SGN 21006 Advanced Signal Processing

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Organization of the course

- Lecturer: Ioan Tabus (office: TF 419, e-mail ioan.tabus@tut.fi)
- Lectures: Mondays 10:15-12:00, Thursdays 10:15-12:00
  SA205 (22.10, 25.10, 29.10, 1.11, 5.11, 12.11, 15.11, 19.11, 22.11, 26.11, 29.11, 3.12)

  Note: 8.11 - no lecture
- Practical sessions for experiments and discussions about the homework assignment:
  1. Group 1: Tuesdays 14:00-16:00 TC 303 First meeting 23.10.2018
  2. Group 2: Fridays 10:00-12:00 TC 303 First meeting 26.10.2018
- Content of the course:
  1. Deterministic and random signals
  2. Optimal filter design
  3. Adaptive filter design
  4. Application areas of Optimal filter design and Adaptive filter design
  5. Spectrum estimation
  6. Nonlinear filters
- Requirements:
  1. Homework assignment and Final examination

Organization of the course

- Ioan Tabus. Lecture slides for the course.
  `http://www.cs.tut.fi/~tabus/`
- Text books:
  1. Sophocles J. Orfanidis. "Optimum Signal Processing: An Introduction", Second Edition,
     `http://www.ece.rutgers.edu/~orfanidi/osp2e`
  2. Simon Haykin, "Adaptive Filter Theory", Prentice Hall International, 2002.
  3. Peter Stoica, Randolph Moses. "Spectral Analysis of Signals", Prentice Hall, 2005. Full book available at
     `http://user.it.uu.se/~ps/SAS-new.pdf`
     The lecture slides for the full book are available at
     `http://www.prenhall.com/~stoica`
- Additional materials:
  1. Danilo Mandic. Slides of the courses "Advanced Signal Processing" and "Spectral Estimation and Adaptive Filtering" given at Department of Electrical and Electronic Engineering, Imperial College, UK.
     `http://www.commsp.ee.ic.ac.uk/~mandic/courses.htm`

Signal Processing: The Science Behind Our Digital Life

- YouTube.com. "What is signal processing?" [Online]. Available: https://youtu.be/EErkgr1MWw0
- "What is signal processing?" https://www.youtube.com/watch?v=YmSvQe2FDKs
- Publications of IEEE Signal Processing Society https://signalprocessingsociety.org/publications-resources/publications

Preview of the course

- Discuss generic applications in the following terms
  1. Problem formulation, scheme of the application, signal flow
  2. Derive or choose the proper algorithm for solving the problem
  3. Check by simulation the performance of the solution
  4. Find theoretical justification of the performance
- Derivation of the main algorithms
- Whenever needed, the necessary mathematical notions are reviewed

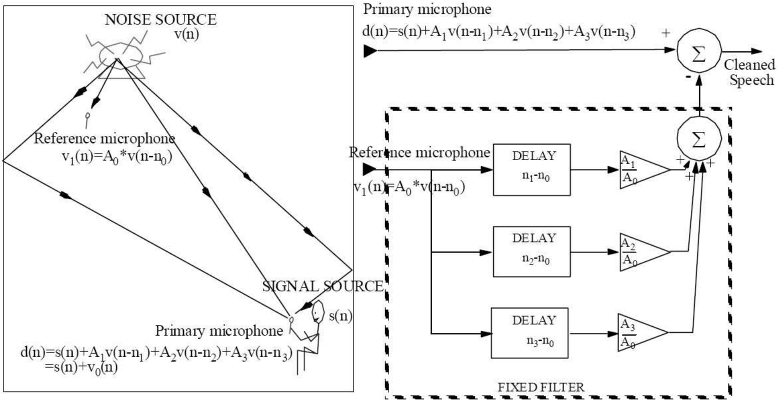
Adaptive noise cancelling

- Problem appearing in many applications:
  - Cancelling 50 Hz interference in electrocardiography (Widrow, 1975);
  - Reduction of acoustic noise in speech (cockpit of a military aircraft: 10-15 dB reduction);
- Two measured inputs, $d(n)$ and $v_1(n)$:
  - $d(n)$ comes from a primary sensor: $d(n) = s(n) + v_0(n)$
    - where $s(n)$ is the information bearing signal;
    - $v_0(n)$ is the corrupting noise:
  - $v_1(n)$ comes from a reference sensor:
- Hypotheses:
  - The ideal signal $s(n)$ is not correlated with the noise sources $v_0(n)$ and $v_1(n)$;

