

Introduction to Pipelining

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- Introduce pipelining, a technique that uses registers to improve the throughput of a circuit
- Examine the tradeoffs of different design styles (combinational, pipelined, multi-cycle) through a multiplier case study

Reminder: Multi-Cycle Computations

- Sequential circuits can implement more computations than combinational circuits
 - Variable amount of input/output
 - Variable number of steps

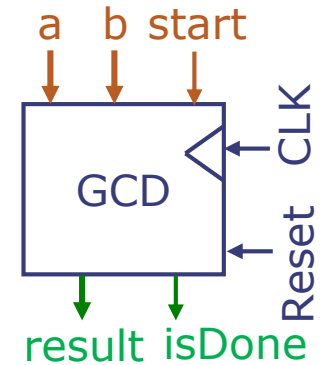
Reminder: Multi-Cycle Computations

- Sequential circuits can implement more computations than combinational circuits
 - Variable amount of input/output
 - Variable number of steps
- Multi-cycle circuits implement a computation over multiple cycles
 - New computation is started by setting inputs at a particular cycle
 - Circuit takes several cycles (possibly a variable number) to finish computation, then makes output available
 - Circuit performs only one computation at a time; a new computation cannot begin until previous one has finished

Reminder: GCD Circuit

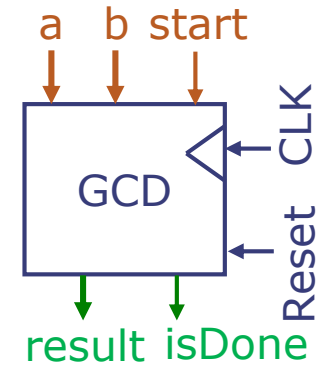
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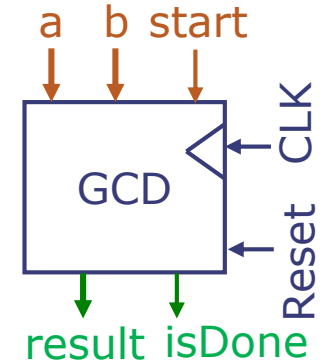
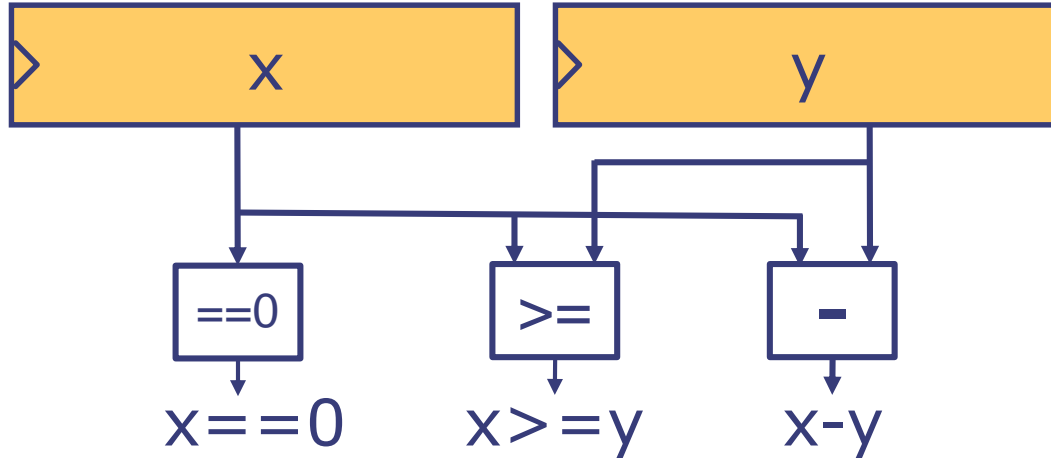
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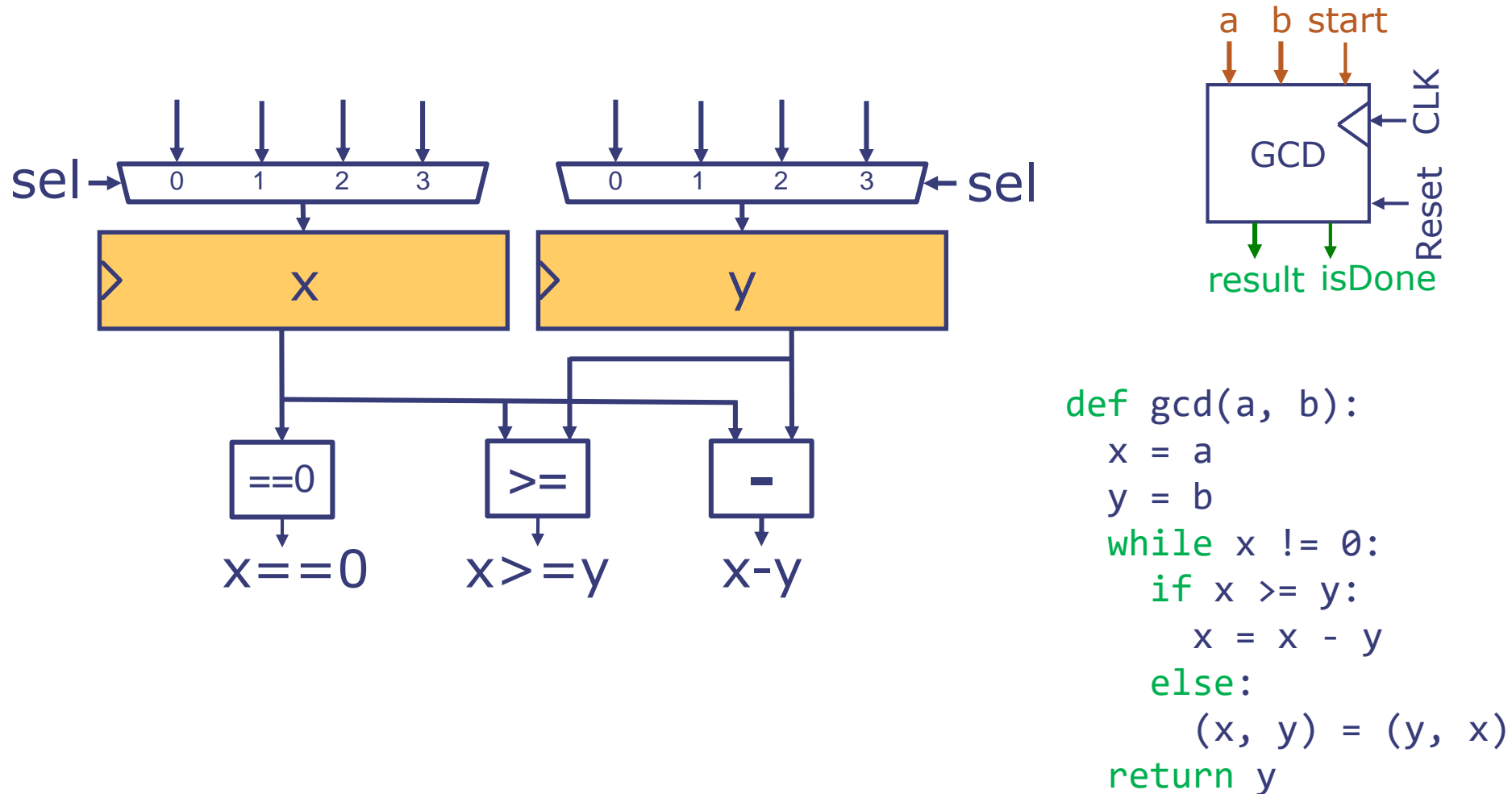
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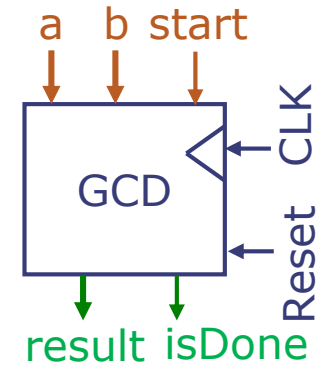
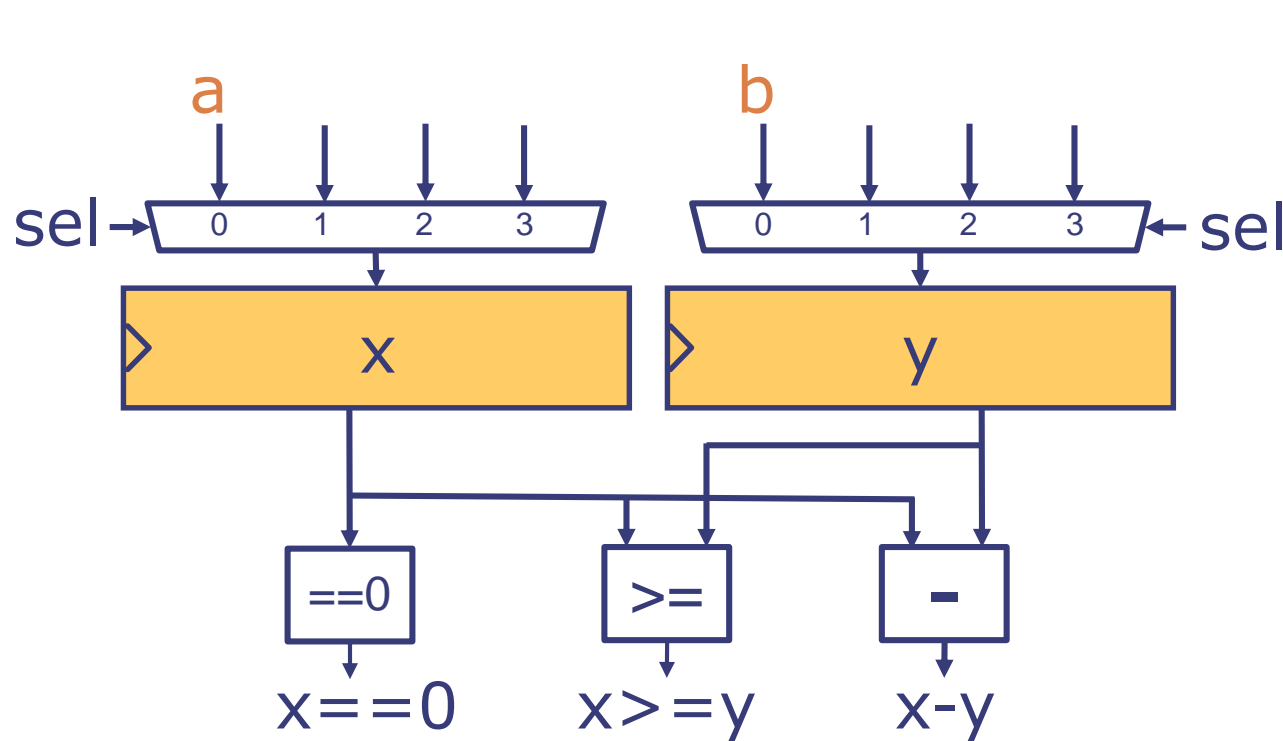


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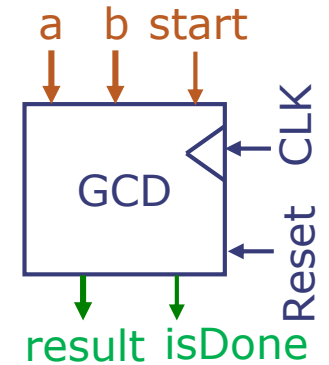
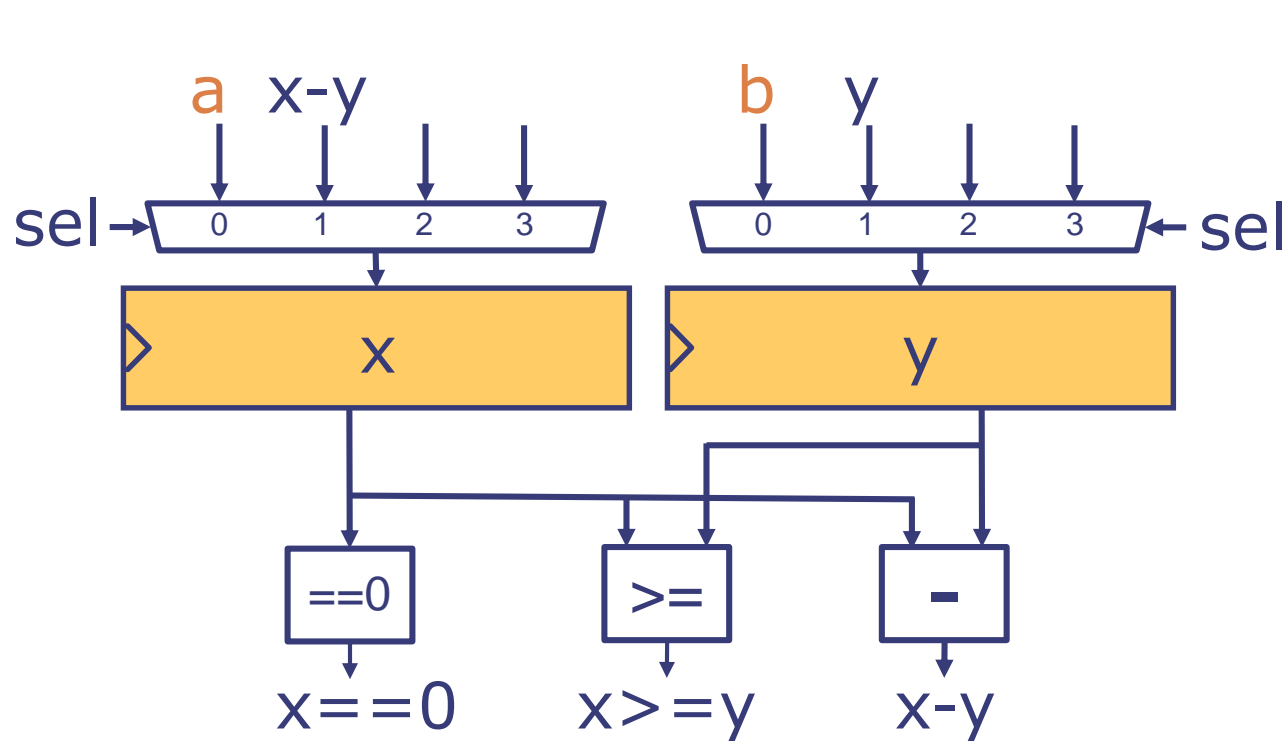


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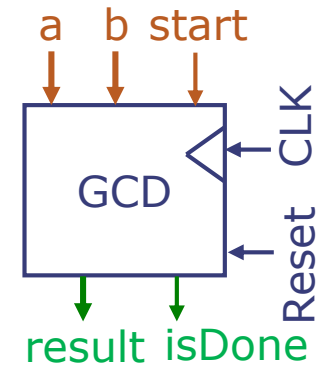
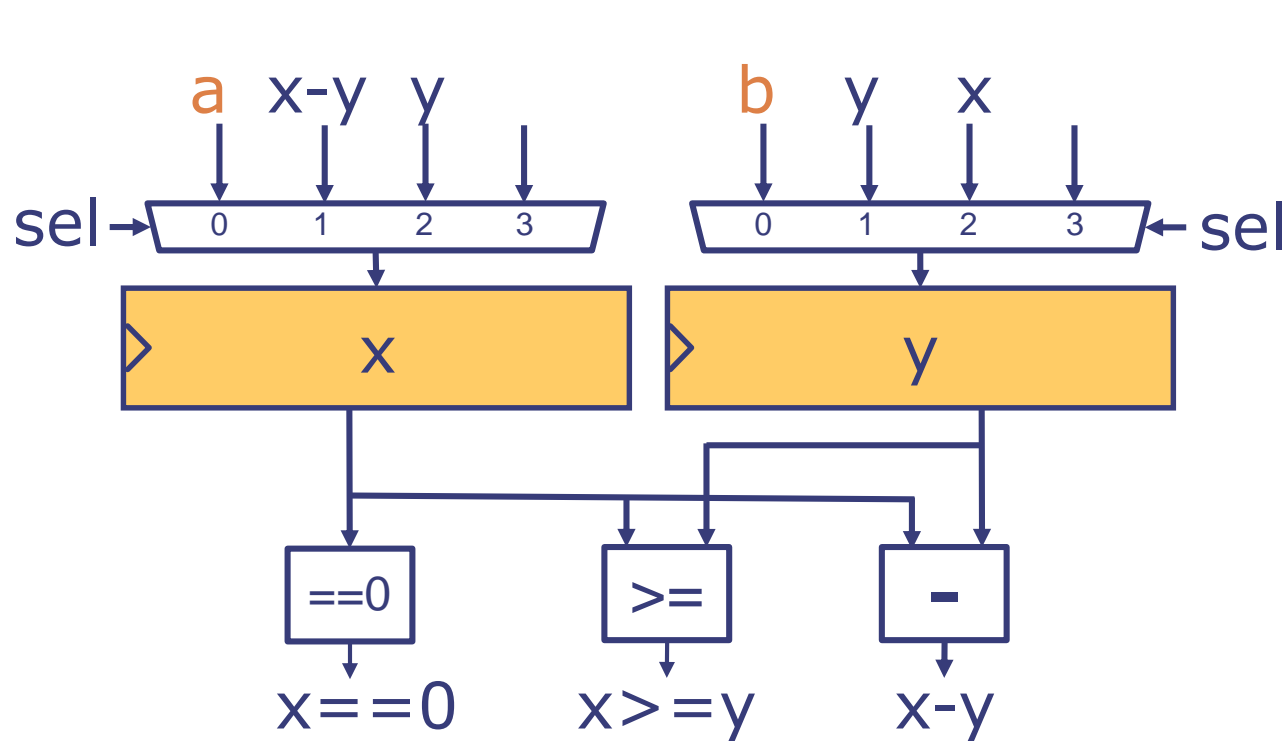
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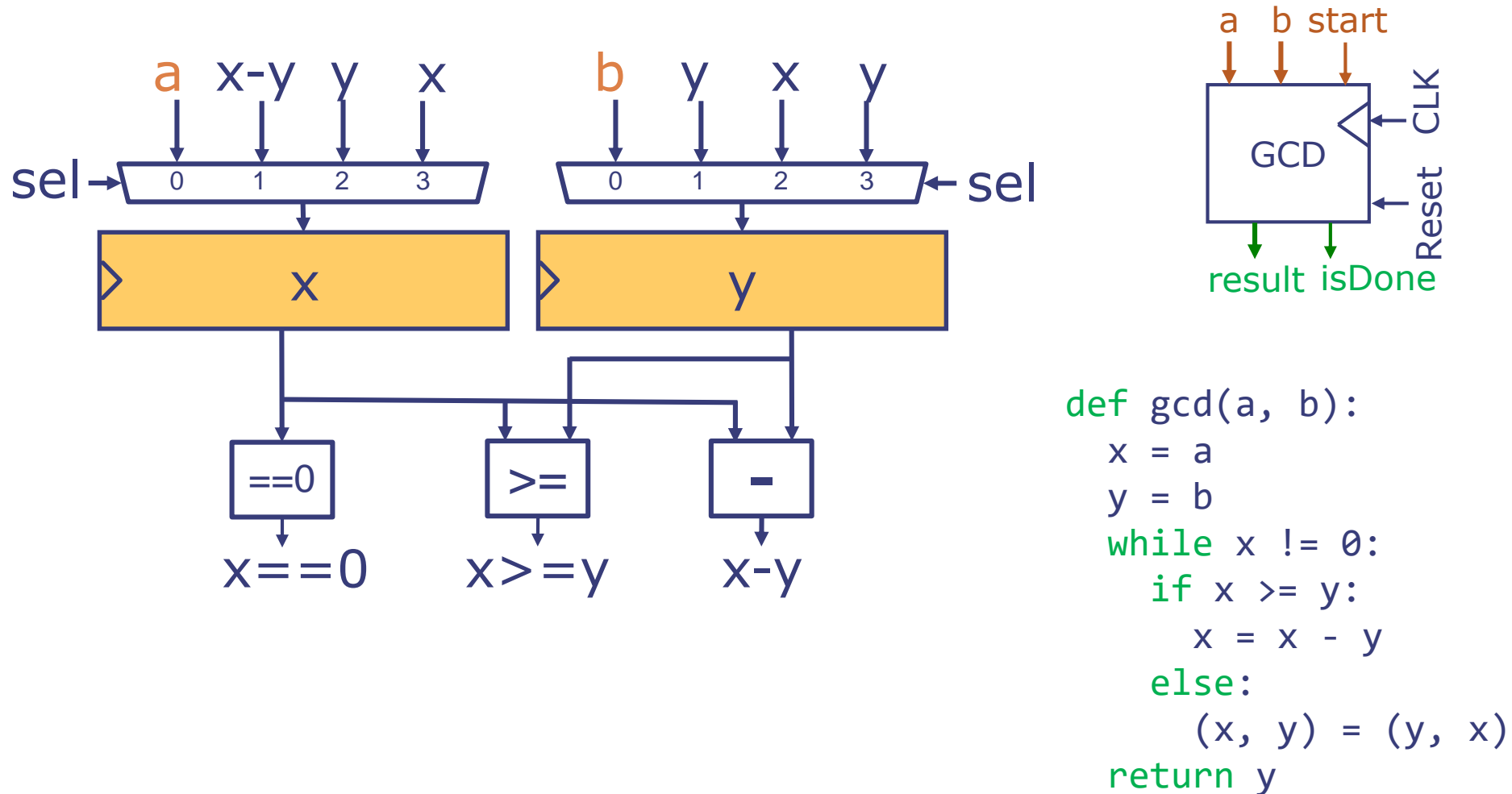
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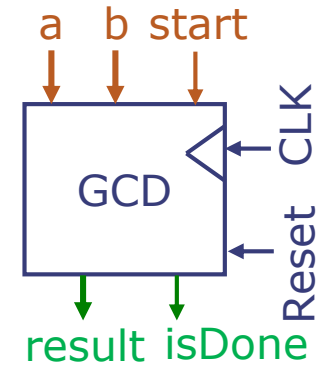
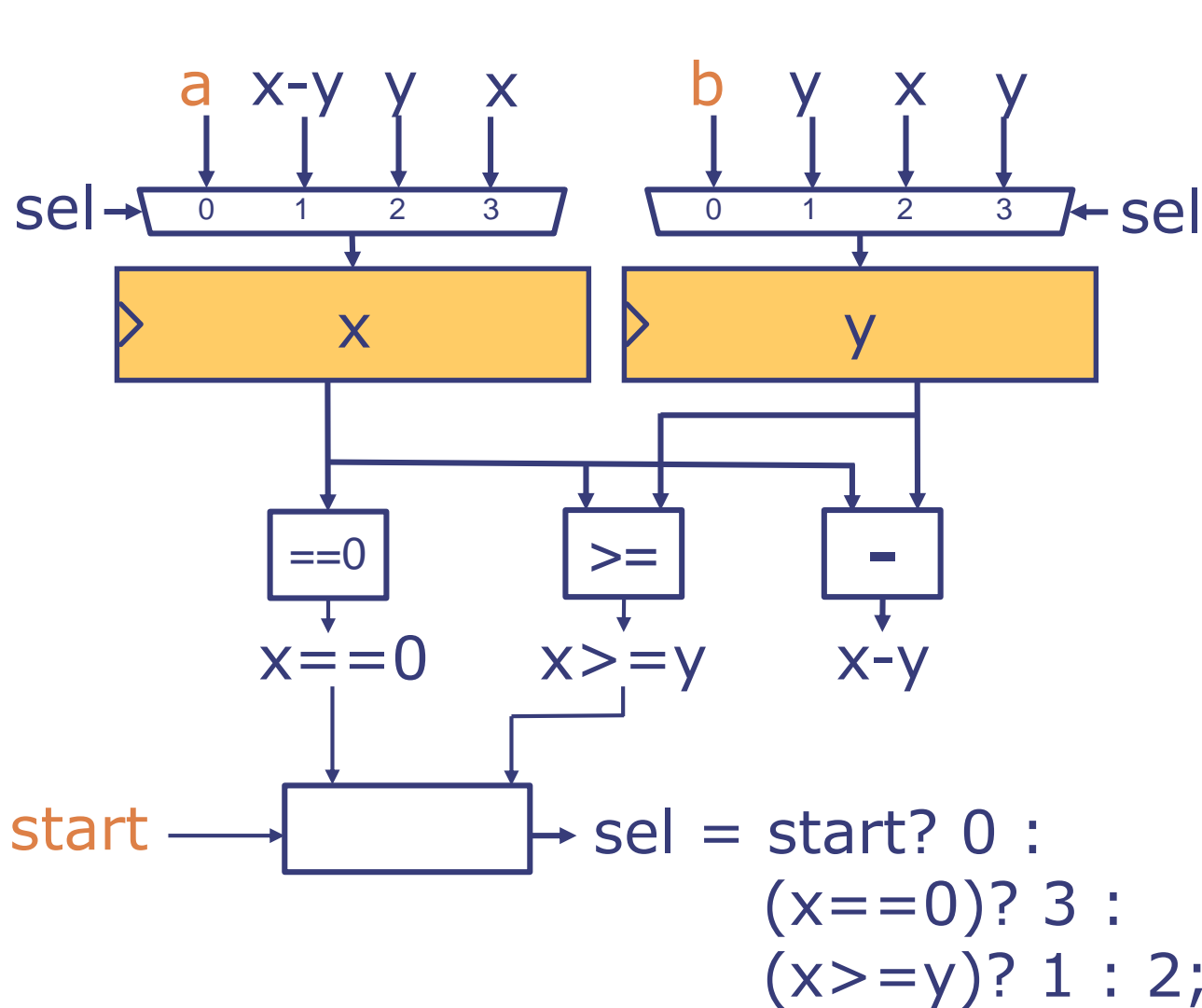


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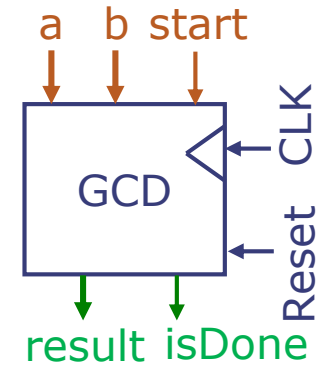
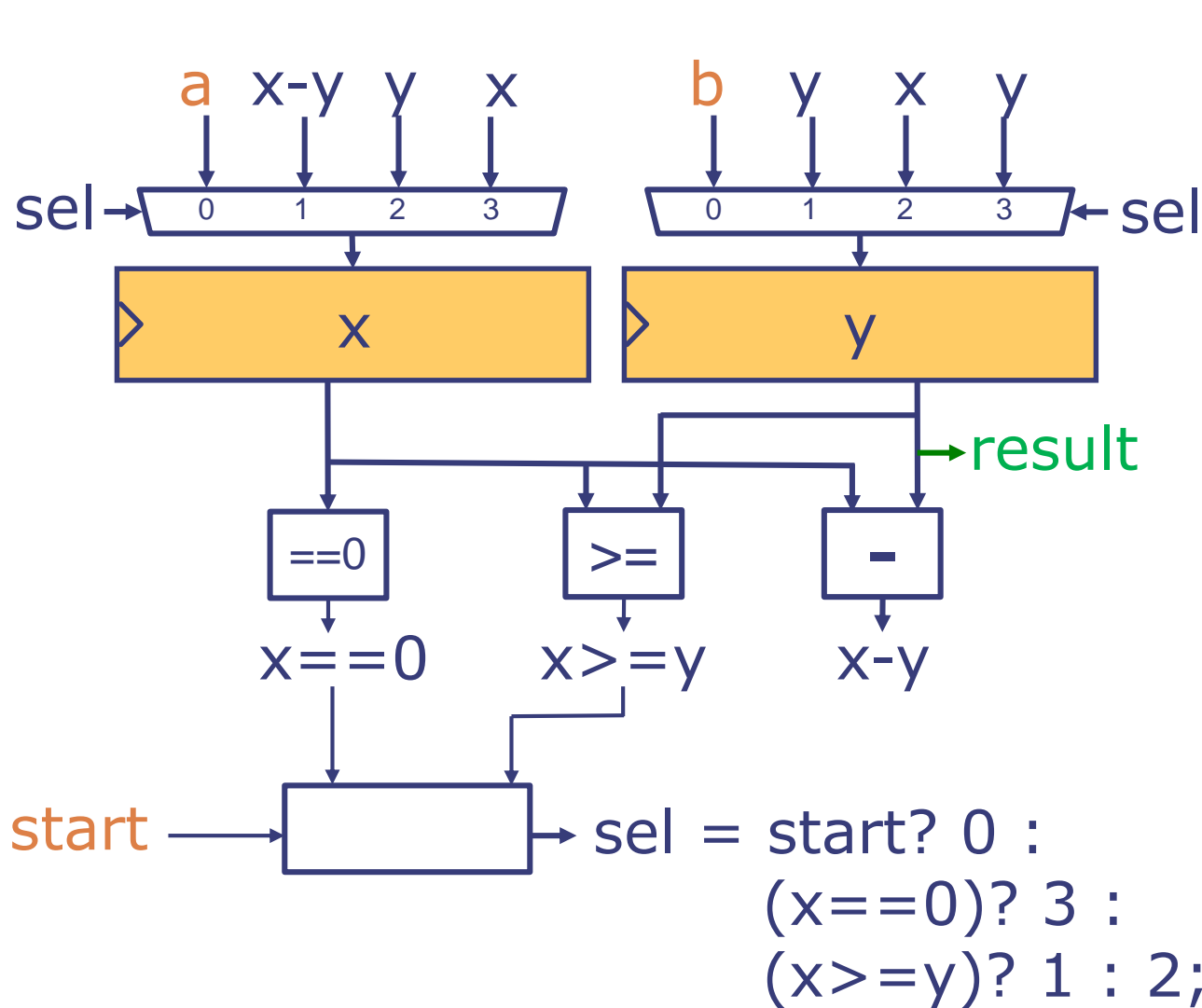


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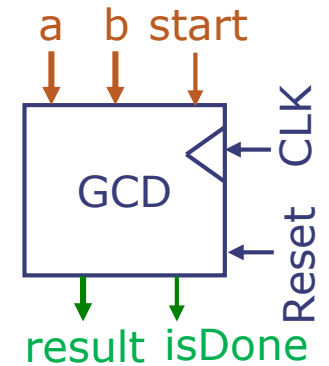
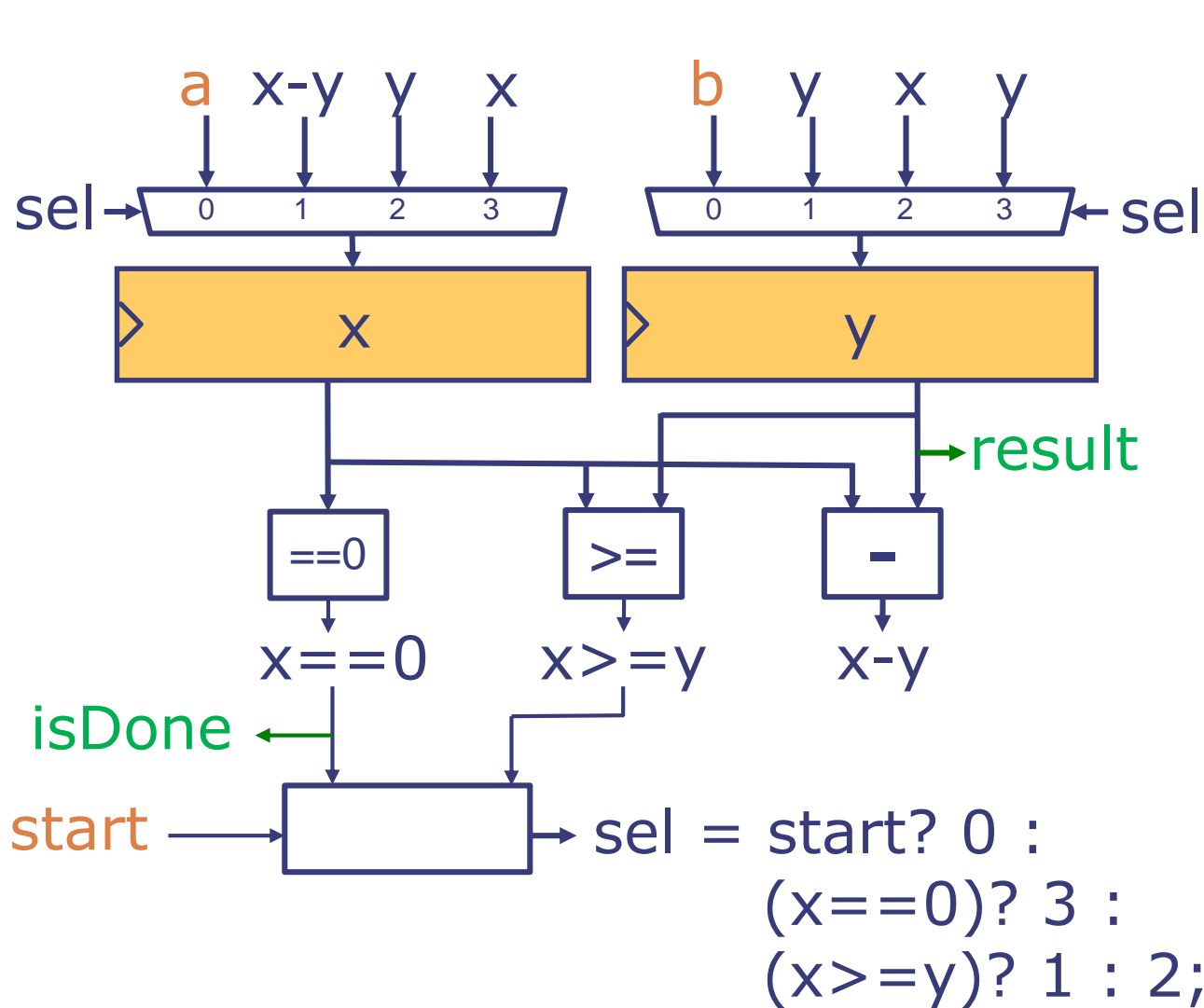
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- New GCD computation is started by setting *start* input to True and passing arguments through inputs *a* and *b*
- Several cycles later, the module signals that it has finished by having *isDone* return True; only then, the *result* method returns the correct result for gcd(a,b)

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 - This requires we learn about one last type...

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```
if (isValid(y))                // check validity
    Word z = fromMaybe(?, y);  // extract valid value
```

Improved GCD Module

Using Maybe Types

```
typedef struct {Word a; Word b;} GCDArgs;  
module GCD;  
  Reg#(Word) x(1);  
  Reg#(Word) y(0);
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endmodule
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Improved GCD Module

Using Maybe Types

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    if (isValid(in)) begin
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Single input and output:

- New GCD computation is started by setting a Valid input *in* (which always includes a and b)
- When GCD computation finishes, *result* becomes a Valid output

Pipelined Circuits

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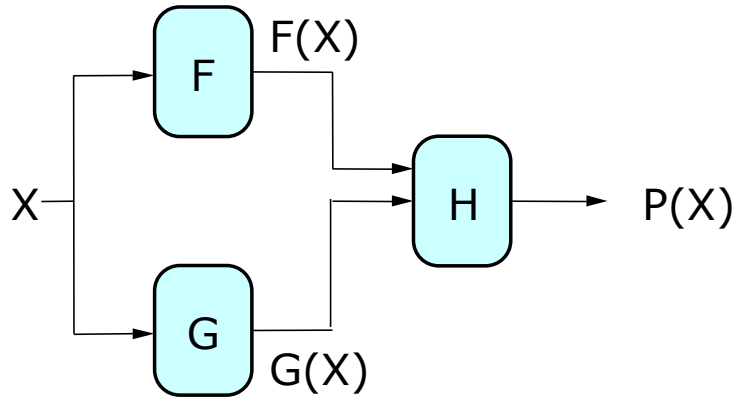
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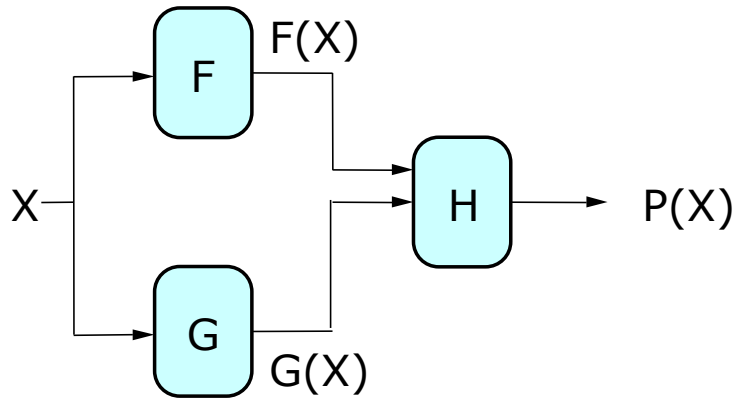
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(maximize instructions/second)

Performance of Combinational Logic



For combinational logic:
latency = t_{PD}
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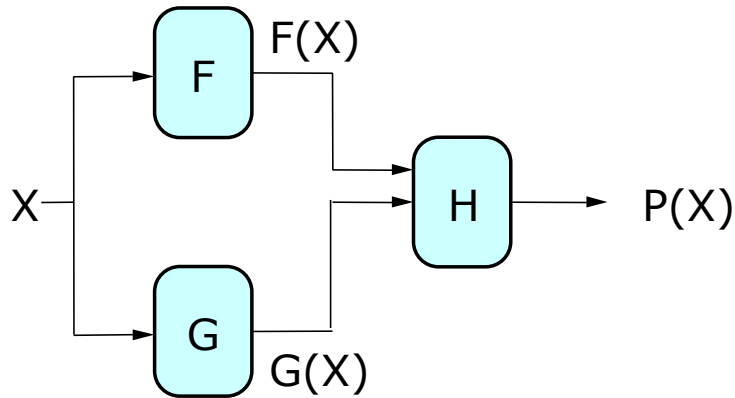
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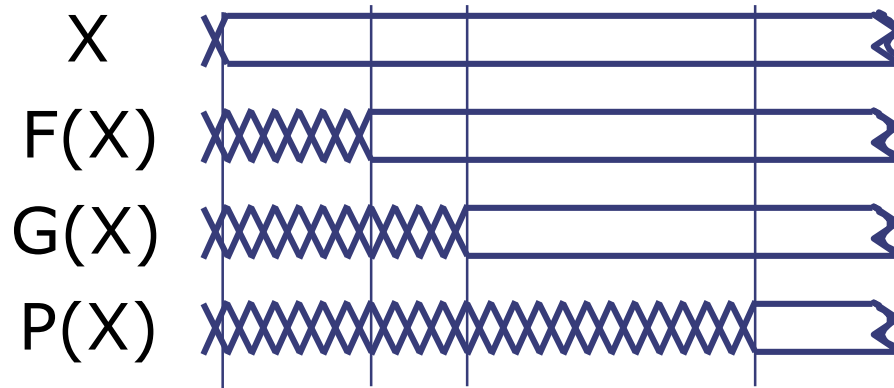


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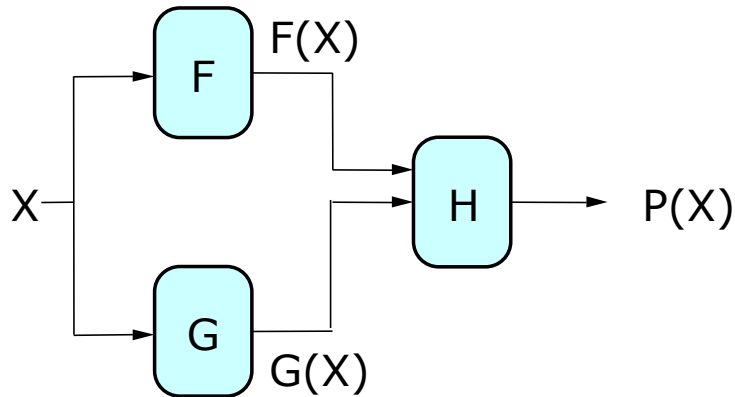
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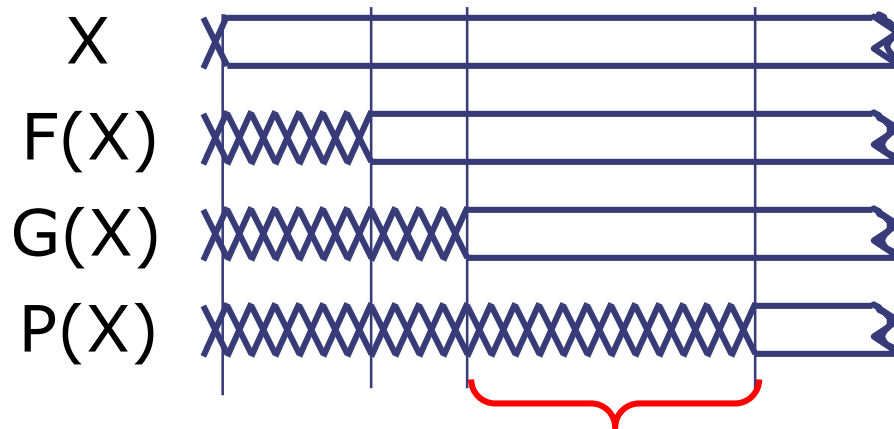


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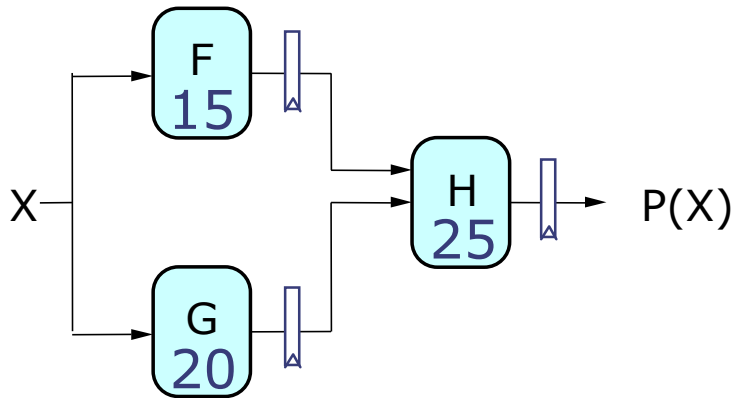
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F & G are "idle", just holding their outputs stable while H performs its computation

Pipelined Circuits

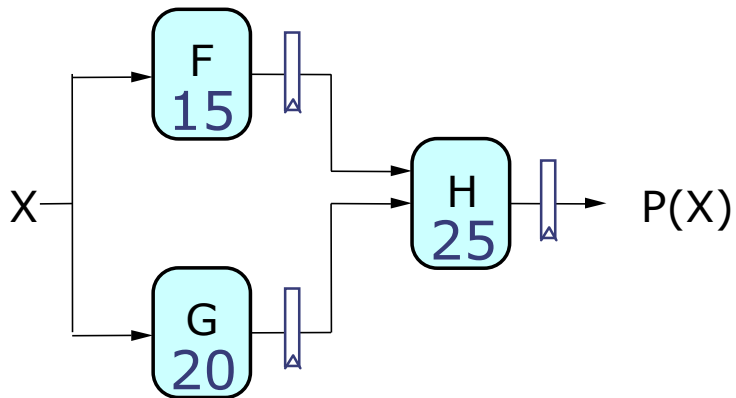
Use registers to hold H's input stable!



