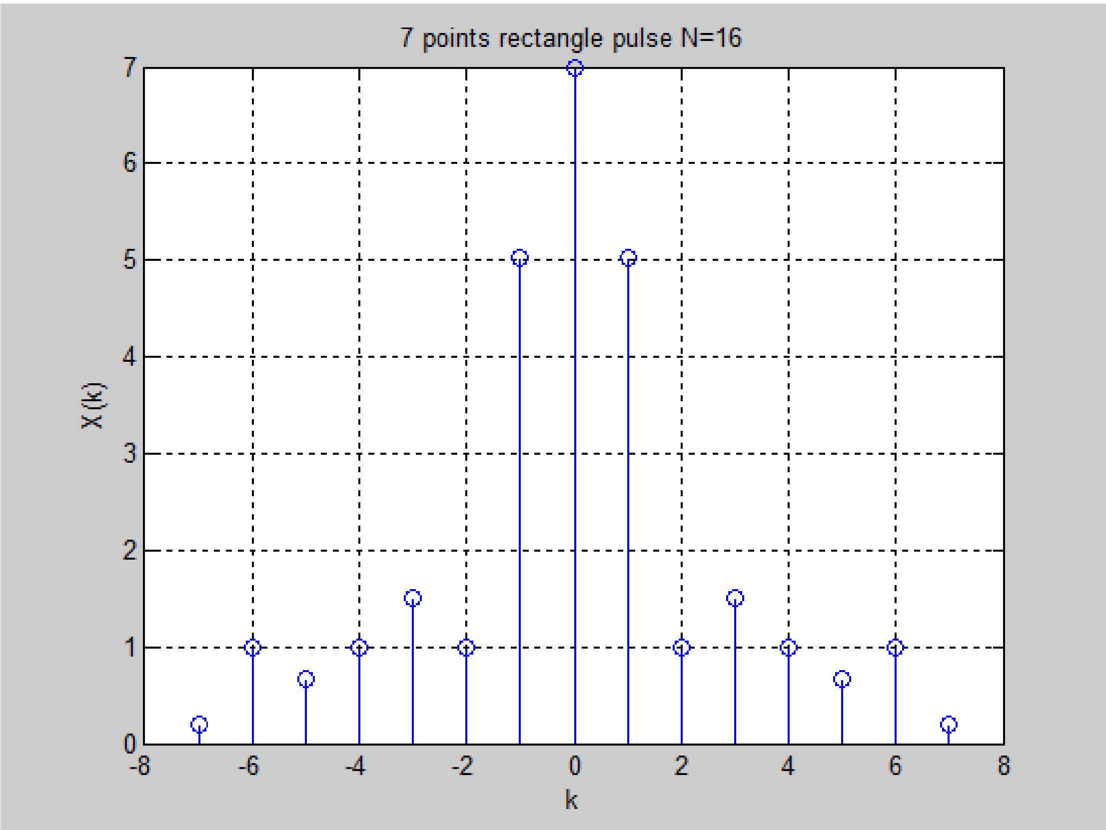
DFT Properties

1. Aliased Sinc Sequence

a. According to the duality, the rectangle pulse and the asinc function are ransform pairs.

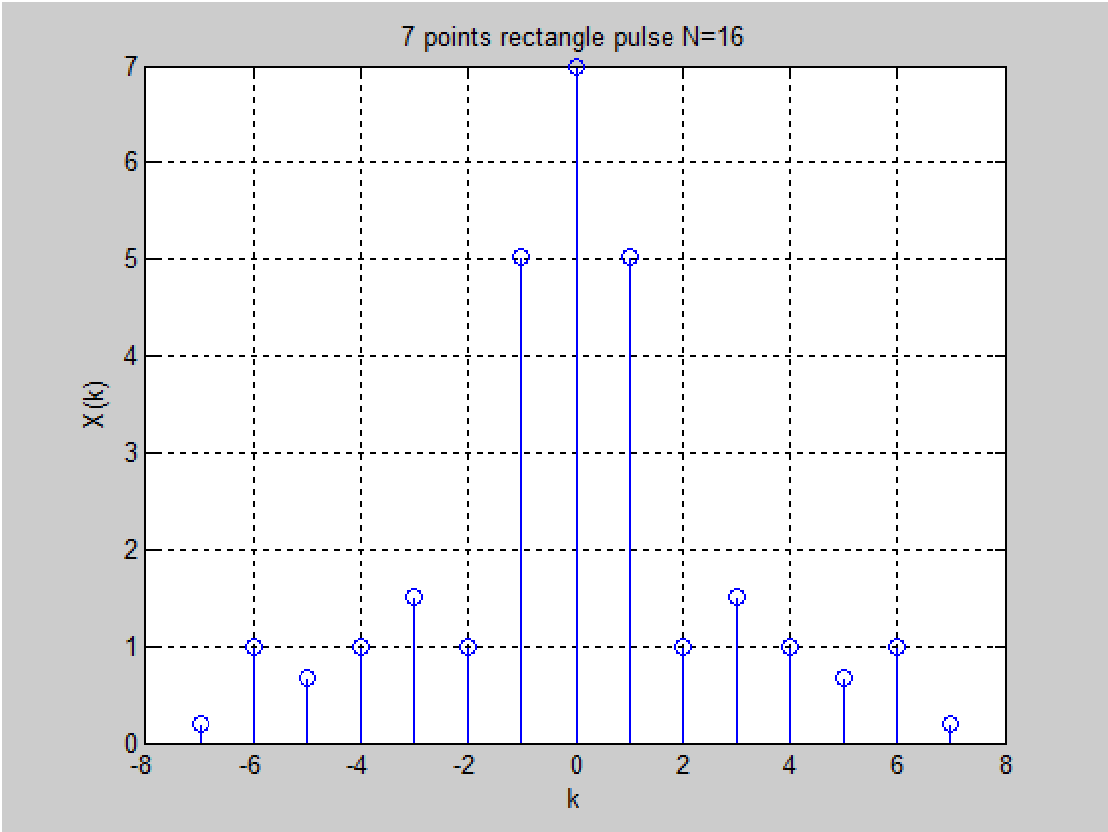
```
n1=-8:7;
x1=boxcar(7);
X1=fft(x1,16);
X1=fftshift(X1);
stem(n1,abs(X1));
title('7 points rectangle pulse N=16'),xlabel('k');ylabel('X(k)');grid on;
```



For an even-symmetric L-point pulse, the result is:

$$R[k] = asinc(\omega, L)|_{\omega=2\pi k/N} = \frac{\sin(\pi k L/N)}{\sin(\pi k/N)}$$

And then, let L=7, N=16, we could get the result:

