Frequency Response

1. Nulling Filters for Rejection

a. Design the filtering system (two cascade FIR filters)

For $\omega = 0.44\pi$:

*fir_1=[1, -2*cos(0.44*pi), 1];*

For $\omega = 0.7\pi$:

*fir_2=[1, -2*cos(0.44*pi), 1];*

b. Generate the input signal:

Input signal is the sum of three sinusoids:

n=0:149;
*x=5*cos(0.3*pi.*n)+22*cos(0.44*pi.*n-pi/3)+22*cos(0.7*pi.*n-pi/4);*

c. Filter the input signal by using the two cascade FIR filter：

fir = firfilt (fir_1, fir_2);
after = firfilt(x, fir);

d. Make plot f the output signal:

The conjugating calculation in time domain:

$$Output[n] = h[n] * x[n] = \sum_{k=-\infty}^{+\infty} h[k] \cdot x[n-k]$$

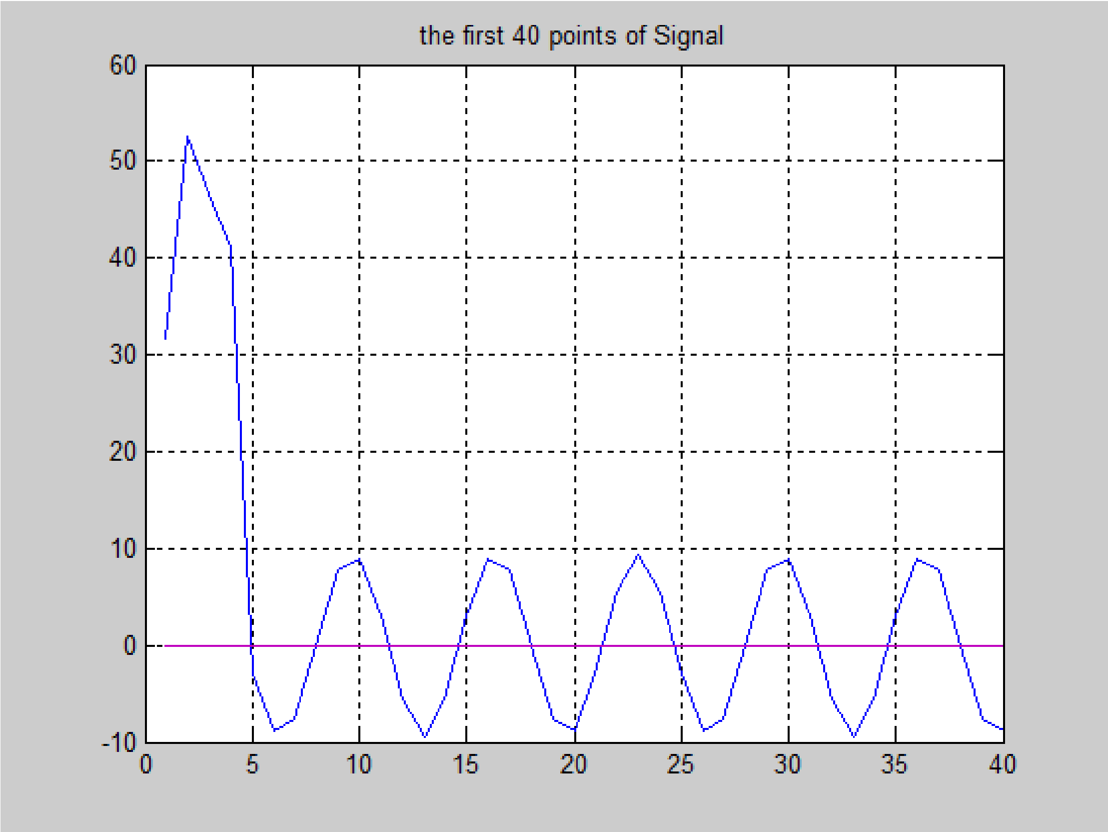
$$= \sum_{k=-\infty}^{+\infty} (h[k] - 2\cos(\omega_n) h[k-1] - h[k-2]) \cdot (5\cos(0.3\pi(n-k)) + 22\cos(0.44\pi(n-k) - \frac{\pi}{3}) + 22\cos(0.7\pi(n-k) - \frac{\pi}{4}))$$

