Complex Exponentials

Exercises:

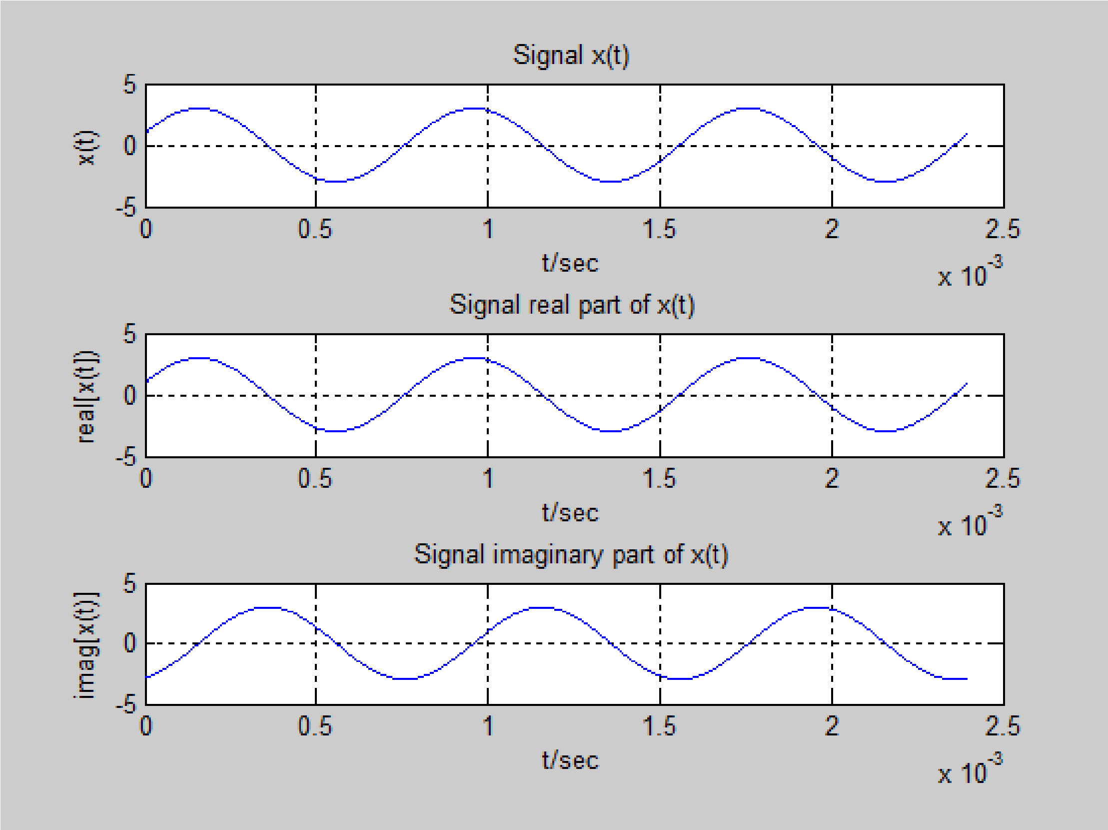
1. Representation of Sinusoids with Complex Exponentials

a. Generate the signal x(t):

```
A = 3;
theta = -0.4*pi;
omega = 2*pi*1250;
T=2*pi/omega;
t= 0:(2*T/1250):3*T;
xt=A*exp(1j*(omega*t+theta));
```

b. Plot the real part and imaginary part versus t respectively:

```
subplot(3,1,1); plot(t,xt); title('Signal x(t)');
subplot(3,1,2); plot(t,real(xt)); title('Signal real part of x(t)');
subplot(3,1,3); plot(t,imag(xt)); title('Signal imaginary part of x(t)');
```