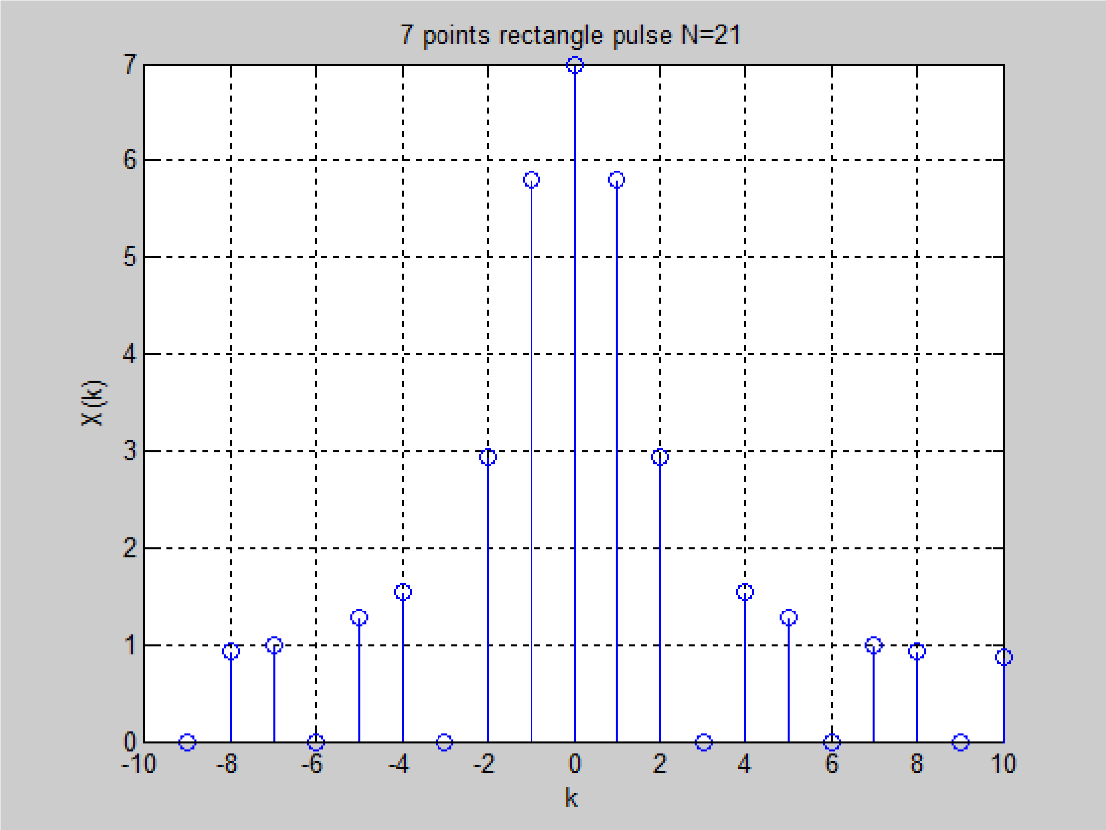


Therefore, this result for this case is the same.

I could change a little bit the code to see the result when it is the 21 points DFT:

```
n1=-10:10;
x1=boxcar(7);
X1=fft(x1,21);
X1=fftshift(X1);
stem(n1,abs(X1));
title('7 points rectangle pulse N=21'),xlabel('k');ylabel('X(k)');grid on;
```

and the result is:

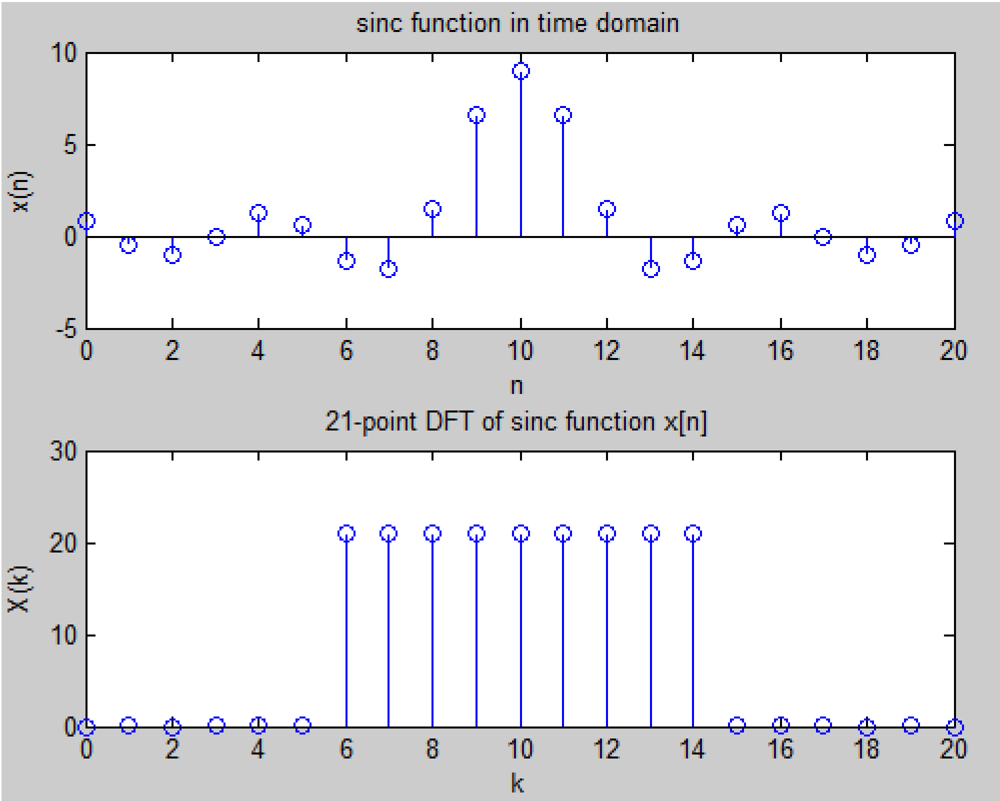


The number of samples from the frequency domain is more than before, so this cauesd there are several zero values. The values on the zero value points, or on the original frequency domain are exactly 0.

b. Sample sequence:

```
n2=0:20;
N2=length(n2);
x2=sin(9*pi.*n2./N2)./sin(pi.*n2./N2);
x2_1=9;
x2=fftshift(x2);
X2=fft(x2);
X2=fftshift(X2);
figure,subplot(2,1,1);stem(n2,x2);
title('sinc function in time domain');xlabel('n'),ylabel('x(n)');
subplot(2,1,2);stem(n2,abs(X2));
title('21-point DFT of sinc function x[n],');xlabel('k'),ylabel('X(k)');
```

and the result of the plot is:

