SGN-21006 Advanced Signal Processing

Exercise 4: Steepest Descent

Introduction

Consider the same application as in the previous exercise, channel equalization (Fig. 1). Instead of solving the Wiener-Hopf equations, you are asked to solve the FIR filter coefficients of the channel equalizer using the steepest descent algorithm. Recall, that given the error signal $e(t) = d(t) - \hat{d}(t)$, the criterion minimized by the Wiener filter is

$$J(\boldsymbol{w}) = \mathrm{E}\left[e^2(t)\right] = \sigma_d^2 - \boldsymbol{w}^T(t)\mathbf{p} - \mathbf{p}^T\boldsymbol{w}(t) + \boldsymbol{w}^T(t)\mathbf{R}\boldsymbol{w}(t).$$

The steepest descent algorithm updates $\boldsymbol{w}$ by moving to the direction of the negative gradient given by

$$\nabla J = -2\mathbf{p} + 2\mathbf{R}\boldsymbol{w}(t).$$

So, given the step size parameter μ, the algorithm can be written as

Input : Autocorrelation matrix $\mathbf{R}$, cross-correlation vector $\mathbf{p}$, initial $\boldsymbol{w}(0)$, step-size μ, max number of iterations N

for $t = 1, \ldots, N$ **do**

$\quad \boldsymbol{w}(t) = \boldsymbol{w}(t-1) + \mu[\mathbf{p} - \mathbf{R}\boldsymbol{w}(t-1)]$

end

Output: The filter $\boldsymbol{w}(t)$ at different time instants t.

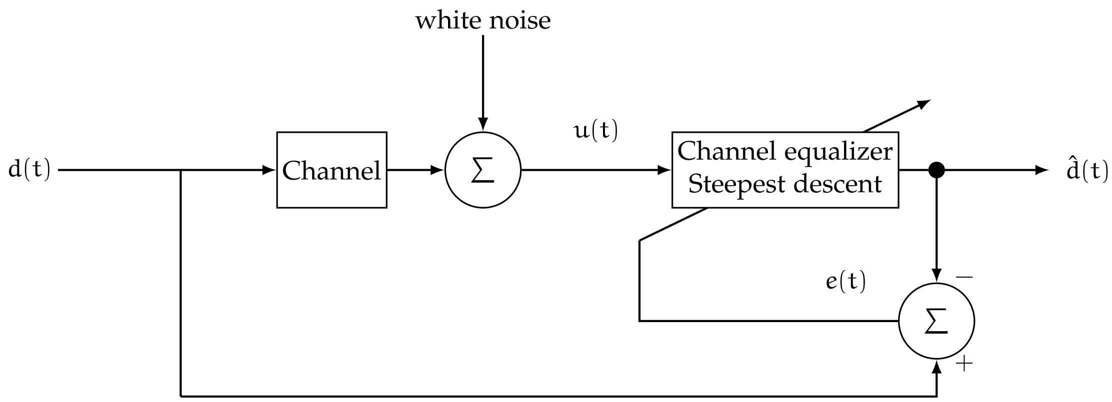

Figure 1: Channel equalization.

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Tasks

1. a.) Write a Matlab function `steepest_descent` that implements the steepest descent algorithm. You are given a template in the file `steepest_descent.m`. Be sure to give the output filter coefficients in the right form!

Test your function, when it is known that the autocorrelation matrix $\mathbf{R} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ and cross-correlation vector $\mathbf{p} = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$. Use $\mu = 0.1$, $N = 1000$ and $\boldsymbol{w}(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. Compare the resulting $\boldsymbol{w}(t)$ to the Wiener filter solution $\boldsymbol{w} = \mathbf{R}^{-1}\mathbf{p}$.

b.) Consider a scenario where the true $\mathbf{R}$ and $\mathbf{p}$ change in time and there is a way to estimate them. Explain, what would be the benefit of using steepest descent instead of the Wiener-Hopf solution to find the vector $\boldsymbol{w}(t)$ at each time instant t? Hint: Think about the computational complexity.

c.) Open the file `test_steepest_descent_with_audio`. It loads the same audio sample as in exercise 2, computes the Wiener filter and uses your implementation of steepest descent with different step size parameters. It plots the mean square error in the filter coefficients compared to the Wiener-Hopf solution. Does increasing the step size parameter increase or decrease the convergence speed? What happens if you add a step size option $\mu = 10^{-6}$?

2. Apply steepest descent again using the same parameters as in Task 1a.

a.) This time, plot the value of the filter coefficients $\boldsymbol{w}$ at different time instants so that the first element w_1 is on the x-axis and the second element w_2 is on the y-axis. One time instance of $\boldsymbol{w}$ if thus a single point in your plot. Also plot in the same figure the Wiener filter solution.

b.) Implement a function that returns the value of the criterion function $J(\boldsymbol{w})$ when given $\boldsymbol{w}$, $\mathbf{R}$, $\mathbf{p}$ and σ_d. Set $\sigma_d = 5$ and evaluate the value of the criterion at points `w_1 = -2:0.2:5` and `w_2 = -2:0.2:5`. Plot the result into the same figure as the steepest descent coefficient values of task 2a using `countour`. The goal is to produce a plot like in Figure 2. Can you see that the Wiener filter solution where steepest descent converges is the minimum of the criterion function?

c.) It is known, that for steepest descent to be stable, the step size should stay within

$$0 \leqslant \mu \leqslant \frac{2}{\lambda_{max}}$$

where λ_{max} is the maximum eigenvalue of $\mathbf{R}$. Compute the eigenvalues of $\mathbf{R}$ using `eig` and play with μ in the area around the maximum value. How does the plot change?

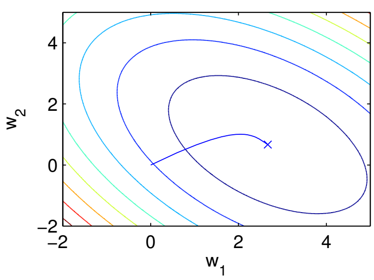

Figure 2: The Wiener solution, the steepest descent path and the value of the criterion function.