c. **Verify the results:**

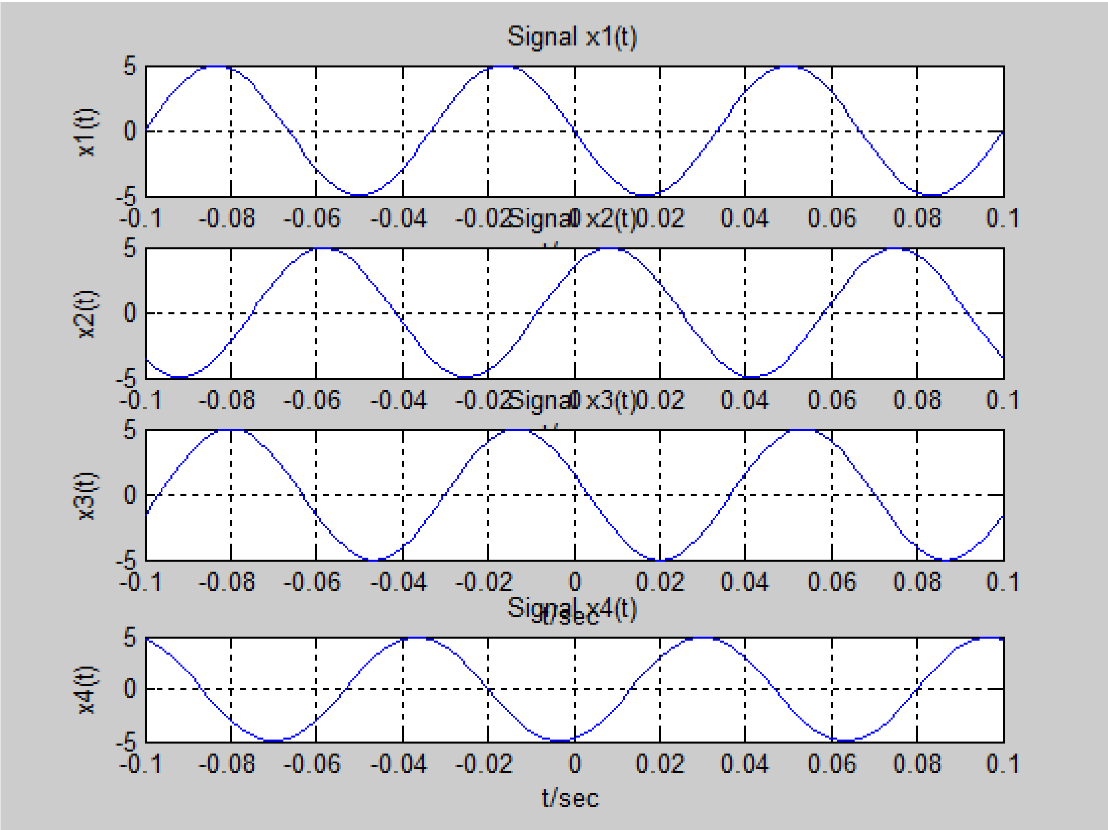
From the plots and the values returned by MATLAB, those showed that the frequencies are all the same, the amplitude are all equaled to 3, and the phase are correct.

2. *Verify Addition of Sinusoids Using Complex Exponentials*

a. **Make a plot of all four signals:**

```
f0=15;
T=1/f0;
A=5;
t=-1.5*T:T/30:1.5*T;
omega=2*pi*15;
theta1=0.5*pi;
theta2=-0.25*pi;
theta3=0.4*pi;
theta4=-0.9*pi;
xt1=A*exp(1j*(omega*t+theta1));
xt2=A*exp(1j*(omega*t+theta2));
xt3=A*exp(1j*(omega*t+theta3));
xt4=A*exp(1j*(omega*t+theta4));
```

then, I would yield the four signals in one plot:

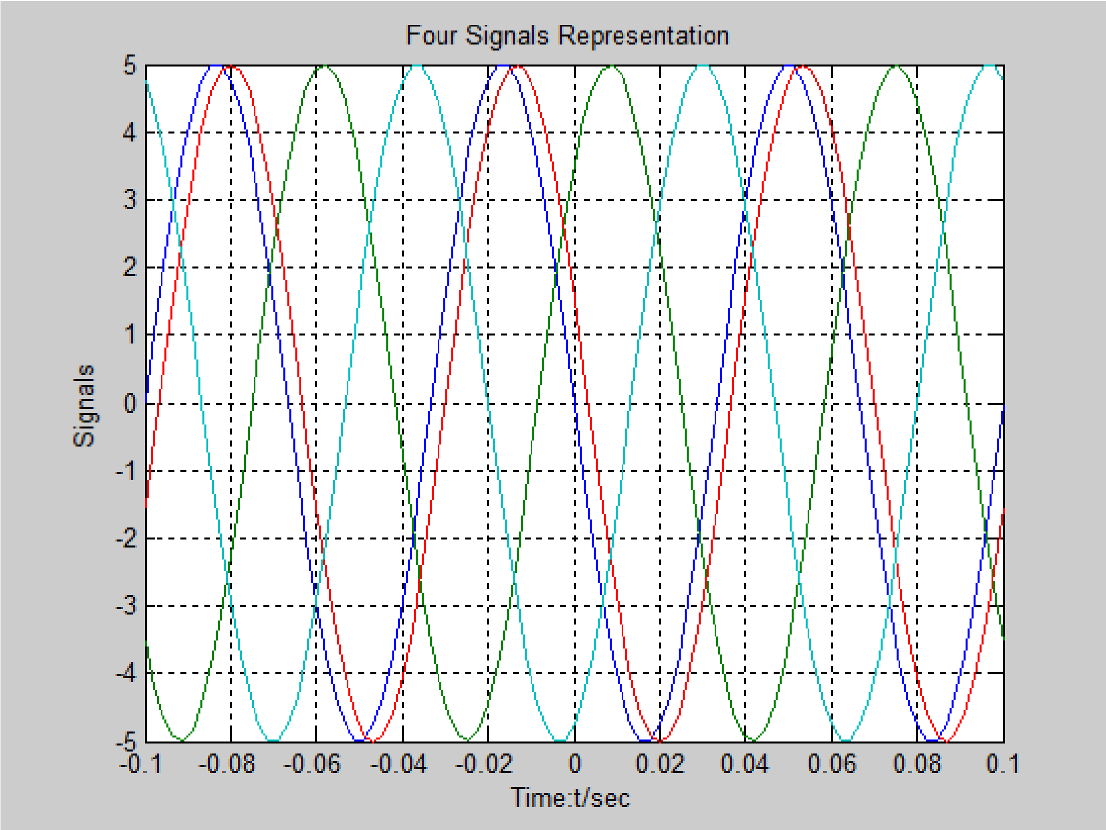


b. **Verify that all the phase of all four signals at t=0, and the maximum amplitude:**

1. Maximum amplitude:

   Four signals in a same plot to present:

   figure,plot(t,xt1,t,xt2,t,xt3,t,xt4);title('Four Signals Representation ');



This figure shows that the signals have the same correct maximum amplitude, A=5.

2. When t=0s, the MATLAB code would calculate the phases for each signal:

```
CPxt1=A*exp(1j*(omega*0+theta1));
CPxt2=A*exp(1j*(omega*0+theta2));
CPxt3=A*exp(1j*(omega*0+theta3));
CPxt4=A*exp(1j*(omega*0+theta4));
phase_1=angle(CPxt1);
phase_2=angle(CPxt2);
phase_3=angle(CPxt3);
phase_4=angle(CPxt4);
```

   then, from the MATLAB prompt:

   phase_1 = 1.5708 ; (0.5*pi, originally)

   phase_2 = -0.7854; (-0.25*pi, originally)

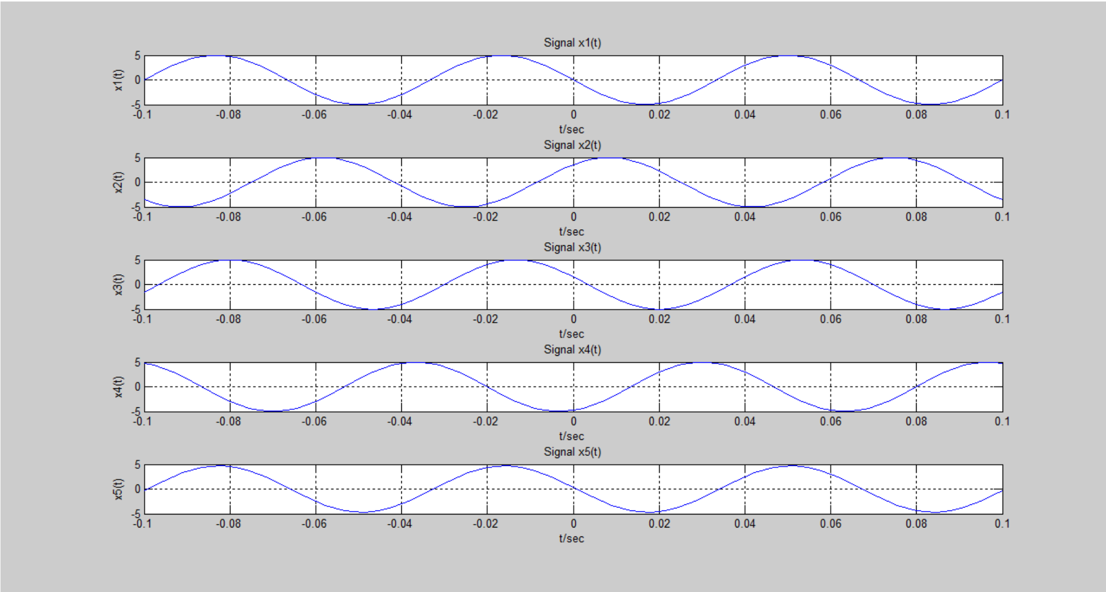
   phase_3 = 1.2566; (0.4*pi, originally)

   phase_4 = -2.8274; (-0.9*pi, originally)

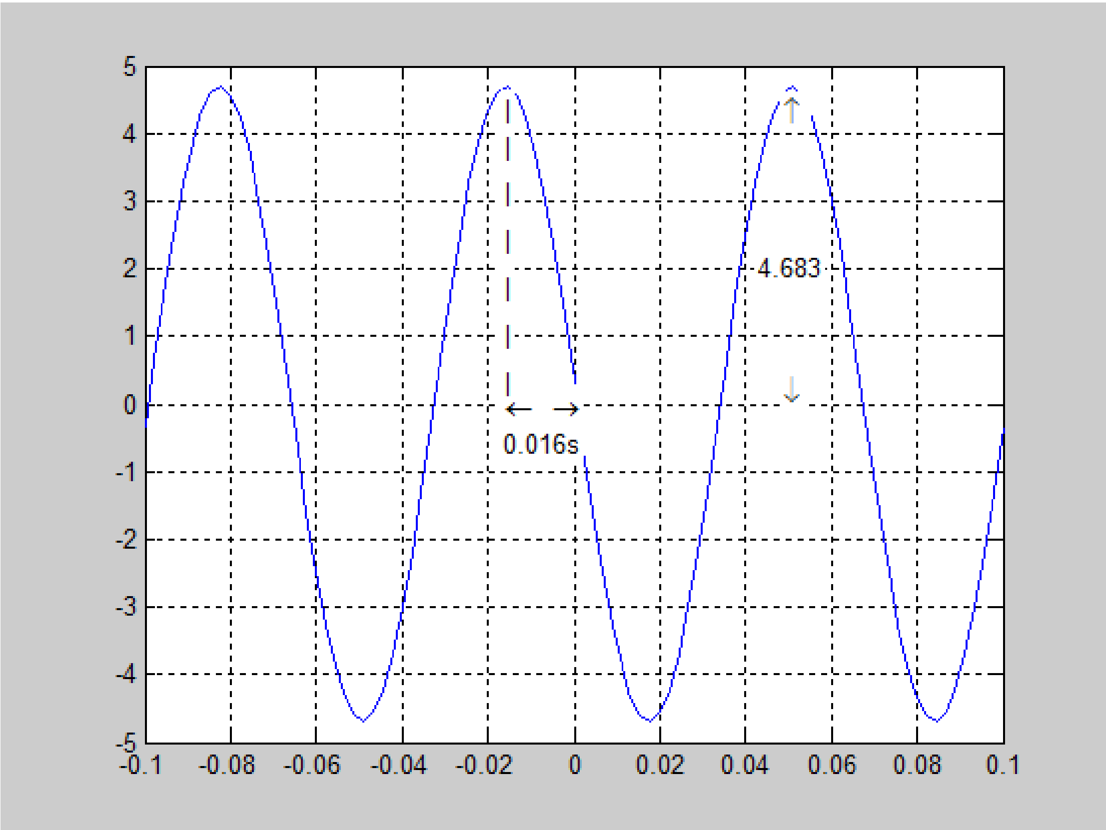
   each phase (t=0) calculated by MATLAB is correspondent with its original phase.

