Final Project Report

Question 1:

1. Index Set:
    1. fc_i = production fixed cost for plant i
    2. svc_i = production variable cost of shoes in plant i
    3. bvc_i = production variable cost of boots in plant i
    4. $distance_{ij}$ = transportation distance from plant i to customer location j
    5. $sdemand_j$ = the projected quarterly sales of shoes for customer location j
    6. $bdemand_j$ = the projected quarterly sales of boots for customer location j
    7. i = plant i, i = 1,2,3,4,5
    8. j = customer location j, j = 1,2,3,4,5,6,7,8

2. Decision Variables

    Y_{in} = binary decision variables for n production line in plant i, n =1,2

    a_i = the number of shoes produced by plant i (Auxiliary variable)

    b_i = the number of boots produced by plant i (Auxiliary variable)

    A_{ij} = the number of shoes sent from plant i to customer location j

    B_{ij} = the number of boots sent from plant i to customer location j

3. Objective Function – Minimize

$$\sum_{i=1}^{5}(fc_i * \sum_{n=1}^{2} Y_{in}) + \sum_{i=1}^{5}(a_i * svc_i) + \sum_{i=1}^{5}(b_i * bvc_i) + 0.03 * \sum distance_{ij} * A_{ij} + 0.04 * \sum distance_{ij} * B_{ij}$$

4. Constraints
    a. Capacity
        i. $\frac{a_i}{15000} + \frac{b_i}{9000} \leq y_{in} * n, n = \text{the number of production line in plant } i$

    b. Binary decision variables

i. $\sum_{n=1}^{2} y_{in} = 1, \quad for\ i = 1, 2 (St.\ louis, Kansas\ City)$

ii. $\sum_{n=1}^{2} y_{in} \leq 1, \quad for\ i = 3, 4, 5 (Columbia, Springfield, Independence)$

c. Demand

i. $\sum_{i=1}^{5} A_{ij} \geq sdemand_j, for\ j = 1,2,3,4,5,6,7,8$

ii. $\sum_{i=1}^{5} B_{ij} \geq bdemand_j, for\ j = 1,2,3,4,5,6,7,8$

5. Answer:
   a. The company should not install a second production line at the Kansas City plant, but open a new plant in Independence with two production lines.
   b. The plant in St. Louis produces 22000 shoes and 4300 boots. The plant in Kansas City only produces 6580 boots. The plant in Independence produces 24800 shoes and 3120 boots.
   c. The plant in St. Louis provides the customer location in St. Louis 1,2,3 with 8000, 6500, 7500 shoes respectively, and provides the customer location in St. Louis 1,3,4 with 2200,1200,900 boots respectively.
      The plant in Kansas City provides the customer location in Kansas City 1 and Springfield with 3380 and 3200 boots respectively.
      The plant in Independence provides the customer location in Kansas City 1, 2, Columbia, and Springfield with 9500, 2300,7000 and 6000 respectively, and provides the customer location in Kansas City 1 and Columbia with 1120 and 2000 boots respectively.

Question 2:

1. Objective Function – Maximize

$$50 * \sum_{i=1}^{5} a_i + 80 * \sum_{i=1}^{5} b_i - \left(\sum_{i=1}^{5} (fc_i * \sum_{n=1}^{2} Y_{in}) + \sum_{i=1}^{5} (a_i * svc_i) + \sum_{i=1}^{5} (b_i * bvc_i) + 0.03 * \sum distance_{ij} * A_{ij} + 0.04 * \sum distance_{ij} * B_{ij} \right)$$

2. Constraints
   - Capacity
     - $\frac{a_i}{15000} + \frac{b_i}{9000} \leq y_{in} * n, n = \textit{the number of production line in plant } i$
   - Binary decision variables
     - $\sum_{n=1}^{2} y_{in} = 1, \quad \textit{for } i = 1, 2 (\textit{St. louis, Kansas City})$
     - $\sum_{n=1}^{2} y_{in} \leq 1, \quad \textit{for } i = 3, 4, 5 (\textit{Columbia, Springfield, Independence})$
   - Demand
     - $\sum_{i=1}^{5} A_{ij} = sdemand_j, \textit{for } j = 1,2,3,4,5,6,7,8$
     - $\sum_{i=1}^{5} B_{ij} = bdemand_j, \textit{for } j = 1,2,3,4,5,6,7,8$