The frequency response:

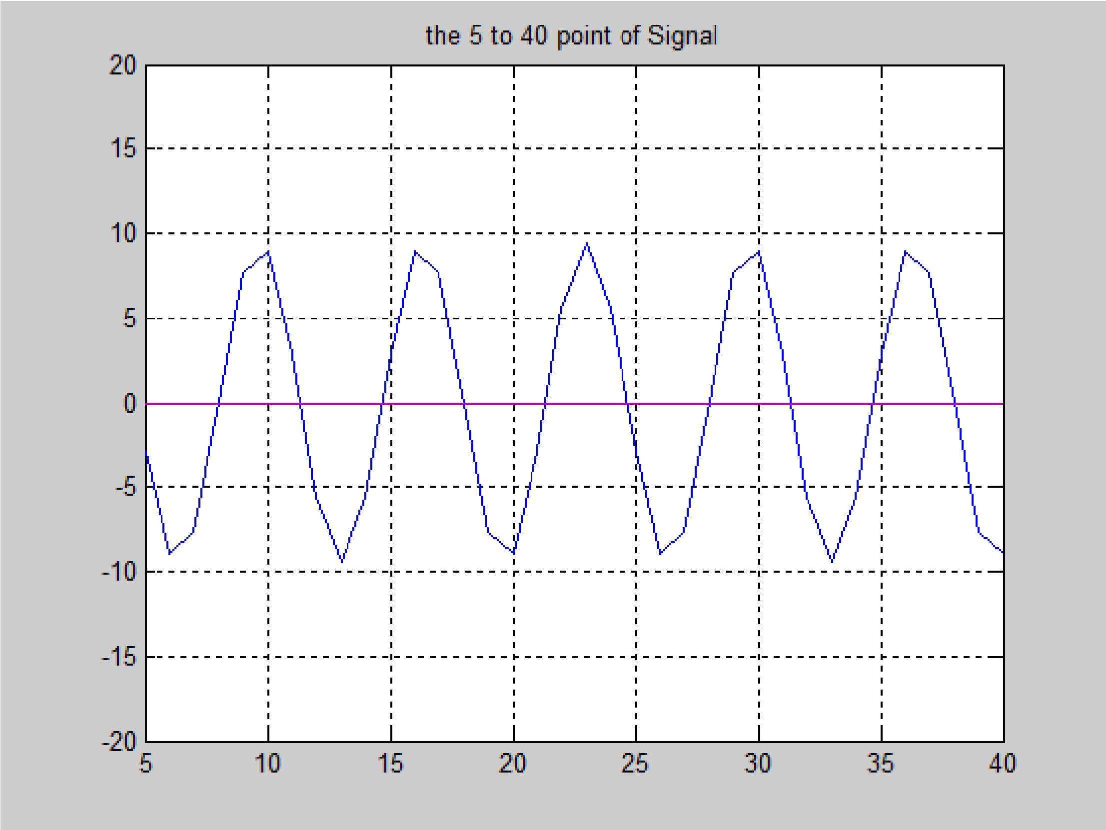
$$Output(e^{j\omega}) = H(e^{j\omega}) \cdot X(e^{j\omega});$$

$$\left|Output(e^{j\omega})\right| = \left|H(e^{j\omega})\right| \cdot \left|X(e^{j\omega})\right|;$$

$$\angle Output(e^{j\omega}) = \angle H(e^{j\omega}) + \angle X(e^{j\omega});$$



e. Plot the output as another range:



According to the mathematical formula, the plot shows the result of the filtering, and it better matches the analysis in the previous analysis, in the range of $n = 5$ to 40.

f. Explain the output signal is different for the first few points:

Because of the length out the overall FIR filter is:

$$Length(overall) = Length(Fir1) + Length(Fir2) - 1$$

So the filter would not be perfectly overlapped and conjugated with the signal from n=0 to the previously points. Therefore, the first a few points of the output signal would be different.

According to the mathematical calculation, there should be 5 points of the “start-up” points.