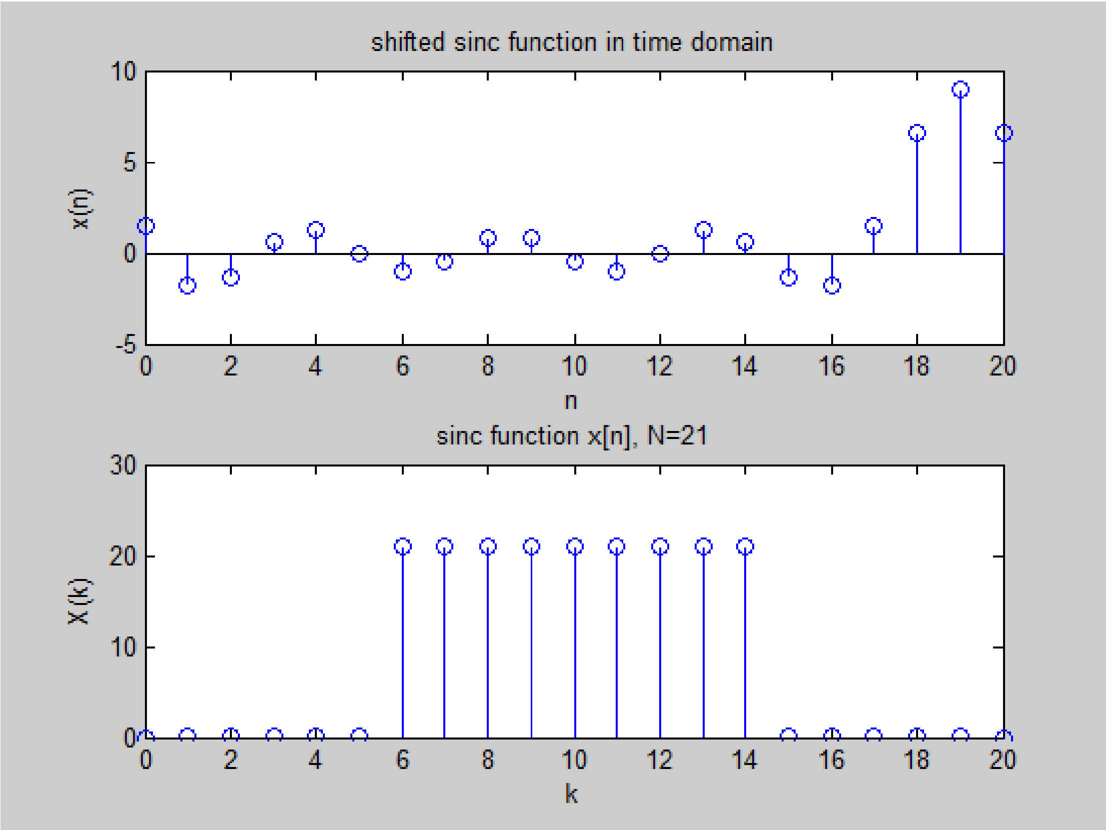


According to the properties of the sin function and the plot above, we can see that if the factor is an even constant, the a0[n] will be an odd function with respect to the base point. So that the function is no longer an even sinc function. So there is so much difference.

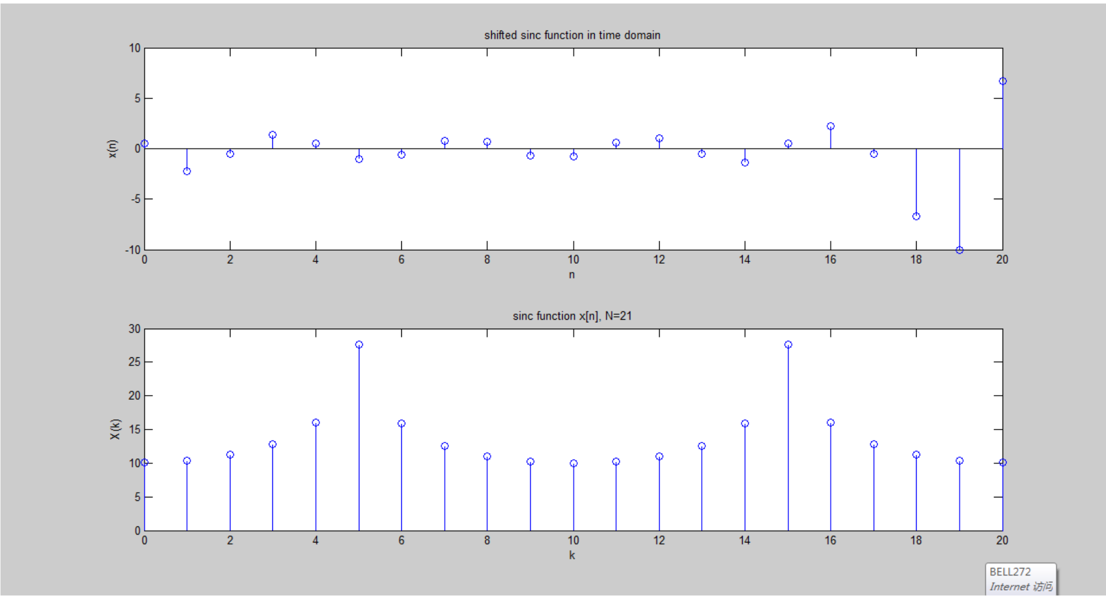
c. N-point DFT of the following shifted asinc sequence:

```
n2=0:20;
N2=length(n2);
x2=sin(9*pi.*(n2+1)./N2)./sin(pi.*(n2+1)./N2);
x2=fftshift(x2);
X2=fft(x2);
X2=fftshift(X2);
x2=fftshift(x2);
figure,subplot(2,1,1);stem(n2,x2);
title('shifted sinc function in time domain');xlabel('n'),ylabel('x(n)');
subplot(2,1,2);stem(n2,abs(X2));
title('sinc function x[n], N=21');xlabel('k'),ylabel('X(k)');
```

and the result of the plot is:



d. I just change a little bit of the code, and I can get the result, and then compute the DFT:

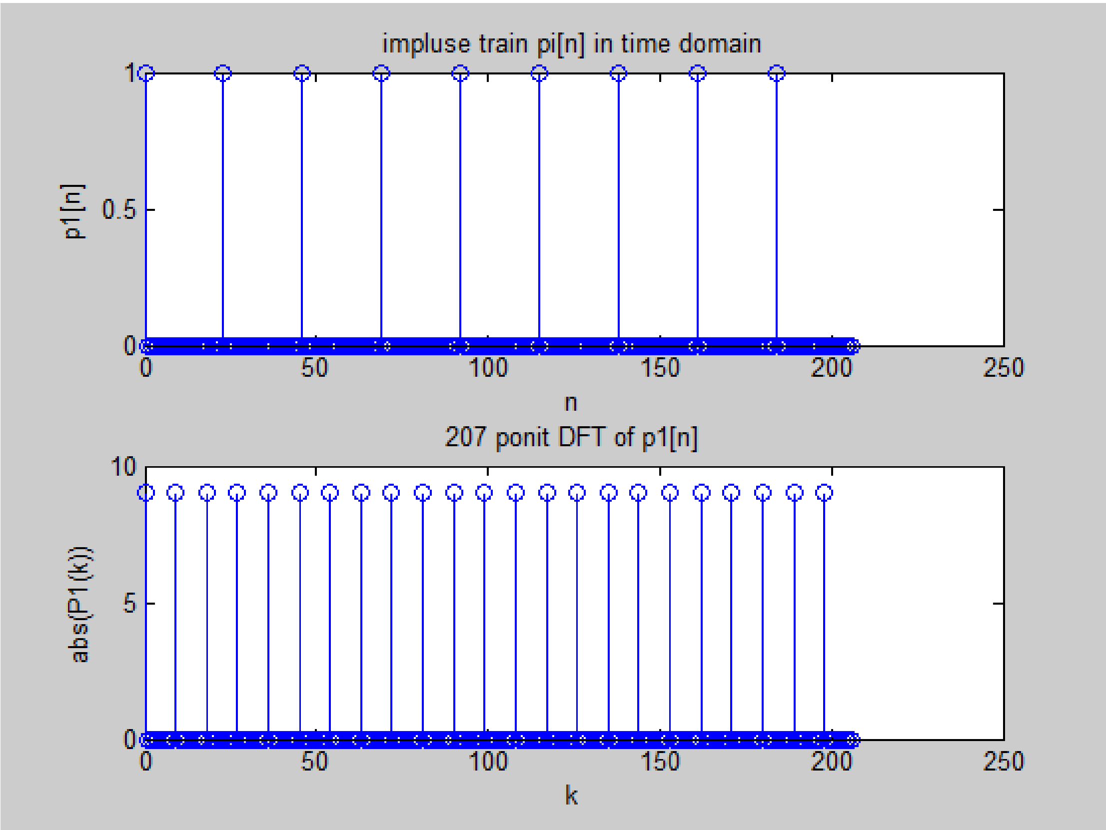


According to the properties of the sin function and the plot above, we can see that if the factor is an even constant, the a0[n] will be an odd function with respect to the base point. So that the function is no longer an even sinc function. So there is so much difference.

2. Impulse Train

A&B

```
n=0:206;
a=[1,zeros(1,22)];
p=repmat(a,1,9);
figure,subplot(2,1,1);stem(n,p);
title('impluse train pi[n] in time domain'),xlabel('n'),ylabel('p1[n]');
P=fft(p);
subplot(2,1,2);stem(n,abs(P));
title('207 ponit DFT of p1[n]');xlabel('k');ylabel('abs(P1(k))');
```



The spacing between impulse in the k domain is 9, which is $\frac{N}{M_0}$.

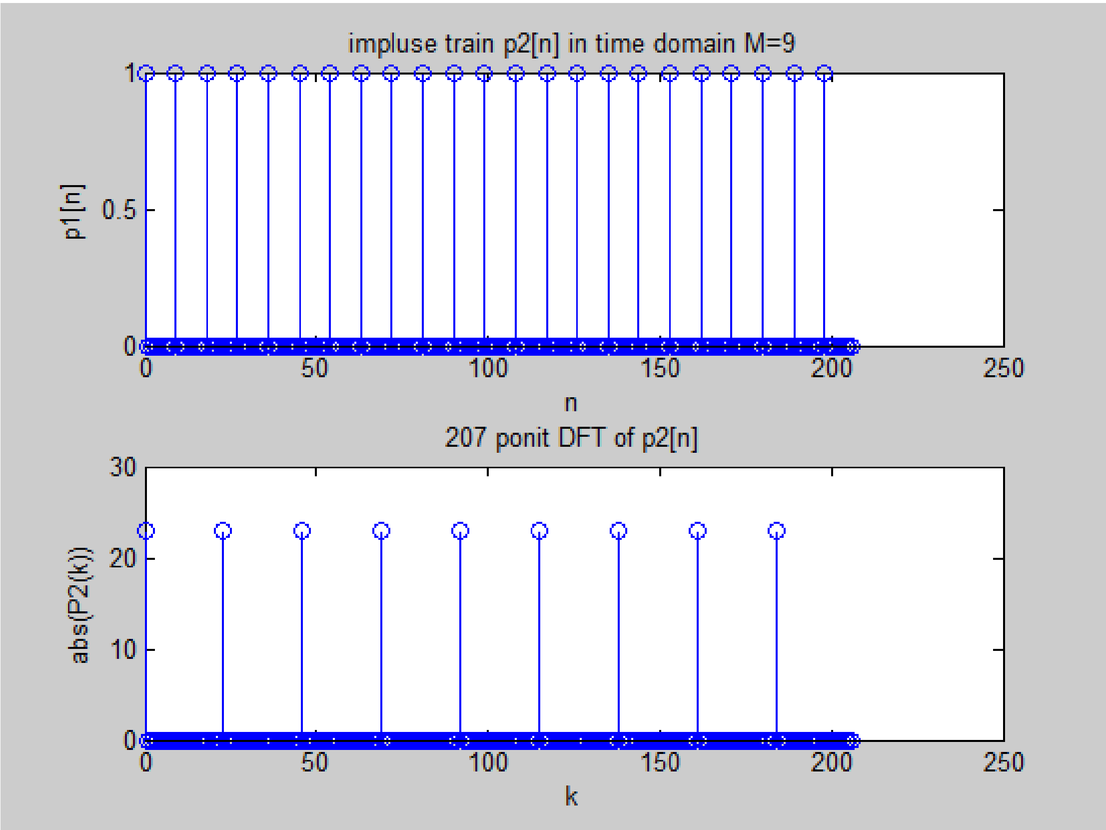
c. The spacing between impulses in the k domain is $\frac{N}{M_0}$ =9. Meanwhile, the magnitude of impulses in the k domain is $\frac{N}{M_0}$ =9.

Therefore, the mathematical form for P[k] is:

$$P[k] = \frac{N}{M_0} \cdot \sum_{l=0}^{M_o-1} \delta[k - l \cdot \frac{N}{M_0}]$$

When the M0 equals to 9, according to the code below:

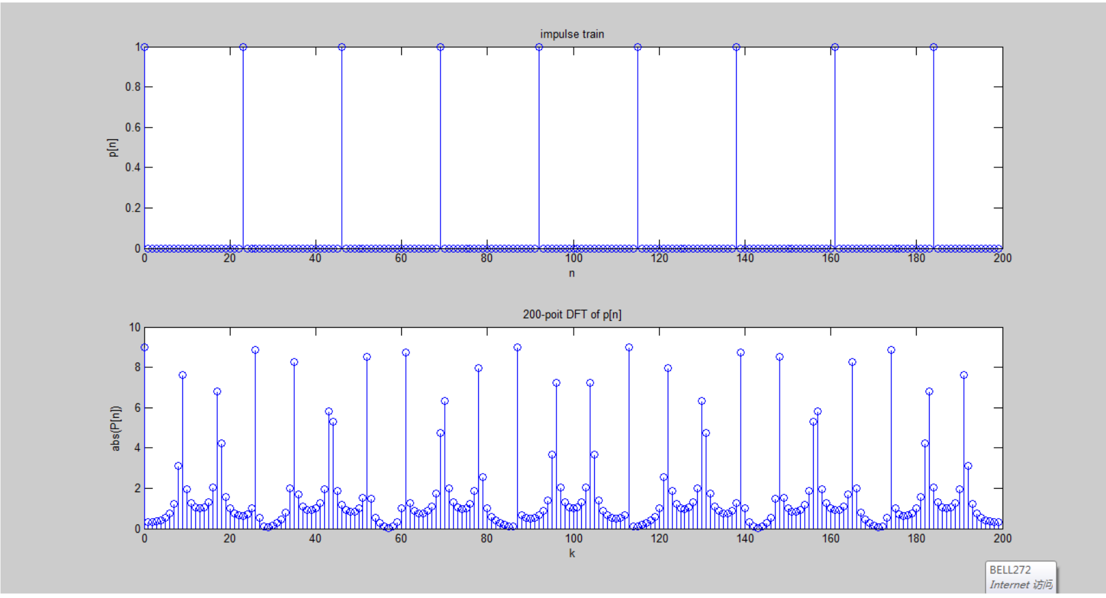
```
n=0:206;
a=[1,zeros(1,8)];
p=repmat(a,1,23);
figure,subplot(2,1,1);stem(n,p);
title('impluse train p2[n] in time domain M=9'),xlabel('n'),ylabel('p1[n]');
P=fft(p);
subplot(2,1,2);stem(n,abs(P));
title('207 ponit DFT of p2[n]');xlabel('k');ylabel('abs(P2(k))');
```



d.

```
n=0:199;
a=[1,zeros(1,22)];
p=[repmat(a,1,8),1,zeros(1,length(n)-8*23-1)];
figure,subplot(2,1,1);stem(n,p);title('impulse train');xlabel('n'),ylabel('p[n]');
P=fft(p);
subplot(2,1,2);stem(n,abs(P));
title('200-poit DFT of p[n]');xlabel('k'),ylabel('abs(P[n])');
```

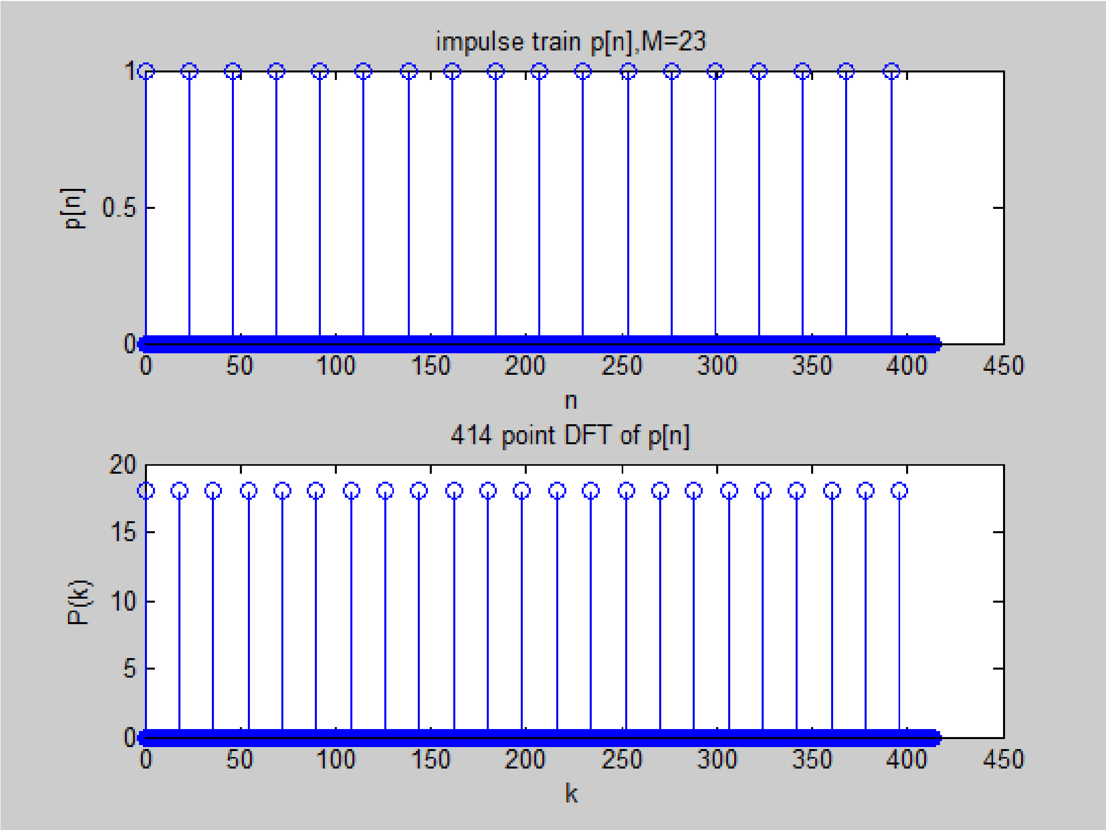
and I can get the MATLAB result:



After 200-point, the signal is no longer a signal with 23 period, 200 instead.

e.

```
n=0:413;
a=[1,zeros(1,22)];
p=repmat(a,1,18);
figure,subplot(2,1,1);stem(n,p);
title('impulse train p[n],M=23');xlabel('n'),ylabel('p[n]');
P=fft(p);
subplot(2,1,2);stem(n,abs(P));
title('414 point DFT of p[n]');xlabel('k'),ylabel('P(k)');
```

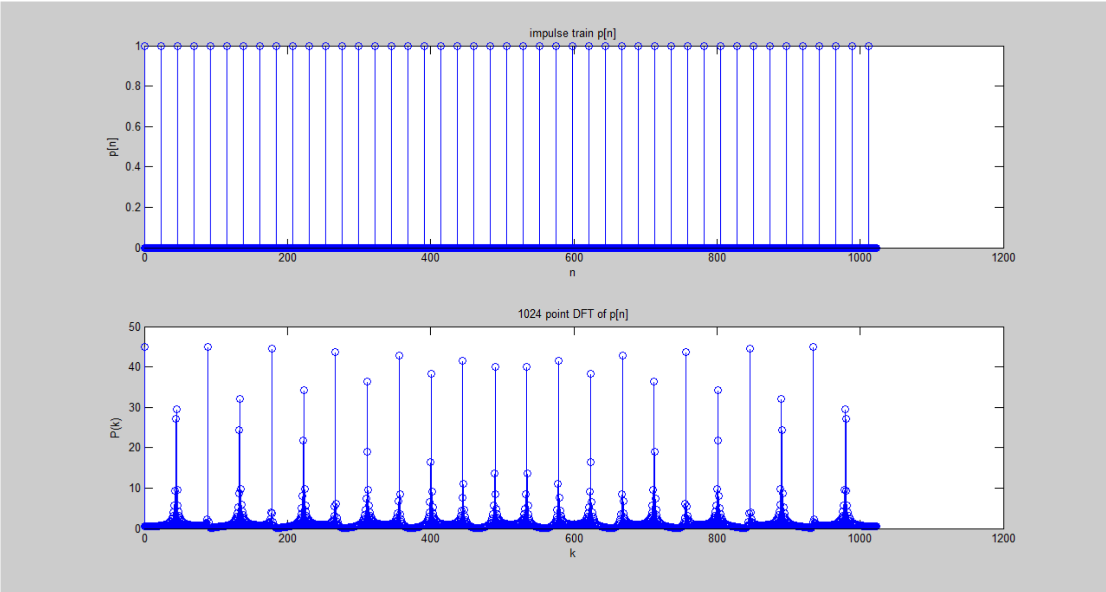


And the space of the impulse in k domain is $\dfrac{N}{M_0} = \dfrac{414}{23}$;