Now F & G can be working on input X_{i+1} while H is performing its computation on X_i . We've created a 2-stage *pipeline*: if we have a valid input X during clock cycle j , $P(X)$ is valid during clock $j+2$.

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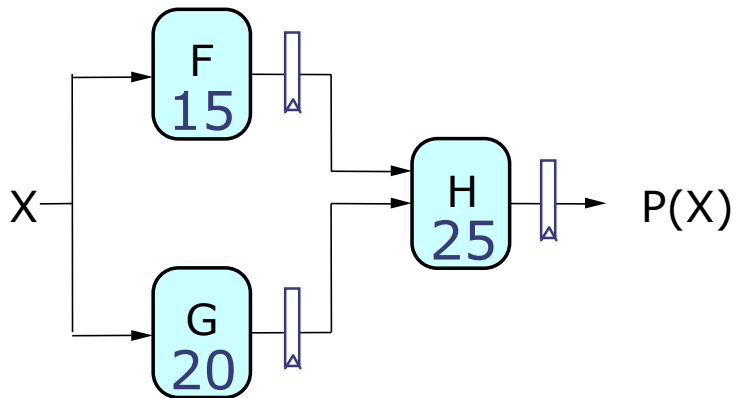
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2-stage pipeline	_____	_____

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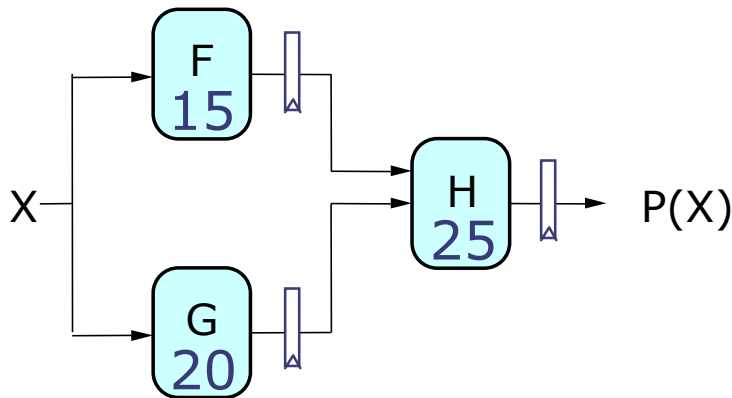
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Pipelined Circuits

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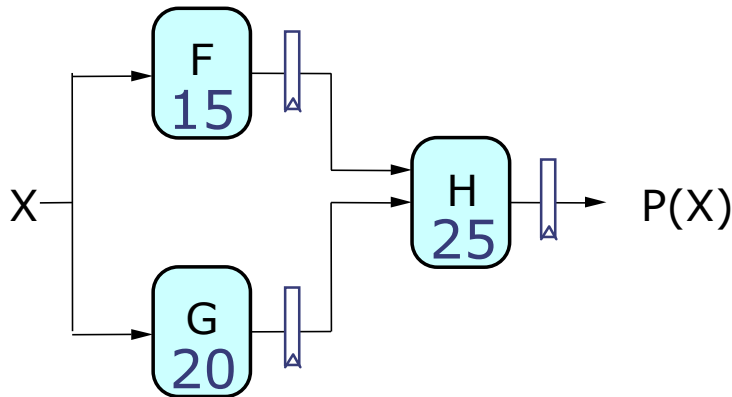
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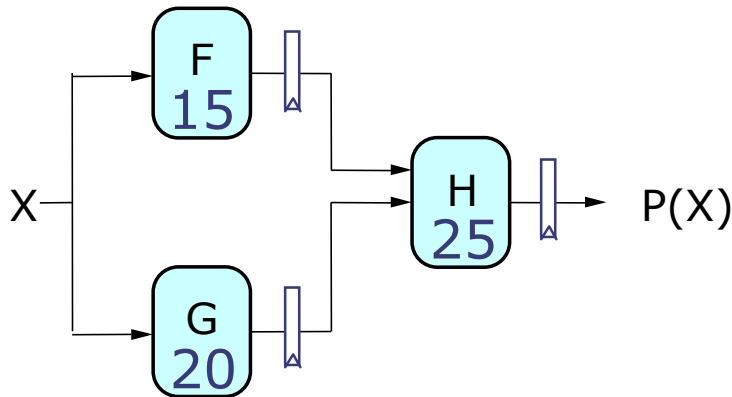
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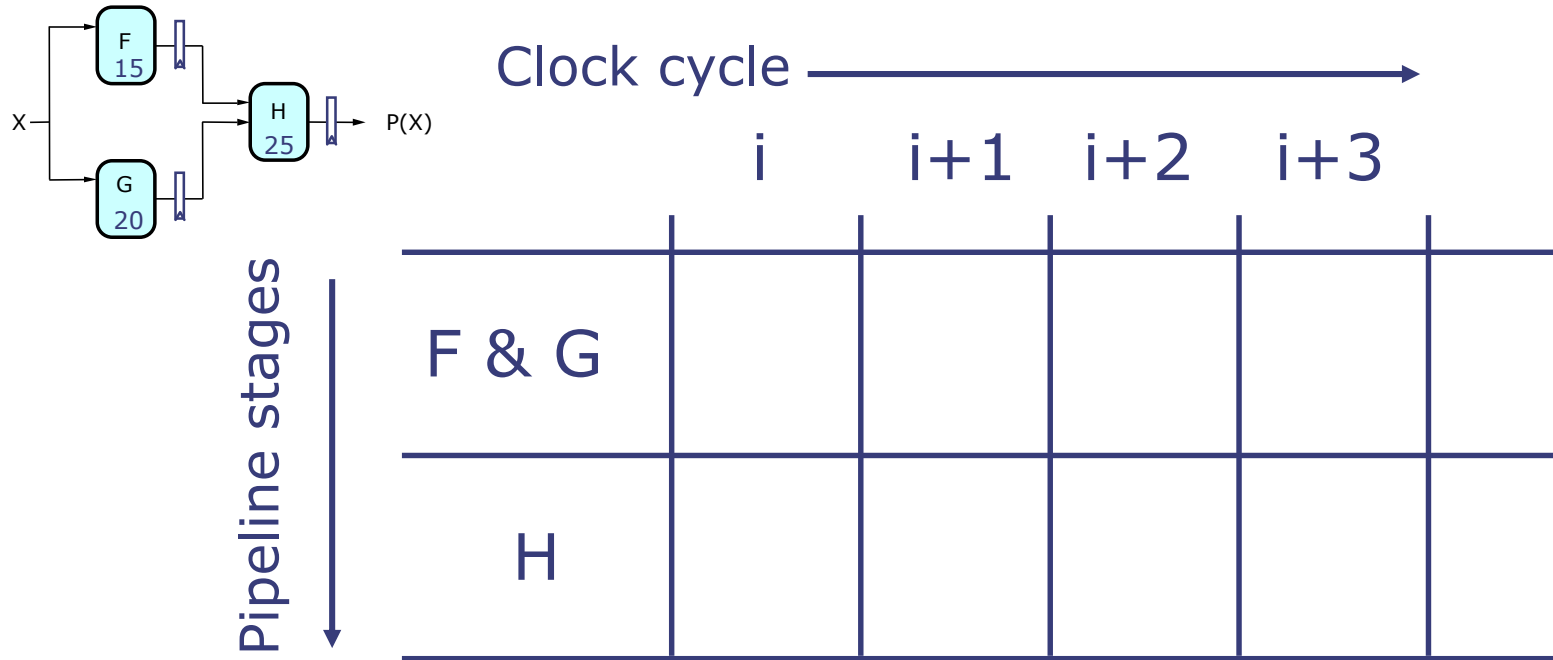


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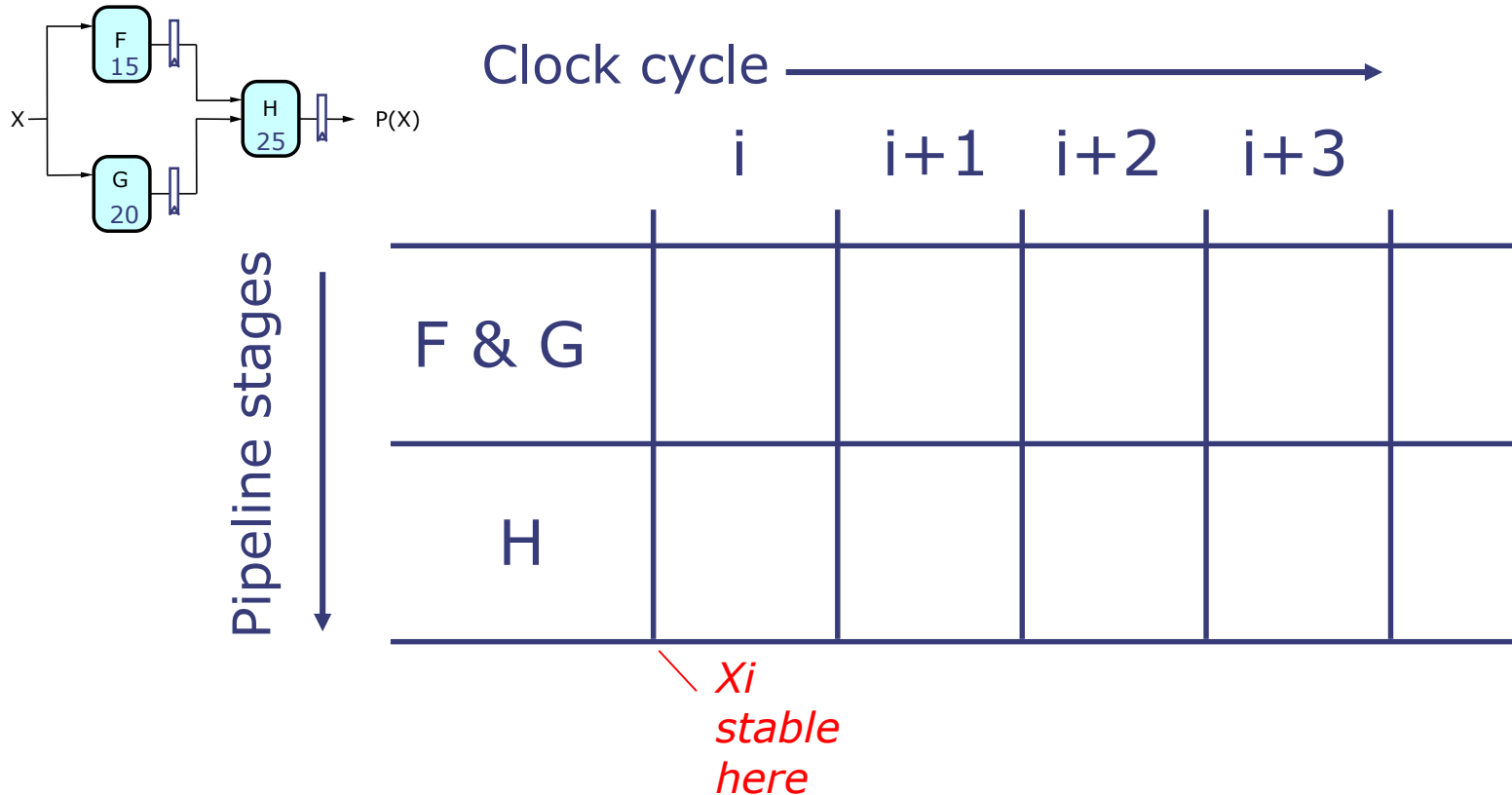
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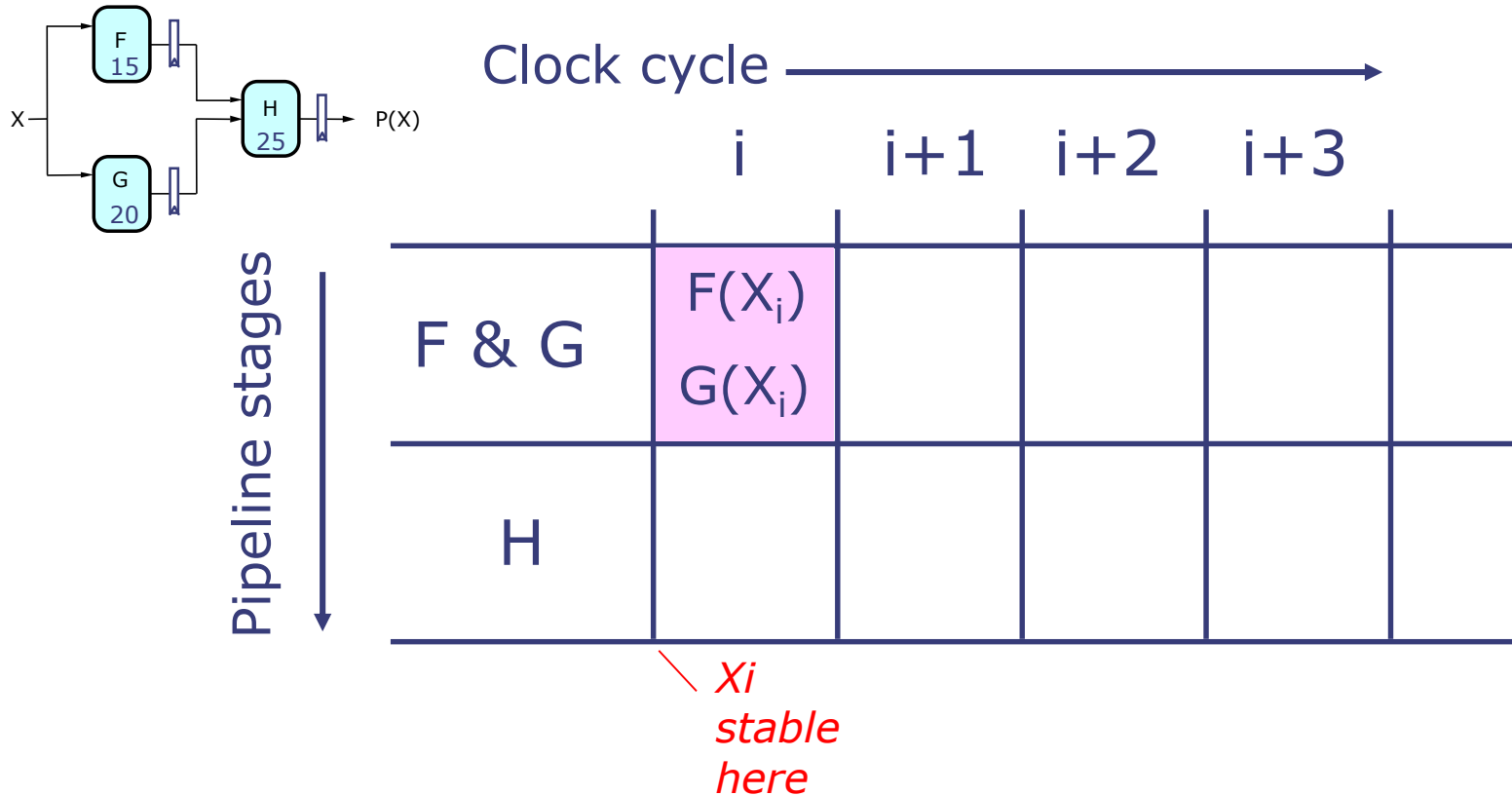
Pipeline Diagrams



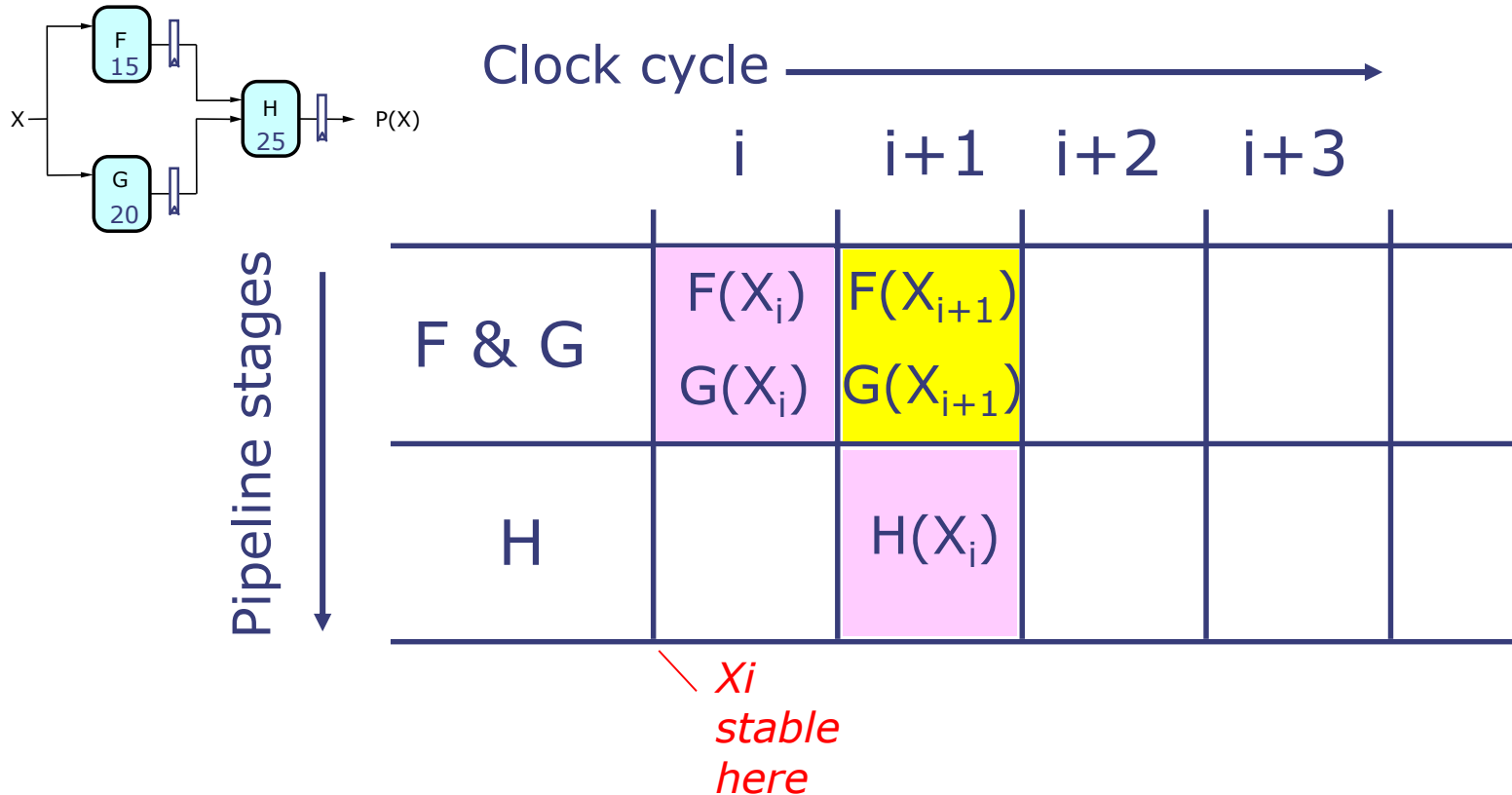
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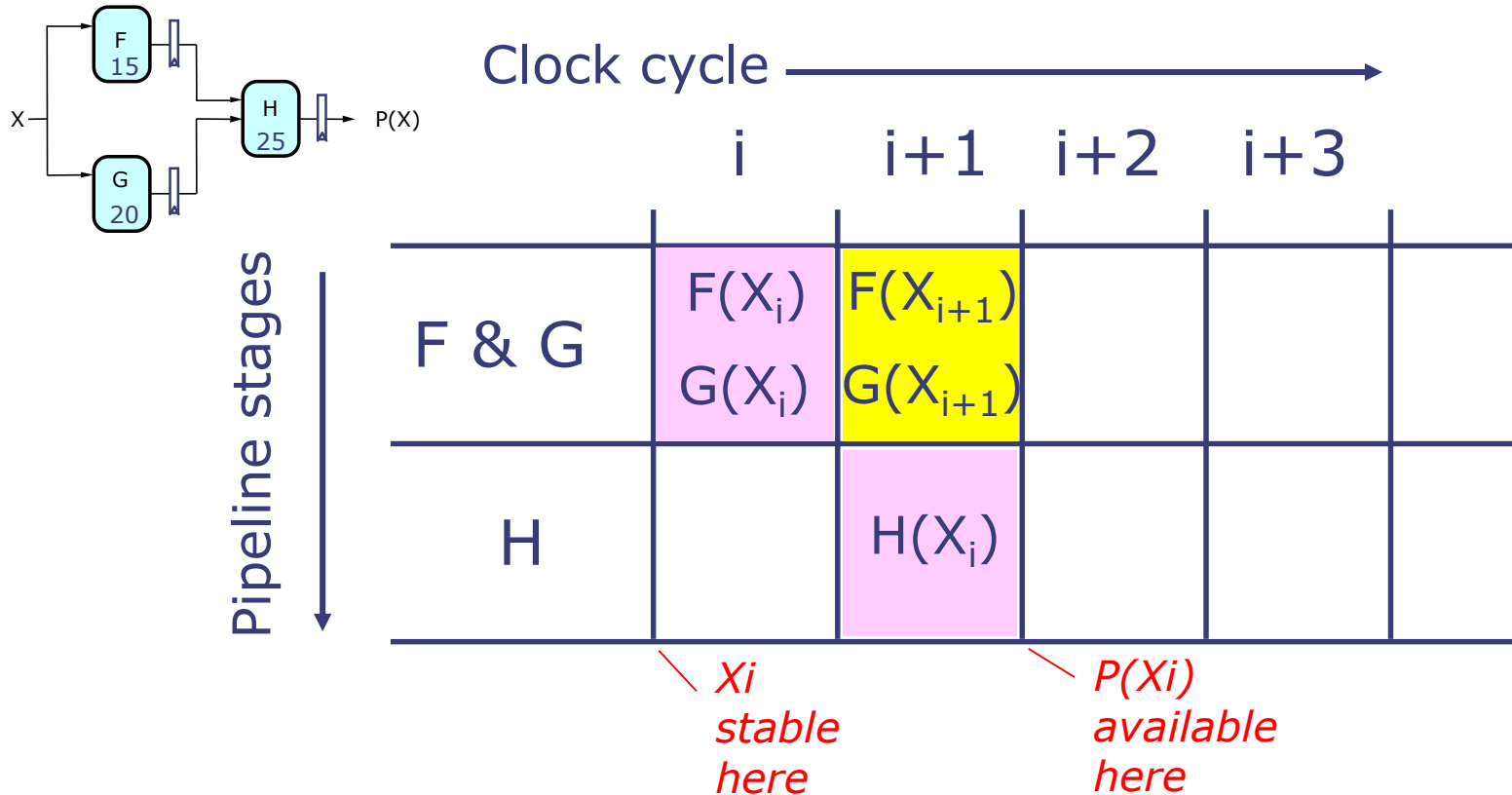
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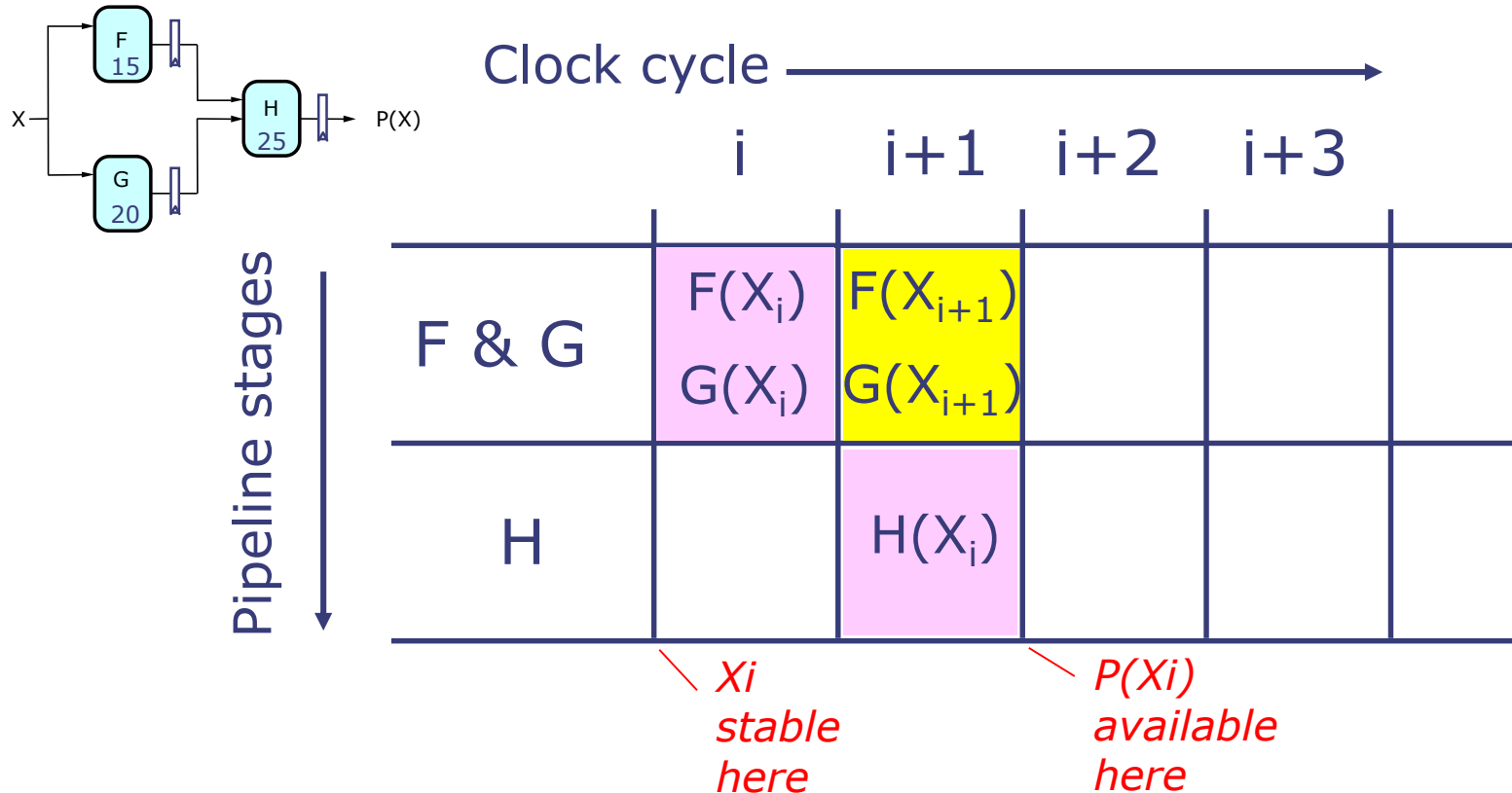
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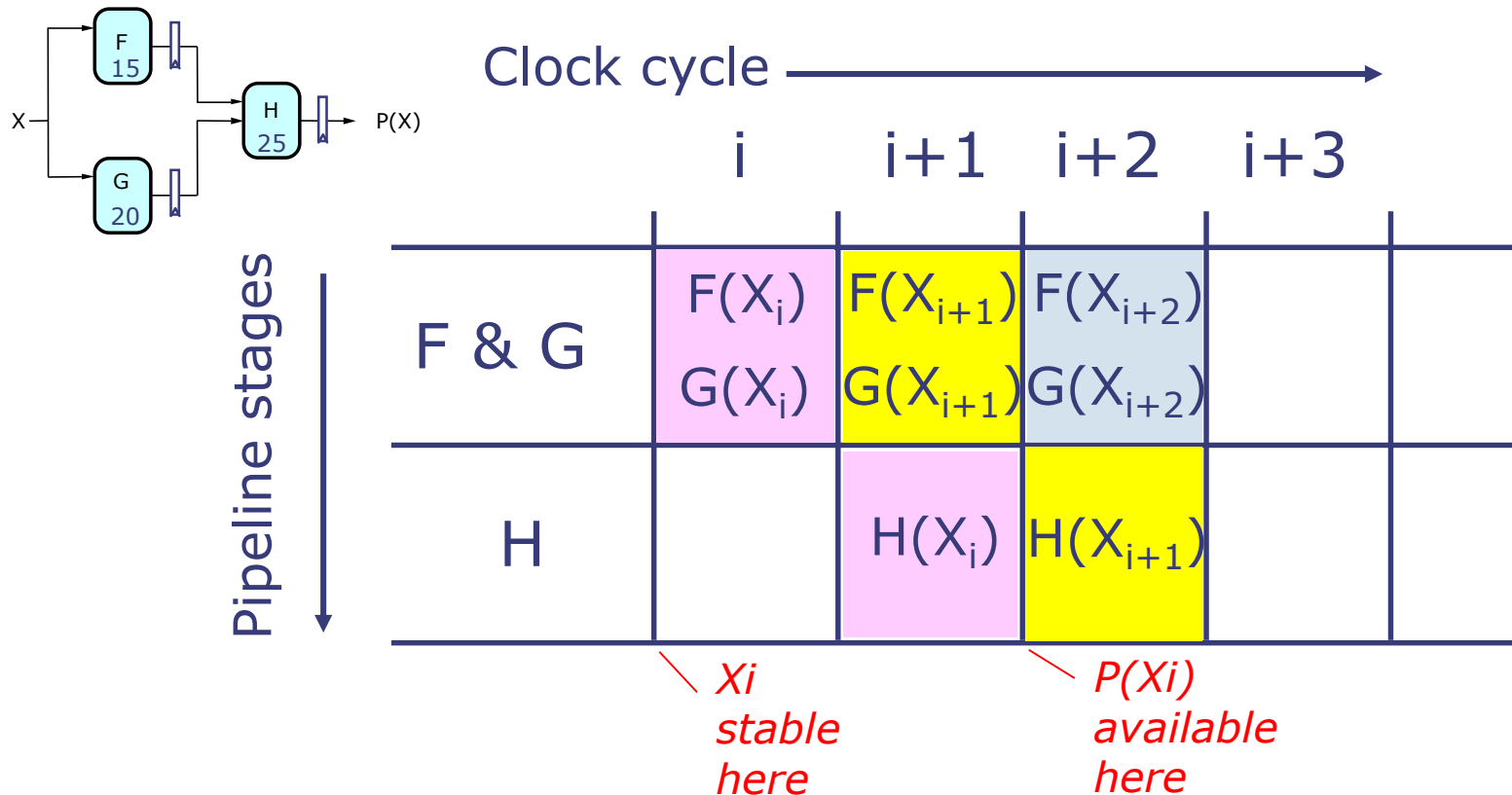


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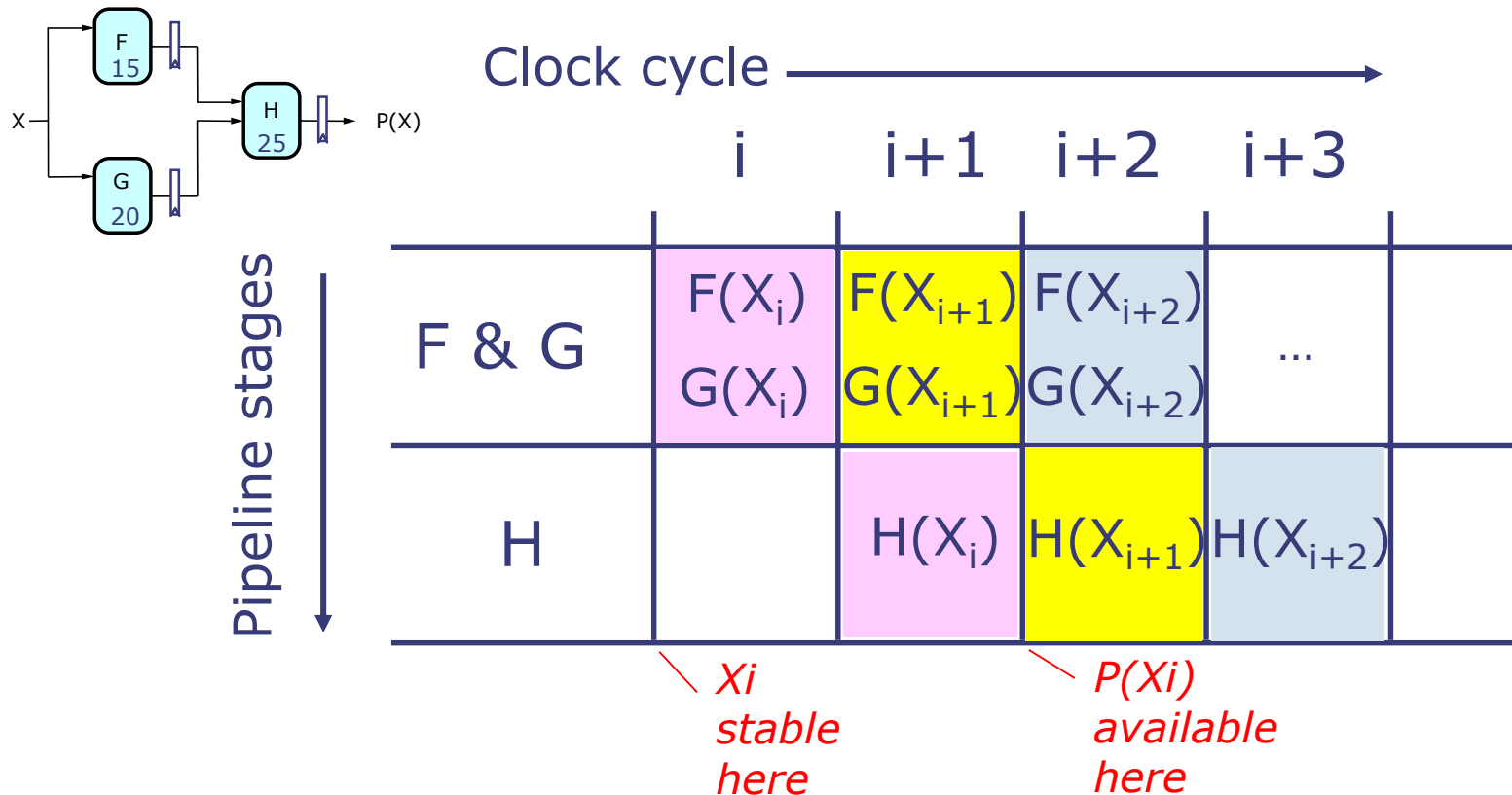
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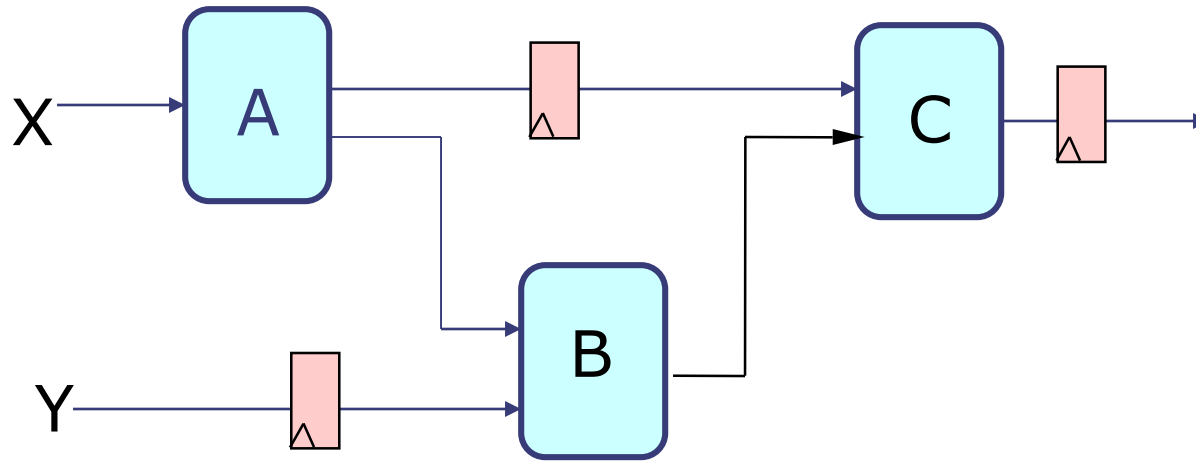
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$$\begin{aligned} \text{K-pipeline latency } L &= K * t_{\text{CLK}} \\ \text{K-pipeline throughput } T &= 1 / t_{\text{CLK}} \end{aligned}$$