c. Create the sum sinusoid:

xt5=xt1+xt2+xt3+xt4;



d. Measure the magnitude and phase of x5:



The time-shift and the magnitude could be presented in the plot:

Magnitude = 4.683;

$$\phi_5 = -\frac{t_5}{T} \cdot 2\pi = -\frac{-0.016}{0,0667} \cdot 2\pi = 1.51 radians$$

Phase = 1.51;

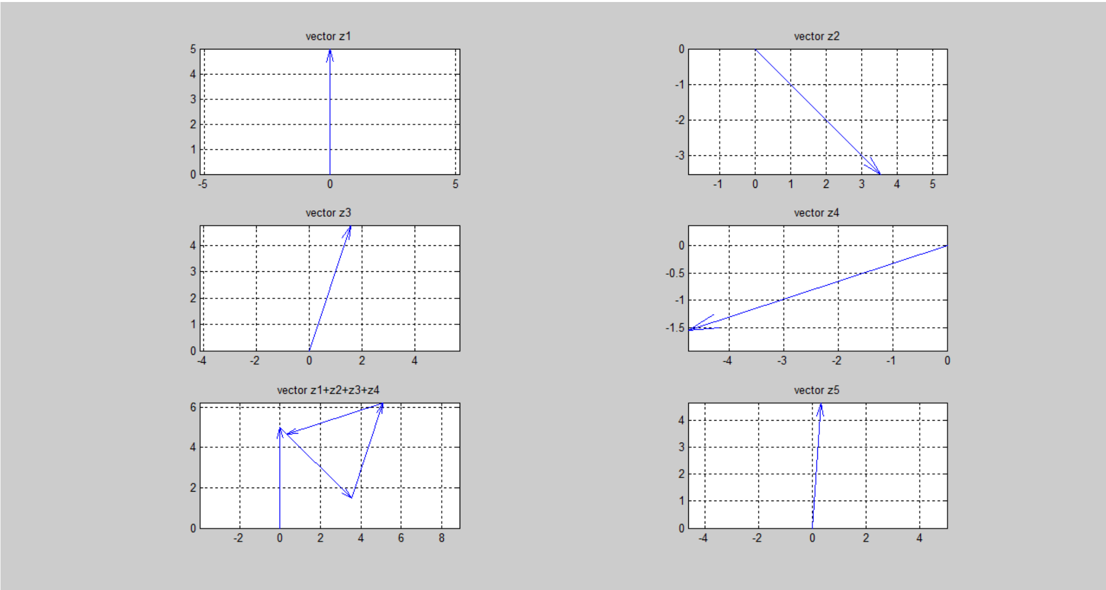
e.　Do the complex arithmetic:

```
z1=5*exp(1j*0.5*pi);
z2=5*exp(1j*(-0.25)*pi);
z3=5*exp(1j*0.4*pi);
z4=5*exp(1j*(-0.9)*pi);
```

f.　Verify :

```
z5=z1+z2+z3+z4;
figure,subplot(3,2,1);zvect(z1);
subplot(3,2,2);zvect(z2);
subplot(3,2,3);zvect(z3);
subplot(3,2,4);zvect(z4);
subplot(3,2,5);zvect(z5);
subplot(3,2,6);zcat([z1,z2,z3,z4]);
```

there is the figure that shows the five complex numbers as vector:



g.　Relate the magnitude and the phase of z5 (using *zprint* function):

```
zprint(z5);
```

then, the results showed on the MATLAB prompt are:

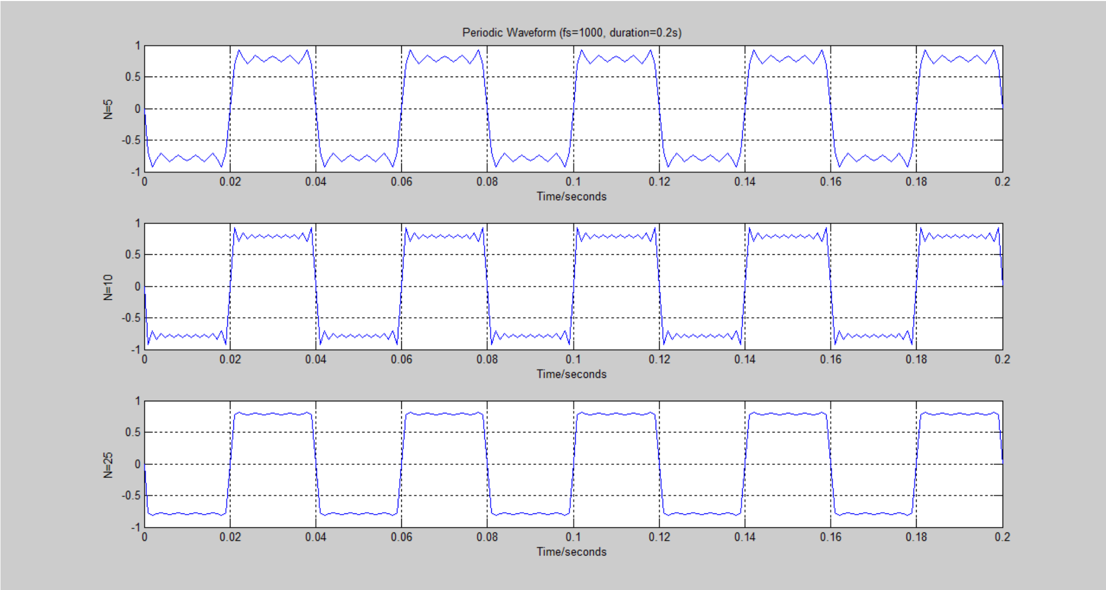
```
Z =     X      +     jY      Magnitude     Phase     Ph/pi    Ph(deg)
     0.3253        4.675         4.686     1.501     0.478     86.02
```

Compared z5 with x5(t), whose measured magnitude and phase are 4.683, 1.51 respectively. These two results are closely matched.