When N=16, we can get $\dfrac{N}{M_0}$=27.

f.:

```
n=0:1023;
a=[1,zeros(1,22)];
p=[repmat(a,1,44),1,zeros(1,length(n)-44*23-1)];
figure,subplot(2,1,1);stem(n,p);
title('impulse train p[n],M=23');xlabel('n'),ylabel('p[n]');
P=fft(p);
subplot(2,1,2);stem(n,abs(P));
title('414 point DFT of p[n]');xlabel('k'),ylabel('P(k)');
```



The spacing is 44.5, the number of peaks is 23. The general relationship between the period of the input signal, p[n], the length of the DFT, and the regular spacing of peaks in the DFT is:

$$Spacing = \frac{length}{period} = \frac{N}{M_0}$$

3. Gaussian

① ***A=0.5, because*** $a \cdot L^2 \leq 100$ ***, we take L=14***

```
L=14;
a=0.5;
n1=-L:L;
g=exp(-a.*1.^2);
```

② ***Fixing the rotated g[n] into v[n];***

```
v=[g(L+1) g(L+2) g(L+3) g(L+4) g(L+5) g(L+6) g(L+7)...
   g(L+8) g(L+9) g(L+10) g(L+11) g(L+12) g(L+13) g(L+14)...
   g(L+15) g(1) g(2) g(3) g(4) g(5) g(6) g(7) g(8) g(9)...
   g(10) g(11) g(12) g(13) g(14)];
```

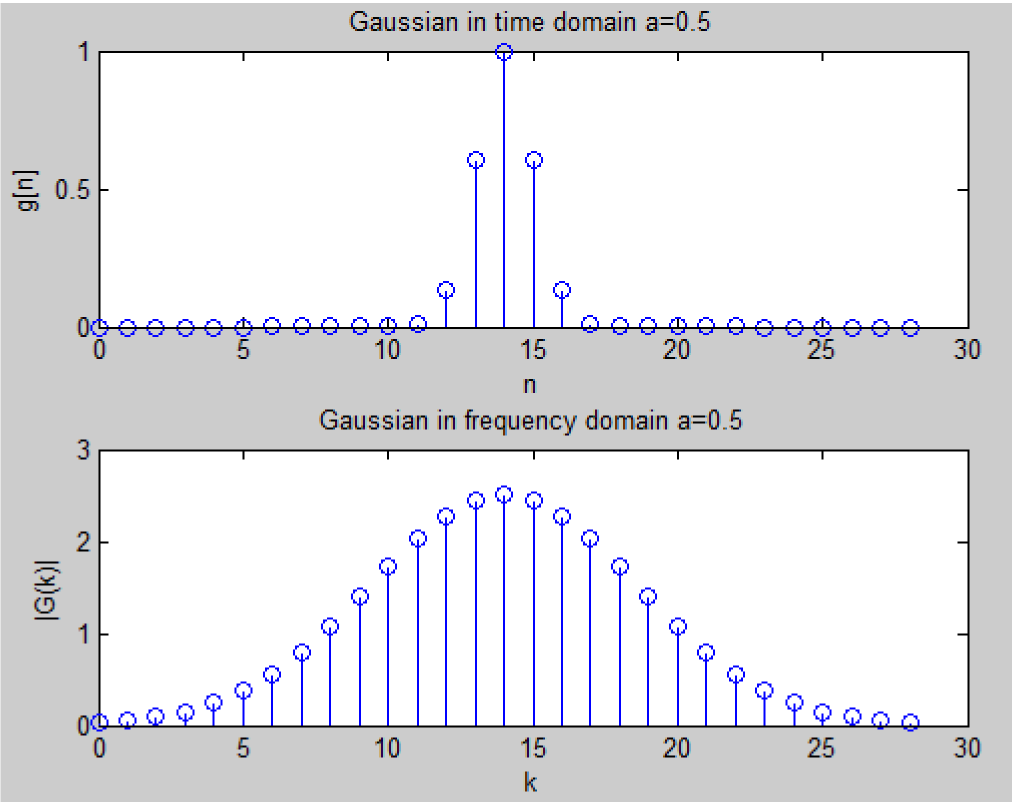
③ ***Computing the DFT of v[n]:***

```
G =

  Columns 1 through 10

    5.6050    4.9843    3.5052    1.9493    0.8572    0.2981    0.0820    0.0178    0.0031    0.0004

  Columns 11 through 20

    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000

  Columns 21 through 29

    0.0004    0.0031    0.0178    0.0820    0.2981    0.8572    1.9493    3.5052    4.9843
```

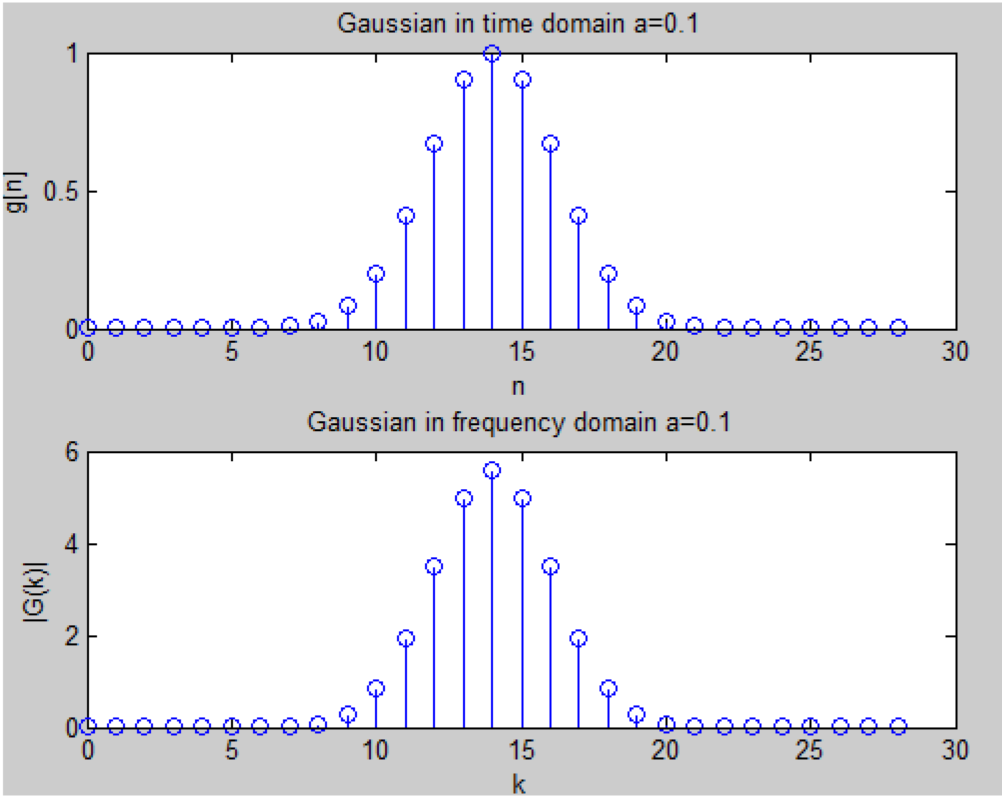
④ *Real part of the DFT, and compassion.*