Ill-Formed Pipelines

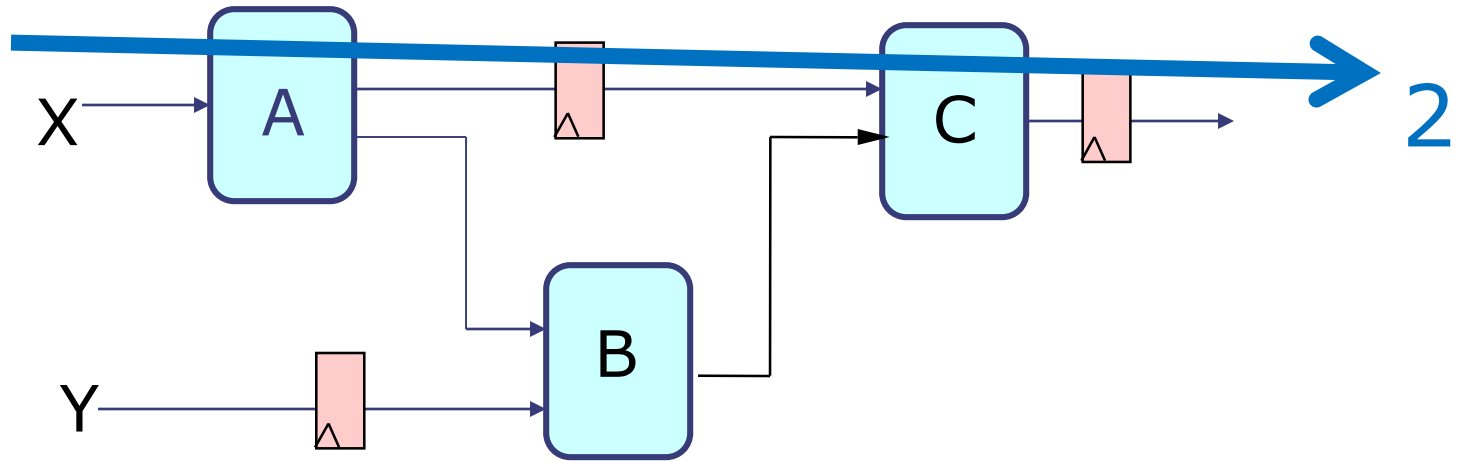
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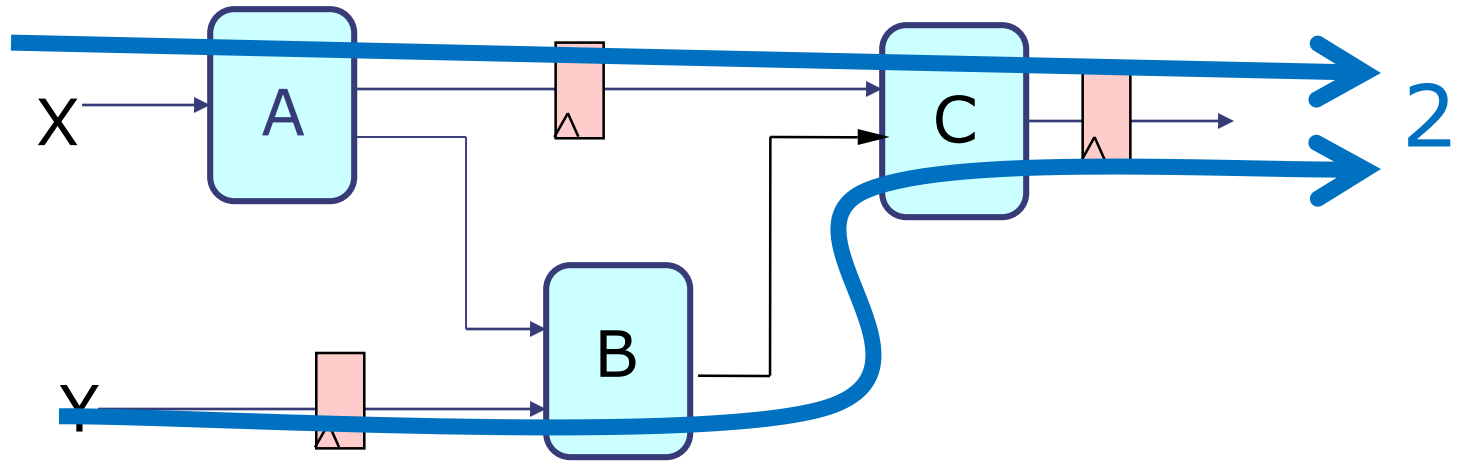
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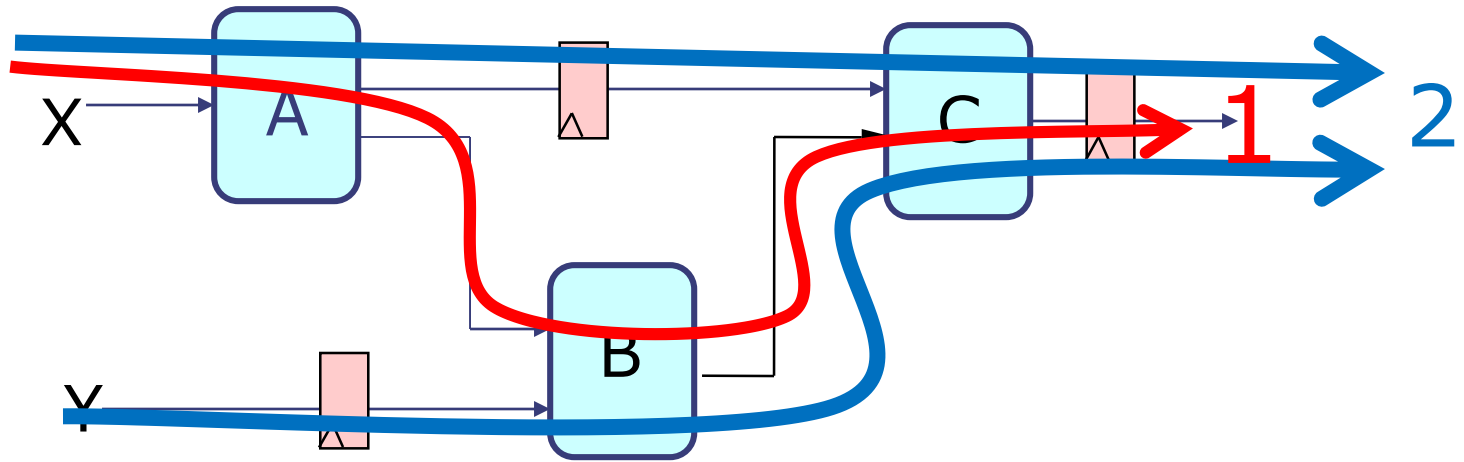
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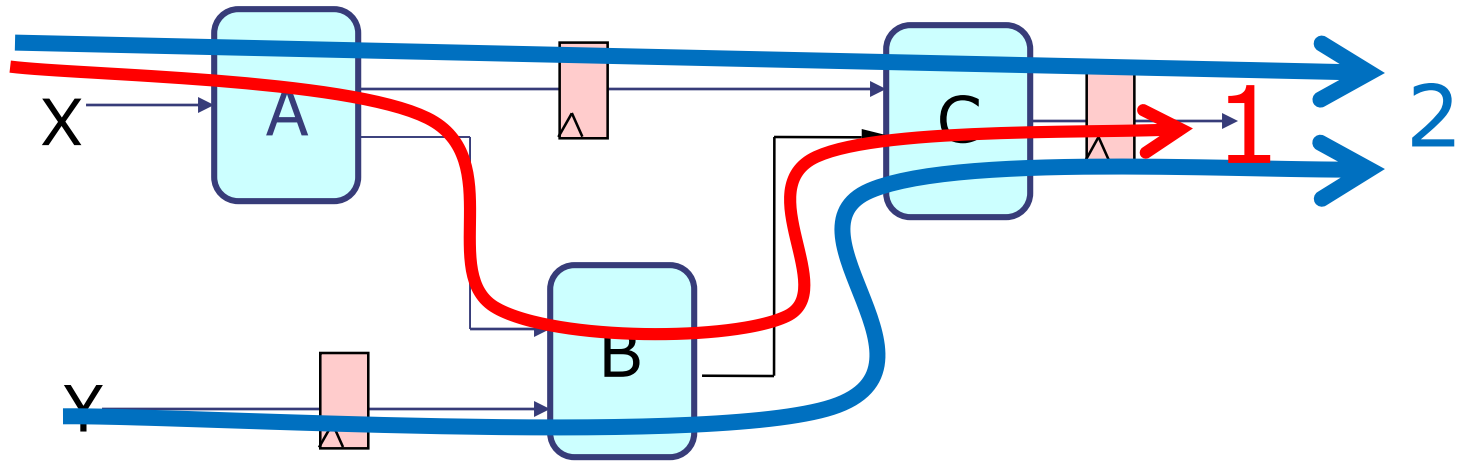
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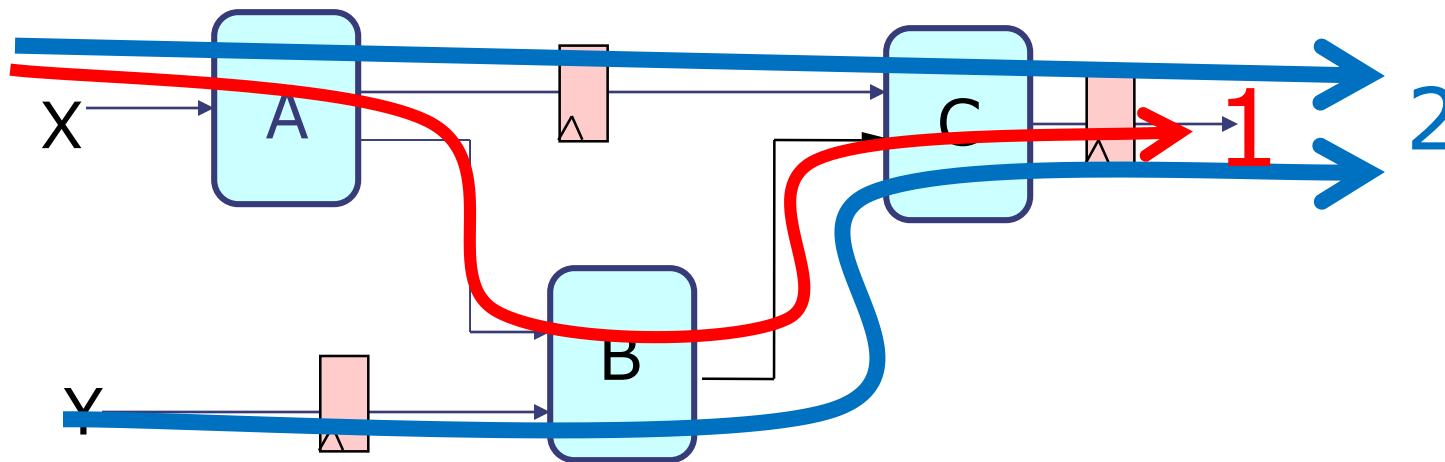
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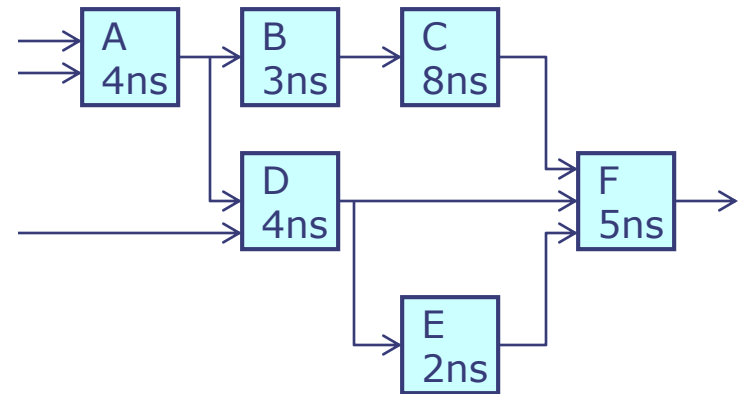
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Problem:

Successive inputs get mixed: e.g., $B(A(X_{i+1}), Y_i)$.
This happens because some paths from inputs to outputs have 2 registers, and some have only 1!

This can't happen in a well-formed K pipeline!

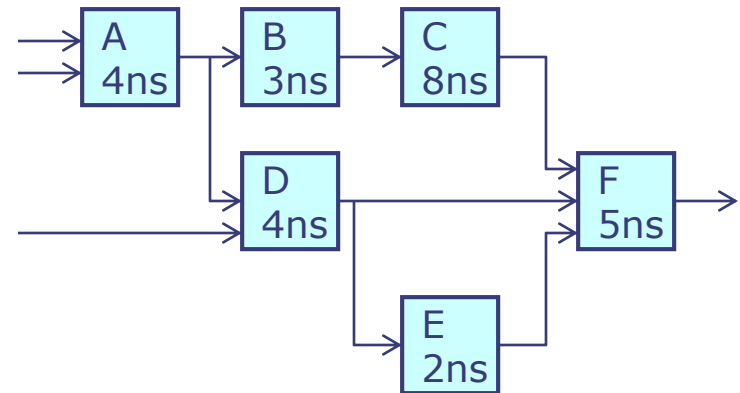
A Pipelining Methodology



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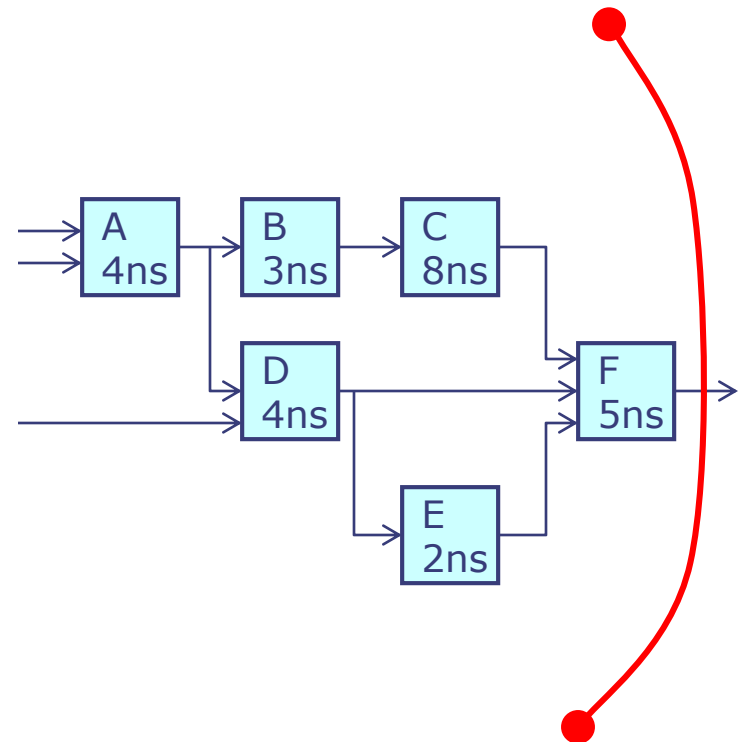
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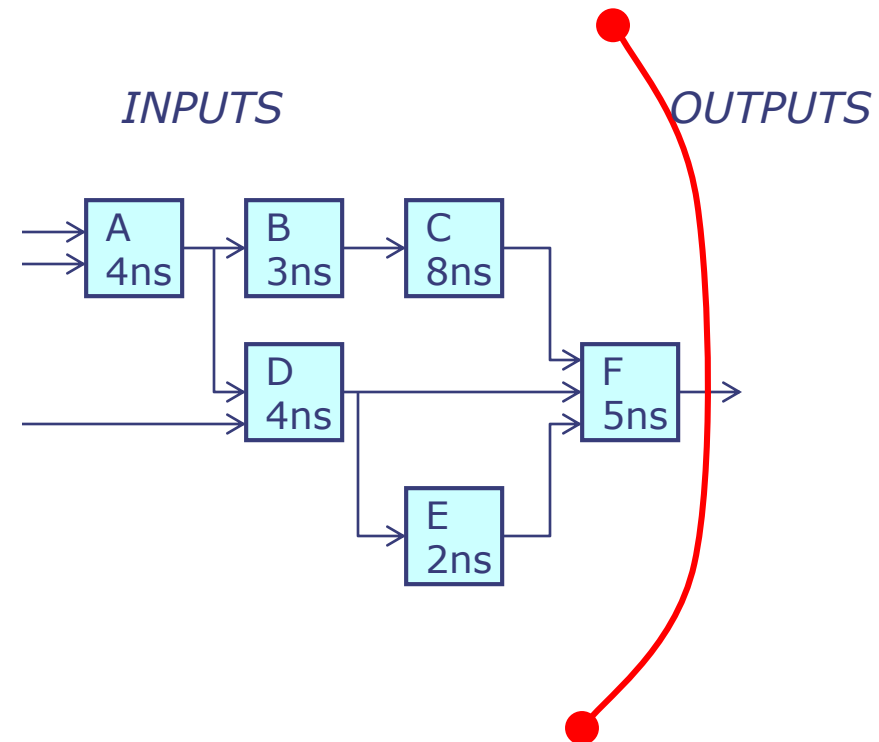
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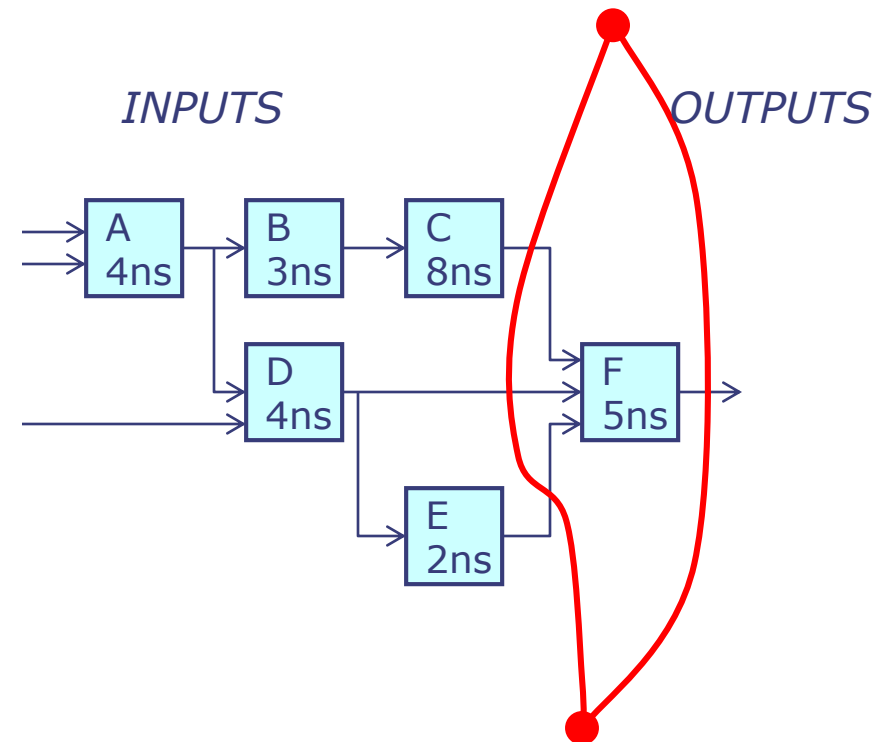
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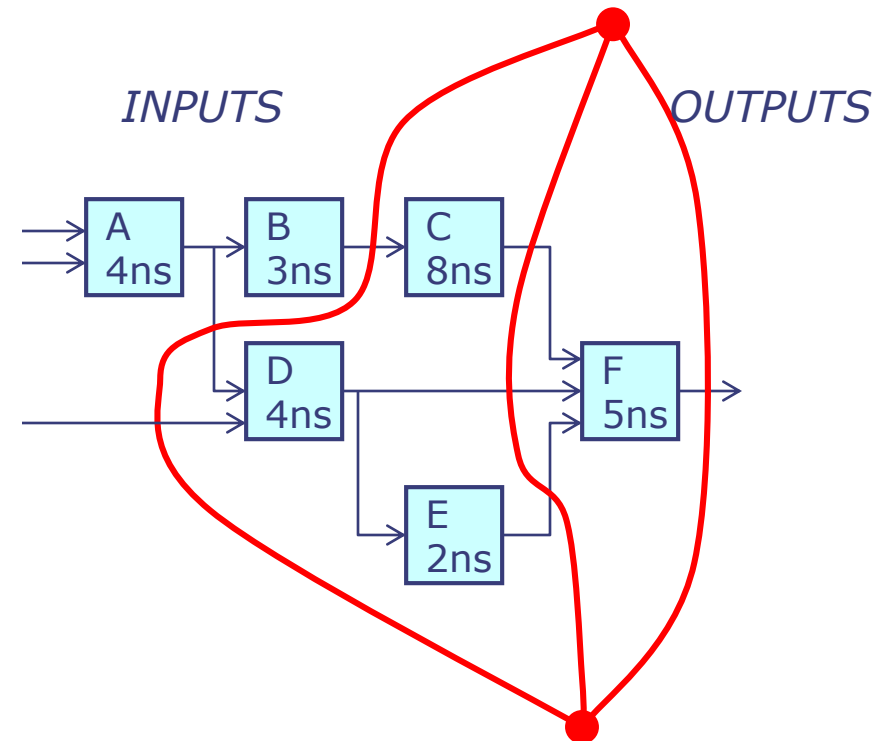
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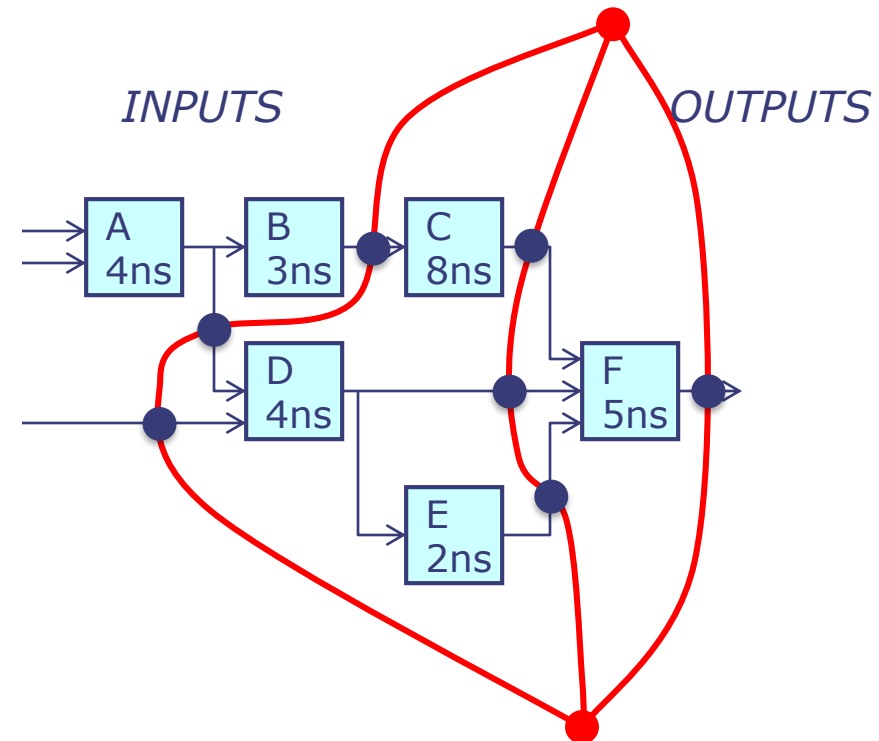
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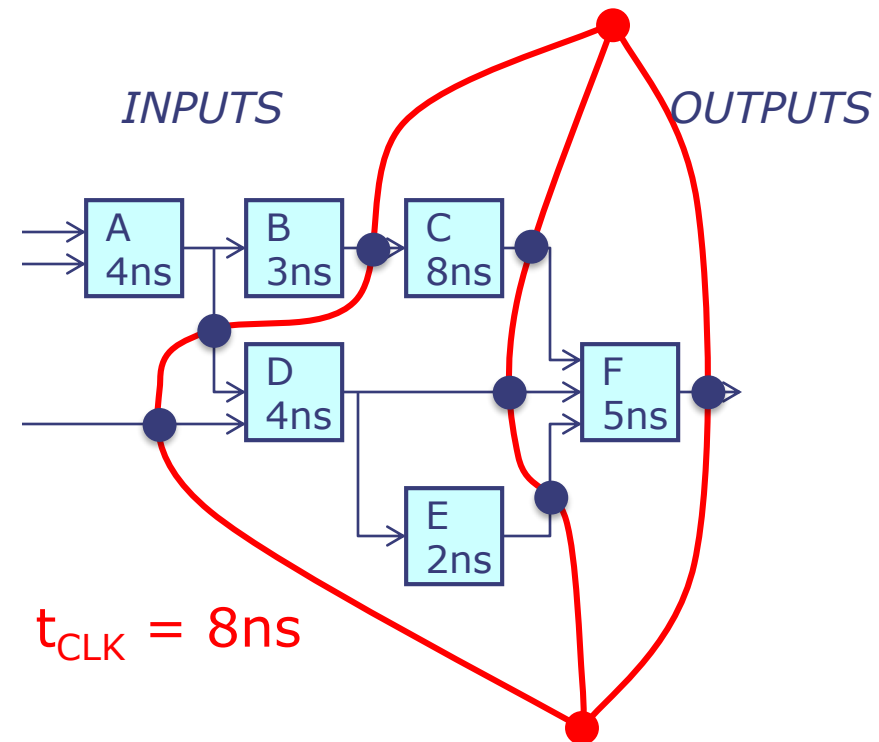
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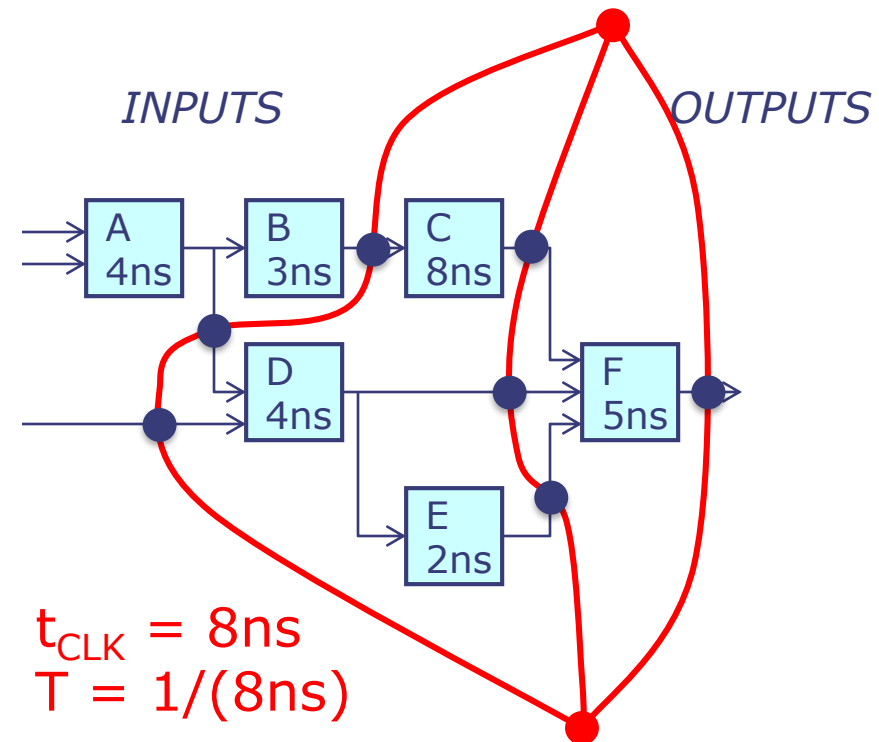
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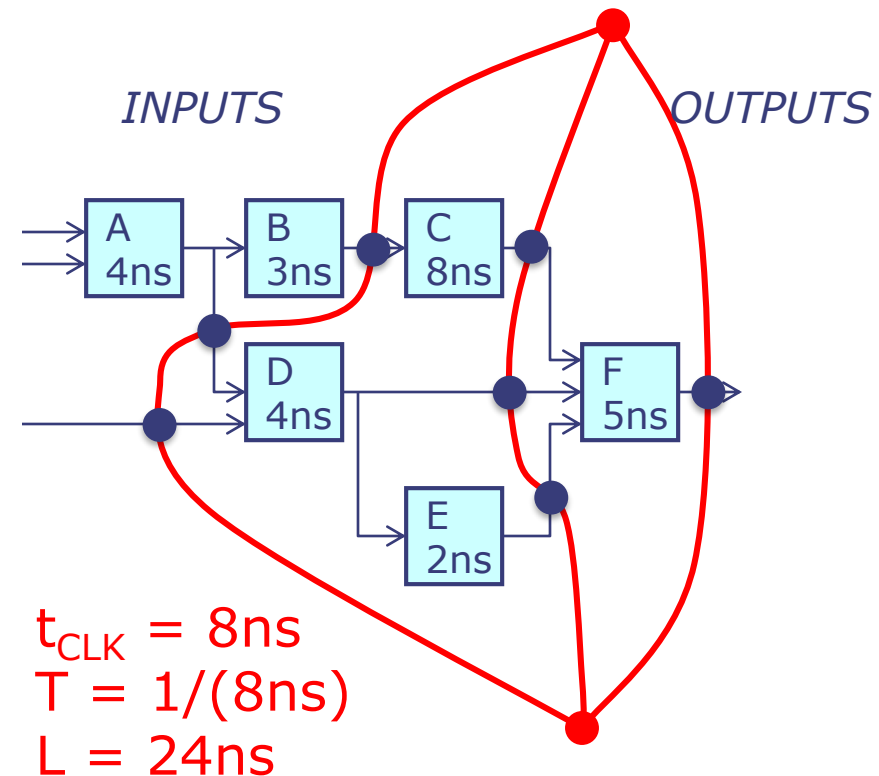
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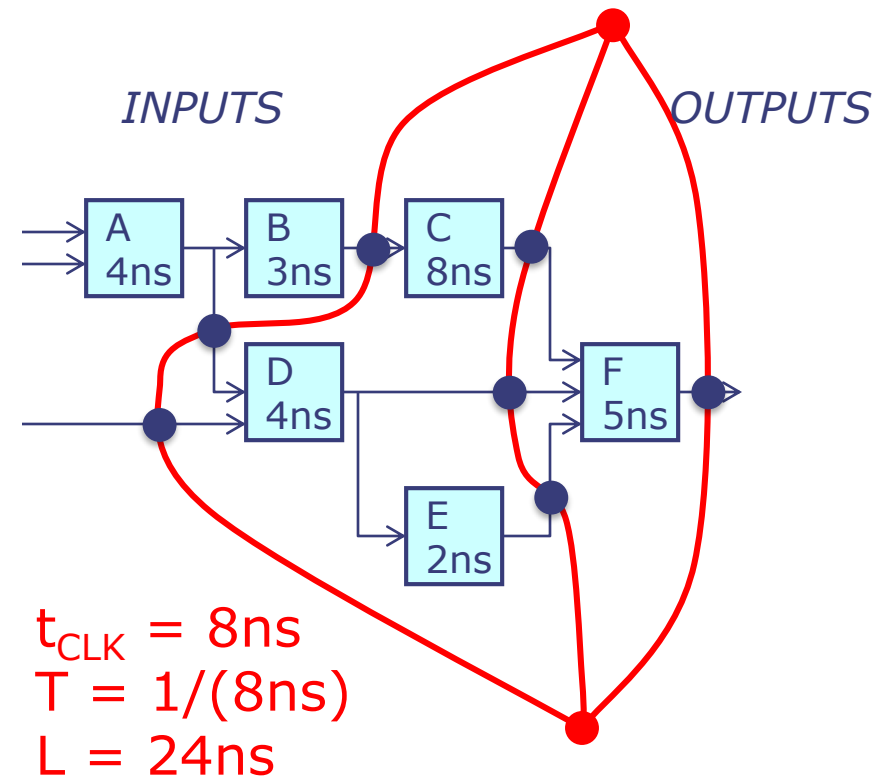
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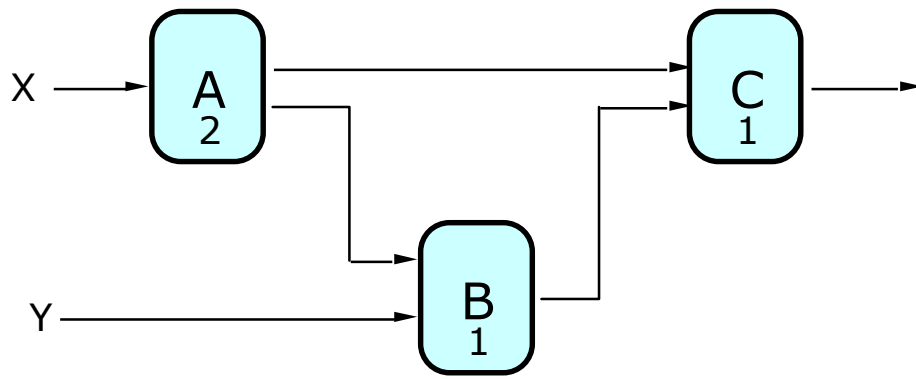
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Strategy:

Focus your attention on placing pipelining registers around the slowest circuit elements (*bottlenecks*).



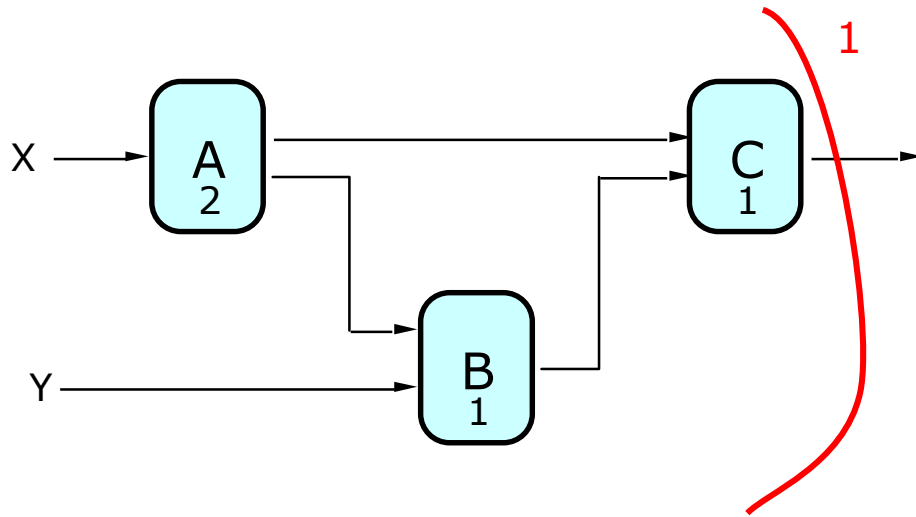
Pipeline Example



OBSERVATIONS:

	LATENCY	THROUGHPUT
0-pipe:	4	1/4
1-pipe:		
2-pipe:		
3-pipe:		

Pipeline Example

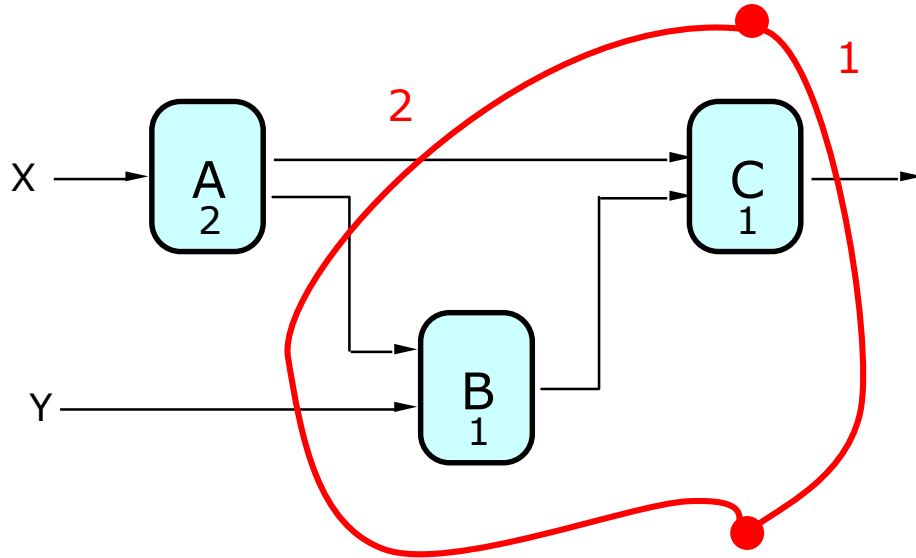


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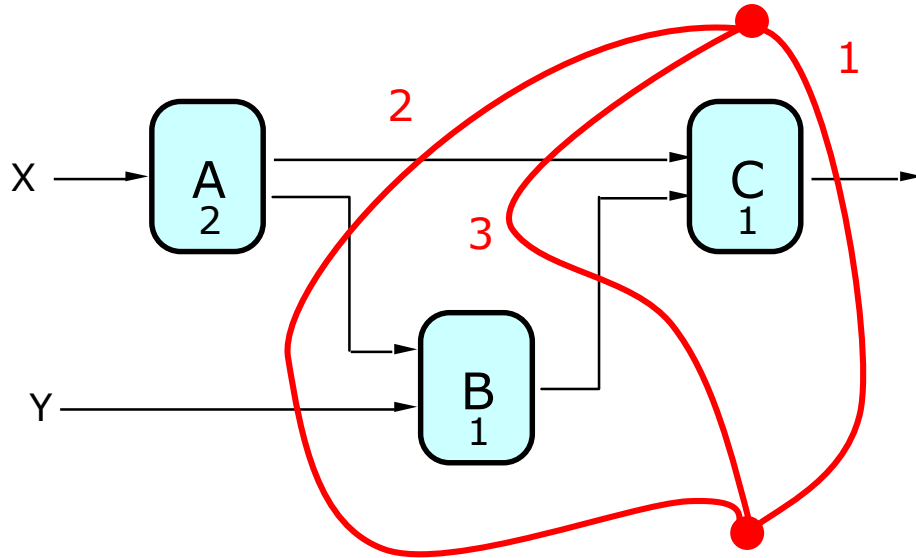


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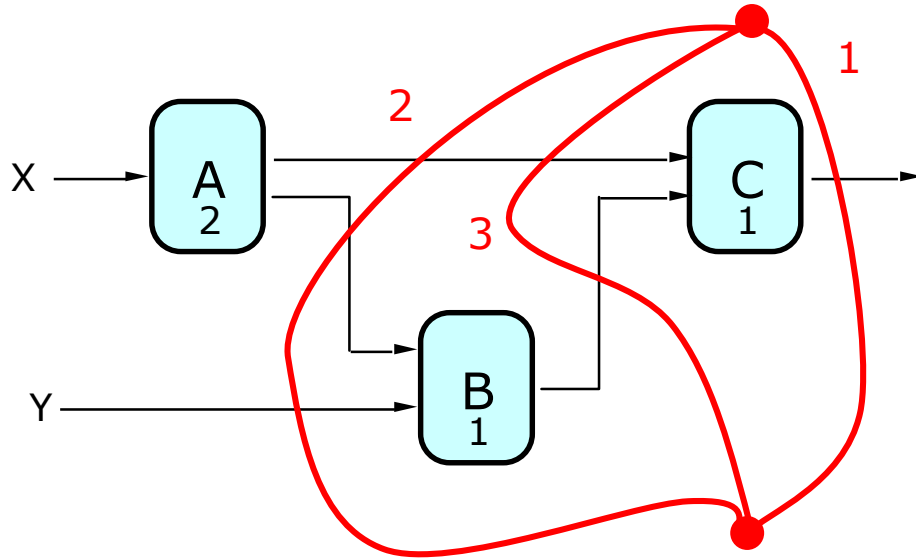


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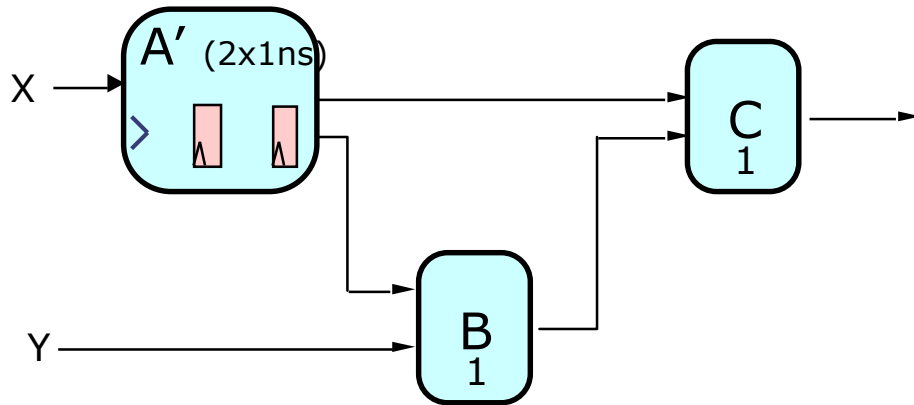


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- Back-to-back registers are sometimes needed to keep pipeline well-formed.