Periodic waveforms

A. Using the *sumcos* function to generate three periodic waveforms

```
fs=1000;
dur=0.2;
f=25;
z=1j;
for N=1:5
    k1=2*N-1;
    f1=25:50:k1*f;
    len1=length(f1);
    z1(1:len1)=1j./(1:2:k1);
end
subplot(3,1,1);x1=sumcos(f1,z1,fs,dur);title('Periodic Waveform (fs=1000, duration=0.2s)');xlabel('Time/seconds'),ylabel('N=5');grid on;
```



a. The plot for three different cases shows that the low frequency signal determines the basic shape of the synthesized signal, while the high frequency signal, which focused on the details of the synthesized signal, could eliminate the vibration of the edge.

b. When N is going to be infinite, the plot would be a rectangle shaped waveform, although there would also be a tiny bit of vibration during the edge area.

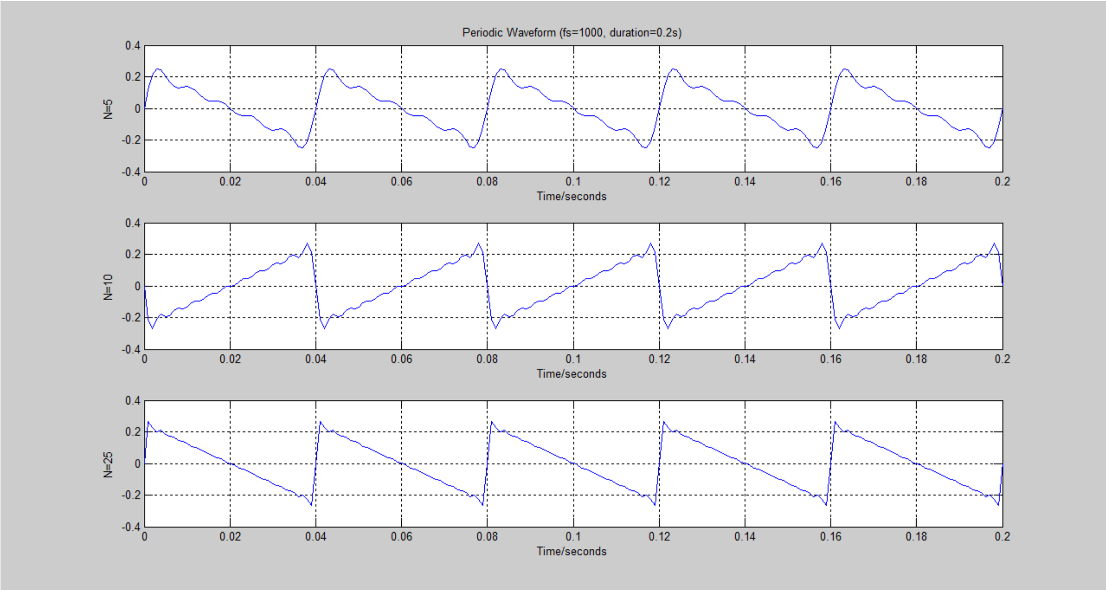
B. Sound function *sound(x,fs)*

As the N is ranging from 1 to 5, while the fundamental frequency is equal to 1kHz. When call the function sound(x,fs), I get the sound: the more N it is, the tone would be higher.

C. Coefficients

```
fs=1000;
dur=0.2;
f=25;
```

```
z=1j;
for N=1:5
    f1=25:25:N*f;
      z1=((-1)^N)*1j./(2*pi:2*pi:2*pi*N);
end
subplot(3,1,1);x1=sumcos(f1,z1,fs,dur);
```



The plot for three different cases shows that the low frequency signal determines the basic shape of the synthesized signal, while the high frequency signal, which focused on the details of the synthesized signal, could eliminate the vibration.

When N is going to be infinite, the plot would be a sawtooth shaped waveform. Moreover, the direction of the waveform would be opposite while the value of k is odd or even.

Conclusion

The goal of this experiment is gain familiarity with complex numbers and their use in representing sinusoidal signals as complex exponential. MATLAB could make plots of phasor diagrams that show the vector addition needed when combining sinusoids. We use the complex exponential representation for complex number, and verify addition of sinusoids.

The periodic waveform could be shaped by the coefficients, and synthesized with many fundamental signals which have different frequency.