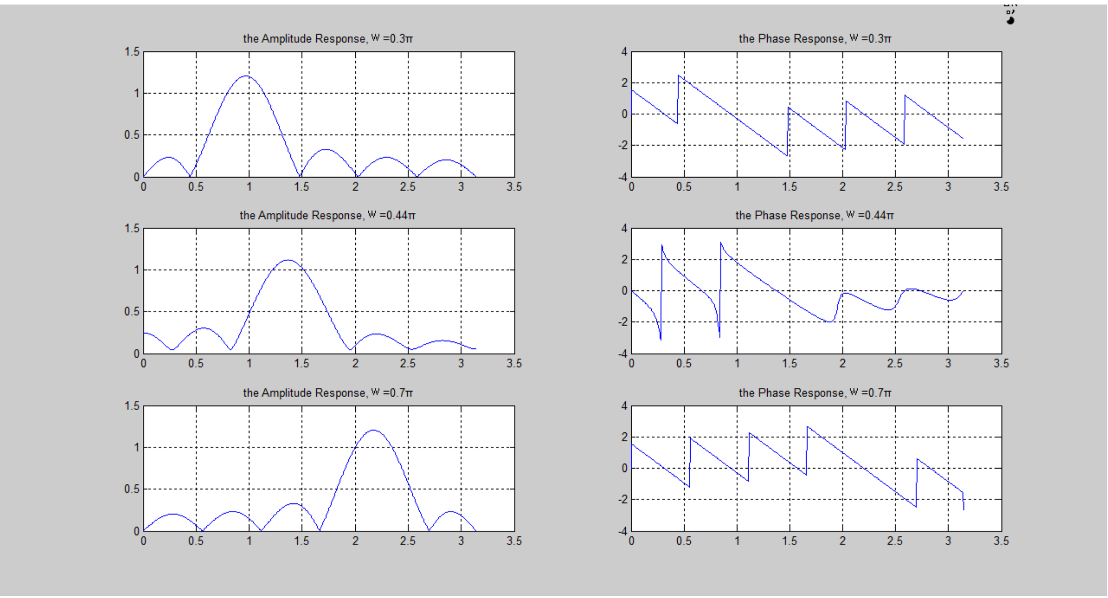
2. Simple Band-pass Filter Deign

The band-pass filter could be explained like the formula:

$$h[n] = \frac{2}{L} \cdot \cos(\omega_c \cdot n) \quad , \quad 0 \le n \le L$$

1. Generate a band pass filter that will pass a frequency :

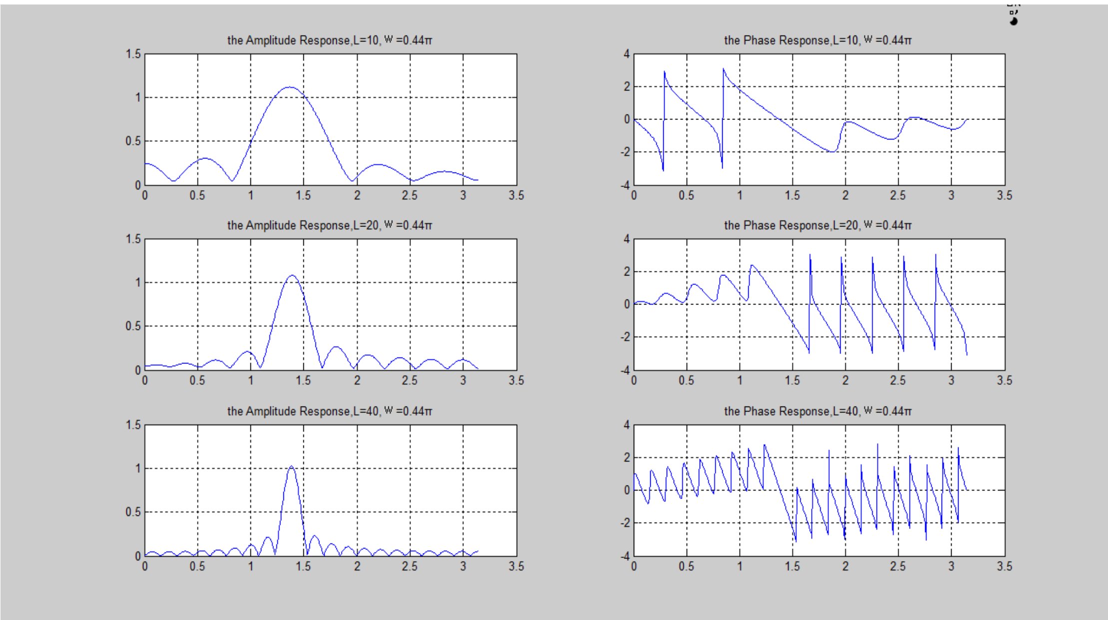
```
omega=0:(pi/500):pi;
L1=10;
n=0:L1;
h_1=0.2*cos(0.44*pi.*n);
h_2=0.2*cos(0.3*pi.*n);
h_3=0.2*cos(0.7*pi.*n);
H_1=freqz(h_1,1,omega);
H_2=freqz(h_2,1,omega);
H_3=freqz(h_3,1,omega);
figure,subplot(3,2,1);plot(omega,abs(H_2));
title('the Amplitude Response, w=0.3π ');grid on;
subplot(3,2,2);plot(omega,angle(H_2));
title('the Phase Response, w=0.3π ');grid on;
subplot(3,2,3);plot(omega,abs(H_1));
title('the Amplitude Response, w=0.44π ');grid on
subplot(3,2,4);plot(omega,angle(H_1));
title('the Phase Response, w=0.44π ');grid on;
subplot(3,2,5);plot(omega,abs(H_3));
title('the Amplitude Response, w=0.7π ');grid on;
subplot(3,2,6);plot(omega,angle(H_3));
title('the Phase Response, w=0.7π ');grid on
```