$$Es(n)v_0(n-k) = 0, \quad Es(n)v_1(n-k) = 0, \quad \text{for all } k$$

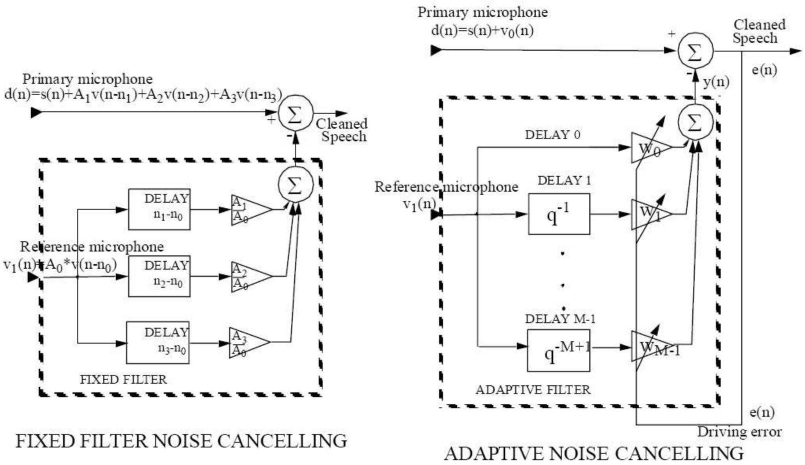
  - The reference noise $v_1(n)$ and the noise $v_0(n)$ are **correlated**, with unknown crosscorrelation $p(k)$, $Ev_0(n)v_1(n-k) = p(k)$

Adaptive noise cancelling



NOISE CANCELLATION WITH A FIXED FILTER

Adaptive noise cancelling



Adaptive noise cancelling

- Description of adaptive filtering operations, at any time instant, n:
  - The reference noise $v_1(n)$ is processed by an adaptive filter, with time varying parameters $w_0(n), w_1(n), \ldots, w_{M-1}(n)$, to produce the output signal

$$y(n) = \sum_{k=0}^{M-1} w_k(n) v_1(n-k)$$

.

  - The error signal is computed as $e(n) = d(n) - y(n)$.
  - The parameters of the filters are modified in an adaptive manner. For example, using the LMS algorithm (the simplest adaptive algorithm)

$$w_k(n+1) = w_k(n) + \mu v_1(n-k) e(n) \qquad (LMS)$$

where μ is the adaptation constant.

Rationale of the method

* $e(n) = d(n) - y(n) = s(n) + v_0(n) - y(n)$
* $Ee^2(n) = Es^2(n) + E(v_0(n) - y(n))^2$ (follows from hypothesis: Exercise)
* $Ee^2(n)$ depends on the parameters $w_0(n), w_1(n), \ldots, w_{M-1}(n)$
* The algorithm in equation (LMS) modifies $w_0(n), w_1(n), \ldots, w_{M-1}(n)$ such that $Ee^2(n)$ is minimized
* Since $Es^2(n)$ does not depend on the parameters $\{w_k(n)\}$, the algorithm (LMS) minimizes $E(v_0(n) - y(n))^2$, thus statistically $v_0(n)$ will be close to $y(n)$ and therefore $e(n) \approx s(n)$, ($e(n)$ will be close to $s(n)$).

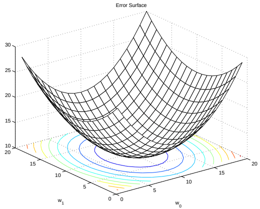
Rationale of the method

* Sketch of proof for Equation (LMS)
  - $e^2(n) = (d(n) - y(n))^2 =$
    $(d(n) - w_0 v_1(n) - w_1 v_1(n-1) - \dots w_{M-1} v_1(n-M+1))^2$

    The square error surface

  - $e^2(n) = F(w_0, \dots, w_{M-1})$

    is a paraboloid.



  - The gradient of square error is
    $\nabla_{w_k} e^2(n) = \frac{de^2(n)}{dw_k} = -2e(n)v_1(n-k)$
  - The method of gradient descent minimization:
    $w_k(n+1) = w_k(n) - \mu_1 \nabla_{w_k} e^2(n) = w_k(n) + \mu_1 2 v_1(n-k) e(n)$
    Rename $\mu = 2\mu_1$.

Rationale of the method

* Checking for effectiveness of Equation (LMS) in reducing the errors

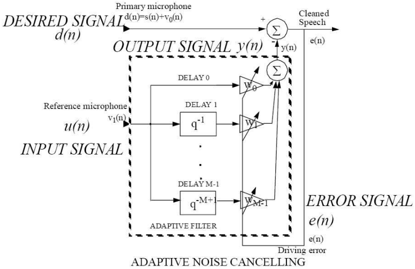
$$\begin{aligned}
\varepsilon(n) &= d(n) - \sum_{k=0}^{M-1} w_k(n+1)v_1(n-k) \\
&= d(n) - \sum_{k=0}^{M-1} (w_k(n) + \mu v_1(n-k)e(n))v_1(n-k) \\
&= d(n) - \sum_{k=0}^{M-1} w_k(n)v_1(n-k) - e(n)\mu \sum_{k=0}^{M-1} v_1^2(n-k) \\
&= e(n) - e(n)\mu \sum_{k=0}^{M-1} v_1^2(n-k) \\
&= e(n)(1 - \mu \sum_{k=0}^{M-1} v_1^2(n-k))
\end{aligned}$$

In order to reduce the error by using the new parameters, $w(n+1)$, the following inequality must hold:

$$|\varepsilon(n)| < |e(n)| \text{ or, equivalently } 0 < \mu < \frac{2}{\sum_{k=0}^{M-1} v_1^2(n-k)}$$

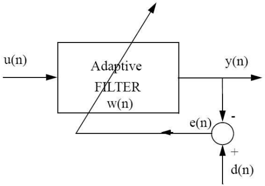
Optimal or Adaptive Linear Filtering Module

* In optimal filtering (or batch, or framewise filtering) the input and desired signals are available for a given time-window, $1, \ldots, N$, and the optimal parameters of the linear filter, and subsequently the filter output for that time window, are computed only once
* In adaptive filtering the input and desired signals are provided to the algorithm sequentially, and at every time instant a set of parameters of the linear filter are computed or updated, and they are used to compute the output of the filter for only that time instance

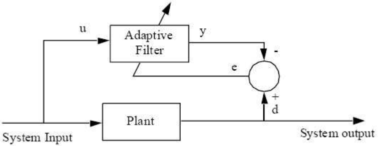


Optimal or Adaptive Linear Filtering Modules in Applications

- In an application one has to figure out the correspondence between the signals in the application and the conventional names of the signals in an optimal/adaptive module:
    - which is the input signal to the filter $u(t)$?
    - which is the desired signal to the filter $d(t)$?
    - which is the output signal of the filter $y(t)$?
- Rarely the filter output $y(t)$ is the interesting resulted signal
- Sometimes the error $e(t)$ is the interesting signal
- Sometimes the vector of parameters $w(t)$ is of interest



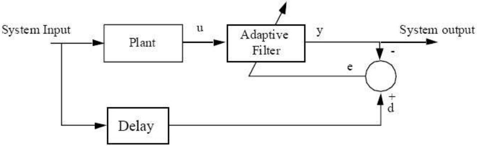
Optimal or Adaptive Linear Filtering Modules in Applications



IDENTIFICATION

SYSTEM IDENTIFICATION

LAYERED EARTH MODELLING

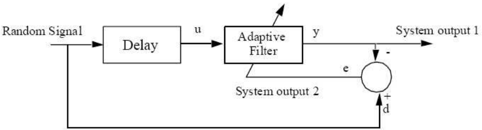


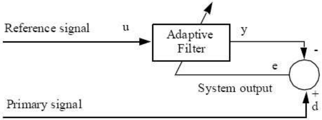
INVERSE MODELLING

PREDICTIVE DECONVOLUTION

ADAPTIVE EQUALIZATION

Optimal or Adaptive Linear Filtering Modules in Applications





PREDICTION

- LINEAR PREDICTIVE CODING
- ADPCM
- AUTOREGRESSIVE SPECTRUM ANALYSIS
- SIGNAL DETECTION

INTERFERENCE CANCELLING

- ADAPTIVE NOISE CANCELLING
- ECHO CANCELLATION
- RADAR POLARIMETRY
- ADAPTIVE BEAMFORMING