82 | 0x83 |
| 1 | 1 | 0x33 | 0xB4 | 0xB5 | 0xB6 | 0xB7 |
| 1 | 0 | 0x95 | 0xC3 | 0xC2 | 0xC1 | 0xC0 |
| 1 | 0 | 0x95 | 0xD3 | 0xD4 | 0xD5 | 0xD6 |
| 1 | 0 | 0x22 | 0x89 | 0x88 | 0x87 | 0x86 |
| 1 | 0 | 0xA0 | 0x92 | 0x93 | 0x94 | 0x95 |
| 1 | 1 | 0x37 | 0xF5 | 0xF6 | 0xF7 | 0xF8 |
| 1 | 1 | 0x18 | 0xA7 | 0xA8 | 0xA9 | 0xAA |

(D) Would a load request to address 0x193C result in a hit or a miss? If it results in a hit, specify what value is returned; if it is a miss, write N/A.

| Hit / Miss : | _ |
|---------------------------------------|---|
| Returned value if hit or N/A if miss: | |

Anne and Ben want to analyze the performance of this cache on the following assembly program, which calculates the first 256 terms in the Fibonacci sequence and stores them in an array A. The base address of array A is 0x3000.

```
// Assume the following registers are initialized:
// x1 = 0 (initial loop index)
// x2 = 256 - 2 = 254 (number of Fibonacci elements to calculate)
// x3 = 0x3000 (base address of array A)
\cdot = 0x100 // The following code starts at address 0x100
fibonacci:
   li x4, 1
                    // x4 = 1 (second element in sequence)
   sw x0, 0(x3)
sw x4, 4(x3)
                    // A[0] = 0
                    // A[1] = 1
loop:
   slli x4, x1, 2 // x4 = byte offset of the ith element
   add x5, x4, x3 // x5 = address of A[i]
   lw x6, 0(x5) // x6 = A[i]
lw x7, 4(x5) // x7 = A[i+1]
   add x6, x6, x7 // x6 = A[i] + A[i+1]
   sw x6, 8(x5) // A[i+2] = x6
   addi x1, x1, 1 // increment i
   blt x1, x2, loop // continue looping
```

Answer the following questions about the behavior of the cache during execution of the above code. Assume that the cache uses a least recently used (LRU) replacement policy, that the cache is initially empty, and that all cache lines in Way 0 are currently the least-recently used.

(E) How many instruction fetches and data accesses occur per iteration of the loop?

| Number of instruction fetches: | |
|--------------------------------|--|
| Number of data accesses: | |

(F) After the program has been running for many loop iterations, what is the steady-state hit ratio for instruction fetches and data accesses? *Hint: Note that in steady state each array element is accessed in multiple loop iterations.*

| Steady-state hit ratio for instruction fetches: | |
|---|--|
| | |
| Steady-state hit ratio for data accesses: | |

Problem 9 (From Past Quizzes)

Dilvina and Saniel are analyzing 4.006 grade statistics and are performing some hefty calculations, so they suspect that a cache could improve their system's performance.

(A) They are considering using a 2-way set-associative cache with a block size of 4 (i.e. 4 words per line). The cache can store a total of 64 words. Assume that addresses and data words are 32 bits wide. To properly make use of locality, which address bits should be used for the block offset, the cache index, and the tag field?

Address bits used for byte offset: A[__1__:___]

Address bits used for tag field: A[___:___]

Address bits used for cache index: A[___:___]

(B) If Dilvina and Saniel instead used a direct-mapped cache with the same total capacity (64 words) and same block size (4 words), how would the following parameters in their system change?

Change in # of cache lines (select one of the choices below):

UNCHANGED ... +1 ... -1 ... 2x ... 0.5x ... CAN'T TELL

Change in # of bits in tag field (select one of the choices below):

UNCHANGED ... +1 ... -1 ... 2x ... 0.5x ... CAN'T TELL

Ultimately, they decided that the 2-way set associative cache would probably have better performance for their application, so the remainder of the problem will be considering a 2-way set associative cache. Below is a snapshot of this cache during the execution of some unknown code. **V** is the valid bit and **D** is the dirty bit of each set.

| | Way 0 | | | | | | | Way 1 | | | | | | | |
|---|-------|---|------|------|------|------|------|-------|---|---|------|------|------|------|------|
| | Word | | | | | | | | | | Word | | | | |
| | V | D | Tag | 0 | 1 | 2 | 3 | | V | D | Tag | 0 | 1 | 2 | 3 |
| 0 | 1 | 0 | 0x32 | 0x0A | 0x1A | 0x2A | 0x3A | 0 | 1 | 0 | 0x33 | 0x80 | 0x81 | 0x82 | 0x83 |
| 1 | 1 | 0 | 0x32 | 0x4B | 0x5B | 0x6B | 0x7B | 1 | 1 | 0 | 0x33 | 0xB4 | 0xB5 | 0xB6 | 0xB7 |
| 2 | 1 | 0 | 0x32 | 0x3C | 0x2C | 0x1C | 0x0C | 2 | 1 | 0 | 0x95 | 0xC3 | 0xC2 | 0xC1 | 0xC0 |
| 3 | 1 | 0 | 0x32 | 0x7D | 0x6D | 0x5D | 0x4D | 3 | 1 | 0 | 0x95 | 0xD3 | 0xD4 | 0xD5 | 0xD6 |
| 4 | 1 | 1 | 0x50 | 0x33 | 0x23 | 0x13 | 0x03 | 4 | 1 | 1 | 0x22 | 0x89 | 0x88 | 0x87 | 0x86 |
| 5 | 1 | 1 | 0x50 | 0x44 | 0x34 | 0x24 | 0x14 | 5 | 1 | 1 | 0xA0 | 0x92 | 0x93 | 0x94 | 0x95 |
| 6 | 0 | 0 | 0x43 | 0x55 | 0x65 | 0x75 | 0x85 | 6 | 0 | 0 | 0x37 | 0xF5 | 0xF6 | 0xF7 | 0xF8 |
| 7 | 1 | 0 | 0x66 | 0x66 | 0x76 | 0x86 | 0x96 | 7 | 1 | 0 | 0x18 | 0xA7 | 0xA8 | 0xA9 | 0xAA |

(C) Would the following memory accesses result in a hit or a miss? If it results in a hit, specify what value is returned; if it is a miss, explain why in a few words or by showing your work.

32-Bit Byte Address: 0x4AB4

| | Line index: | |
|--|---------------|--|
| | Tag: 0x | |
| | Block offset: | |
| Returned value if hit / Explanation if miss: | | |
| 32-Bit Byte Address: 0x21E0 | | |
| | | |
| | | |
| | Line index: | |
| | Tag: 0x | |
| | Block offset: | |

Returned value if hit / Explanation if miss: _____

Dilvina and Saniel want to analyze the performance of this same cache on the following assembly program, which averages the quiz 2 scores for the 225 students in 4.006. The students' scores are stored in an array A whose base address is at 0x3000.

```
// Assume the following registers are initialized:
// x1 = 0 (loop index)
// x2 = 225 (number of 4.006 students)
// x3 = 0x3000 (base address of array A)
// x6 = 0 (used for summing)
\cdot = 0x0 // The following code starts at address 0x0
sum loop:
     slli x4, x1, 2
                         // x4 = 4*i
     divide by n:
     // divide by 225 here
```

Answer the following questions about the behavior of the cache during execution of the above code. Assume that the cache uses a least recently used (LRU) replacement policy, that the cache is initially empty, and that all cache lines in Way 0 are currently the least-recently used.

(

| accesses occur per iteration of the loop? |
|---|
| r of instruction fetches per loop iteration: |
| umber of data accesses per loop iteration: |
| sses (both instruction fetches and data accesses) the |
| First loop iteration hit ratio: |
| |

(F) After the program has been running for many loop iterations, what is the steady-state hit ratio for all memory accesses (both instruction fetches and data accesses)?

| Steady-state | hit | ratio: | |
|--------------|-----|--------|--|
| | | | |

(G) Dilvina and Saniel want to use the cache above in their memory hierarchy between the CPU and main memory. The cache takes 5 cycles to determine if a memory access is a miss or a hit, and the main memory in their system takes an additional 140 cycles for each memory access that reaches it. What is the steady-state AMAT of their memory system during the test that they ran above?

| AMAT: | | | |
|--------------|--|--|--|
| | | | |