8. Prove (from the definition of a limit of a sequence) that if the sequence $\{a_n\}_{n=1}^{\infty}$ tends to limit L as n → 1, then for any fixed number M > 0, the sequence $\{Ma_n\}_{n=1}^{\infty}$ tends to the limit ML.

Proof:
The claim can be re-written as: $\lim_{n\to\infty} a_n = L \ \rightarrow \ \lim_{n\to\infty} Ma_n = ML$
To prove $\lim_{n\to\infty} Ma_n = ML$, we need to prove that, $(\forall\varepsilon > 0, \exists n_0 \in \mathrm{N}, \forall \mathrm{n} > n_0)[|Ma_n - ML| < \varepsilon]$
Clearly, $[|Ma_n - ML| < \varepsilon] \leftrightarrow [|a_n - L| < \varepsilon/M]$, because M>0.

Because according to the assumption, $\lim_{n\to\infty} a_n = L$
$\forall\varepsilon > 0$, let $\varepsilon_0 = \frac{\varepsilon}{M} > 0$, according to the definition of sequence limit, $\exists n_0 \in \mathrm{N}, \forall \mathrm{n} > n_0$, such that,

$$|a_n - L| < \varepsilon_0 \ \rightarrow \ |a_n - L| < \varepsilon/M \ \rightarrow \ |Ma_n - ML| < \varepsilon$$

Then, by definition, we get: $\lim_{n\to\infty} Ma_n = ML$, as required by the claim.

Q.E.D