2. **Make a plot about the frequency responses of the different filter:**

```
L2=20; n2=0: L2; L3=40; n3=0:L3;
h_4=0.1*cos(0.44*pi.*n2);
h_5=0.05*cos(0.44*pi.*n3);
H_4=freqz(h_4,1,omega);
H_5=freqz(h_5,1,omega);
```

```
figure,subplot(3,2,1);plot(omega,abs(H_1));
title('the Amplitude Response,L=10, w=0.44π ');grid on;
subplot(3,2,2);plot(omega,angle(H_1));
title('the Phase Response,L=10, w=0.44π ');grid on;
subplot(3,2,3);plot(omega,abs(H_4));
title('the Amplitude Response,L=20, w=0.44π ');grid on;
subplot(3,2,4);plot(omega,angle(H_4));
title('the Phase Response,L=20, w=0.44π ');grid on;
subplot(3,2,5);plot(omega,abs(H_5));
title('the Amplitude Response,L=40, w=0.44π ');grid on;
subplot(3,2,6);plot(omega,angle(H_5));
title('the Phase Response,L=40, w=0.44π ');grid on;
max_1=max(abs(H_1)); Ha=abs(H_1);
x1=1:length(Ha);
a1=find( round ( Ha(x1) ) == ceil(0.707*max_1) );
max_4=max(abs(H_4)); Hb=abs(H_4);
x2=1:length(Hb);
a2=find( round ( Hb(x2) ) == ceil(0.707*max_4) );
max_5=max(abs(H_5)); Hc=abs(H_5);
x3=1:length(Hc);
a3=find( round ( Hc(x3) ) == ceil(0.707*max_5) );
```



And from the MATLAB surface, we could get the length of the band pass:

When L=10, the band width is 116 points;

When L=20, the band width is 60 points;

When L=40, the band width is 30 points;

3. Comment on the selectivity of L=10, and analyze the filter:

The frequency response is shown in the previous plot. When $\omega = 0.44\pi$, the amplitude $|H(e^{j\omega})|$ is around 0.9 to 1, which means the frequency corresponding to approximate 0.44π would pass through the filter. But for the frequency corresponding to 0.3π or 0.7π, the amplitude $|H(e^{j\omega})|$ is around 0.2 or 0.3, which means that

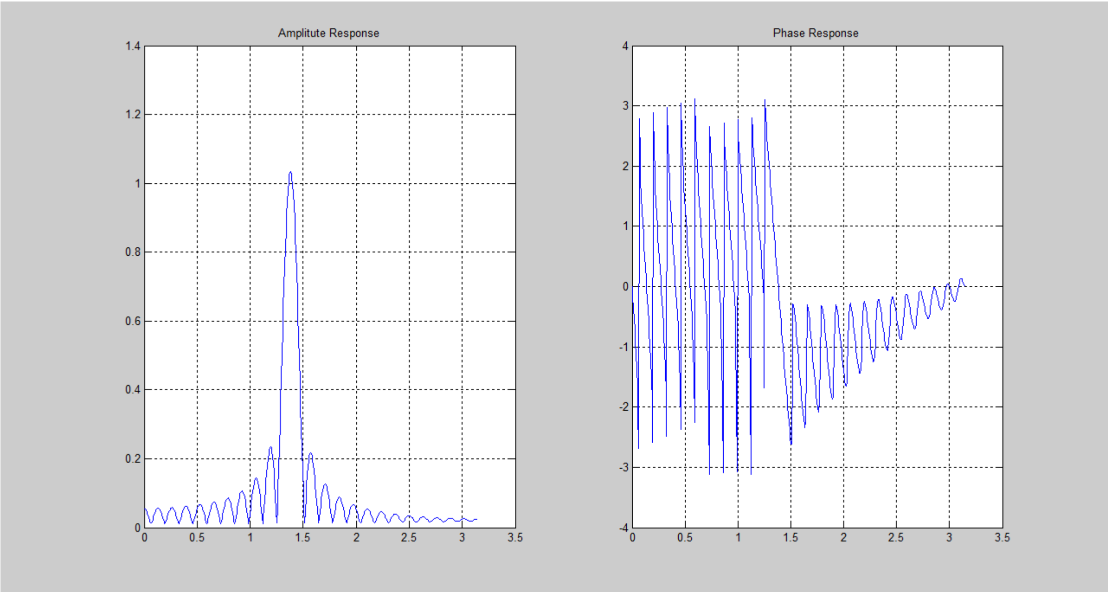
4. Generate a band pass filter that will pass the certain frequency:

The frequency component $|\omega| \le 3\pi$ would be reduced by a factor of 10;

The frequency component $0.7\pi \le |\omega| \le \pi$ would be reduced by a factor of 10.

```
LL=?;
n_n=0:LL;
hh=(2/LL)*cos(0.44*pi.*n_n);
HH=freqz(hh,1,omega);
figure,subplot(1,2,1);plot(omega,abs(HH));title('Amplitute Response');grid on;
subplot(1,2,2);plot(omega,angle(HH));title('Phase Response');grid on;
```

As the code shown above, we did several experiments with different value of LL to measure the amplitude in each case. For better meeting the requirement in that part, I finally choose LL=46 and its frequency response is:

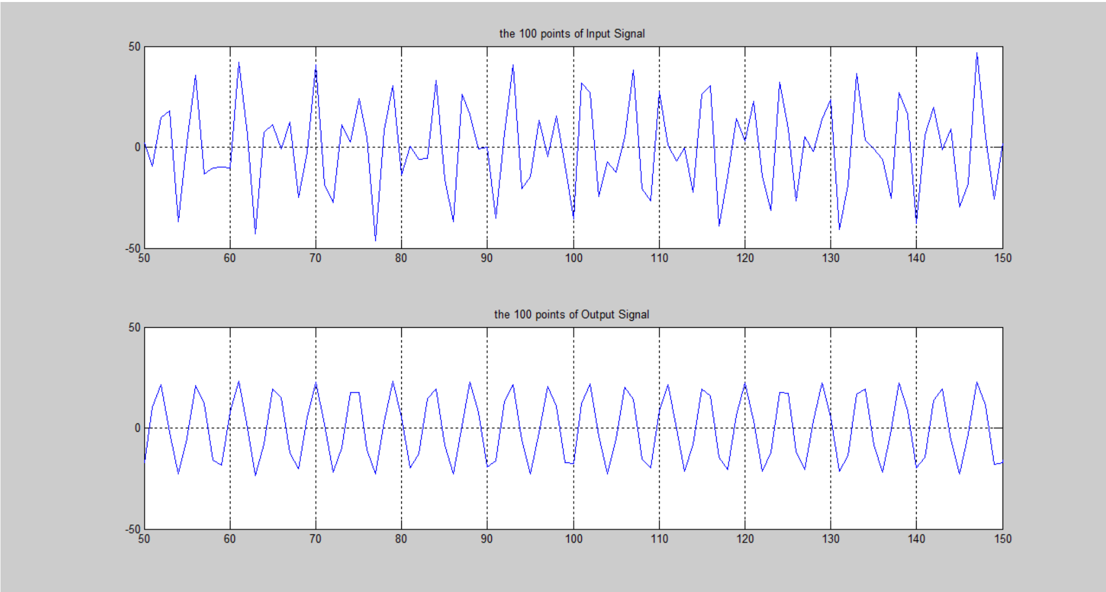


5. Use the filter the signal:

```
nv=0:149;
x=5*cos(0.3*pi.*nv)+22*cos(0.44*pi.*nv-pi/3)+22*cos(0.7*pi.*nv-pi/4);
after=conv(hh,x);
```

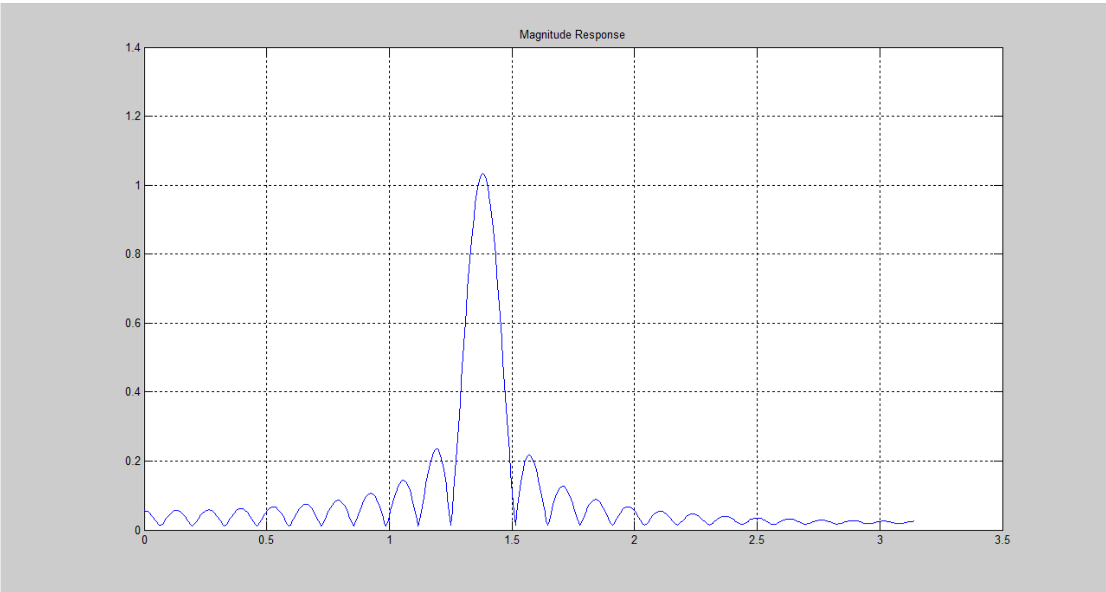
figure,subplot(2,1,1);plot(x);axis([50,150,-50,50]);
title('the 100 points of Input Signal');grid on;
subplot(2,1,2);plot(after);axis([50,150,-50,50]);
title('the 100 points of Output Signal');grid on;

and the input signal and the output signal is shown in the figure below:



The filter has filtered the two frequency of signal by conjugating the input signal with the FIR filter. The filter is like a window, and let the corresponding signal pass, while dramatically reduce the other signal with low gain. Therefore, we could get the filtering signal which is corresponding to the filter's character.

6. Frequency response:



$h[n]=\frac{2}{L}\cdot\cos(\omega_c\cdot n)$, the plot demonstrated the $\left|H(e^{j\omega})\right|$.

$$\left|output[e^{j\omega}]\right|=\left|H[e^{j\omega}]\right|\cdot\left|X(e^{j\omega})\right|$$

Therefore, from the mathematical explanation: the different component of the frequency ω_x of the input signal, there would be a corresponding magnitude response for it. The output of each component frequency is the multiplication. Overall, we can get the conclusion that $H(e^{j\omega})$ would determine the relative size of each sinusoidal component in the output signal.