3. Answer:
   a. The company should not install a second production line at the Kansas City plant but open a new plant in Independence with two production lines.
   b. The plant in St. Louis produces 22000 shoes and 4300 boots. The plant in Kansas City only produces 6580 boots. The plant in Independence produces 24800 shoes and 3120 boots.
   c. The plant in St. Louis provides the customer location in St. Louis 1,2,3 with 8000, 6500, 7500 shoes respectively, and provides the customer location in St. Louis 1,3,4 with 2200,1200,900 boots respectively.

      The plant in Kansas City provides the customer location in Kansas City 1 and Springfield with 3380 and 3200 boots respectively.

      The plant in Independence provides the customer location in Kansas City 1, 2, Columbia, and Springfield with 9500, 2300, 7000, and 6000 respectively, and provides the customer location in Kansas City 1 and Columbia with 1120 and 2000 boots respectively.

4. We know revenue that MSC gets from selling its products and want to find out the maximized total profit. Total profit = Total revenue - total cost, so we also need the minimum total cost to calculate the maximum total profit. Because other conditions don't change, the optimal expansion plan under this scenario has the same minimum total cost as question 1, which is $5990884. So the optimal expansion doesn't change, the company should not install a second production line at the Kansas City plant but open a new plant in Independence with two production lines.

Question 3:

1. Objective Function – Maximize

$$50 * \sum_{i=1}^{5} a_i + 80 * \sum_{i=1}^{5} b_i - (\sum_{i=1}^{5} (fc_i * \sum_{n=1}^{2} Y_{in}) + \sum_{i=1}^{5} (a_i * svc_i) + \sum_{i=1}^{5} (b_i * bvc_i) + 0.03 * \sum distance_{ij} * A_{ij} + 0.04 * \sum distance_{ij} * B_{ij})$$

2. Constraints
   a. Capacity
      - $\frac{a_i}{15000} + \frac{b_i}{9000} \leq y_{in} * n,\ n = \text{the number of production line in plant } i$

   b. Binary decision variables
      - $\sum_{n=1}^{2} y_{in} = 1, \quad for\ i = 1, 2 (St.\ louis, Kansas\ City)$
      - $\sum_{n=1}^{2} y_{in} \leq 1, \quad for\ i = 3, 4, 5 (Columbia, Springfield, Independence)$

   c. Demand
      - $\sum_{i=1}^{5} A_{ij} \leq sdemand_j,\ for\ j = 1,2,3,4,5,6,7,8$
      - $\sum_{i=1}^{5} B_{ij} \leq bdemand_j,\ for\ j = 1,2,3,4,5,6,7,8$

3. Answer:
   a. The company should neither install a second production line at the Kansas City plant nor open a new plant in other cities.

b. The plant in St. Louis produces 22000 shoes and 4800 boots. The plant in Kansas City produces 7500 shoes and 4500 boots.
c. The plant in St. Louis provides the customer location in St. Louis 1,2,3 with 8000, 6500, 7500 shoes respectively, and provides the customer location in St. Louis 1,3,4 and Columbia with 2200,1200,900 and 500 boots respectively.
   The plant in Kansas City provides the customer location in Kansas City 1, 2 with 5200 and 2300 shoes respectively and provides the customer location in Kansas City 1 with 4500 boots.

4. We also want to get the maximum total profit in this question, but now MSC does not need to meet the retailers' projected demand in its entirety. So that in this scenario, we have a smaller total cost than before, which become $4346270. Therefor, the optimal expansion plan in this scenario change to the company should neither install a second production line at the Kansas City plant nor open a new plant in other cities.