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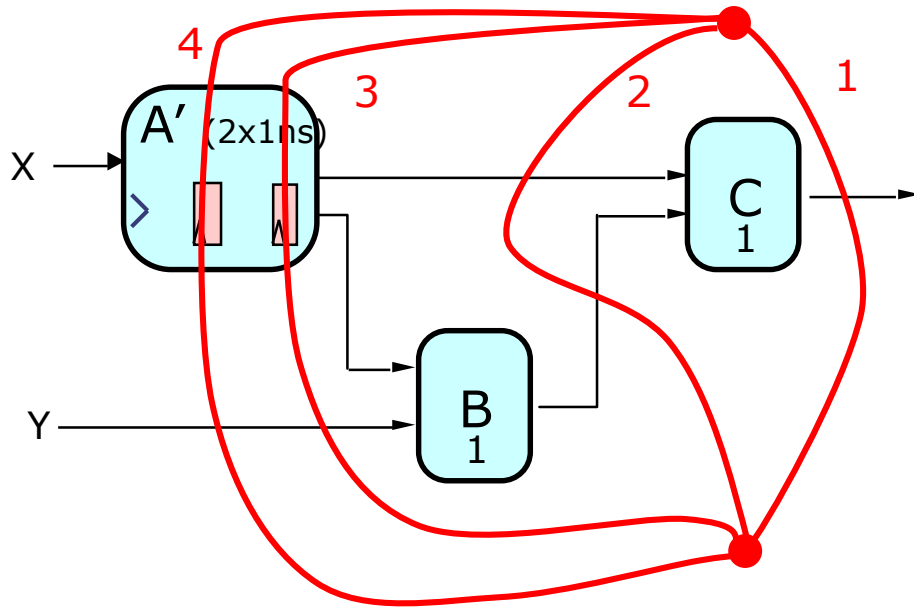
Pipelined Components



Pipelined systems can be hierarchical:

- Replacing a slow combinational component with a k -pipe version may let us decrease the clock period

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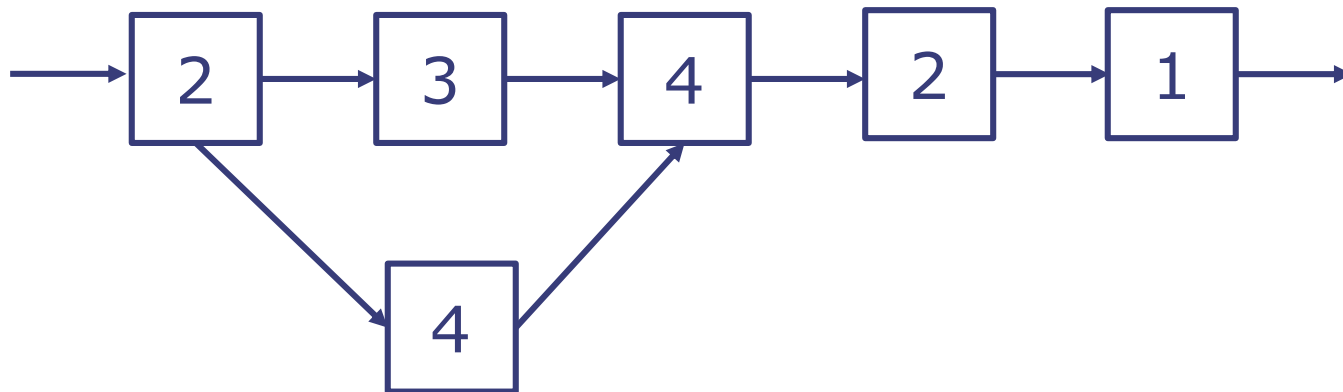
4-stage pipeline, throughput=1

Pipelined systems can be hierarchical:

- Replacing a slow combinational component with a k -pipe version may let us decrease the clock period
- Must account for new pipeline stages in our plan

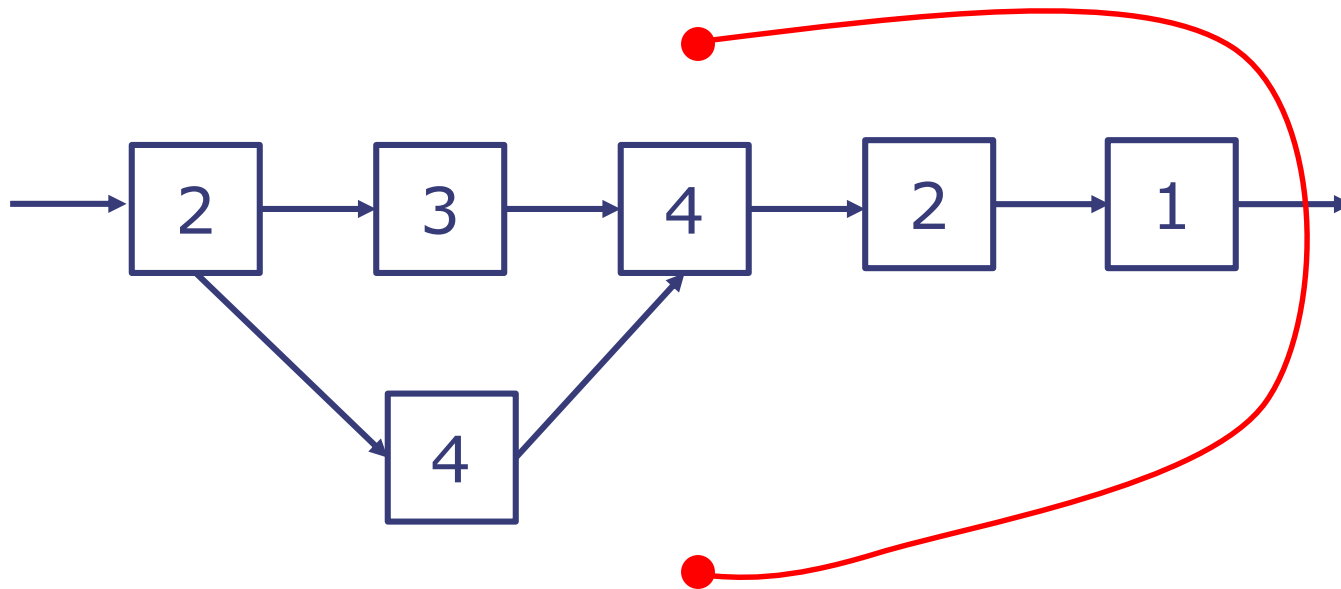
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- Pipeline the following circuit for maximum throughput while minimizing latency. The number in each module is the module's latency.



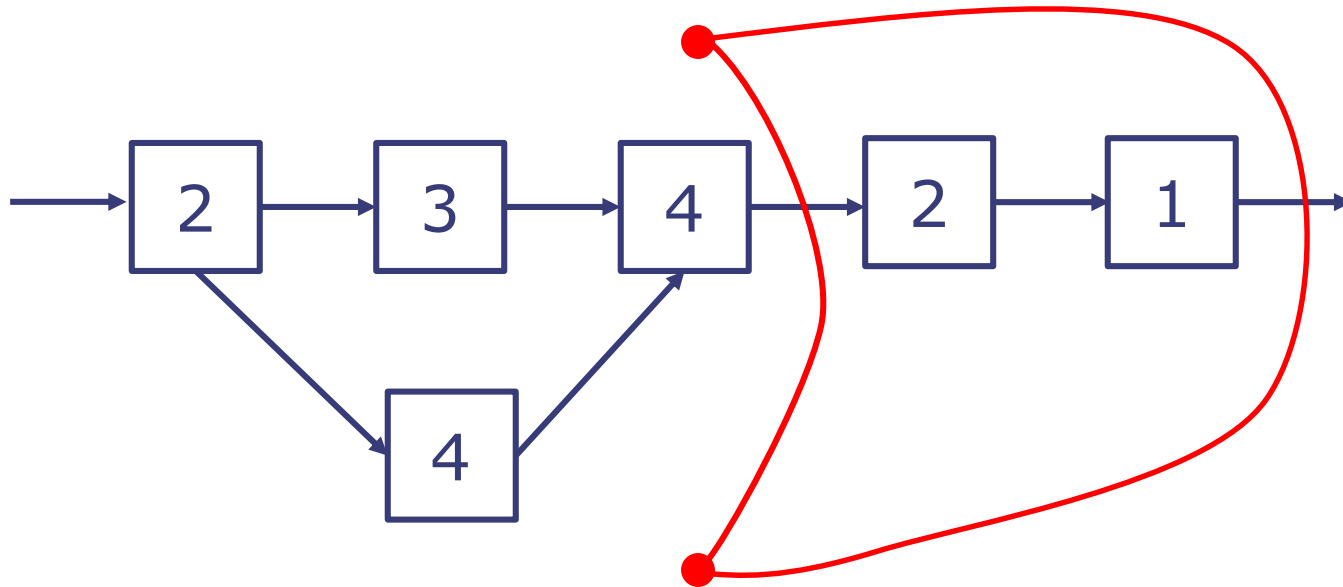
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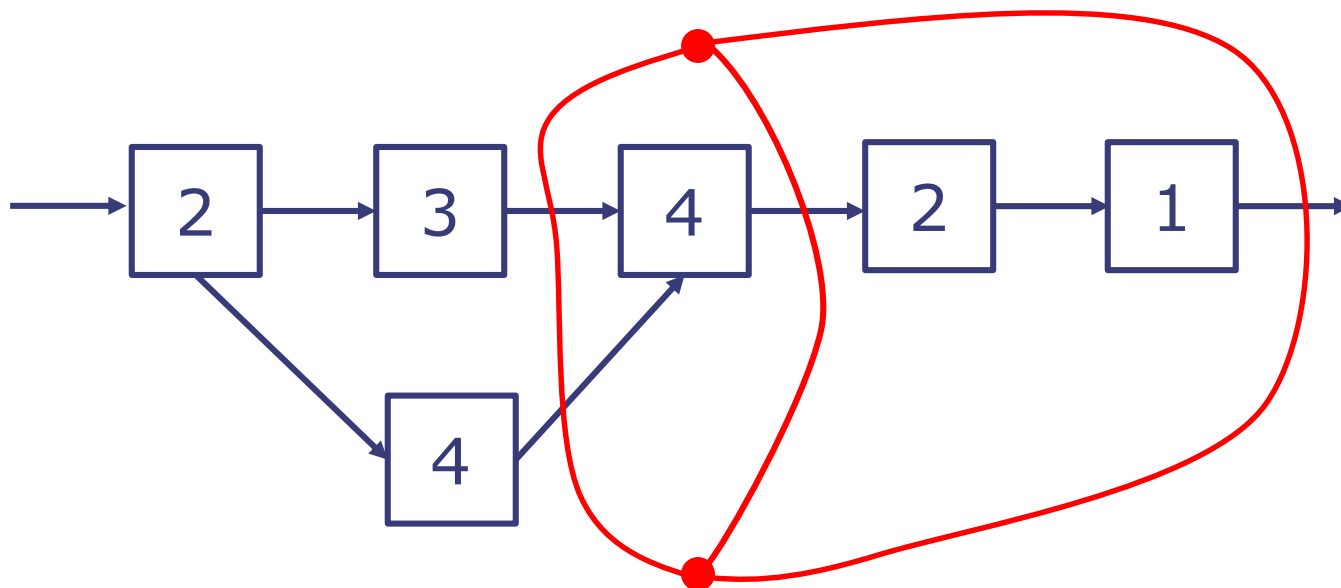
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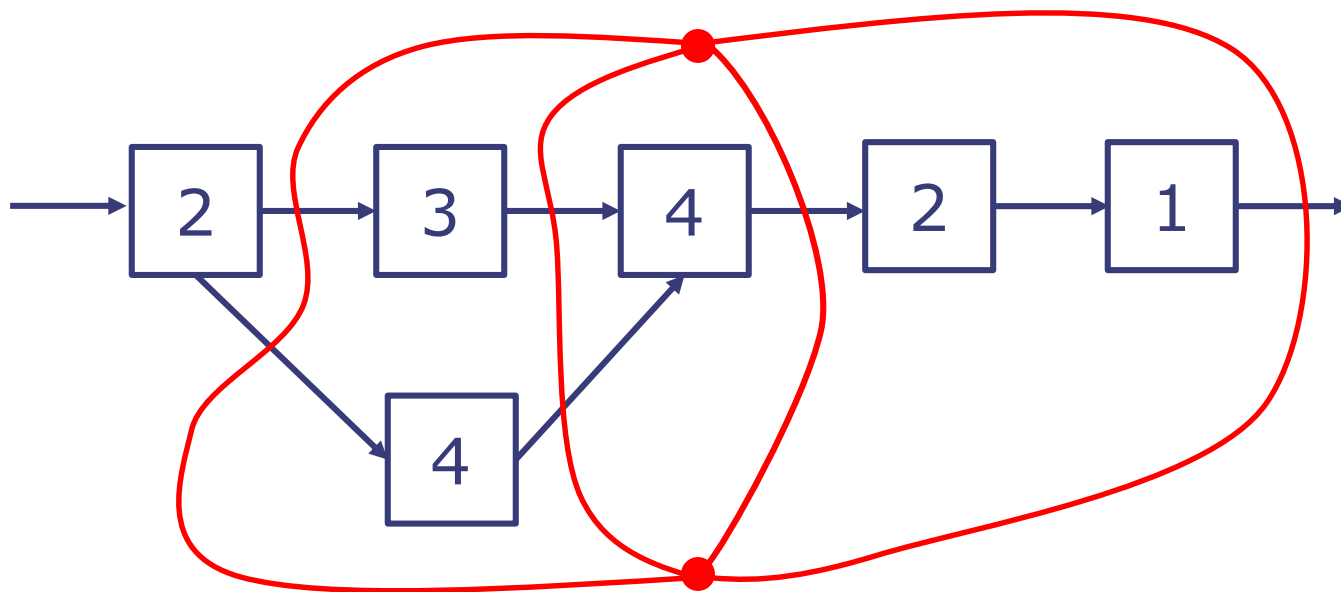
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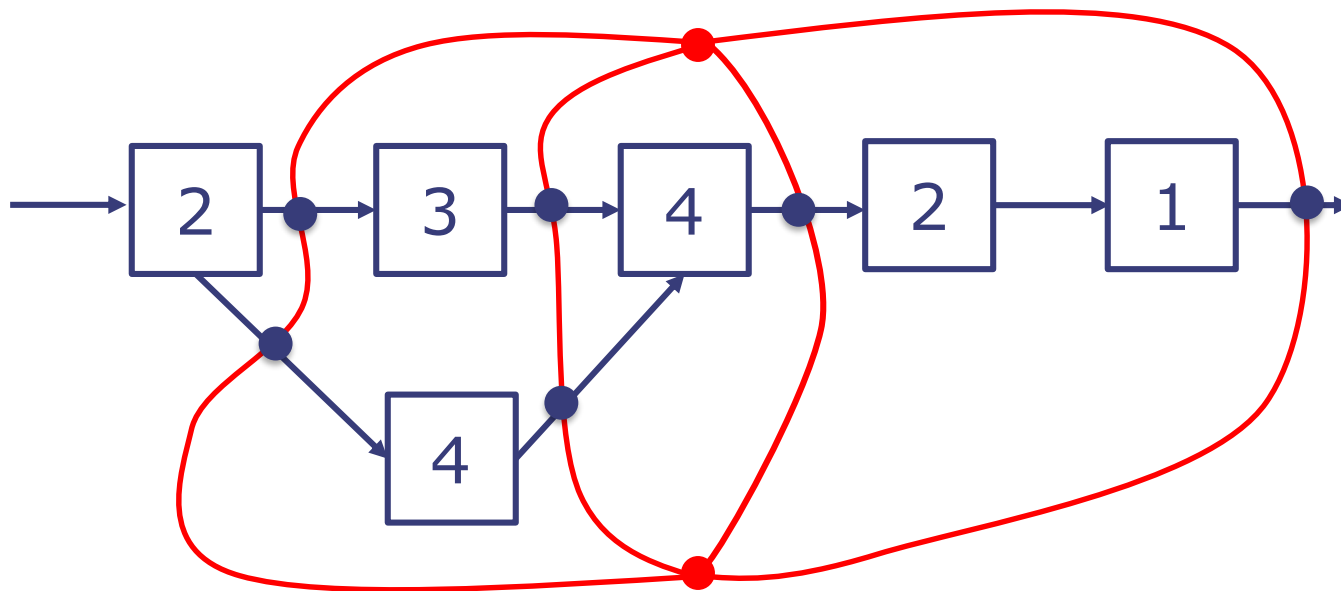
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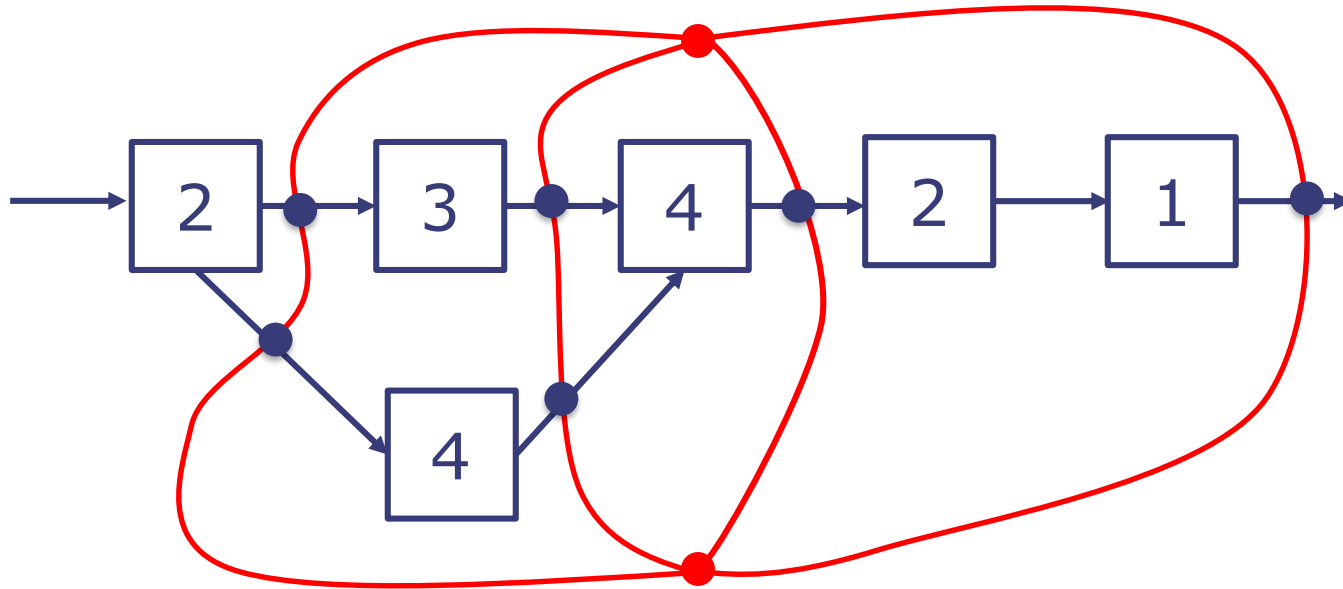
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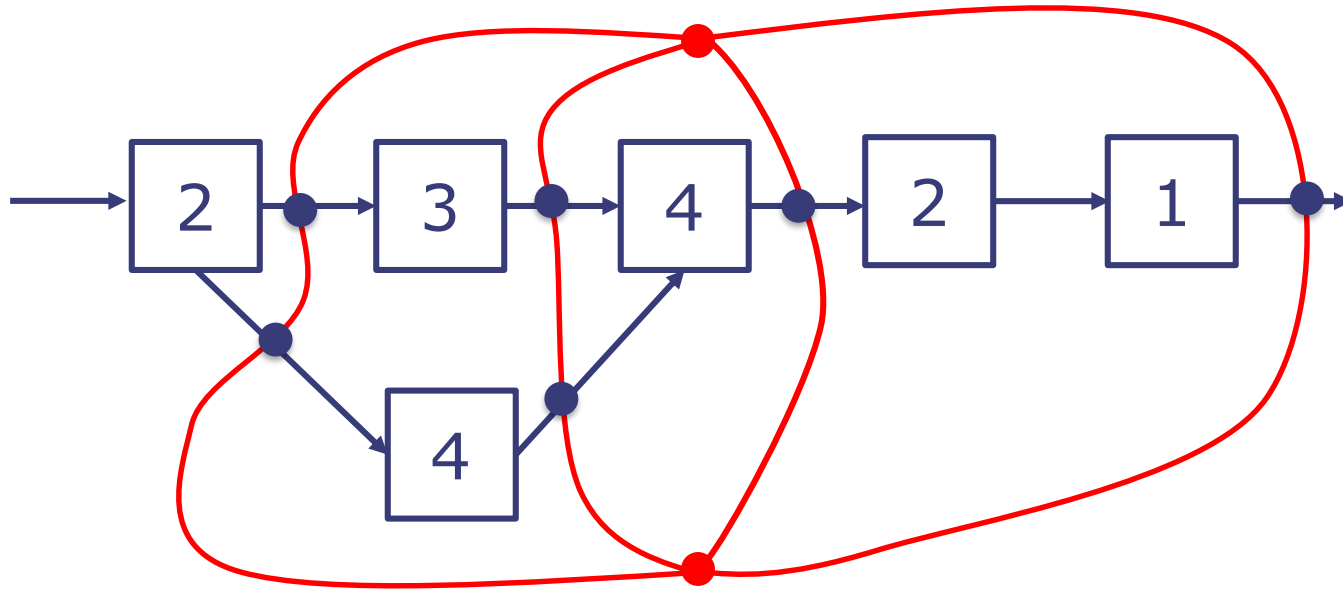
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$$\begin{aligned}t_{\text{CLK}} &= 4 \\T &= 1/(4) \\L &= 4*4 = 16\end{aligned}$$

Design Tradeoffs

Introduction:

Multiplier Case Study

Multiplication by repeated addition

b Multiplicand 1101 (13)
a Multiplier * 1011 (11)

At each step we add either
b (1101) or 0 to the result
depending upon a bit in the
multiplier

tp	0000	
m0	+ 1101	
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Note the first addition is unnecessary because it simply yields m0

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m3	+	1101	
tp		10001111	(143)

At each step we add either b (1101) or 0 to the result depending upon a bit in the multiplier

$$m_i = (a[i] == 0) ? 0 : b;$$

We also shift the result by one position at every step

Note the first addition is unnecessary because it simply yields m0

Also note that these are unsigned binary numbers.

Multiplication by repeated addition

Combinational circuit

b Multiplicand 1101 (13)
a Multiplier * 1011 (11)

tp		0000	
m0	+	1101	
tp		01101	
m1	+	1101	
tp		100111	
m2	+	0000	
tp		0100111	
m3	+	1101	
tp		10001111	(143)

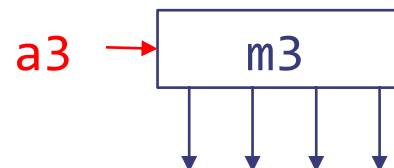
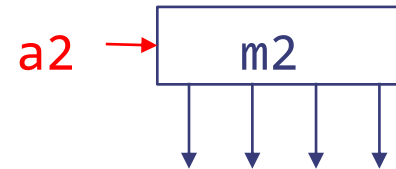
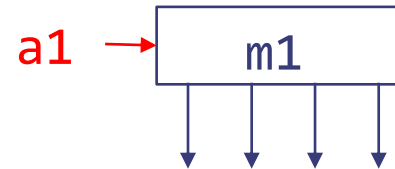
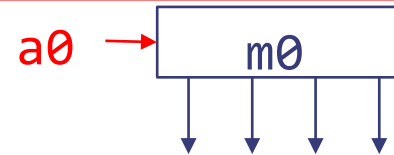
$$m_i = (a[i] == 0) ? 0 : b;$$

Multiplication by repeated addition

Combinational circuit

b Multiplicand 1101 (13)
 a Multiplier * 1011 (11)

tp		0000	
m0	+	1101	
tp		01101	
m1	+	1101	
tp		100111	
m2	+	0000	
tp		0100111	
m3	+	1101	
tp		10001111	(143)



$m_i = (a[i]==0)? 0 : b;$

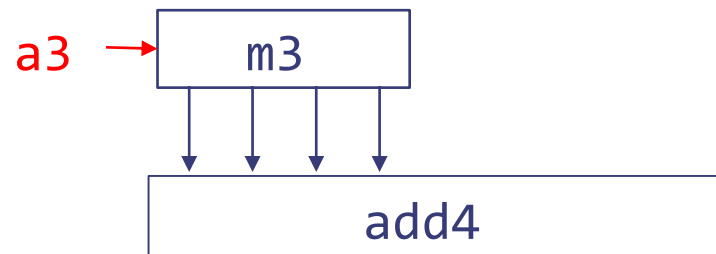
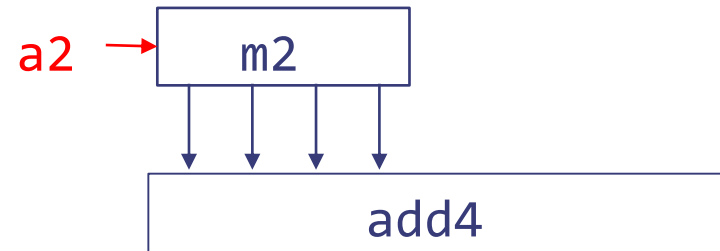
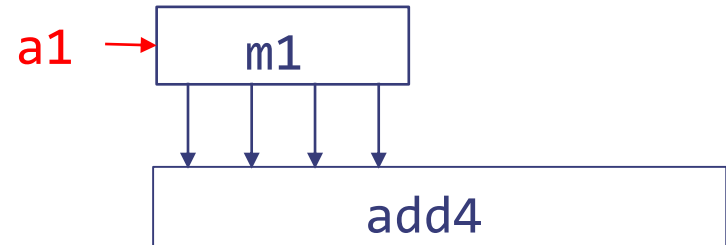
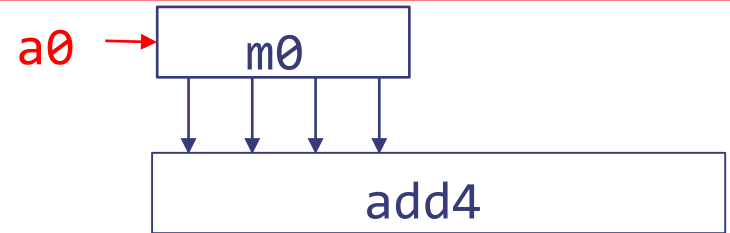
Multiplication by repeated addition

Combinational circuit

b Multiplicand 1101 (13)
a Multiplier * 1011 (11)

tp		0000	
m0	+	1101	
tp		01101	
m1	+	1101	
tp		100111	
m2	+	0000	
tp		0100111	
m3	+	1101	
tp		10001111	(143)

$m_i = (a[i] == 0) ? 0 : b;$



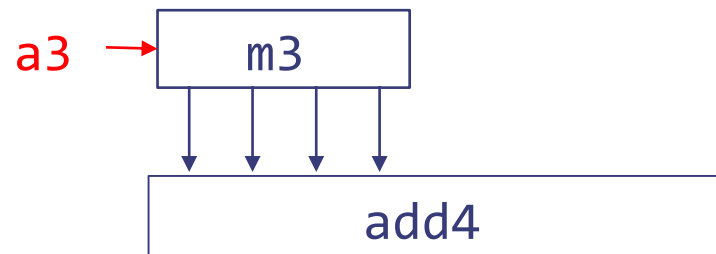
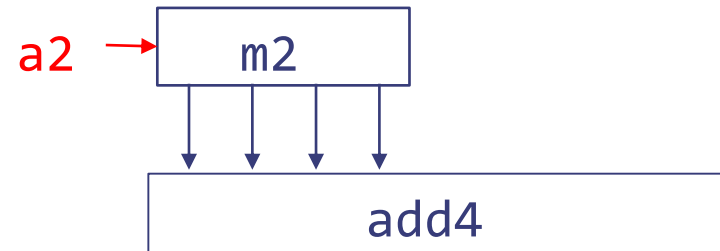
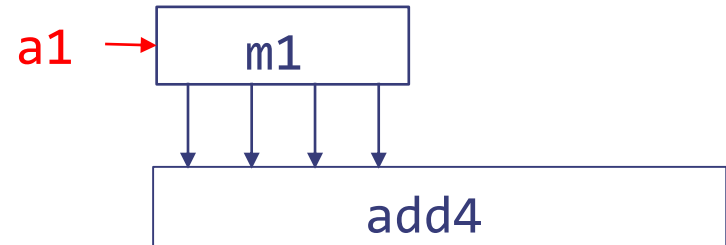
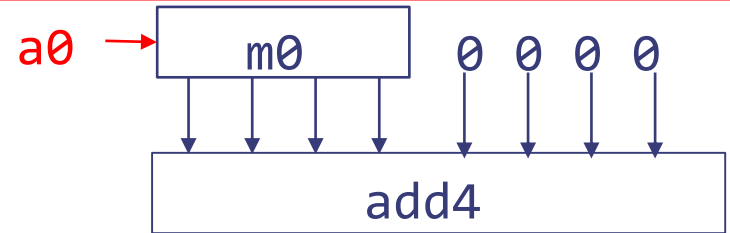
Multiplication by repeated addition

Combinational circuit

b Multiplicand 1101 (13)
a Multiplier * 1011 (11)

tp		0000	
m0	+	1101	
tp		01101	
m1	+	1101	
tp		100111	
m2	+	0000	
tp		0100111	
m3	+	1101	
tp		10001111	(143)

$m_i = (a[i] == 0) ? 0 : b;$



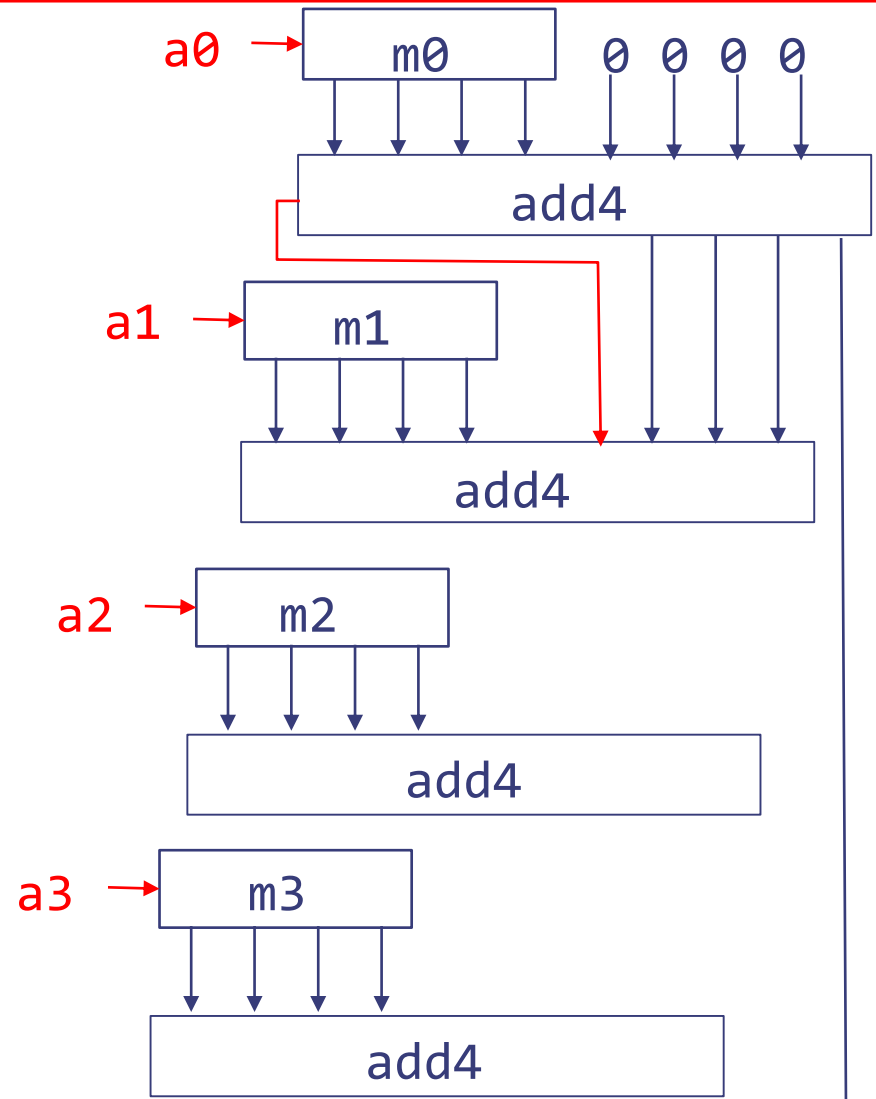
Multiplication by repeated addition

Combinational circuit

b Multiplicand 1101 (13)
a Multiplier * 1011 (11)

tp		0000	
m0	+	1101	
tp		01101	
m1	+	1101	
tp		100111	
m2	+	0000	
tp		0100111	
m3	+	1101	
tp		10001111	(143)

$m_i = (a[i] == 0) ? 0 : b;$



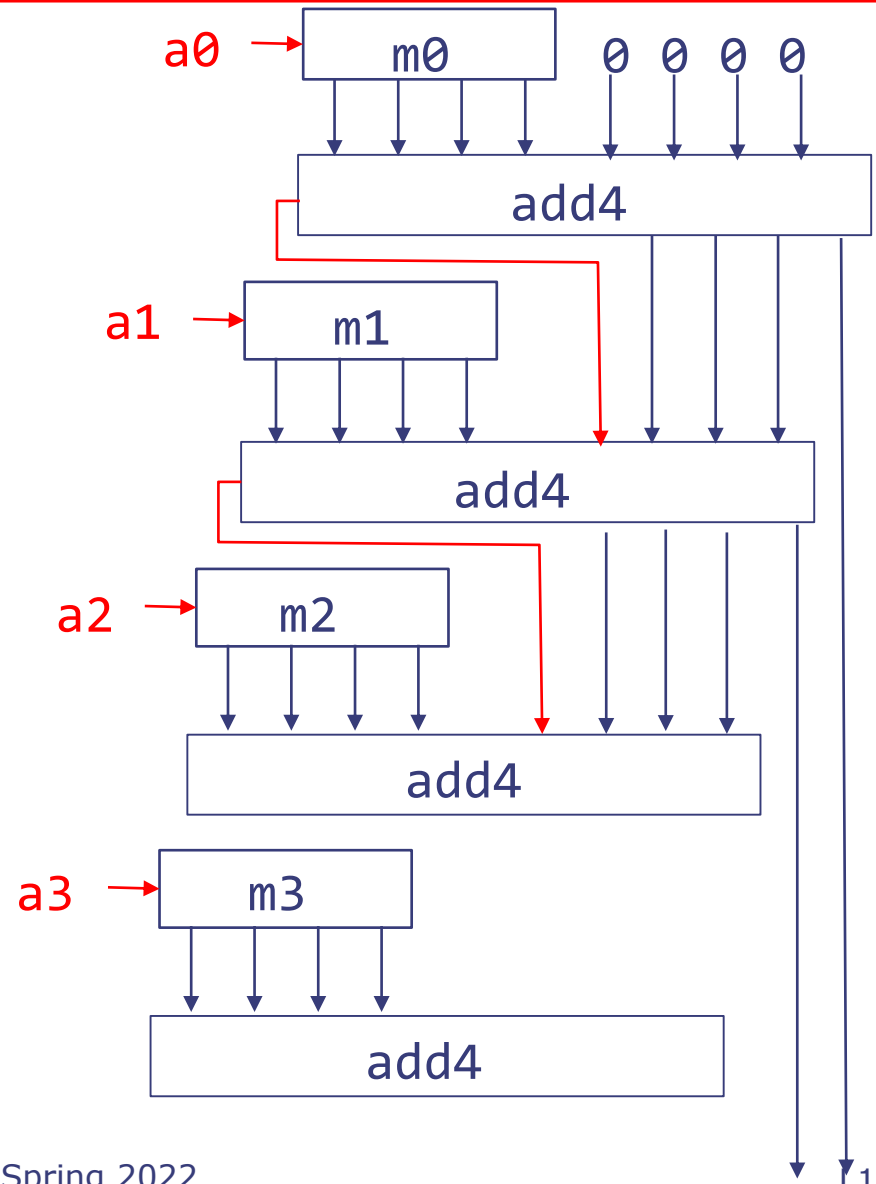
Multiplication by repeated addition

Combinational circuit

b Multiplicand 1101 (13)
a Multiplier * 1011 (11)

tp		0000	
m0	+	1101	
tp		01101	
m1	+	1101	
tp		100111	
m2	+	0000	
tp		0100111	
m3	+	1101	
tp		10001111	(143)

$m_i = (a[i] == 0) ? 0 : b;$



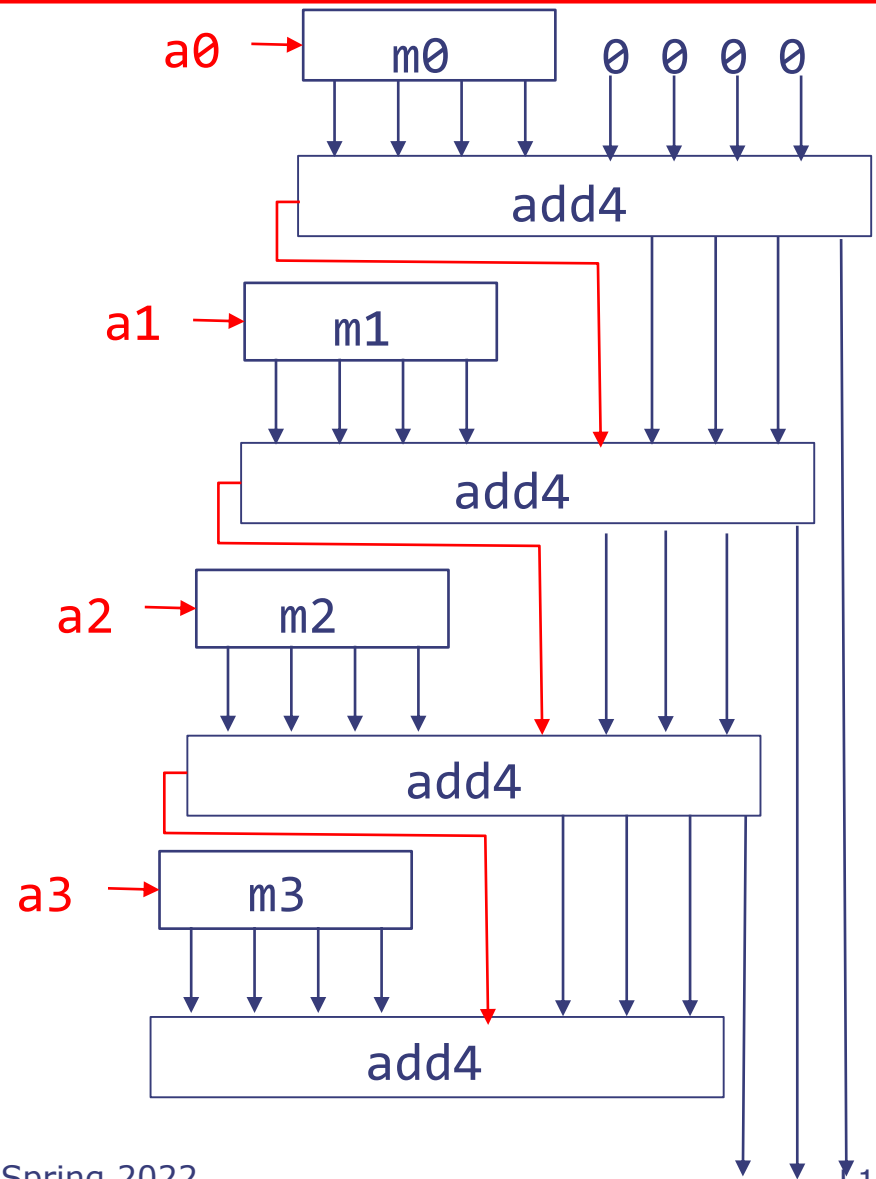
Multiplication by repeated addition

Combinational circuit

b Multiplicand 1101 (13)
 a Multiplier * 1011 (11)

tp		0000	
m0	+	1101	
tp		01101	
m1	+	1101	
tp		100111	
m2	+	0000	
tp		0100111	
m3	+	1101	
tp		10001111	(143)

$m_i = (a[i] == 0) ? 0 : b;$



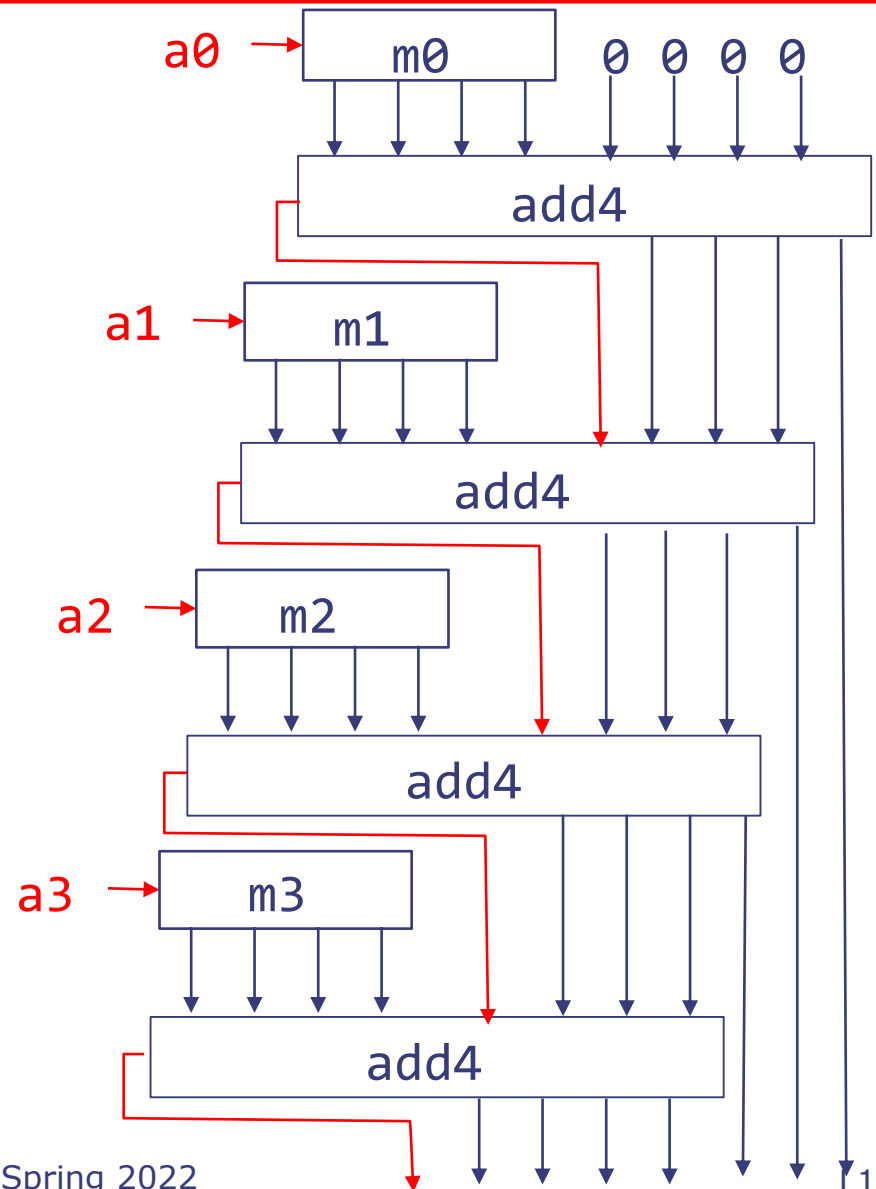
Multiplication by repeated addition

Combinational circuit

b Multiplicand 1101 (13)
a Multiplier * 1011 (11)

tp		0000	
m0	+	1101	
tp		01101	
m1	+	1101	
tp		100111	
m2	+	0000	
tp		0100111	
m3	+	1101	
tp		10001111	(143)

$m_i = (a[i] == 0) ? 0 : b;$



Multiplication by repeated addition

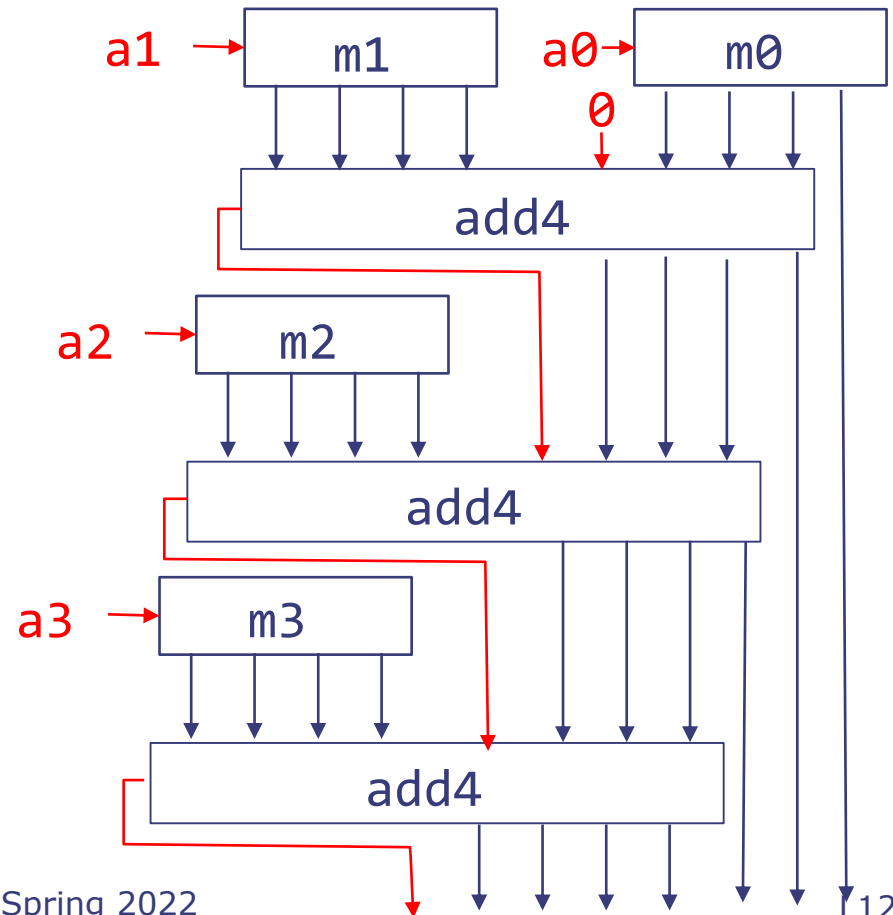
Combinational circuit

b Multiplicand 1101 (13)
a Multiplier * 1011 (11)

tp		0000	
m0	+	1101	
tp		01101	
m1	+	1101	
tp		100111	
m2	+	0000	
tp		0100111	
m3	+	1101	
tp		10001111	(143)

$m_i = (a[i] == 0) ? 0 : b;$

First adder is unnecessary
(always yields m0)



Implementation of mi

$$\begin{array}{rcccc} & & B_3 & B_2 & B_1 & B_0 \\ & & A_3 & A_2 & A_1 & A_0 \\ \times & A_3 & & & & \\ \hline & & B_3A_0 & B_2A_0 & B_1A_0 & B_0A_0 \\ & B_3A_1 & B_2A_1 & B_1A_1 & B_0A_1 & \\ & B_3A_2 & B_2A_2 & B_1A_2 & B_0A_2 & \\ + & B_3A_3 & B_2A_3 & B_1A_3 & B_0A_3 & \\ \hline \end{array}$$

Implementation of mi

$mi = (a[i] == 0) ? 0 : b;$

		B_3	B_2	B_1	B_0	
	\times	A_3	A_2	A_1	A_0	
<hr style="border: 0.5px solid blue;"/>						
		B_3A_0	B_2A_0	B_1A_0	B_0A_0	$m0$
		B_3A_1	B_2A_1	B_1A_1	B_0A_1	$m1$
		B_3A_2	B_2A_2	B_1A_2	B_0A_2	$m2$
$+$	B_3A_3	B_2A_3	B_1A_3	B_0A_3		$m3$
<hr style="border: 0.5px solid blue;"/>						

Implementation of mi

The “Binary”
Multiplication
Table

*	0	1
0	0	0
1	0	1

$mi = (a[i]==0)? 0 : b;$

$$\begin{array}{rcccc}
 & & B_3 & B_2 & B_1 & B_0 \\
 \times & A_3 & A_2 & A_1 & A_0 & \\
 \hline
 & & B_3A_0 & B_2A_0 & B_1A_0 & B_0A_0 & m0 \\
 & & B_3A_1 & B_2A_1 & B_1A_1 & B_0A_1 & m1 \\
 & & B_3A_2 & B_2A_2 & B_1A_2 & B_0A_2 & m2 \\
 + & B_3A_3 & B_2A_3 & B_1A_3 & B_0A_3 & & m3 \\
 \hline
 \end{array}$$

Implementation of mi

The “Binary”
Multiplication
Table

*	0	1
0	0	0
1	0	1

= AND

$mi = (a[i] == 0) ? 0 : b;$

$$\begin{array}{r}
 \begin{array}{cccc}
 & B_3 & B_2 & B_1 & B_0 \\
 \times A_3 & A_2 & A_1 & A_0 & \\
 \hline
 & B_3A_0 & B_2A_0 & B_1A_0 & B_0A_0 \\
 & B_3A_1 & B_2A_1 & B_1A_1 & B_0A_1 \\
 & B_3A_2 & B_2A_2 & B_1A_2 & B_0A_2 \\
 + & B_3A_3 & B_2A_3 & B_1A_3 & B_0A_3 \\
 \hline
 \end{array}
 \end{array}
 \begin{array}{l}
 m0 \\
 m1 \\
 m2 \\
 m3
 \end{array}$$

Implementation of mi

The “Binary”
Multiplication
Table

*	0	1
0	0	0
1	0	1

= AND

$mi = (a[i] == 0) ? 0 : b;$

BA_i called a “partial product” \longrightarrow

	B_3	B_2	B_1	B_0	
	$\times A_3$	A_2	A_1	A_0	
	B_3A_0	B_2A_0	B_1A_0	B_0A_0	$m0$
	B_3A_1	B_2A_1	B_1A_1	B_0A_1	$m1$
	B_3A_2	B_2A_2	B_1A_2	B_0A_2	$m2$
+	B_3A_3	B_2A_3	B_1A_3	B_0A_3	$m3$

Implementation of mi

The “Binary”
Multiplication
Table

*	0	1
0	0	0
1	0	1

= AND

$mi = (a[i]==0)? 0 : b;$

$$\begin{array}{rcccc}
 & B_3 & B_2 & B_1 & B_0 \\
 \times & A_3 & A_2 & A_1 & A_0 \\
 \hline
 BA_i \text{ called a "partial product"} \longrightarrow & B_3A_0 & B_2A_0 & B_1A_0 & B_0A_0 & m0 \\
 & B_3A_1 & B_2A_1 & B_1A_1 & B_0A_1 & m1 \\
 & B_3A_2 & B_2A_2 & B_1A_2 & B_0A_2 & m2 \\
 + & B_3A_3 & B_2A_3 & B_1A_3 & B_0A_3 & m3 \\
 \hline
 \end{array}$$

Multiplying N-digit number by M-digit number gives (N+M)-digit result

Implementation of mi

The “Binary”
Multiplication
Table

*	0	1
0	0	0
1	0	1

= AND

$$mi = (a[i]==0)? 0 : b;$$

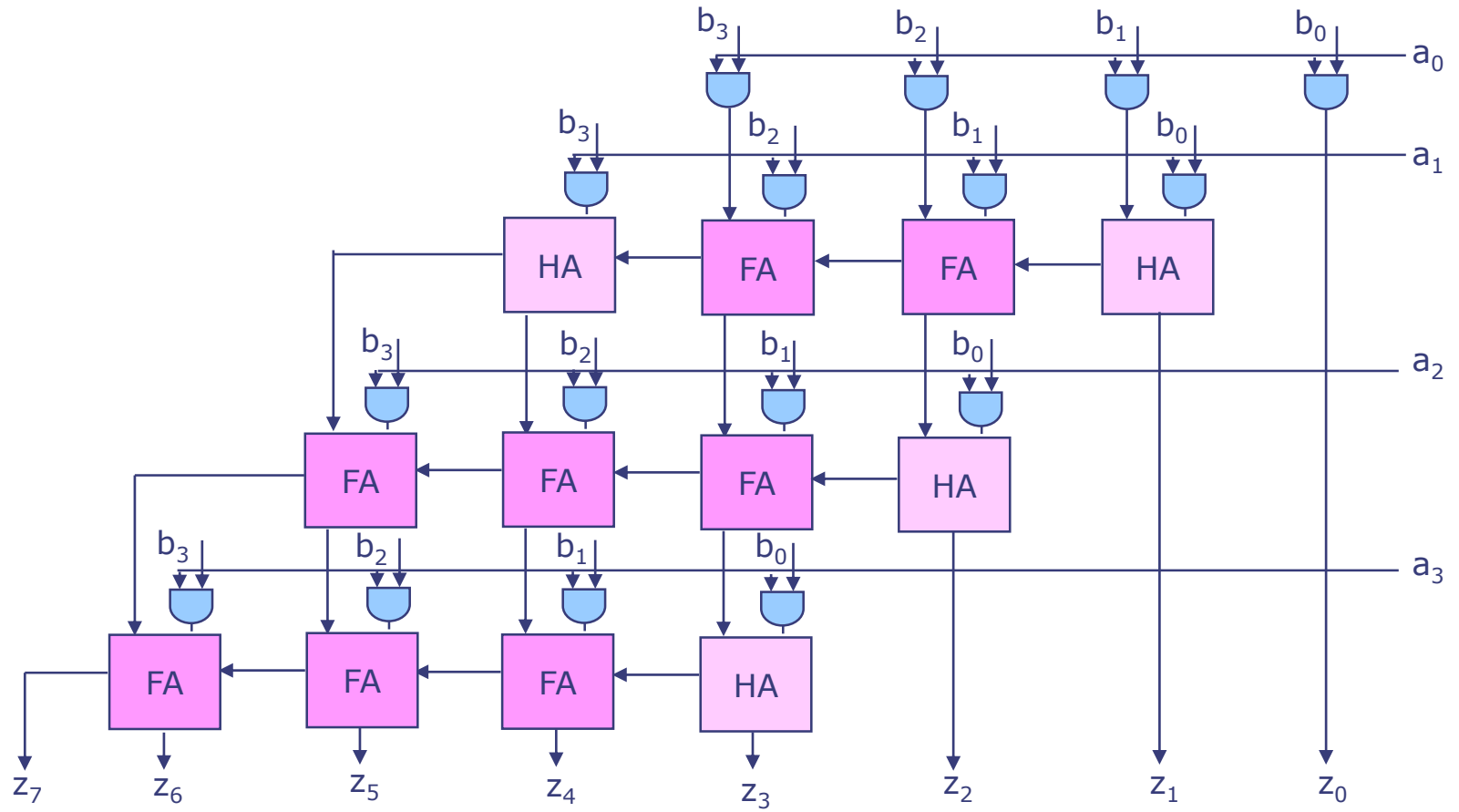
$$\begin{array}{rcccc}
 & B_3 & B_2 & B_1 & B_0 \\
 \times & A_3 & A_2 & A_1 & A_0 \\
 \hline
 BA_i \text{ called a "partial product"} \longrightarrow & B_3A_0 & B_2A_0 & B_1A_0 & B_0A_0 & m0 \\
 & B_3A_1 & B_2A_1 & B_1A_1 & B_0A_1 & m1 \\
 & B_3A_2 & B_2A_2 & B_1A_2 & B_0A_2 & m2 \\
 + & B_3A_3 & B_2A_3 & B_1A_3 & B_0A_3 & m3 \\
 \hline
 \end{array}$$

Multiplying N-digit number by M-digit number gives (N+M)-digit result

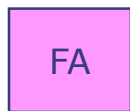
Easy part: forming partial products (bunch of AND gates)

Hard part: adding M N-bit partial products

Combinational Multiplier Redrawn

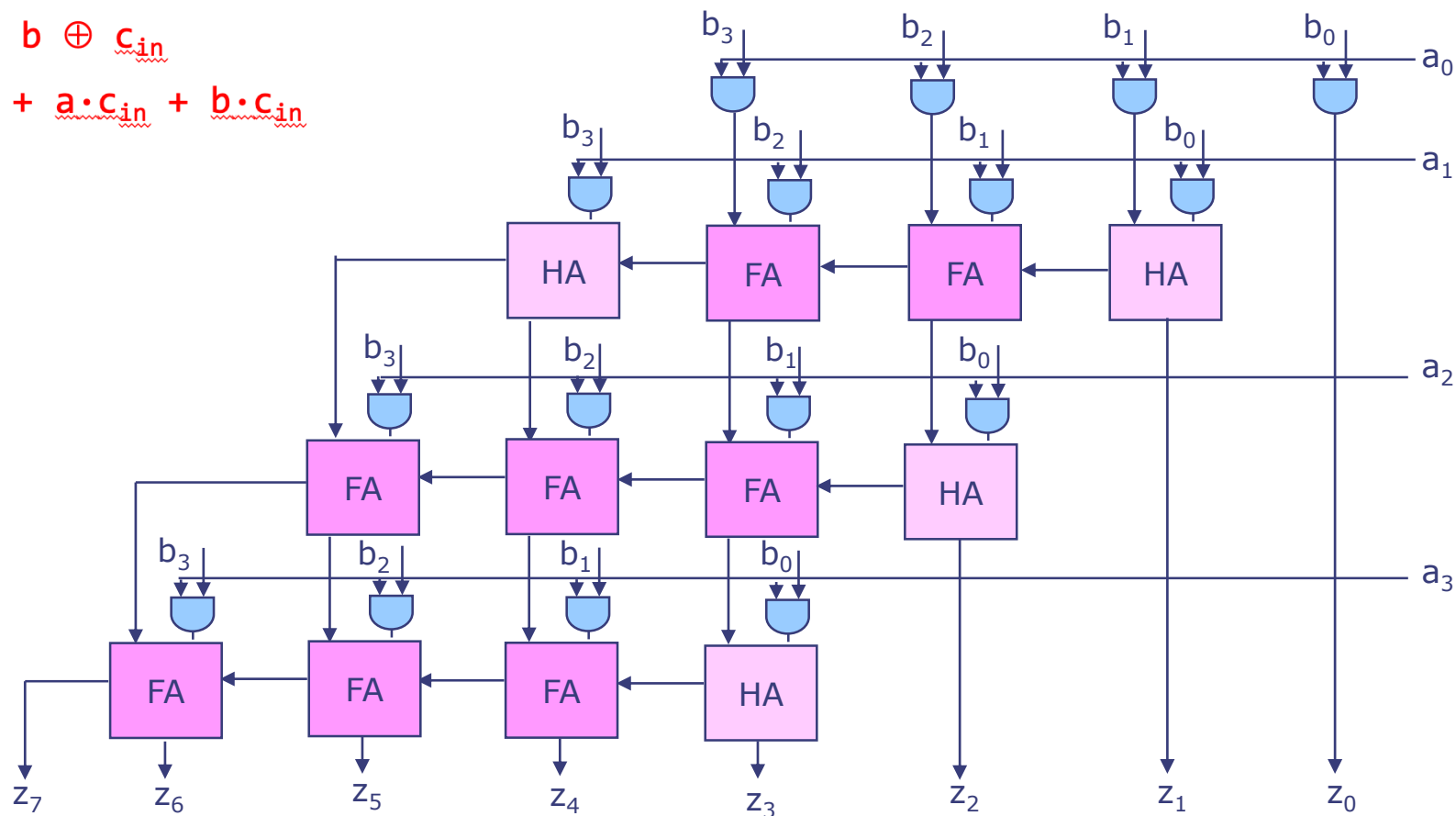


Combinational Multiplier Redrawn



$$s = a \oplus b \oplus c_{in}$$

$$c_{out} = a \cdot b + a \cdot c_{in} + b \cdot c_{in}$$

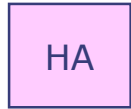


Combinational Multiplier Redrawn



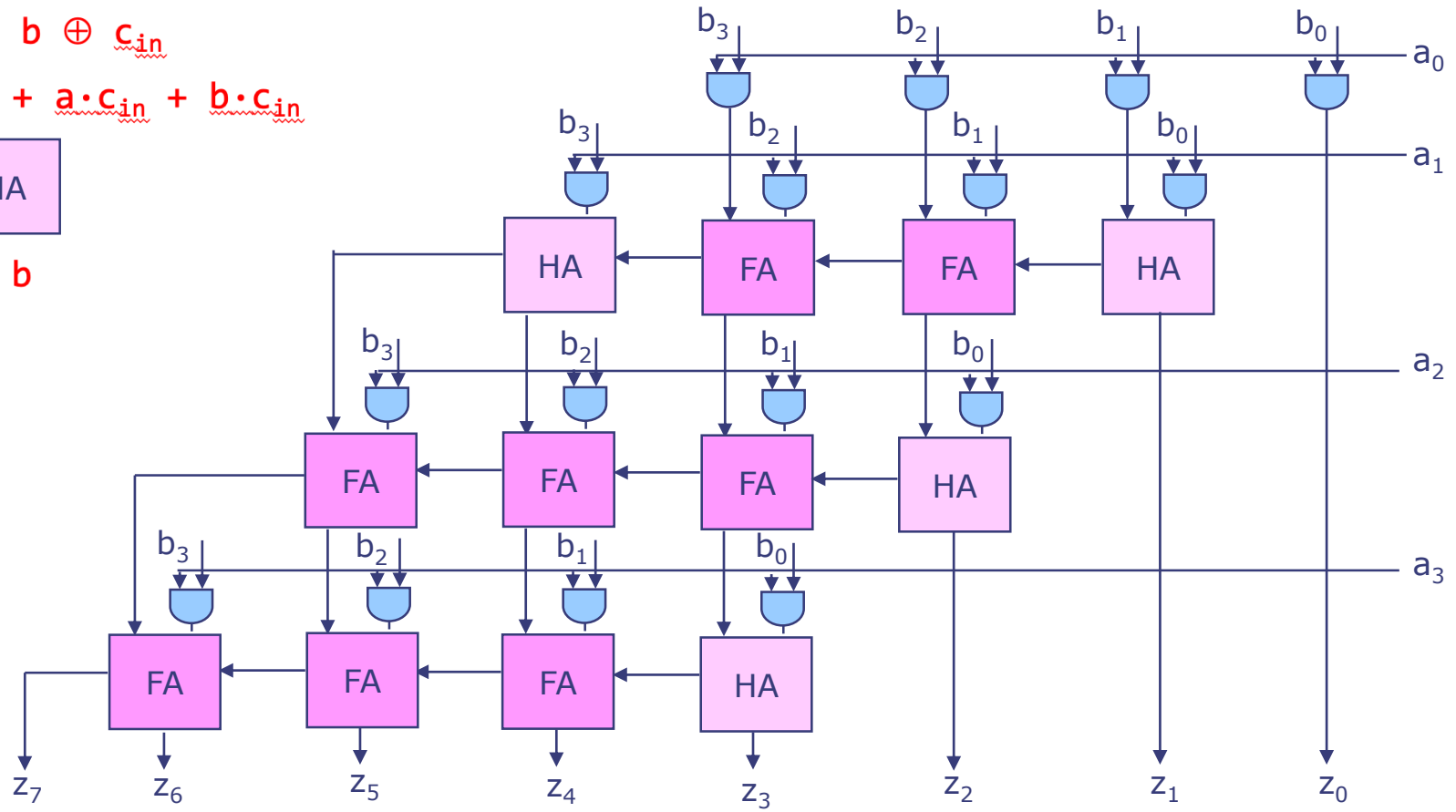
$$s = a \oplus b \oplus c_{in}$$

$$c_{out} = a \cdot b + a \cdot c_{in} + b \cdot c_{in}$$



$$s = a \oplus b$$

$$c_{out} = a \cdot b$$

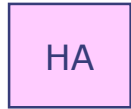


Combinational Multiplier Redrawn



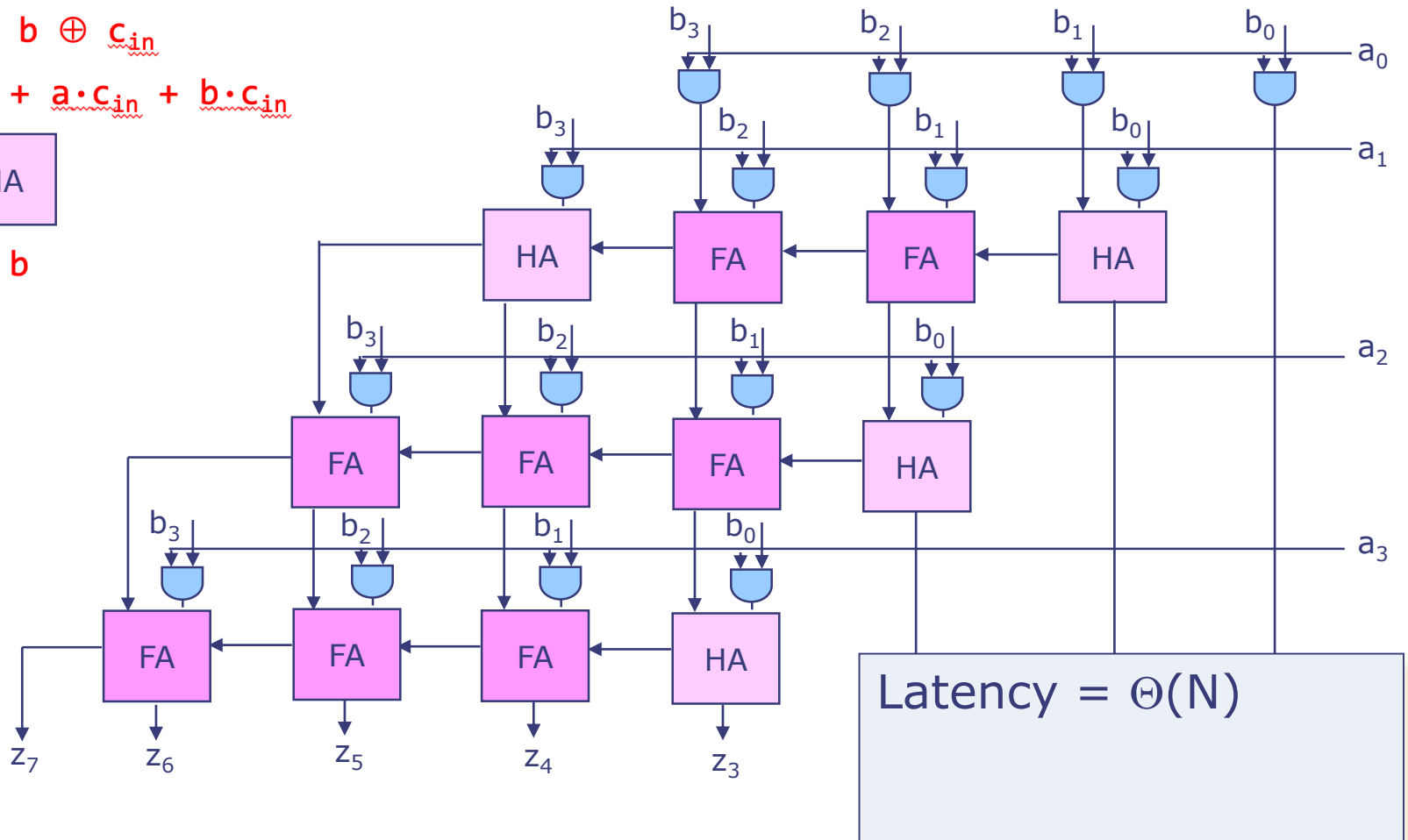
$$s = a \oplus b \oplus c_{in}$$

$$c_{out} = a \cdot b + a \cdot c_{in} + b \cdot c_{in}$$



$$s = a \oplus b$$

$$c_{out} = a \cdot b$$

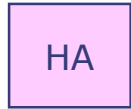


Combinational Multiplier Redrawn



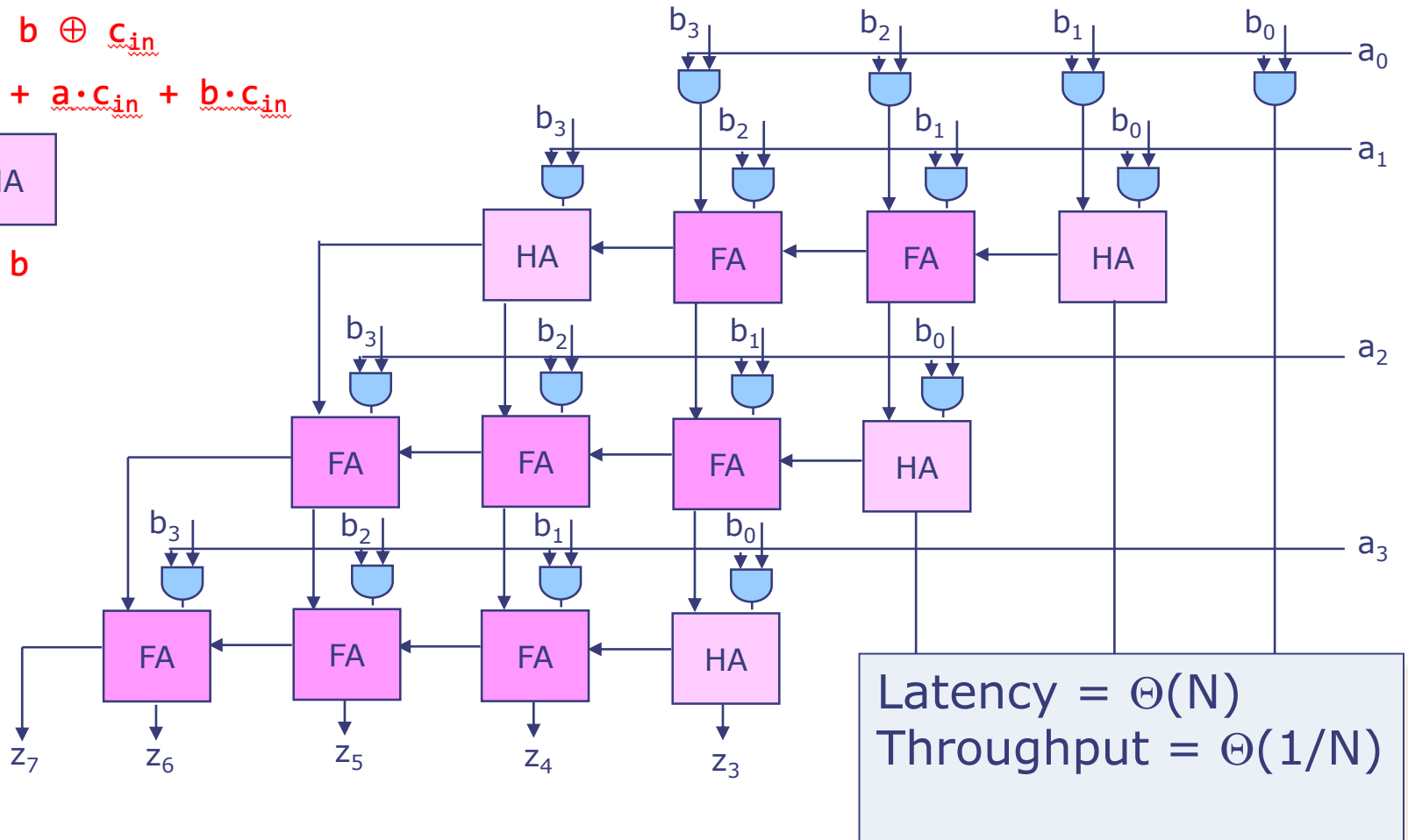
$$s = a \oplus b \oplus c_{in}$$

$$c_{out} = a \cdot b + a \cdot c_{in} + b \cdot c_{in}$$



$$s = a \oplus b$$

$$c_{out} = a \cdot b$$

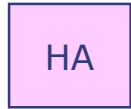


Combinational Multiplier Redrawn



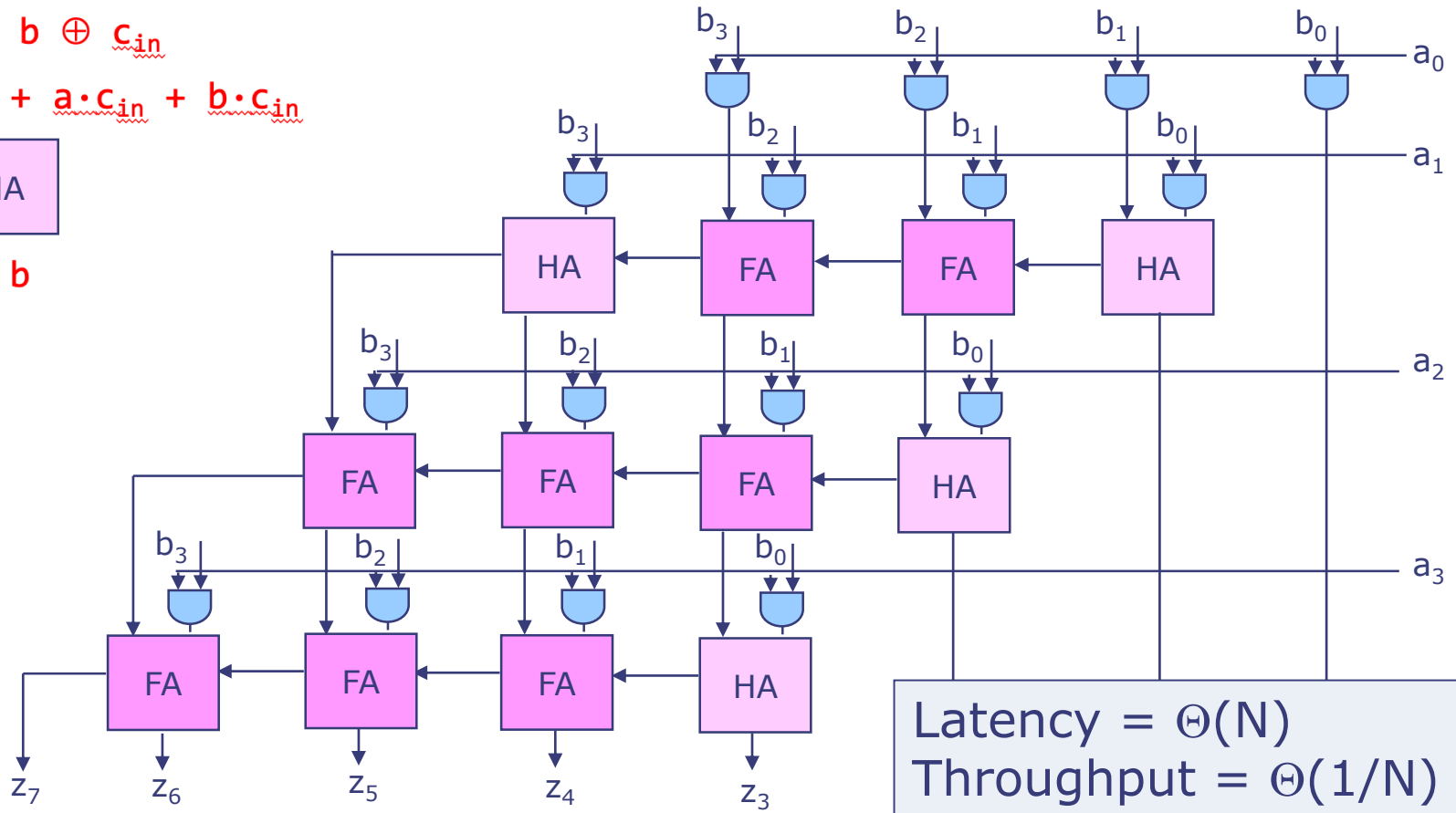
$$s = a \oplus b \oplus c_{in}$$

$$c_{out} = a \cdot b + a \cdot c_{in} + b \cdot c_{in}$$



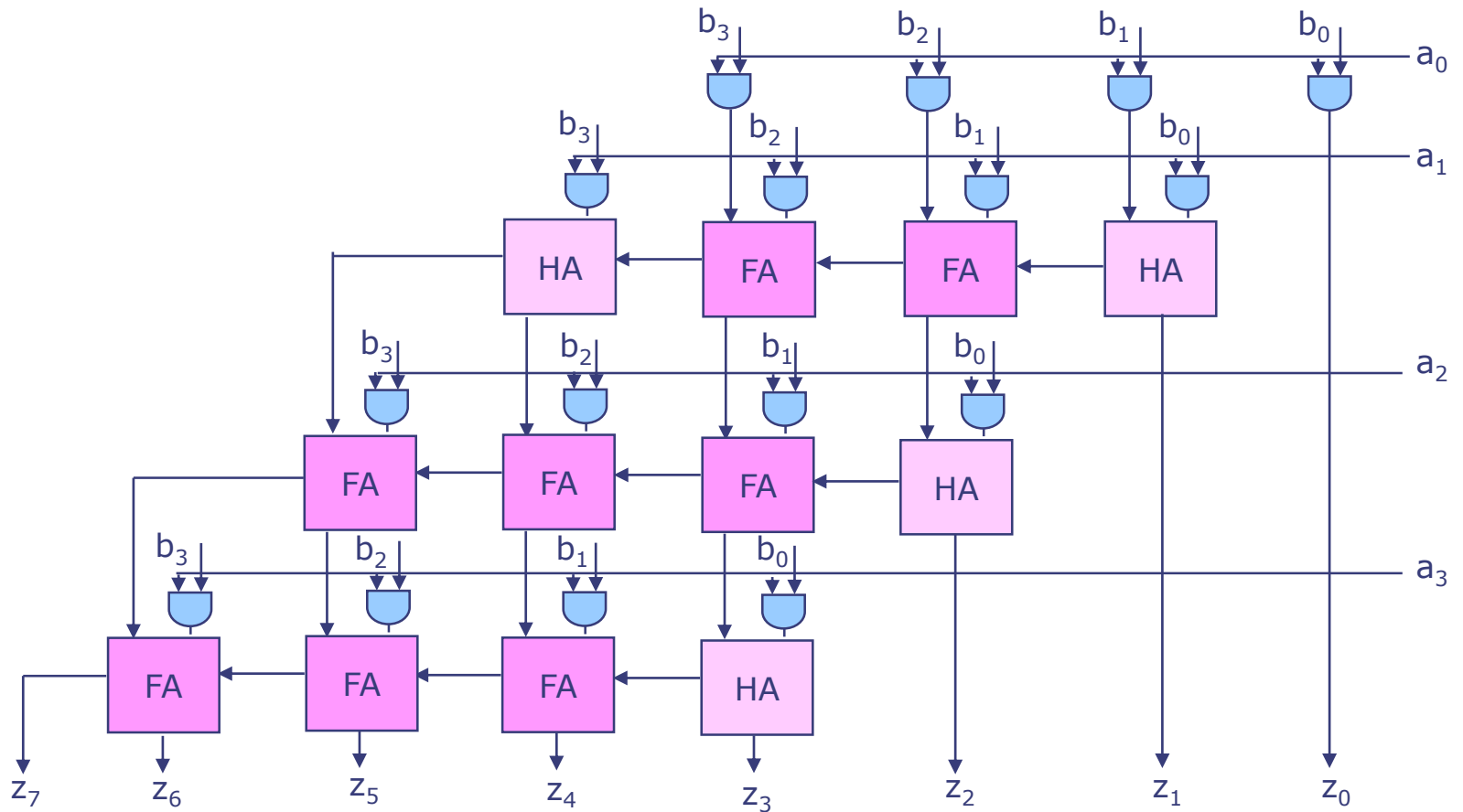
$$s = a \oplus b$$

$$c_{out} = a \cdot b$$



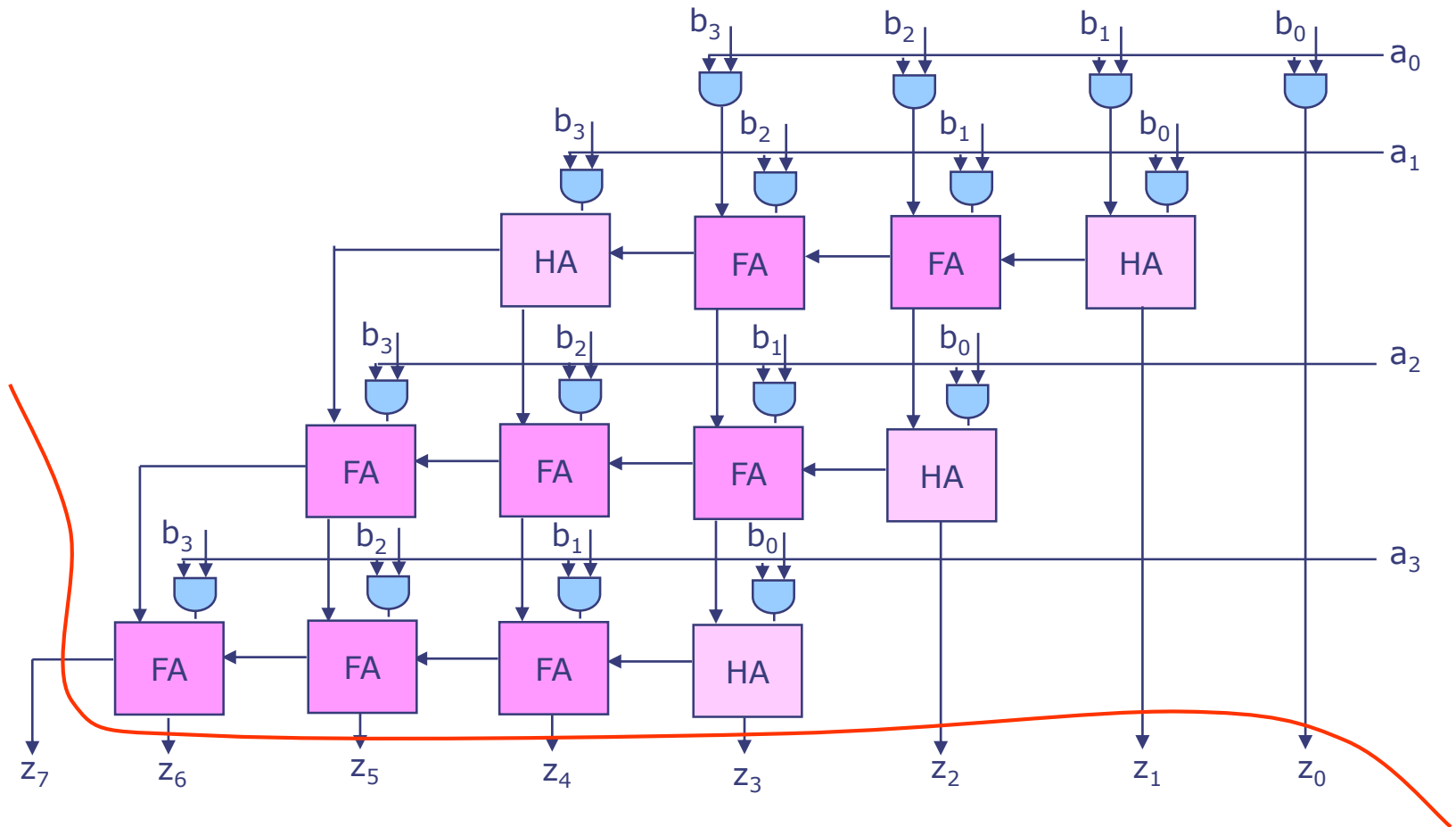
Pipelining to Increase Throughput

First Attempt



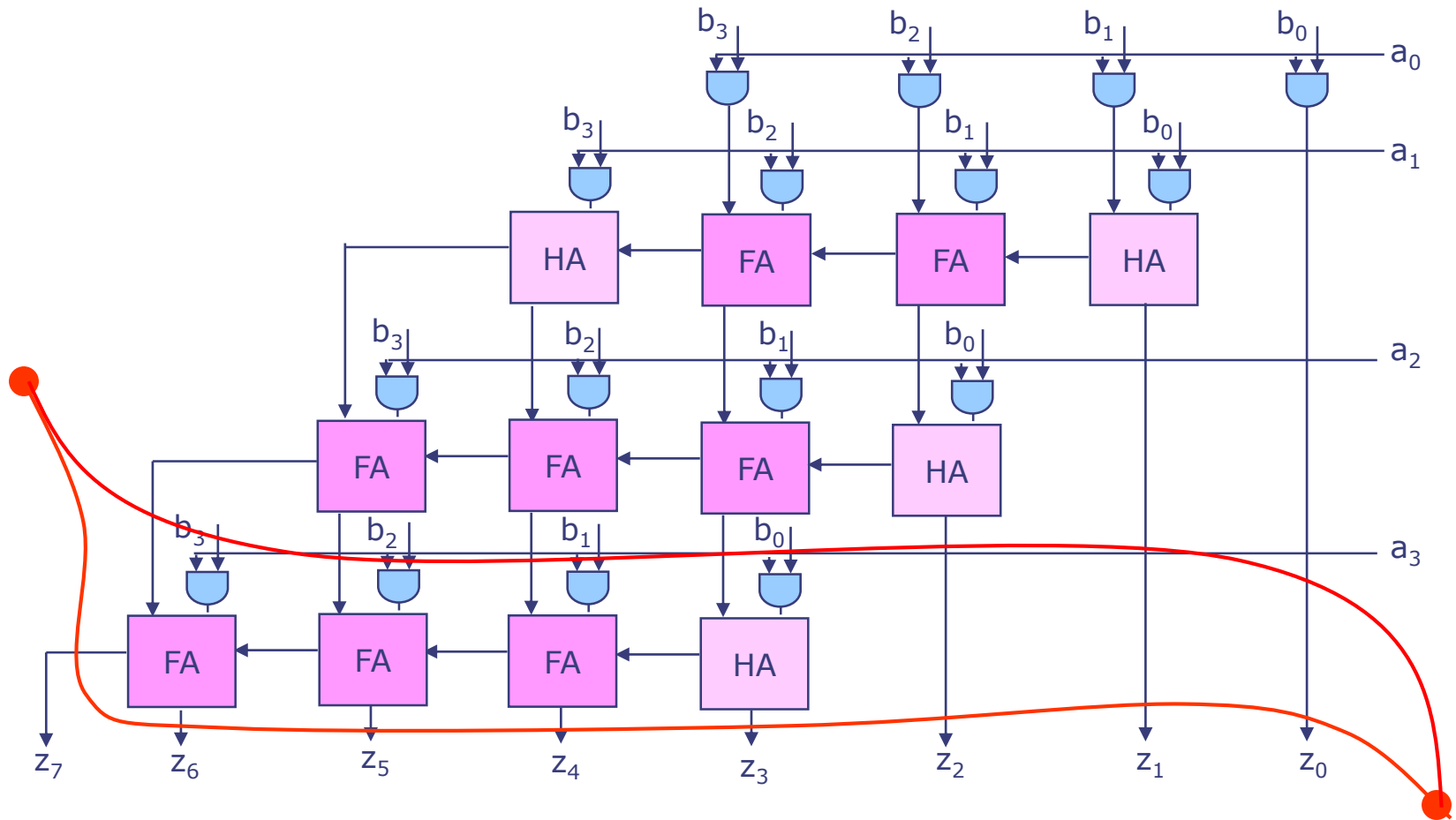
Pipelining to Increase Throughput

First Attempt



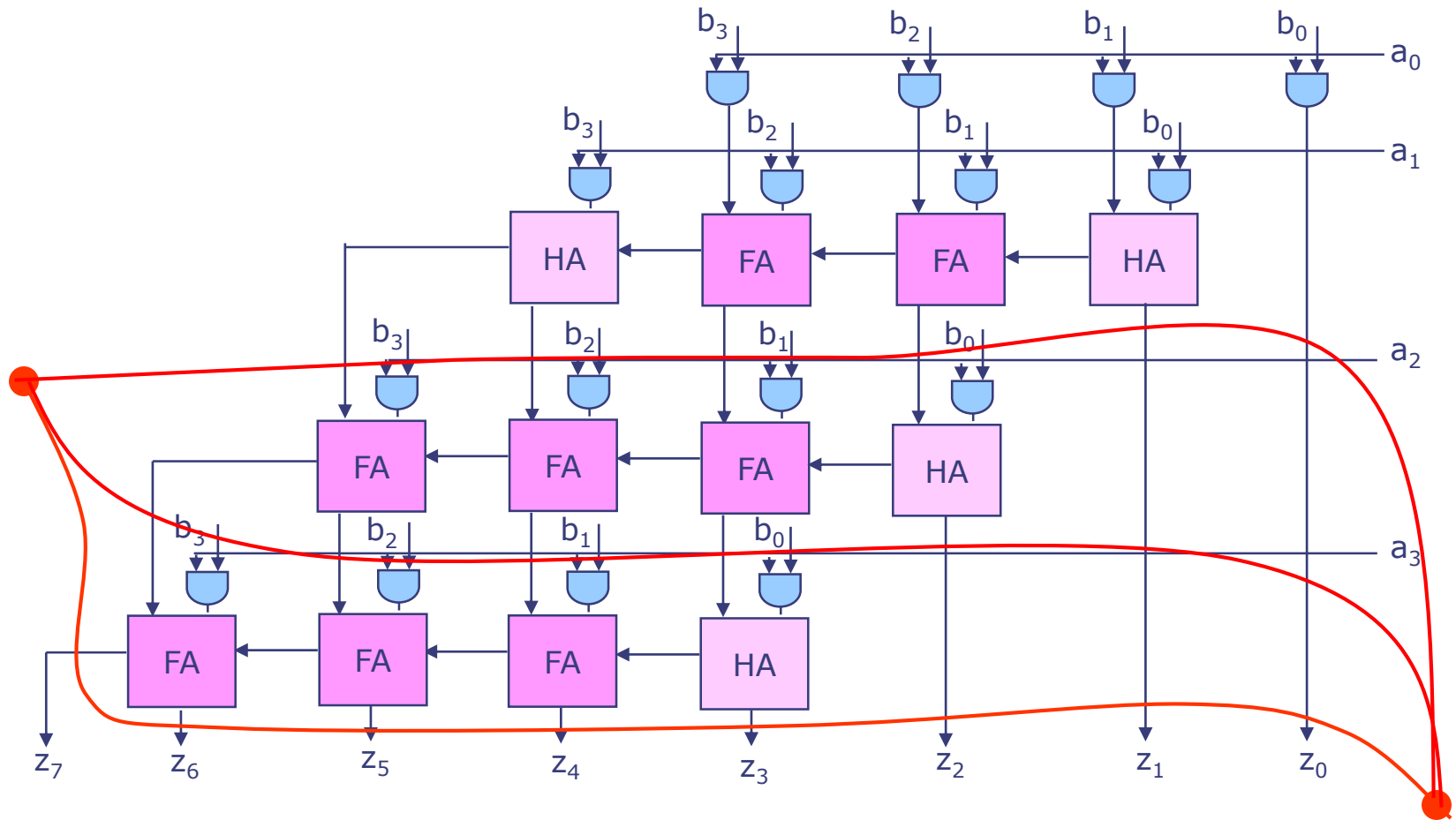
Pipelining to Increase Throughput

First Attempt



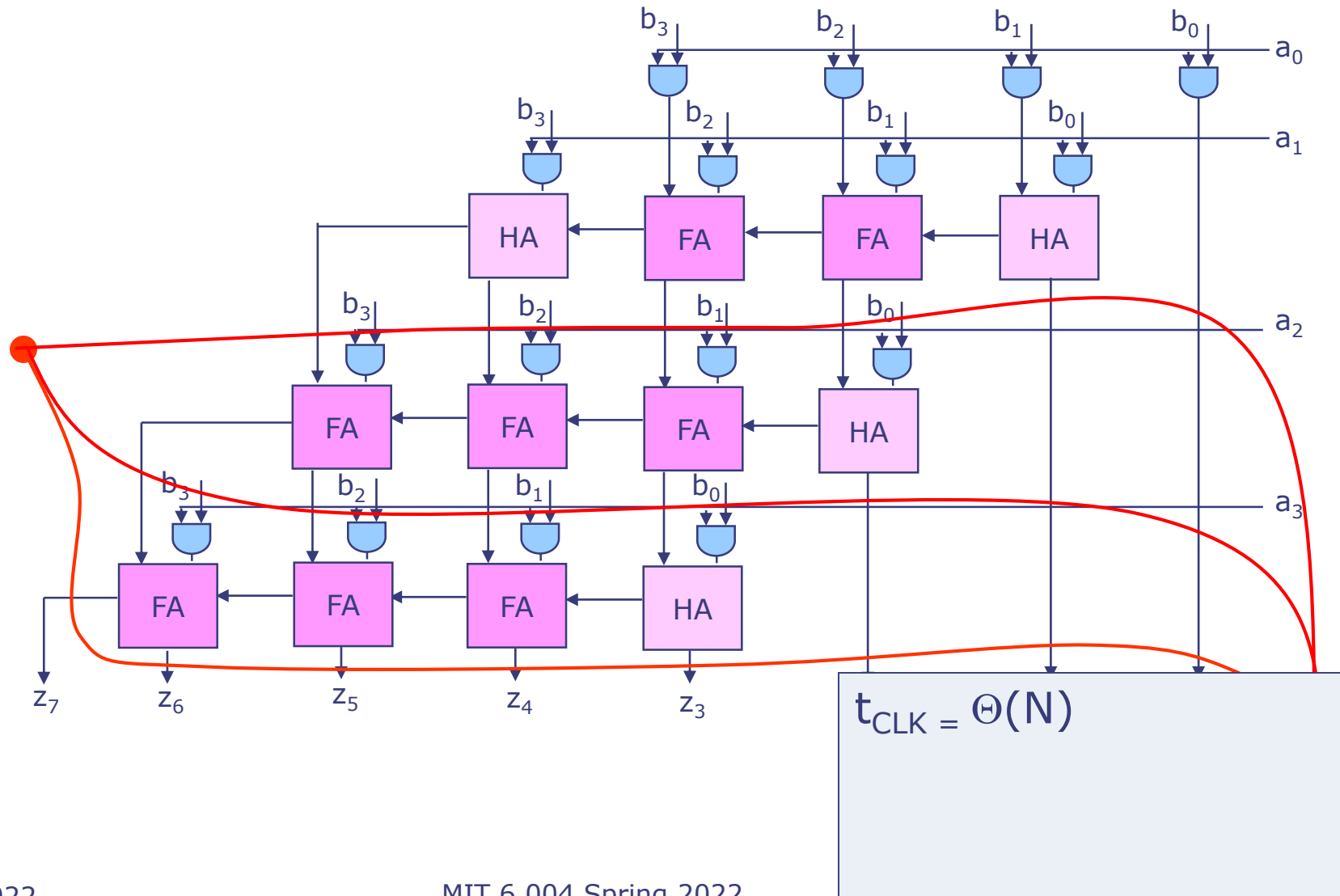
Pipelining to Increase Throughput

First Attempt



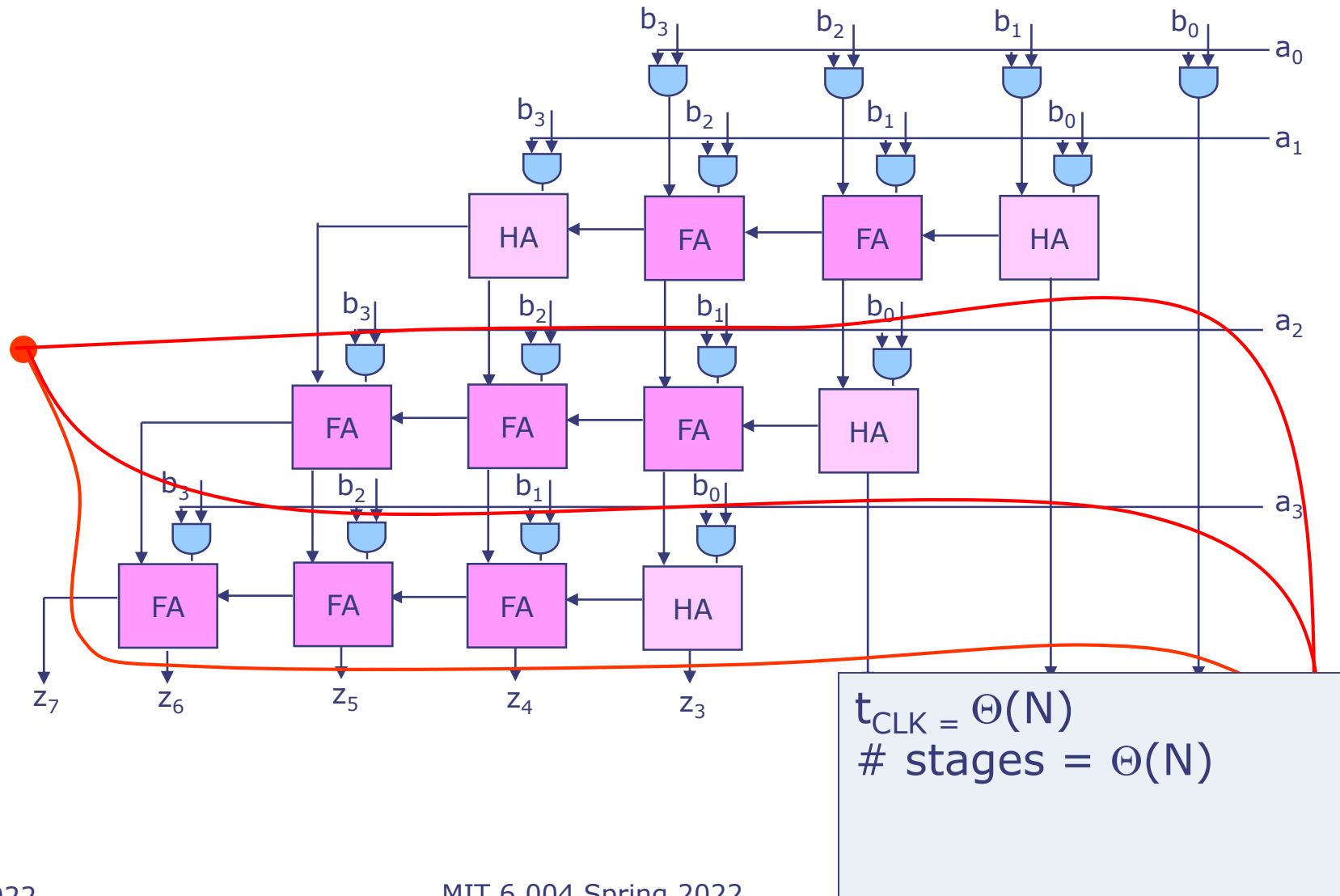
Pipelining to Increase Throughput

First Attempt



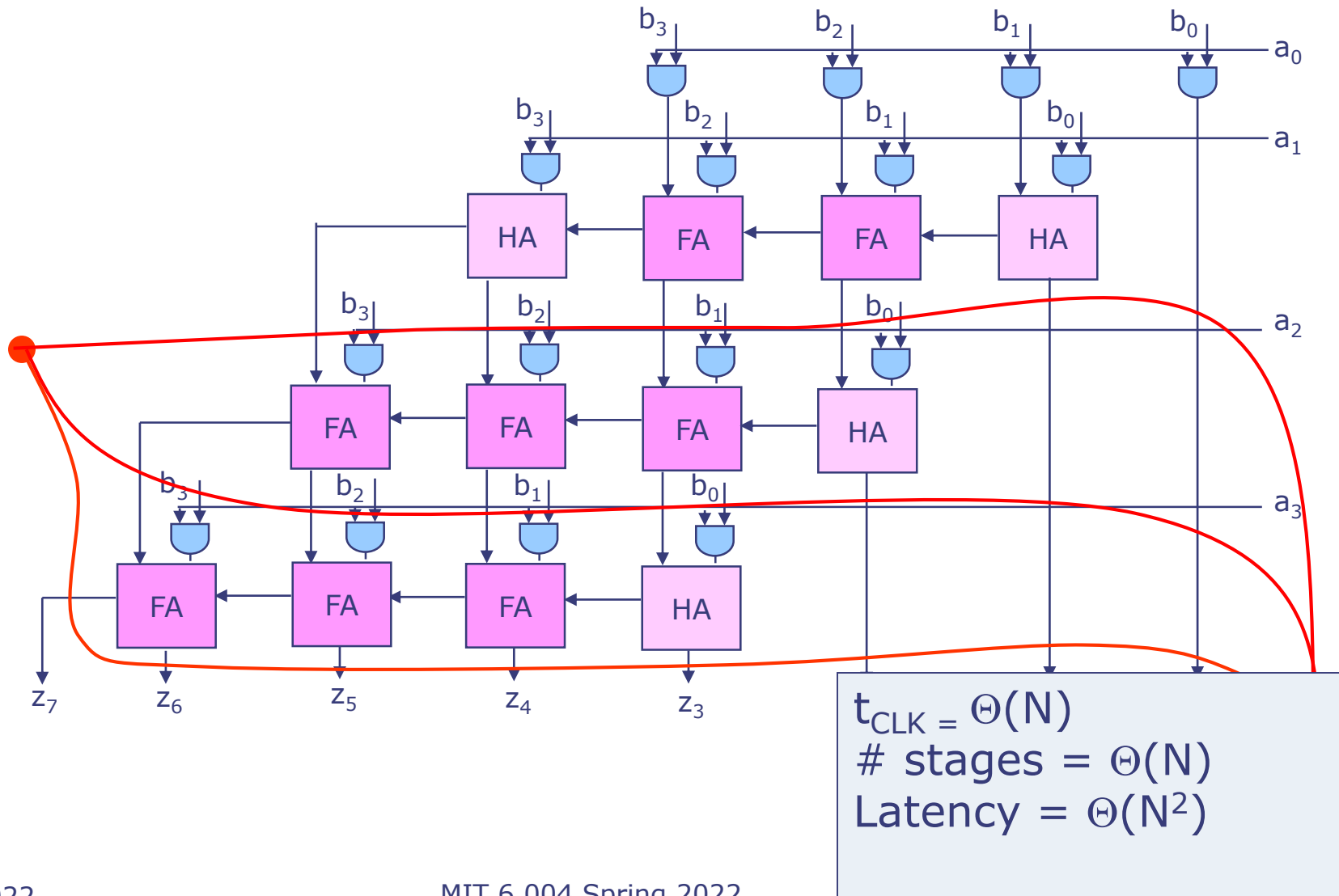
Pipelining to Increase Throughput

First Attempt



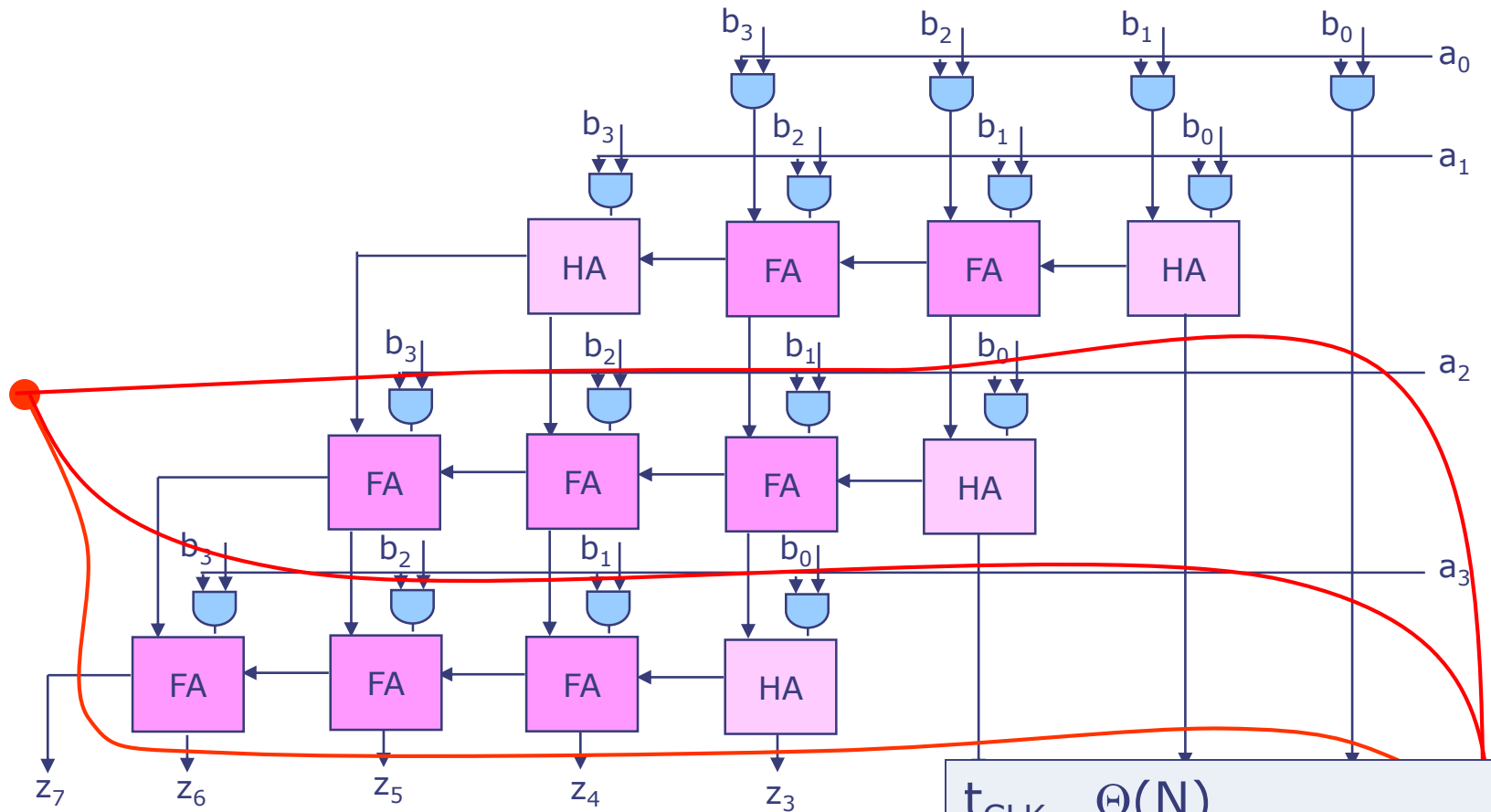
Pipelining to Increase Throughput

First Attempt



Pipelining to Increase Throughput

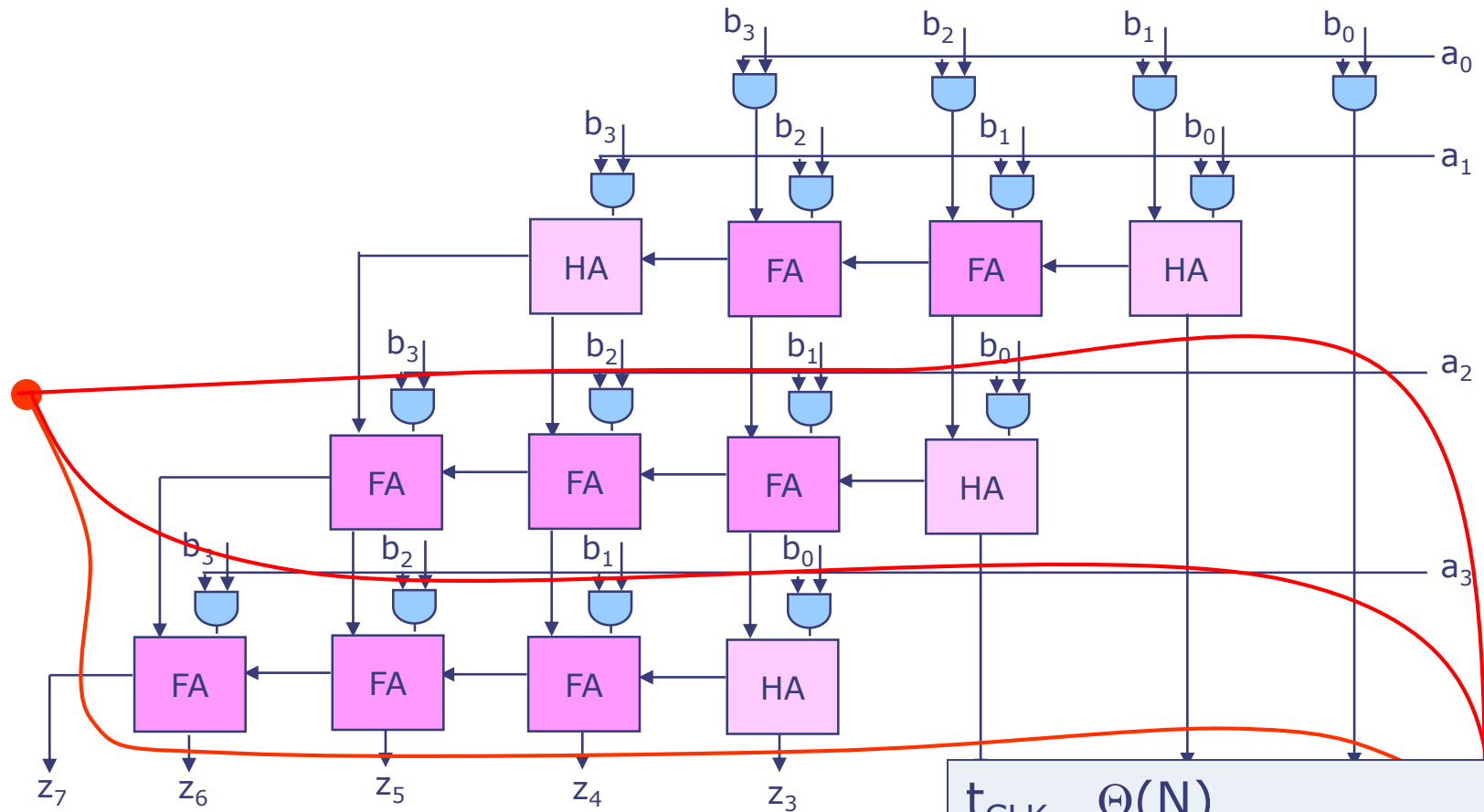
First Attempt



$t_{\text{CLK}} = \Theta(N)$
stages = $\Theta(N)$
Latency = $\Theta(N^2)$
Throughput = $\Theta(1/N)$

Pipelining to Increase Throughput

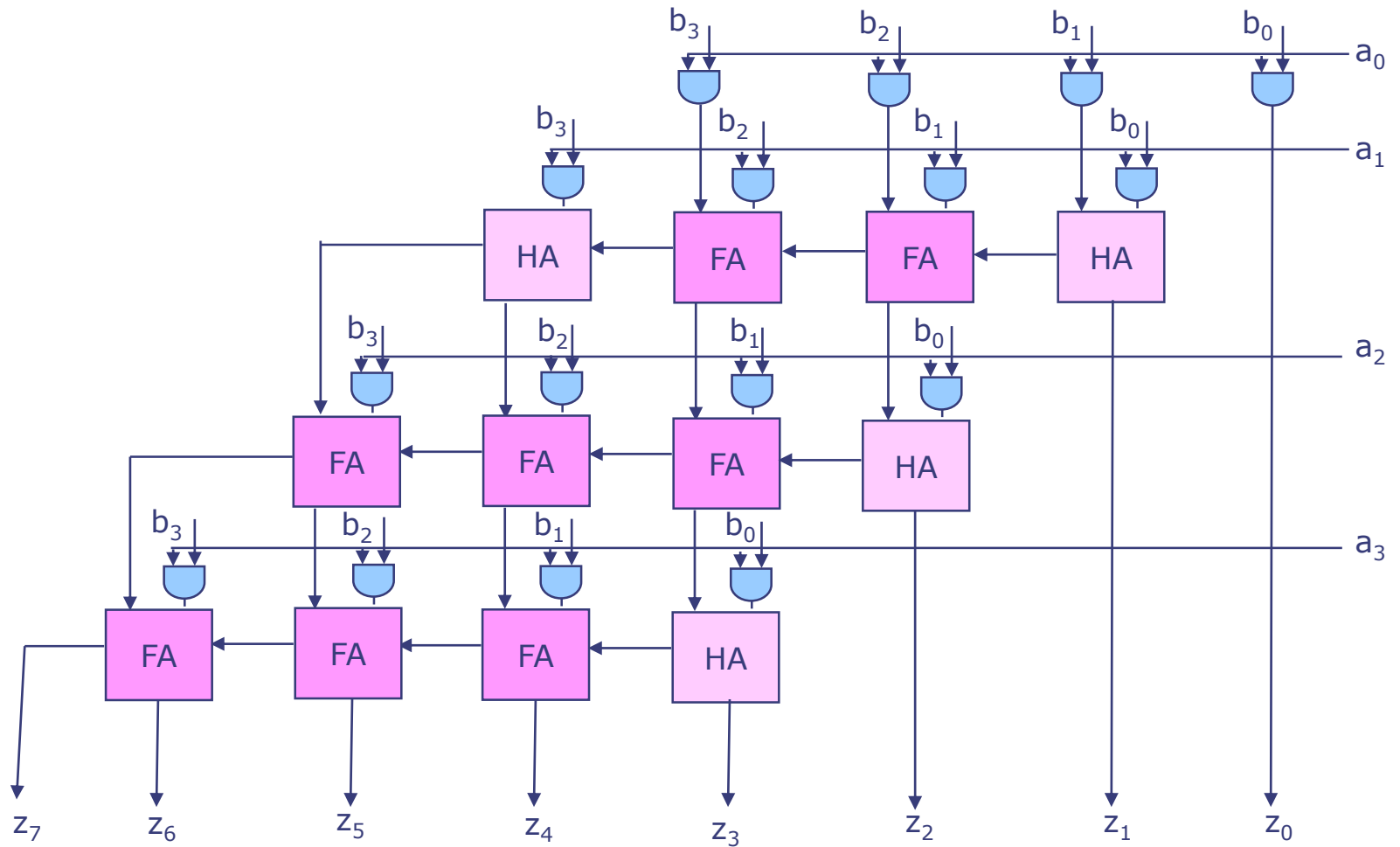
First Attempt



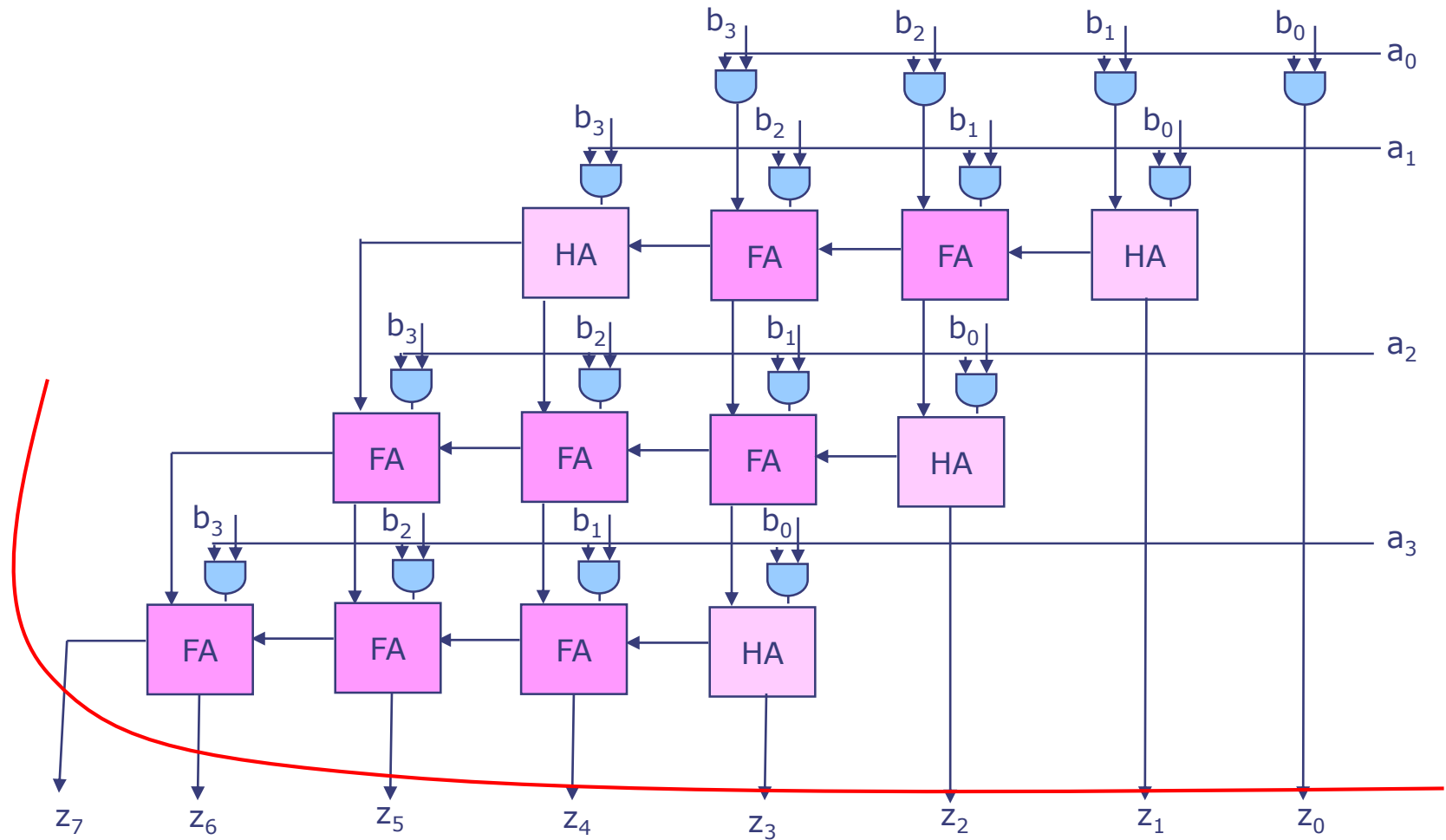
Need to break carry chain

$t_{\text{CLK}} = \Theta(N)$
stages = $\Theta(N)$
Latency = $\Theta(N^2)$
Throughput = $\Theta(1/N)$

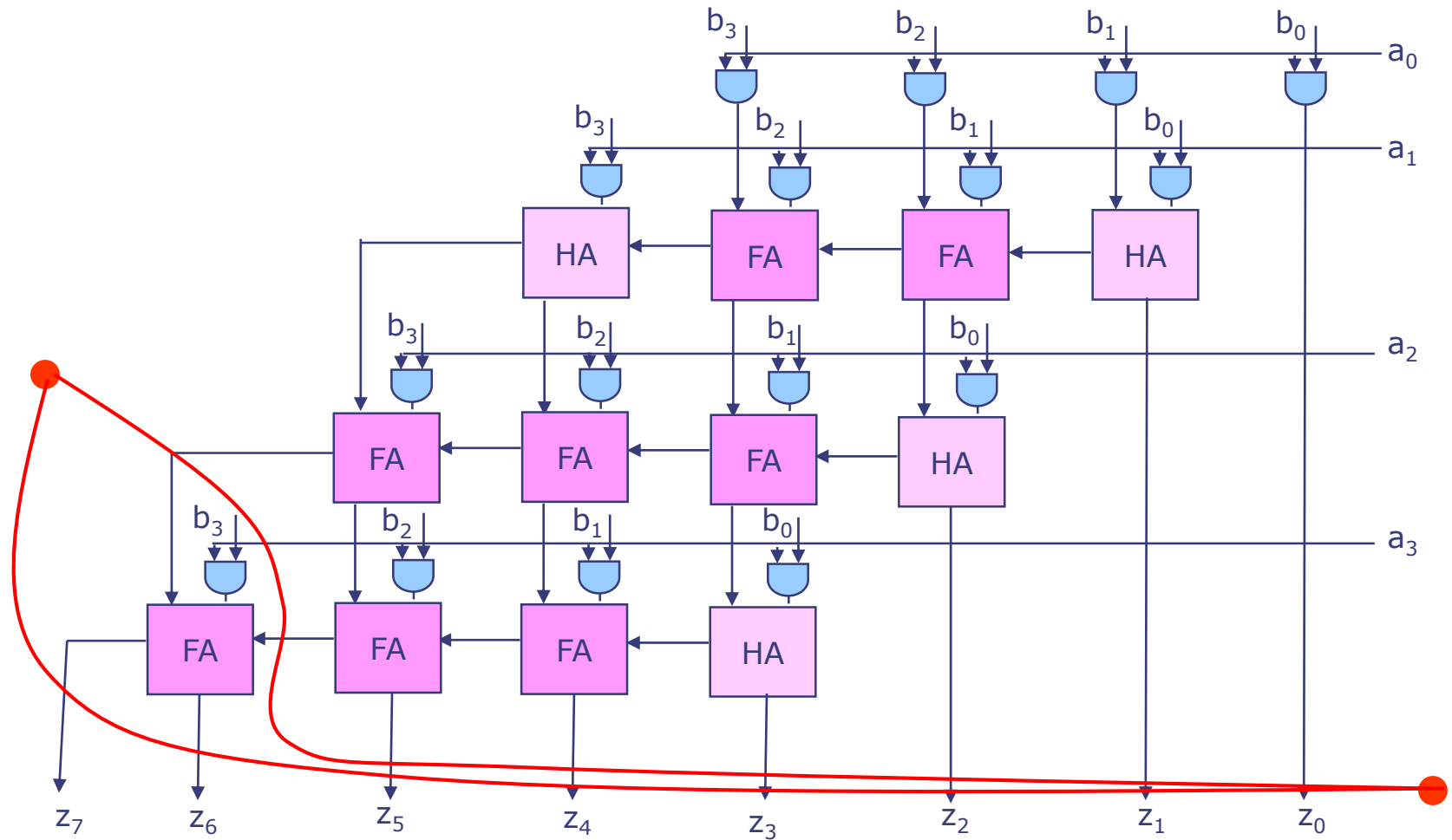
Pipelining to Increase Throughput



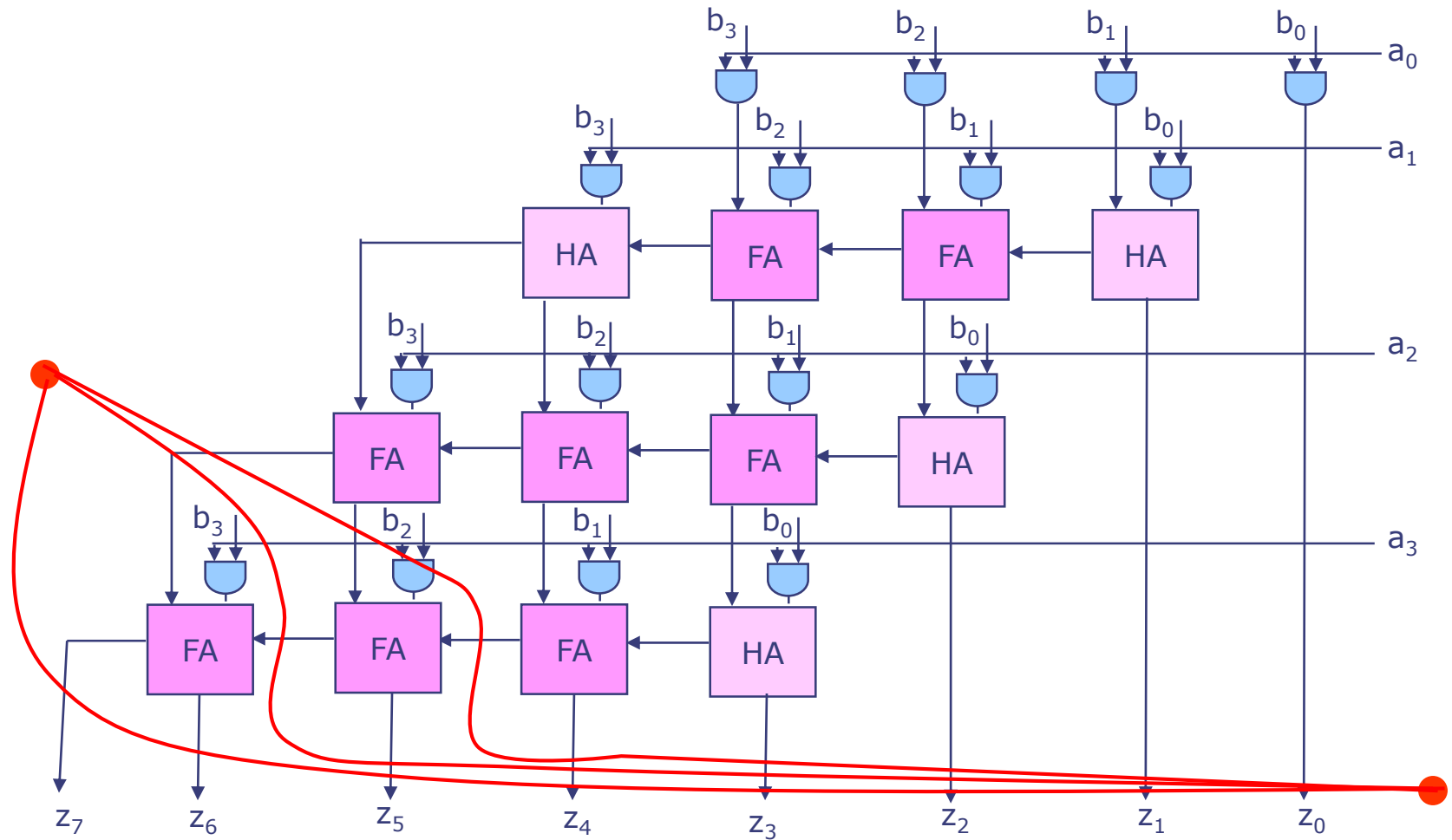
Pipelining to Increase Throughput



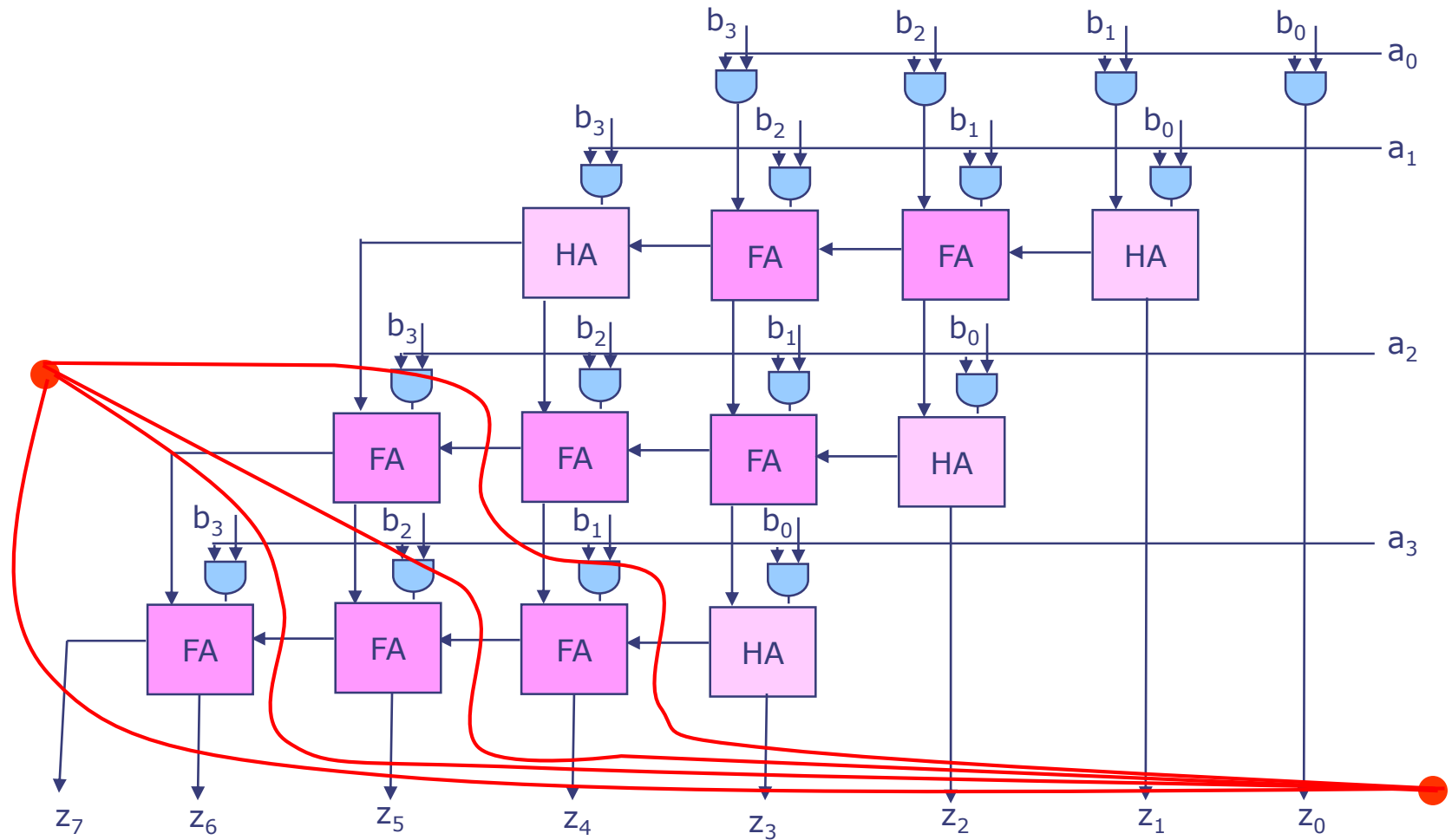
Pipelining to Increase Throughput



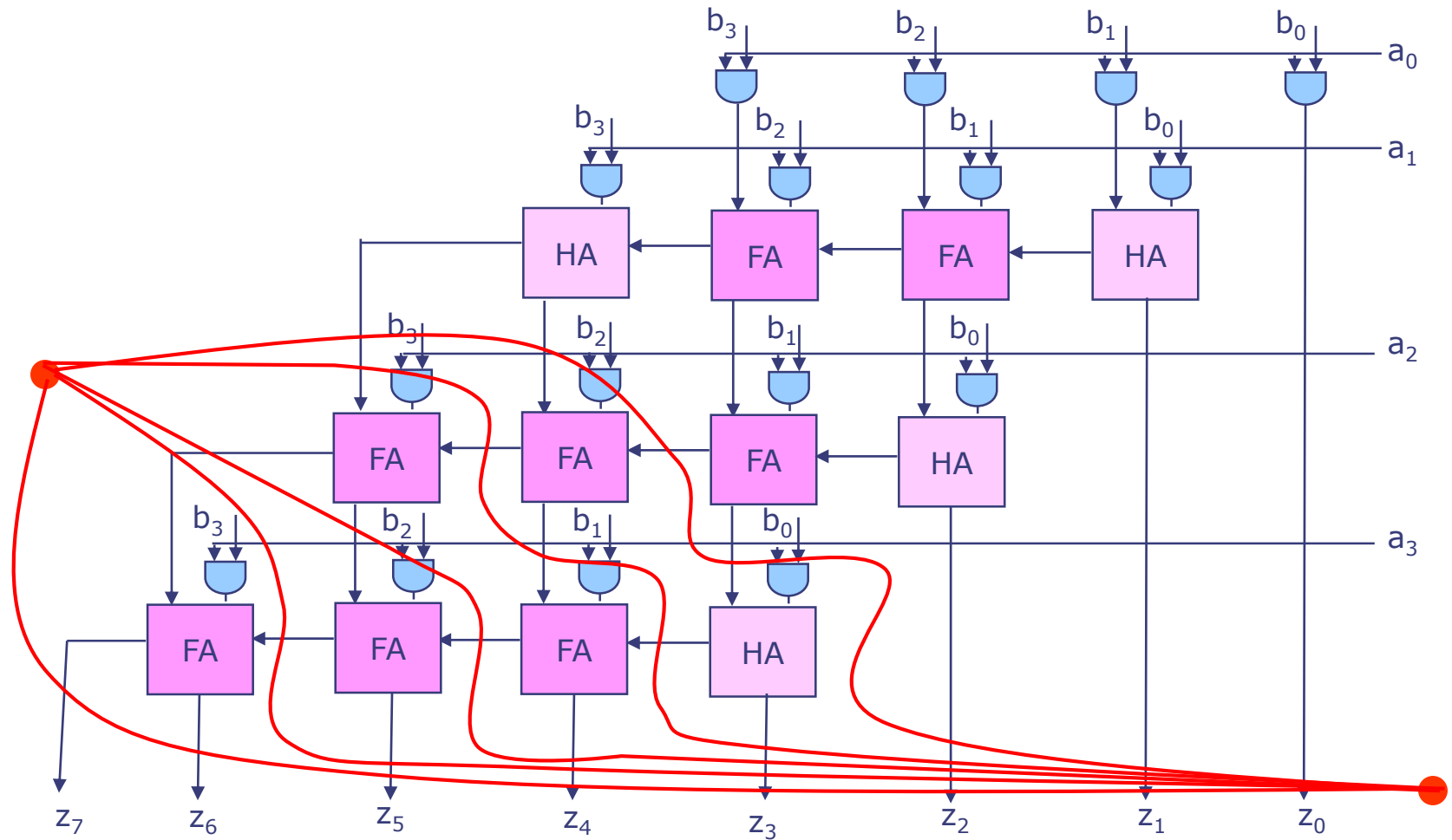
Pipelining to Increase Throughput



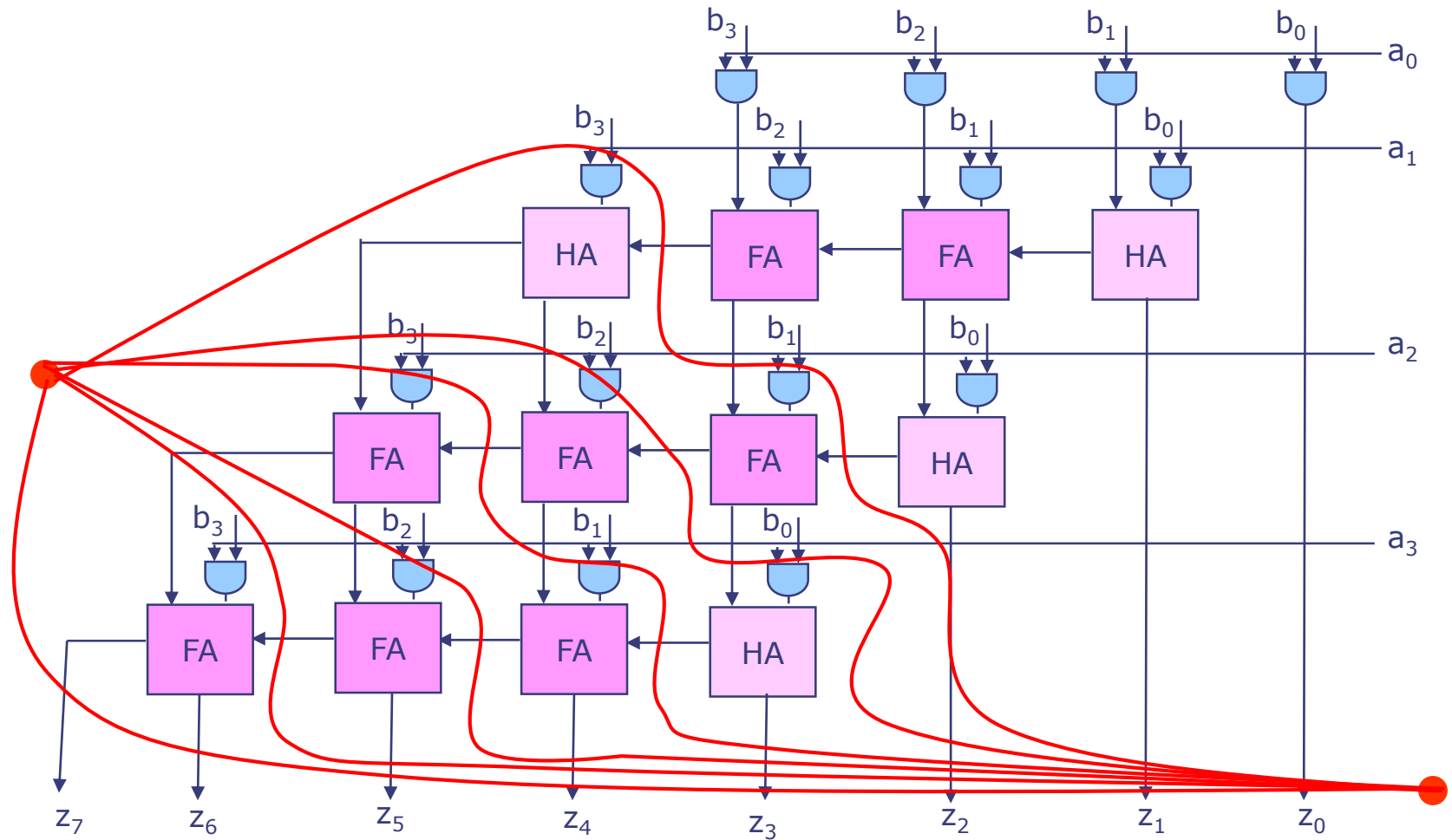
Pipelining to Increase Throughput



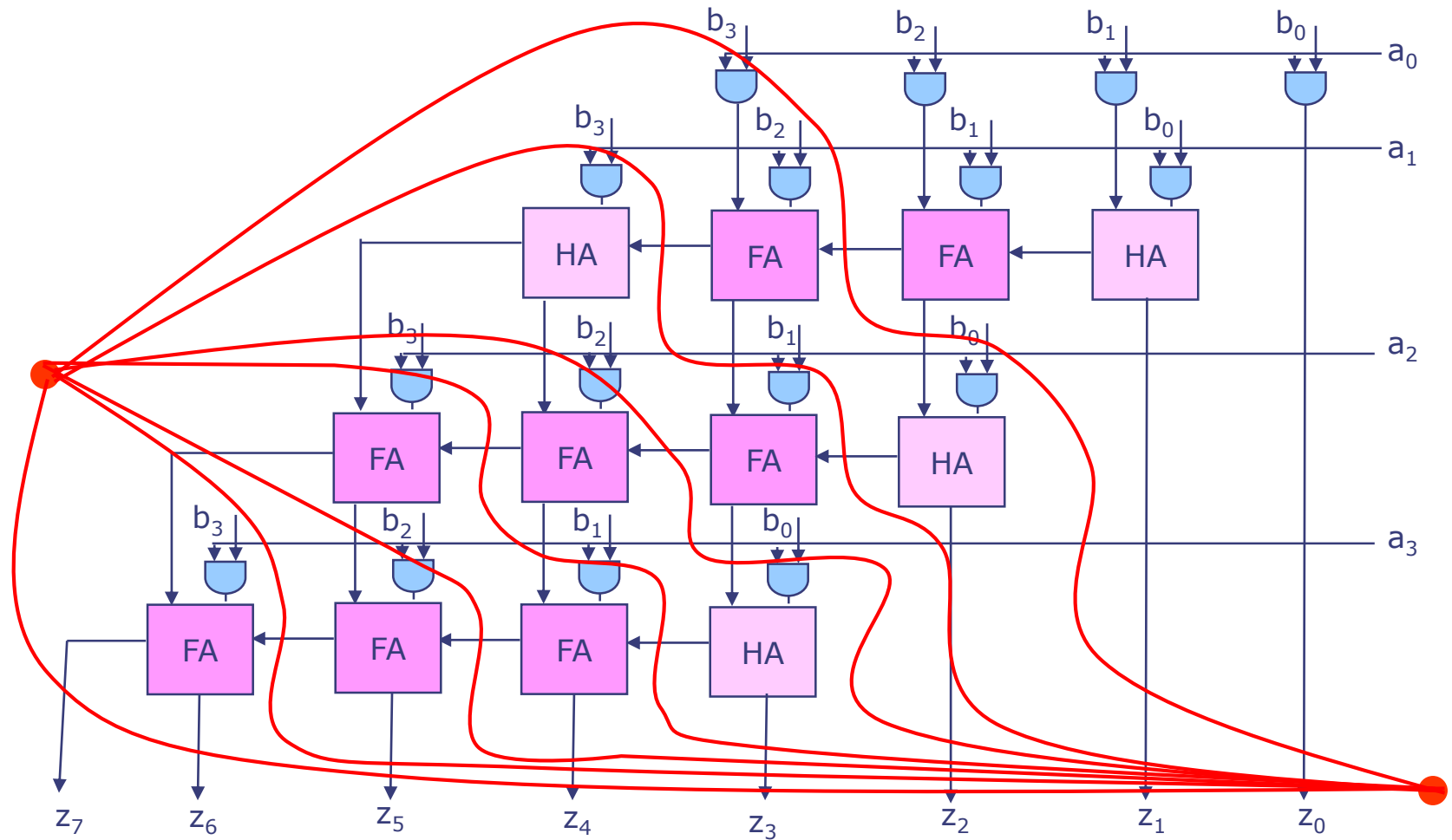
Pipelining to Increase Throughput



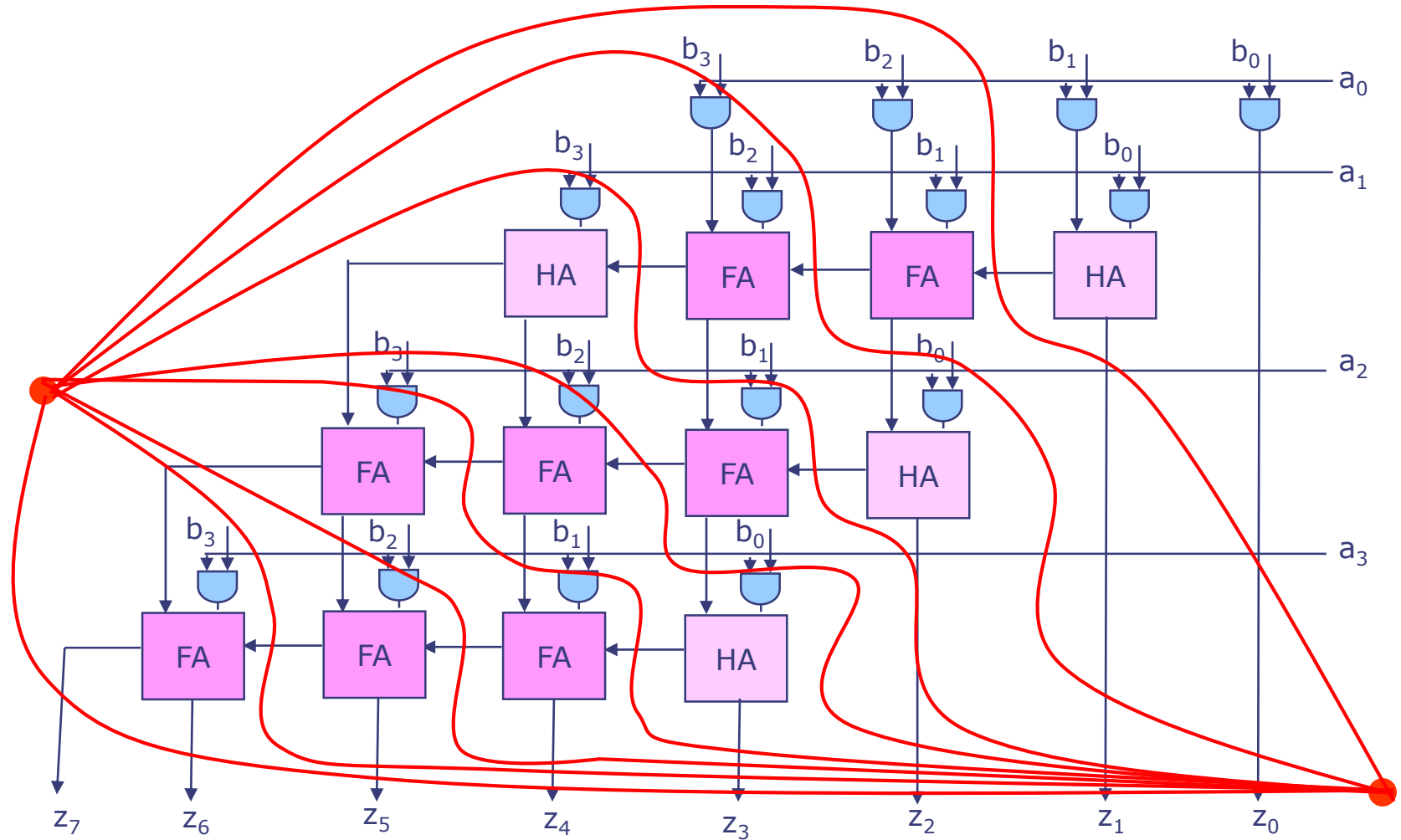
Pipelining to Increase Throughput



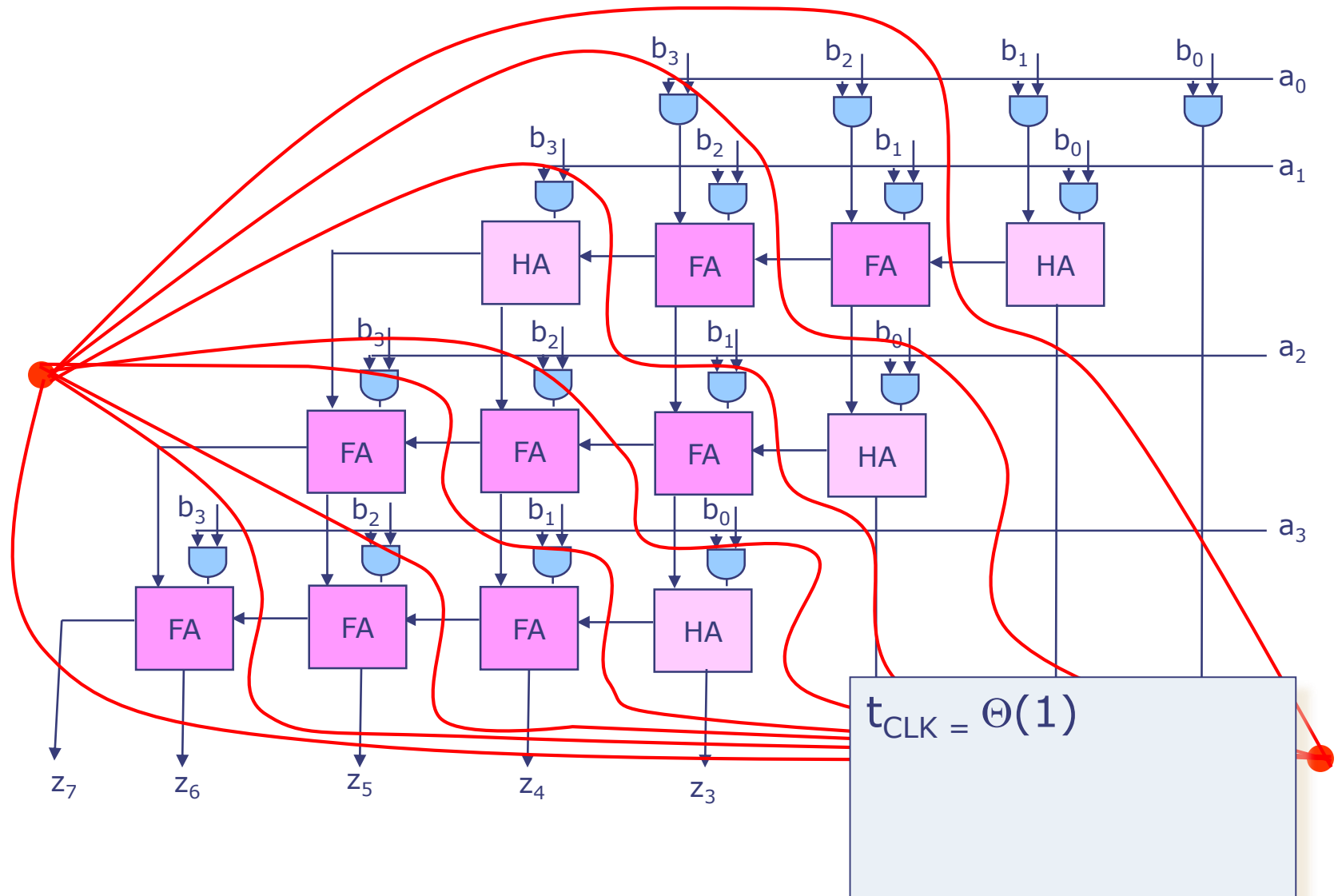
Pipelining to Increase Throughput



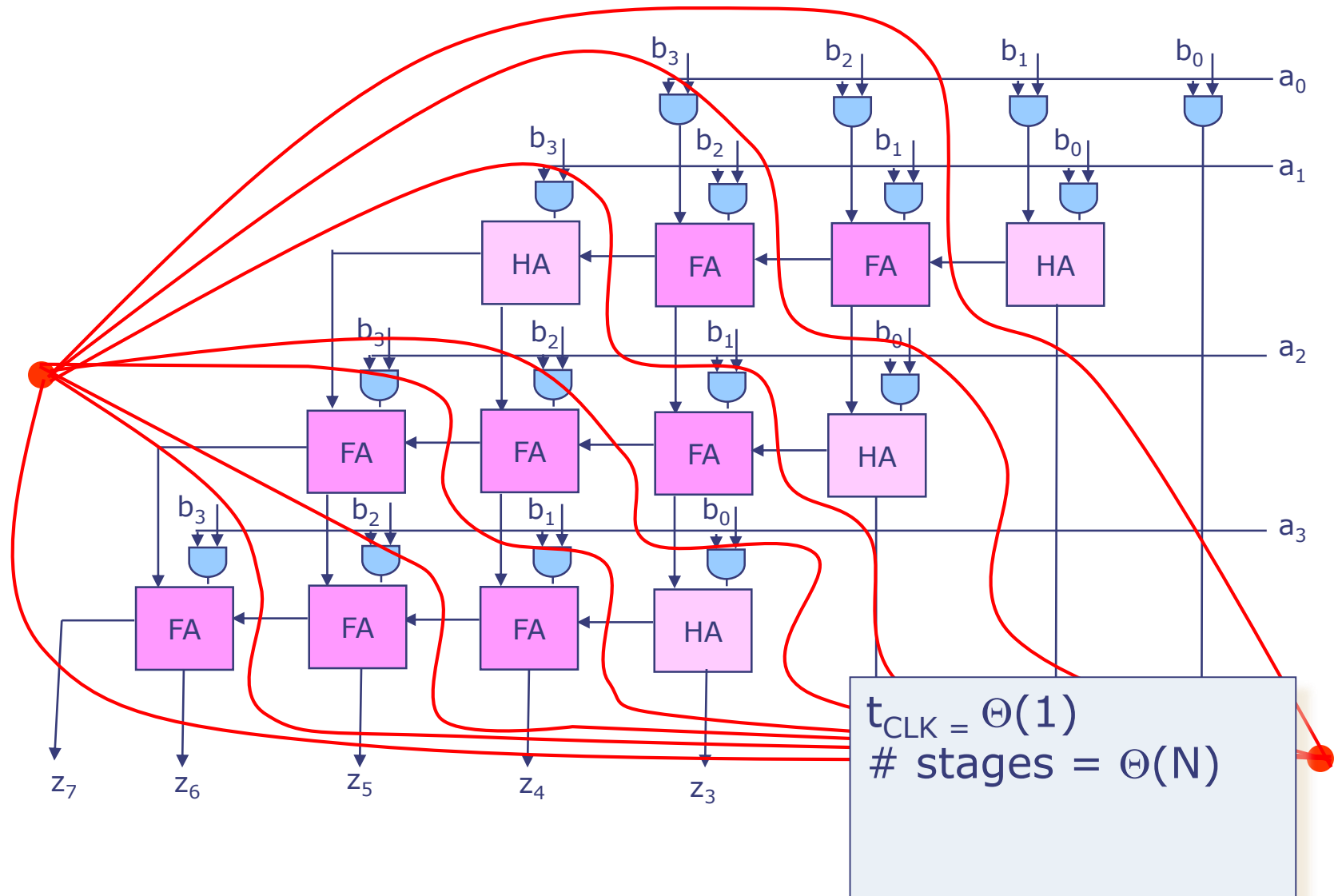
Pipelining to Increase Throughput



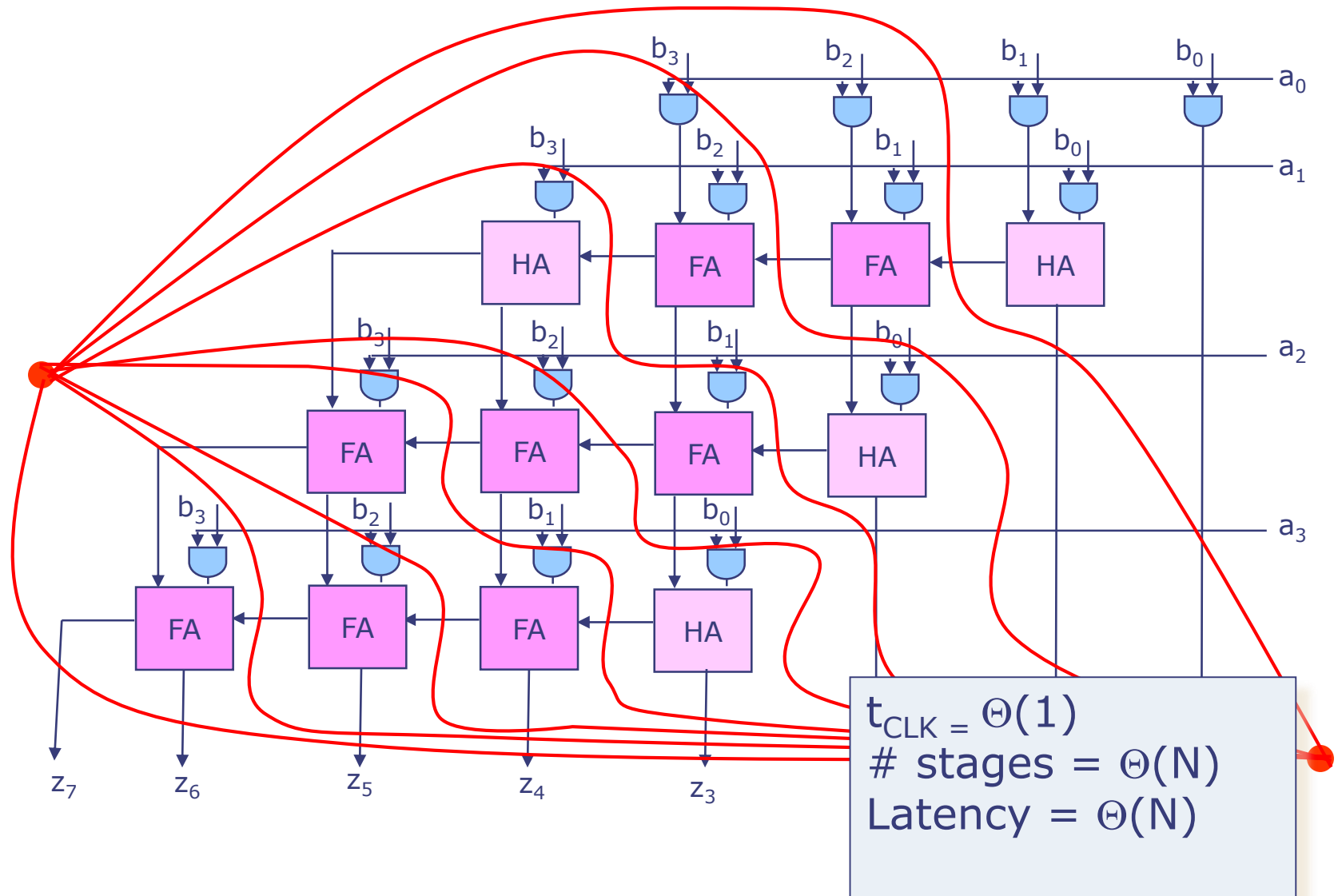
Pipelining to Increase Throughput



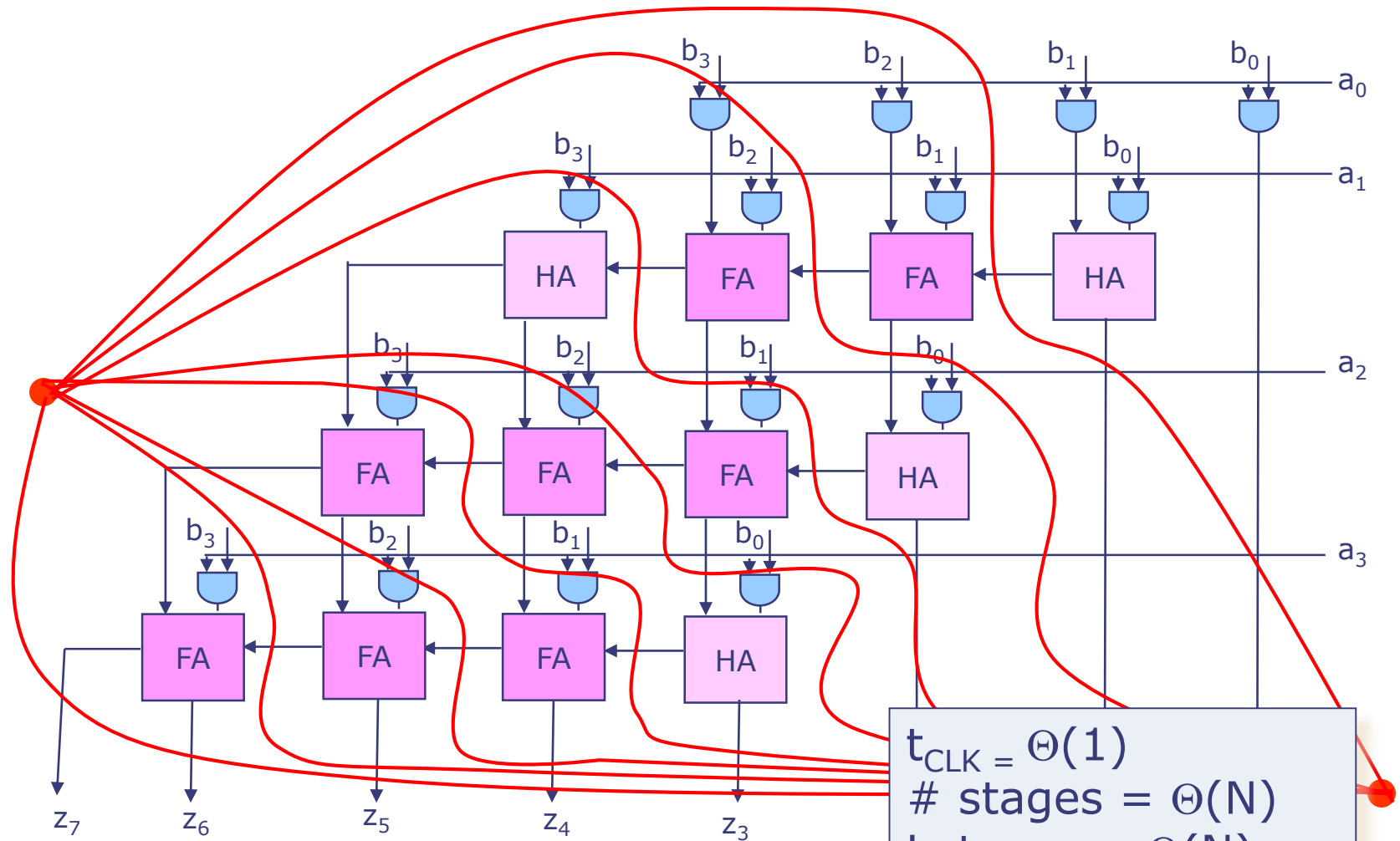
Pipelining to Increase Throughput



Pipelining to Increase Throughput



Pipelining to Increase Throughput



$t_{\text{CLK}} = \Theta(1)$
stages = $\Theta(N)$
Latency = $\Theta(N)$
Throughput = $\Theta(1)$

Folded (Multi-Cycle) Multiplier

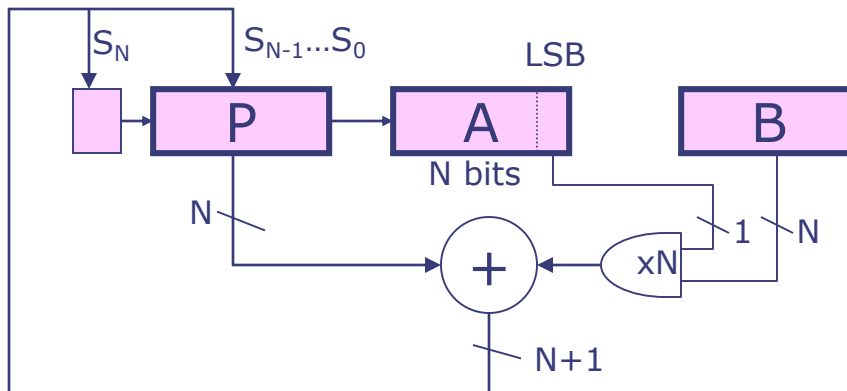
- Combinational circuits often have repetitive logic
 - Example: N -bit multiplier has $N-1$ adders

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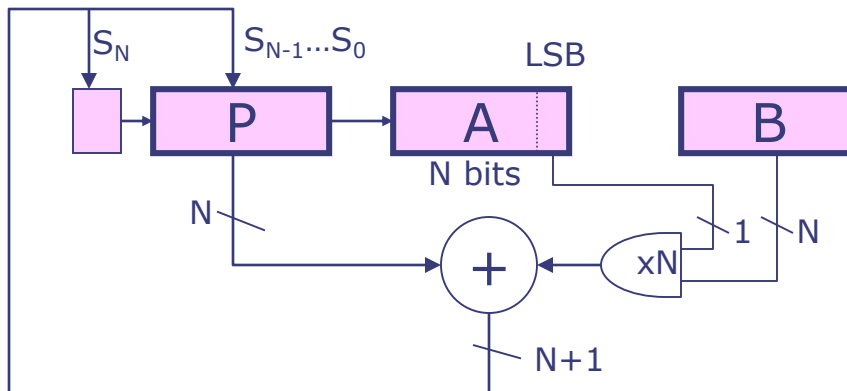
Init: $P \leftarrow 0$, load A&B

```
Repeat N times {  
     $P \leftarrow P + (A_{\text{LSB}} == 1 ? B : 0)$   
    shift  $S_N, P, A$  right one bit  
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Done: 2N-bit result in P,A

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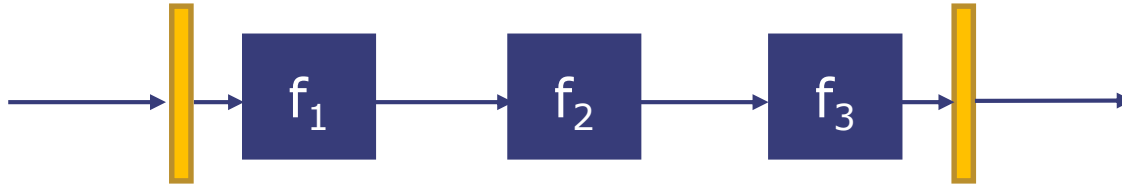
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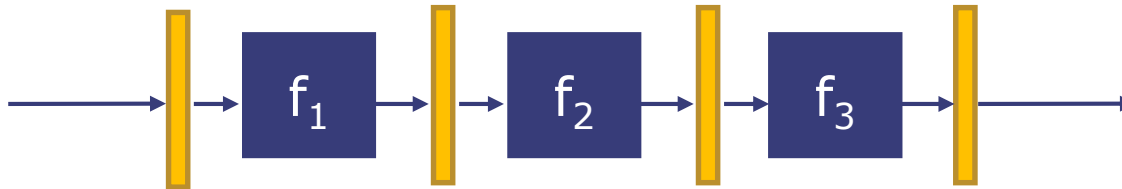
Tradeoff: reduced area, but lower throughput

Summary: Design Alternatives

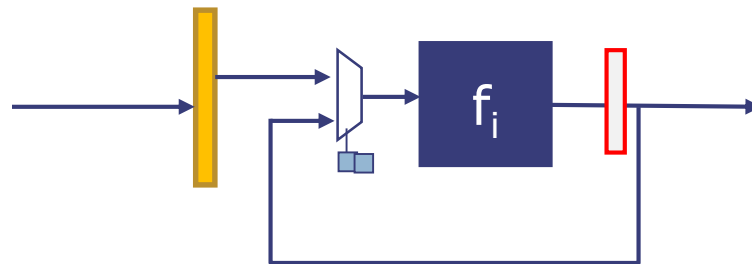
Several combinational modules in one pipeline stage (A)



One module per pipeline stage (B)

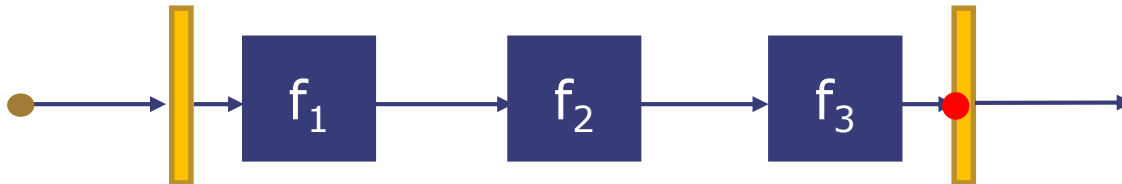


Folded reuse a block, multi-cycle (C)

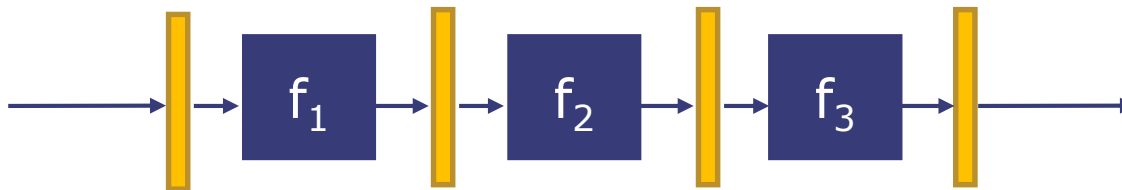


Summary: Design Alternatives

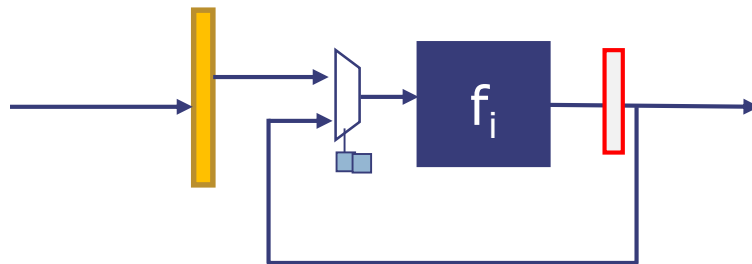
Several combinational modules in one pipeline stage (A)



One module per pipeline stage (B)

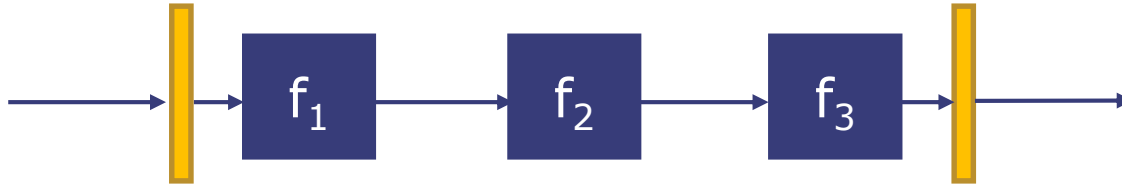


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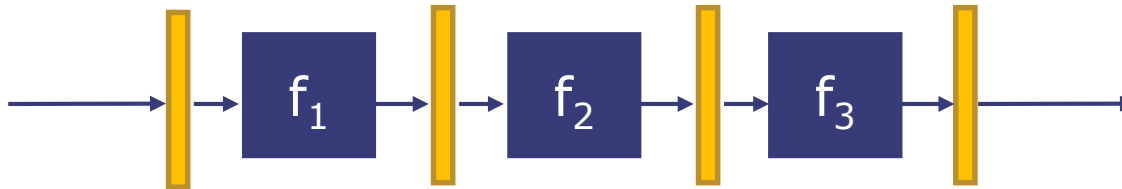


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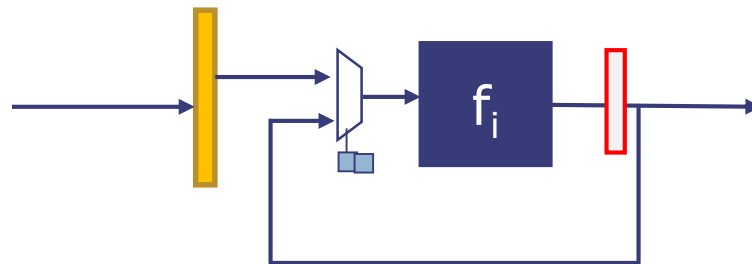
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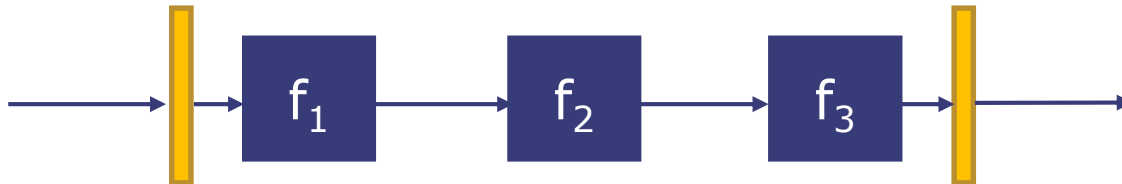


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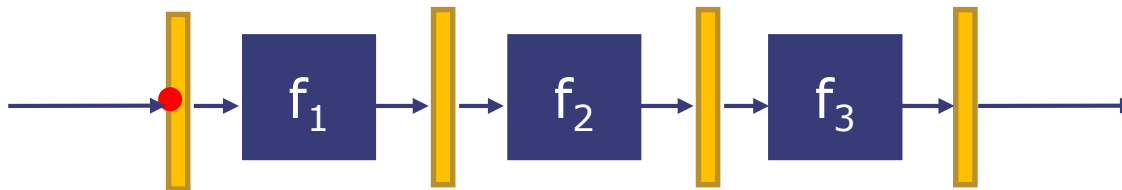


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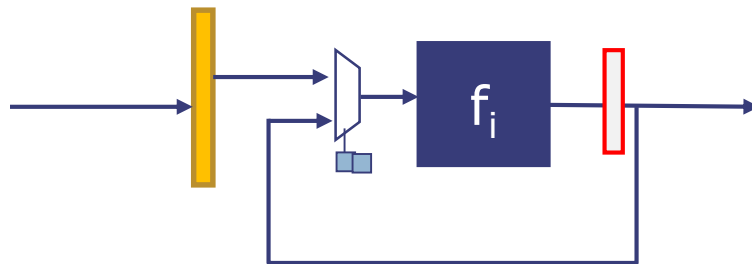
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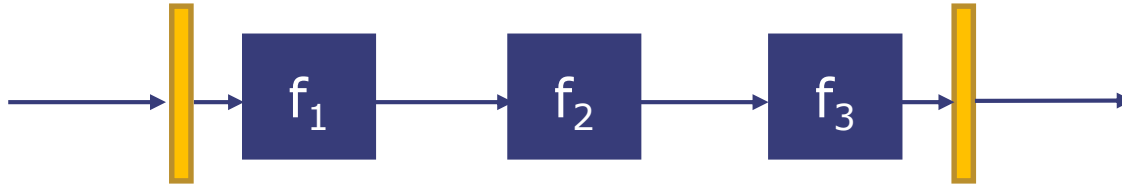


Folded reuse a block, multi-cycle (C)

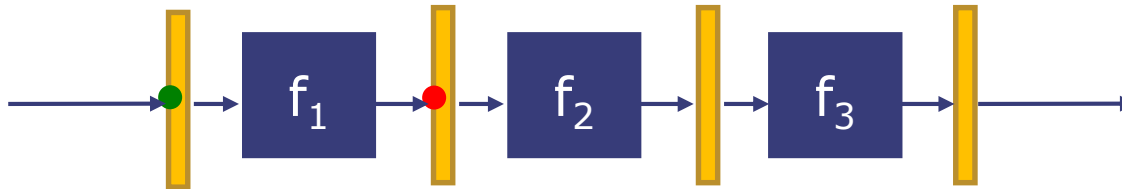


Summary: Design Alternatives

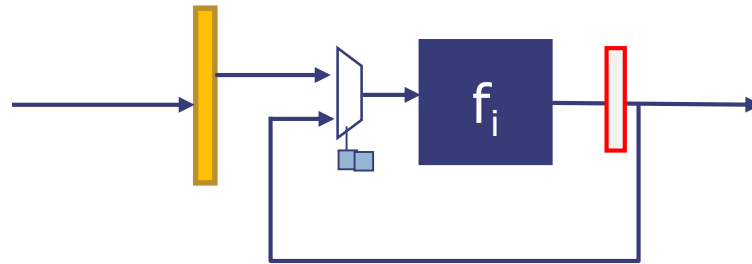
Several combinational modules in one pipeline stage (A)



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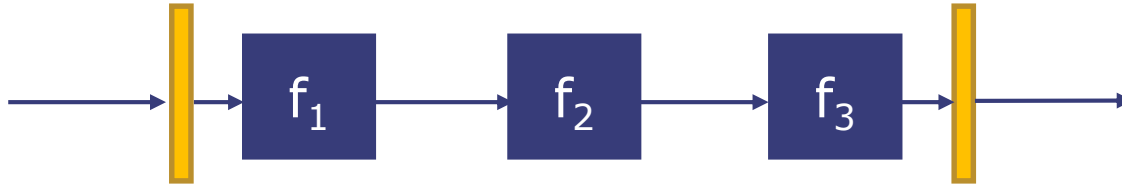


Folded reuse a block, multi-cycle (C)

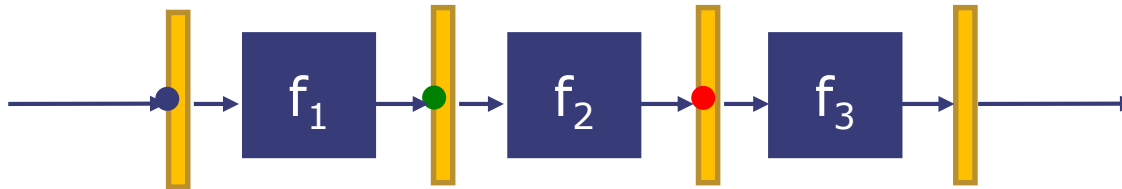


Summary: Design Alternatives

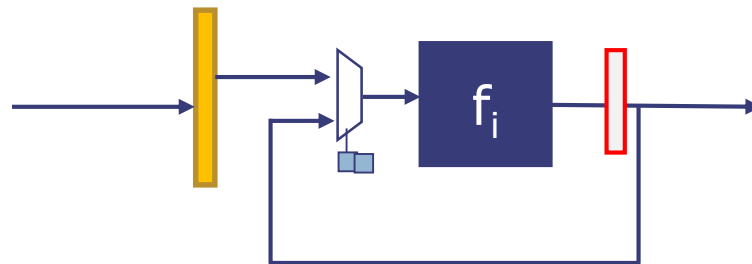
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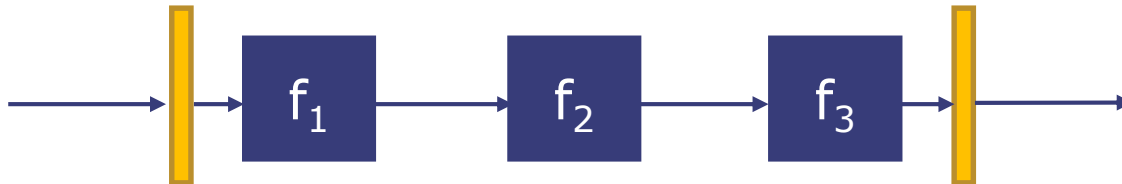


Folded reuse a block, multi-cycle (C)

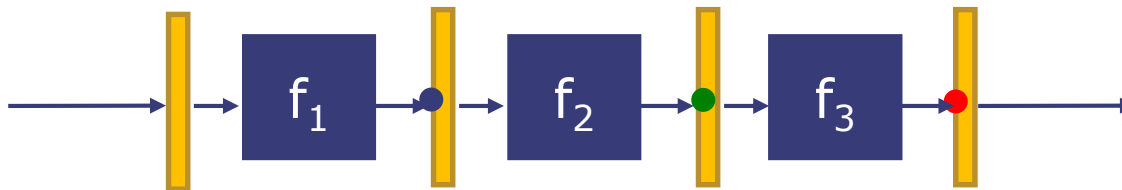


Summary: Design Alternatives

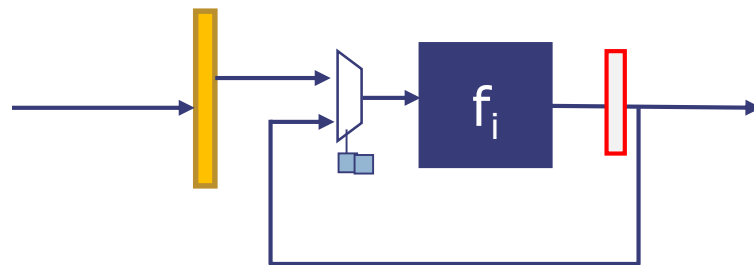
Several combinational modules in one pipeline stage (A)



One module per pipeline stage (B)

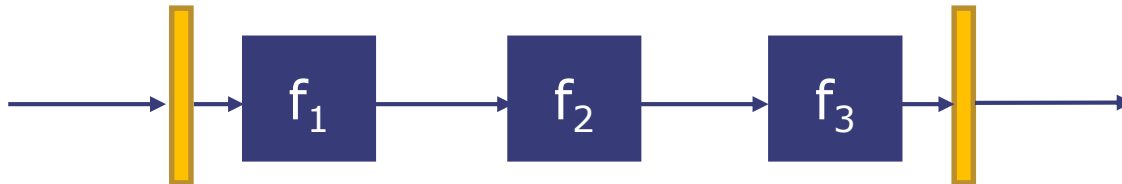


Folded reuse a block, multi-cycle (C)

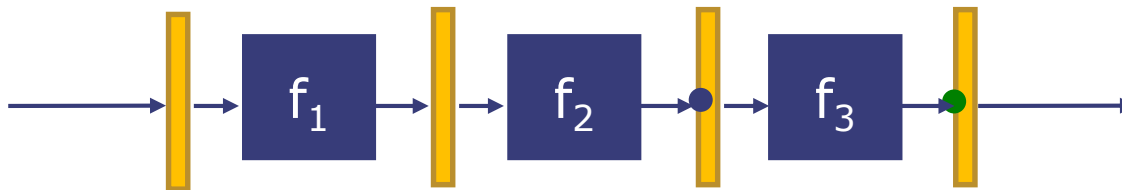


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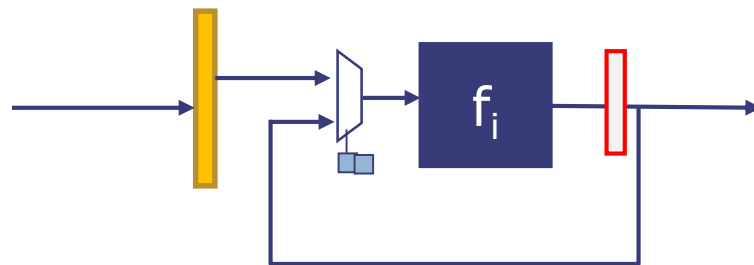
Several combinational modules in one pipeline stage (A)



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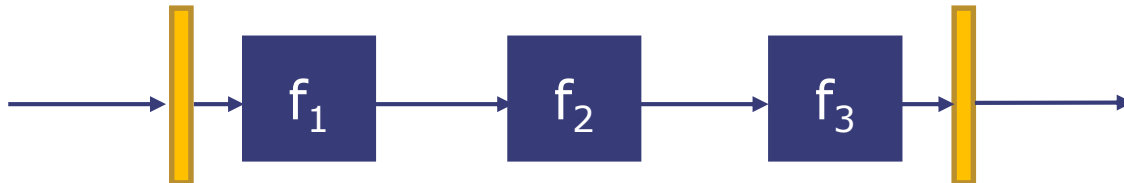


Folded reuse a block, multi-cycle (C)

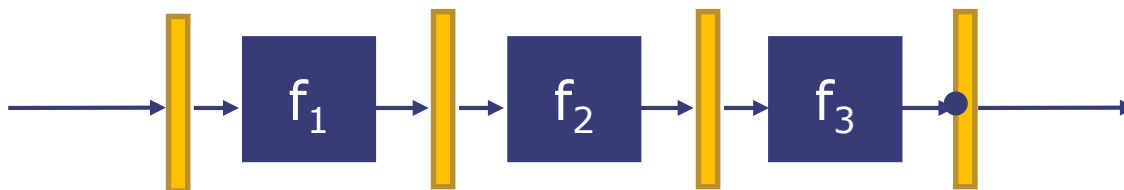


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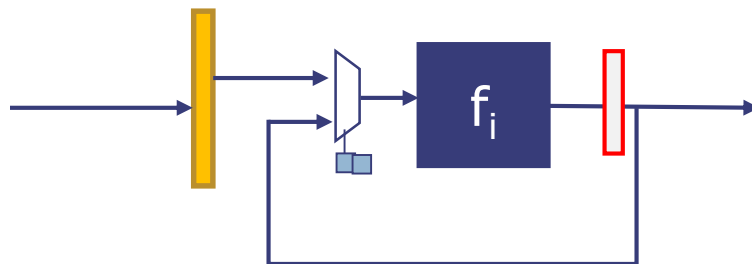
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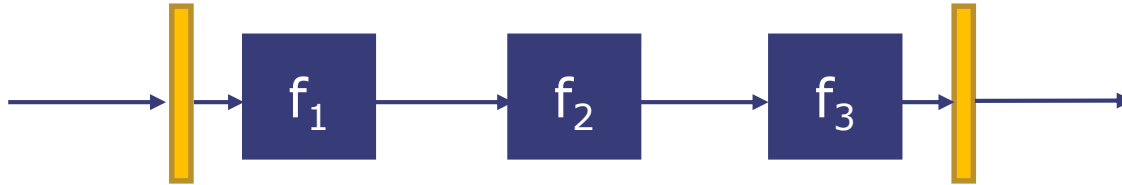


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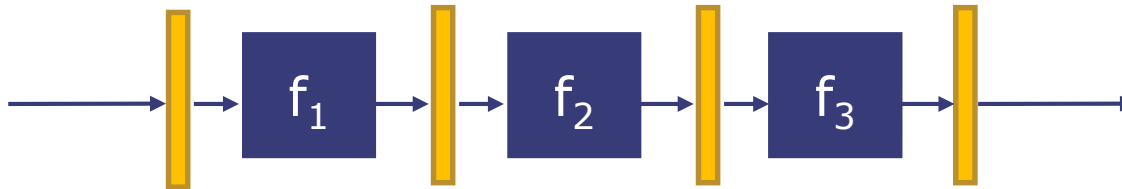


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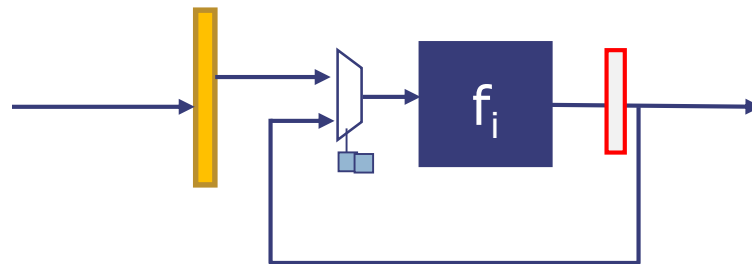
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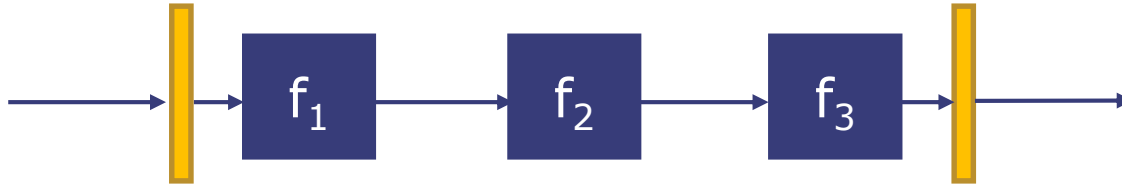


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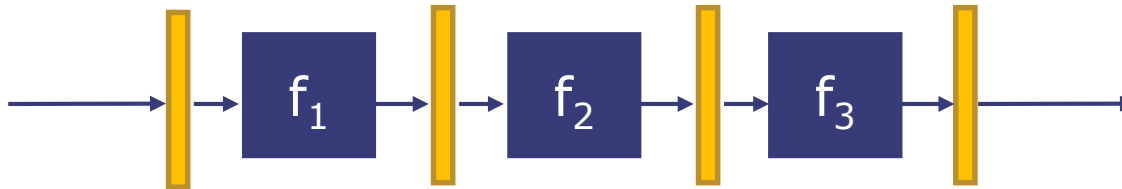


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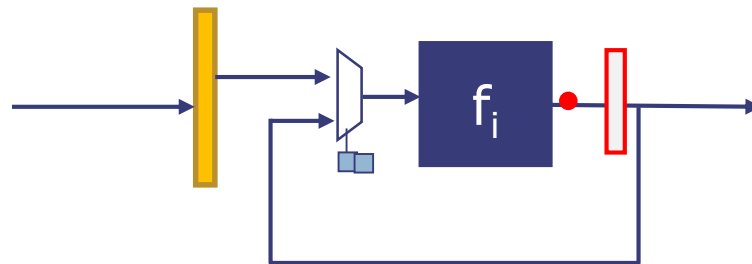
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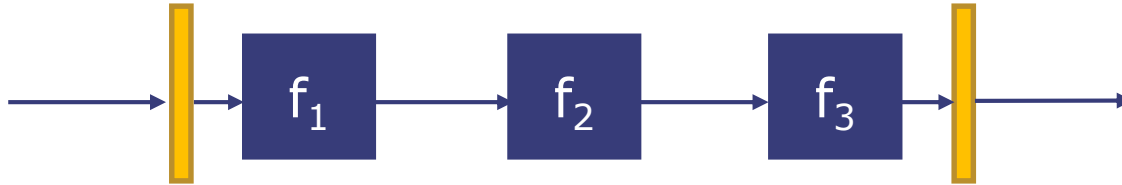


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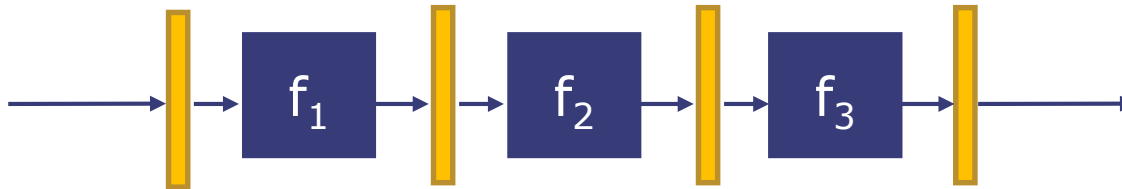


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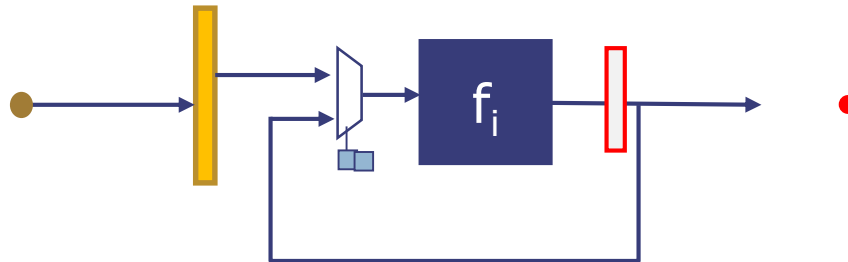
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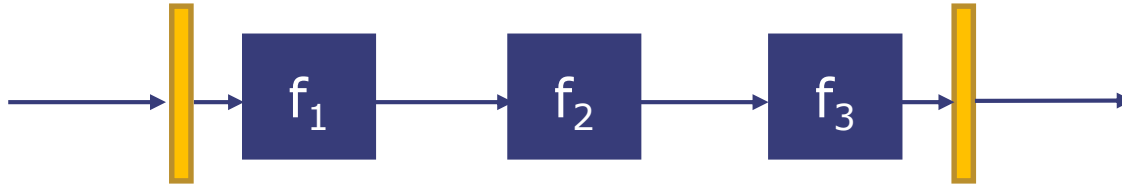


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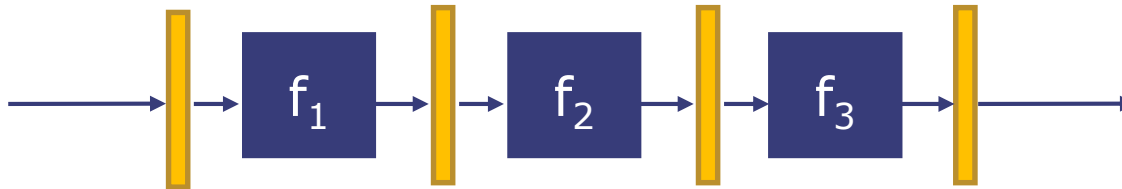


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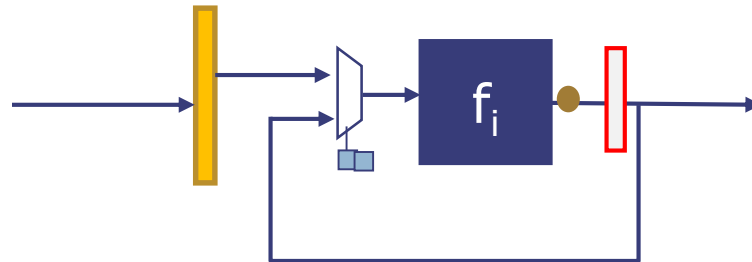
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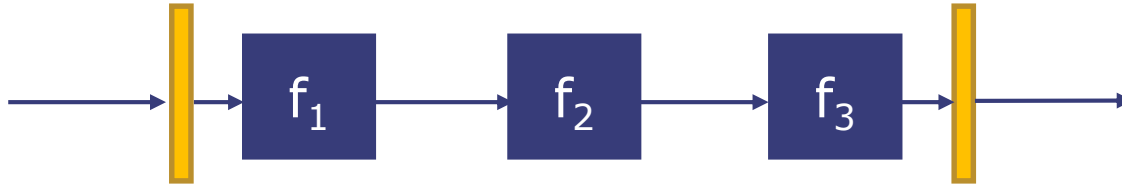


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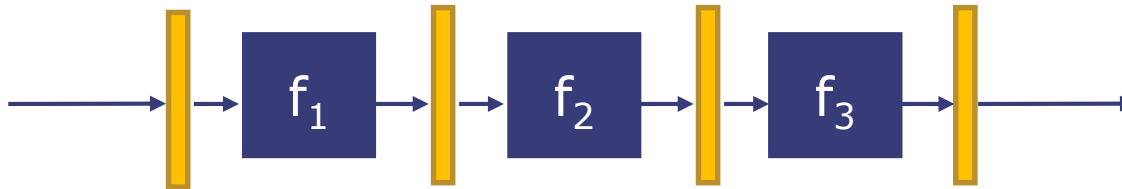


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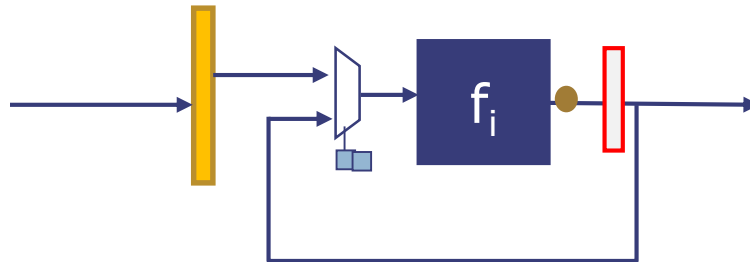
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Clock?

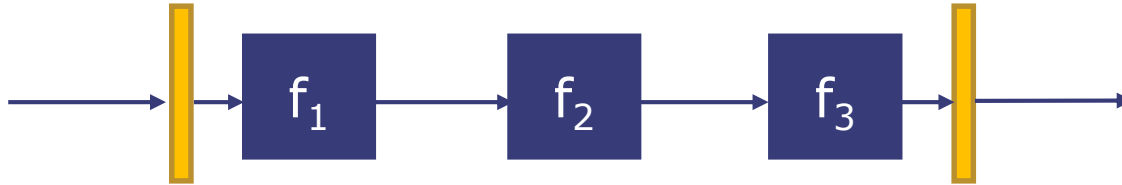
Latency?

Area?

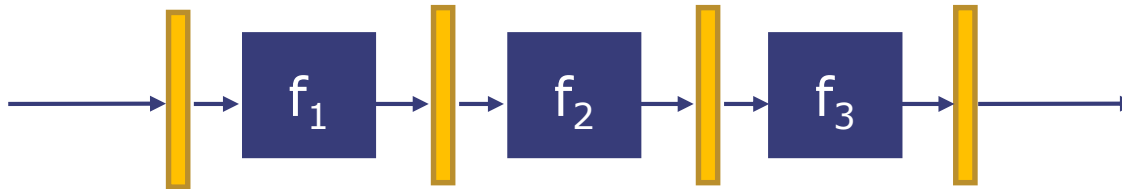
Throughput?

Summary: Design Alternatives

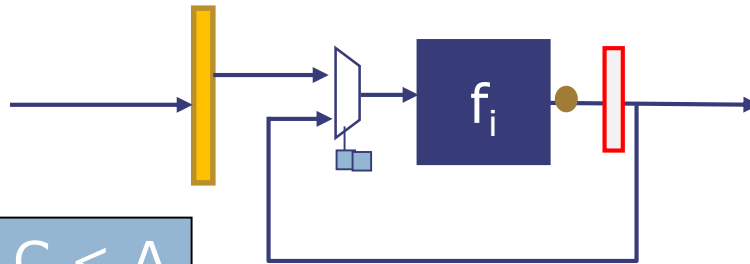
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One module per pipeline stage (B)



Folded reuse a block, multi-cycle (C)



Clock: $B \approx C < A$

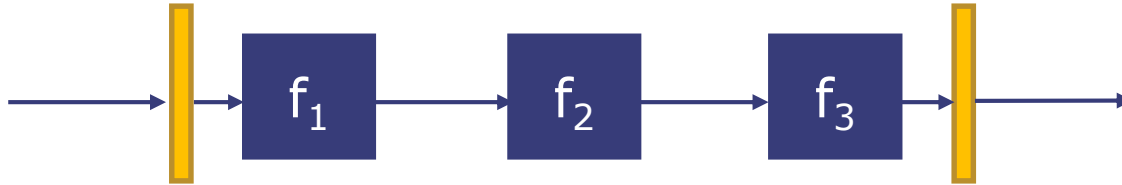
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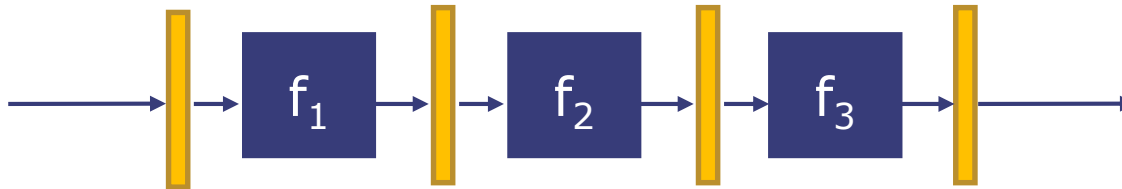
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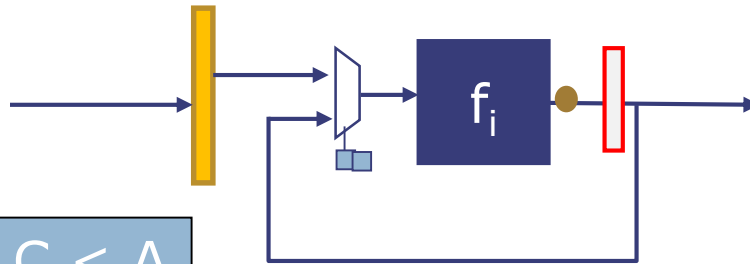
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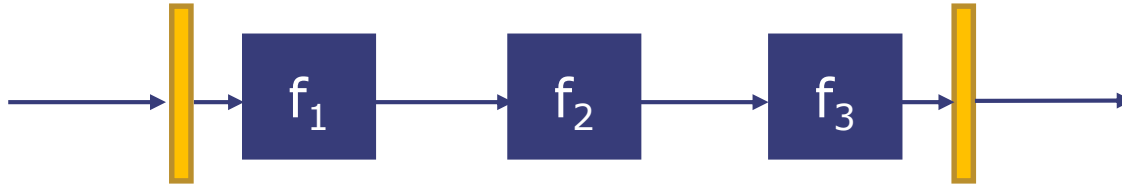
Area: $C < A < B$

Latency?

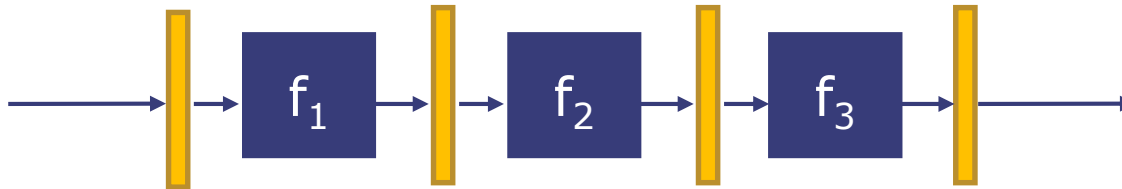
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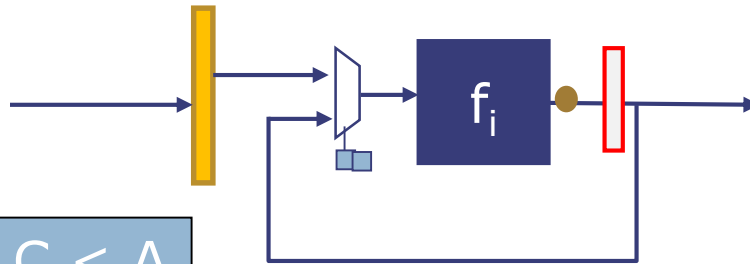
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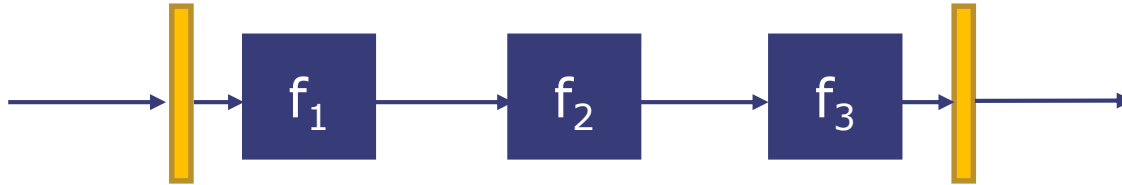
Area: $C < A < B$

Latency: $A < B < C$

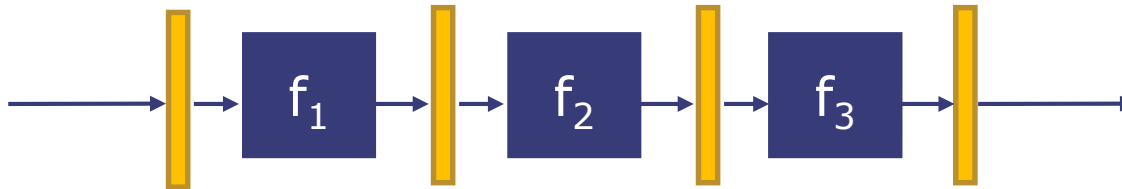
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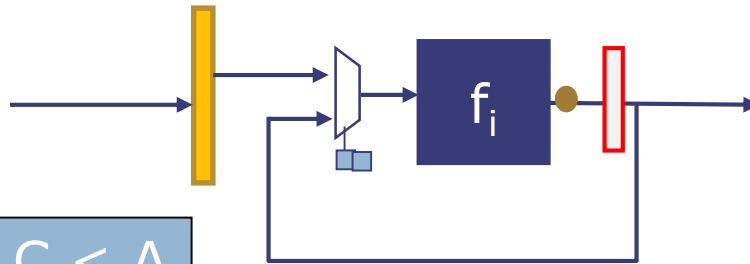
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Folded reuse a block, multi-cycle (C)



Clock: $B \approx C < A$

Area: $C < A < B$

Latency: $A < B < C$

Throughput: $C < A < B$

Thank You!

Next Lecture:
Design Tradeoffs in Sequential Circuits