G=fftshift(G);
Figure,sunplot(2,1,1);stem(n2,g);
Title('Gaussian'),xlabel('n'),ylabel('g[n]');
N2=0:n-1;
Subplot(2,1,2);stem(n2,real(G));
Title('Gaussian in frequency'),xlabel('k'),ylabel('G(k)');

⑤ *Result of plot*



⑥ ***a=0.1, then we plot:***



when a=0.1, the width of the Gaussian is the same in both the time and frequency domains.

4. Real Exponential

① . use this code to generate a finite portion of an exponential signal:

N=32;

n=0:N-1;

x=0.9.^n;

② the implementation of the program:

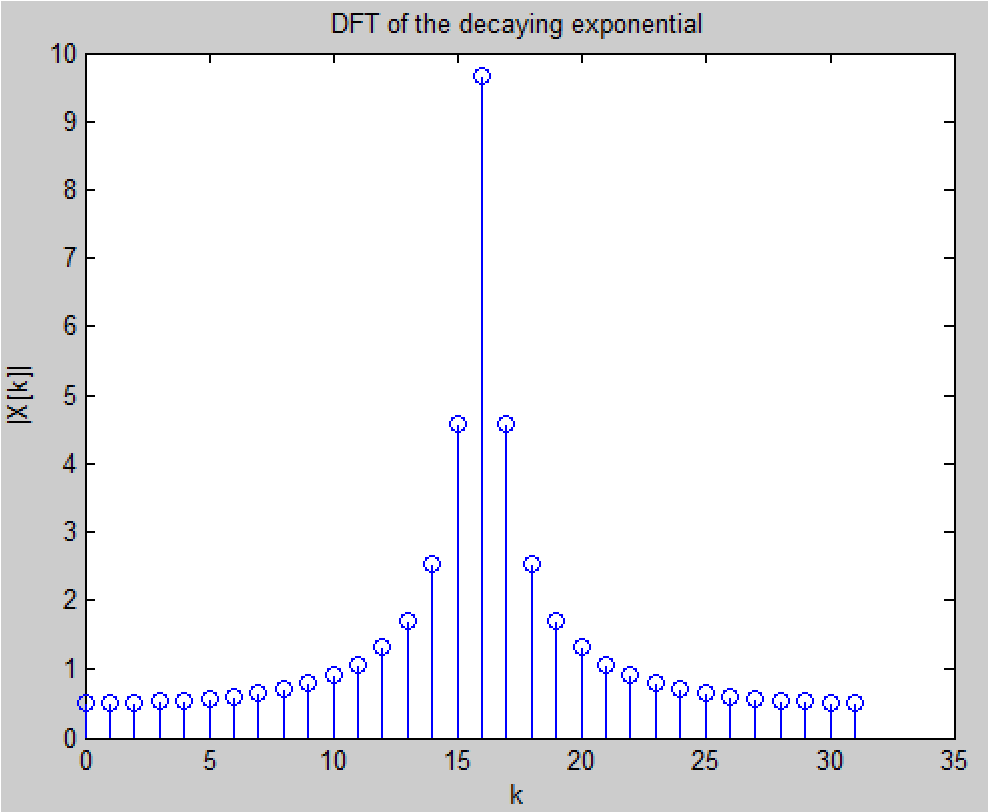
X=fft(x);

X=fftshift(X);

Figure,subplot(2,1,1);stem(n,abs(X));

Title('DFT of the decaying exponential');xlabel('k'),ylabel('X[k]');

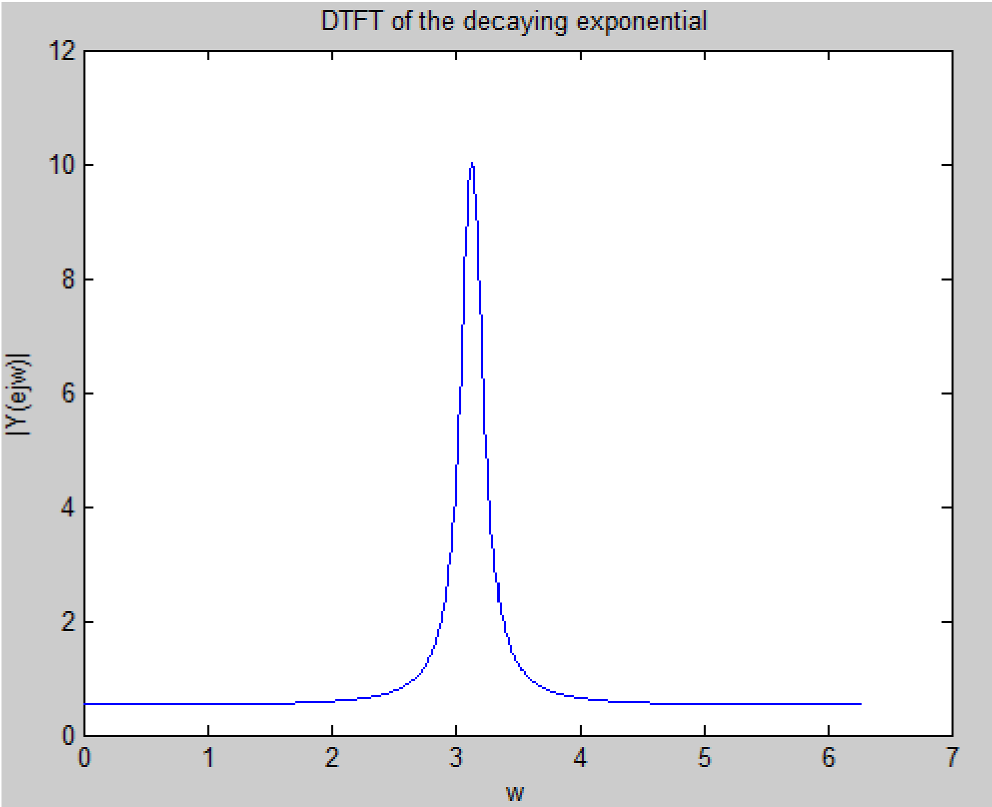
③ the result is shown as:

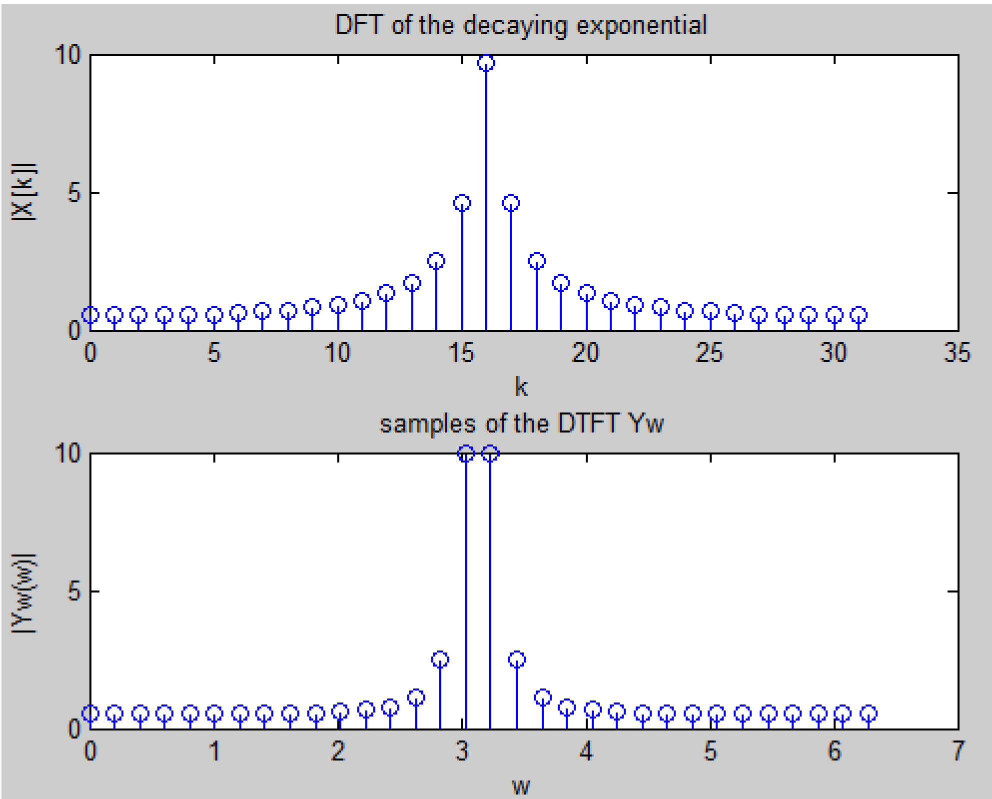


④ $\left|Y(e^{jw})\right| = \left|\dfrac{1}{1-0.9e^{-jw}}\right|$, there would be 32 samples between 0 and 2π .

*w=0:0.01:2*pi;*
*Yw=1./(1-0.9.*exp(-j.*w));*
Yw=fftshift(Yw);
Figure,plot(w,Yw);title('DFT of decay expomemtial');xalbel('w'),ylabel('abs(Y(e^jw))');
n1=0:0.20258:6.28;
*Y=1./(1-0.9.*exp(-j.*n1));*
Y=ffyshift(Y);
Subplot(2,1,2);stem(n1,Y);
Ylim([0 10.01]);
Title('samples of Yw');xlabel('w'),ylabel('Yw(w)');

⑤ The result of the plot:

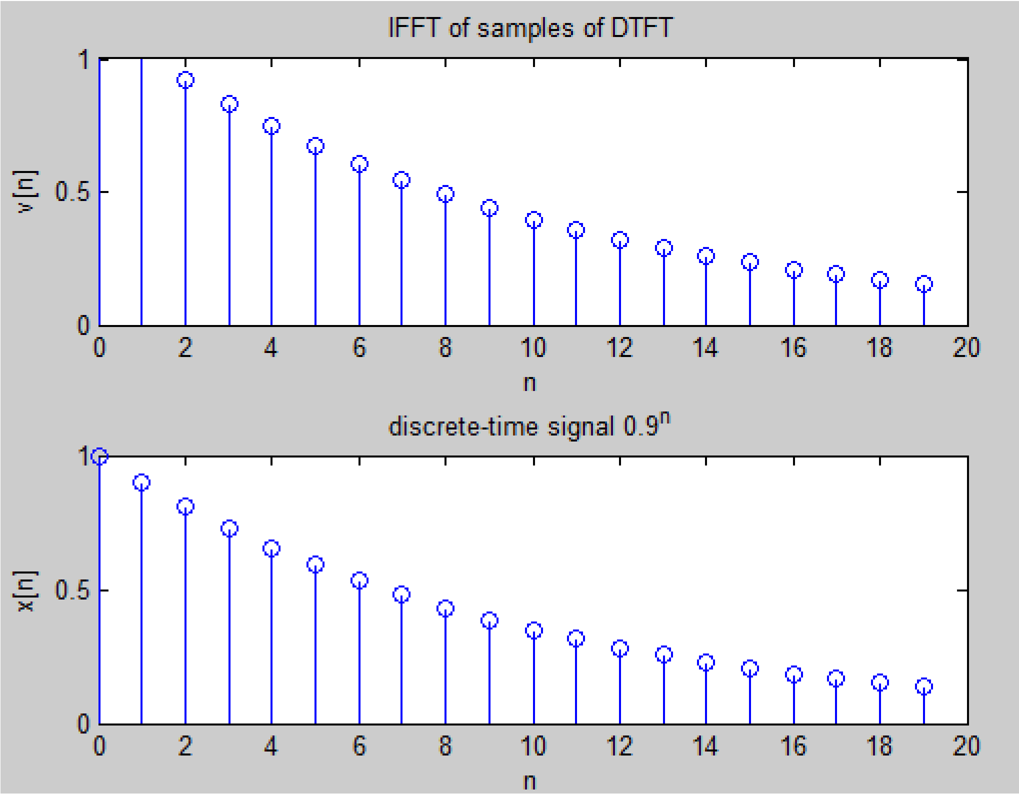

DTFT of the decaying exponential
|Y(ejw)|
w
0
2
4
6
8
10
12
1
3
5
7


DFT of the decaying exponential
|X[k]|
k
0
5
10
15
20
25
30
35
samples of the DTFT Yw
|Yw(w)|
w
1
2
3
4
6
7

⑥ Another sampling the DTFT $V[k] = Y(e^{j\omega})|_{\omega=(2\pi/N)k}$

```
N=100;
k=0:N-1;
n=0:N-1;
V=1./(1-0.9.*exp(-j.*(2*pi/N).*k));
Figure, subplot(2,1,1);stem(k,ifft(V));ylim([0 1.01]);
Title('IFFI of samples of DIFT'); xlabel('n'),ylabel('v[n]');
Subplot(2,1,2);stem(n,0.9.^n);
Title('discrete-time signal 0.9^n'); xlabel('n'),ylabel('x[n]');
```

If N equals to 20:



And if N equals to 100: