DSP Crib Sheet

$|y(n)| = |\sum x \cdot h| \le \sum |x| \cdot |h| \le M \sum |h|$

Properties of Signal

Memoryless: if output at time n only depend on input at time n

Time Invariance: System behaves same way regardless of what input is applied

Stability: A relaxed system is BIBO stable iff the input x[n] is bounded and y[n] is bounded.

to feed a u(n) and check y(n) bound.

LTI System

x[n] → H → h[n] h[n]: impulse function

For 2 Linear system in series, the τ_{eff} is also linear

For 2 TI system in series, the τ_{eff} is also TI

For 2 causal system in series, the τ_{eff} is may not be causal

For 2 LTI system in series, the τ_{eff} is not LTI

For 2 LTI system in series, the τ_{eff} order can be swapped

For 2 TV system in series, τ_{eff} order cannot be swapped

For 2 non-L sys in series, the τ_{eff} is linear

For 2 stable system in series, the τ_{eff} is stable

A total system can be causal but not the individual system

If sys is Parallel, you sum.

Summation Index Shifting

$\sum_{k=1}^{2} a^k \to \sum_{k=0}^{1} a^{k+1}$ — $\sum_{k=1}^{2} a^k \to \sum_{k=2}^{2} a^{k-1}$

$\sum_{k=0}^{n-1} a^k = \frac{1-a^n}{1-a}$ — $\sum_{k=0}^{\infty} a^k = \frac{1}{1-a}$

Fourier Series

$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jkw_o t}$ — $a_k = \frac{1}{T}\int_T x(t)e^{-jkw_o t}$

Time shift: x(t-τ) is —$e^{-jkw_o\tau a_k}$

Time reversal: x(-t) is —a_{-k}

Time Scaling: x(at), a>0 (periodic $\frac{T}{a}$) is —a_k

Conjugation: x*(t) is —a^*_{-k}

Symmetry x(t) real is —$a_k = a^*_{-k}$

Differentiation $\frac{d}{dt}x(t)$ is — $jkw_o a_k$

Integration $\int_{-\infty}^{t} x(t)dt, a_0 = 0$ is — $\frac{a_k}{jkw_0}$

Convolution $\int_T h(\tau) \circledast x(t-\tau)d\tau$ is —Ta_kb_k

Multiplication x(t)y(t) is —$\sum_{m=-\infty}^{\infty} a_m b_{k-m}$

Parseval $\frac{1}{T}\int_T |x(t)|^2 dt$ is —$\sum_{k=-\infty}^{\infty} |a_k|^2$

Fourier series only exist if the signal is a continuous

Fourier Transform

$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(jw)e^{jwt}dw$	$X(jw) = \int_{-\infty}^{\infty} x(t)e^{-jwt}dt$
Time Scaling x(at)	$\frac{1}{\lvert a\rvert}X(jw/a)$
Differentiation $\frac{d}{dt}x(t)$	$jwX(jw)$
Integration $\int_{-\infty}^{t} x(t)dt$	$\frac{1}{jw}X(jw) + \pi X(0)\delta(w)$
Multiplication x(t)y(t)	$\frac{1}{2\pi}\int_{-\infty}^{\infty} X(ju)Y(jw-ju)du$
Delta $\delta(t)$	1
One 1	$2\pi\delta(w)$
Exponent e^{jw_ot}	$2\pi\delta(w-w_0)$
Cosine $\cos(w_0t)$	$\pi[\delta(w-w_o)+\delta(w+w_0)]$
Sin $\sin(w_0t)$	$\frac{\pi}{j}[\delta(w-w_o)+\delta(w+w_0)]$
Unit Step u(t)	$\frac{1}{jw}+\pi\delta(w)$
Decaying Step $u(t)e^{-at}, a>0$	$\frac{1}{a+jw}$
Parseval $\int_{-\infty}^{\infty}\lvert x(t)\rvert^2dt$	$\frac{1}{2\pi}\int_{2\pi}\lvert X(jw)\rvert^2dw$

A BIBO LTI → z-trans's Roc contain unit circle

General Notes

Shift, Flip, Scale, break down signal to even and odd to find transform. $x_e(n) = \frac{x(n)+x(-n)}{2}$, $x_o(n) = \frac{x(n)-x(-n)}{2}$

$e^{j\pi} = -1$ $e^{j2\pi} = 1$ $e^{j\frac{\pi}{2}} = j$ $e^{j\frac{3\pi}{2}} = -j$

If x(n) is real and even, then the $\angle H(W)$ is at 0 if $X(w) \ge 0$ and it would be at π if $X(w) \le 0$

$\int_{-\pi}^{\pi} X(w)dw = 2\pi x(0)$

Frequency Response

Given $y(n) + Ay(n-1) + \cdots = x(n) + Bx(n-1) + \cdots$

$H(Z) = \frac{Y(Z)}{X(Z)}$ $Z = e^{jw}$ Use Z-transform then sub the exponential back in to solve further then.

The impulse response is the inverse transform of H(Z)

Once the frequency response is known, then $y(n) = H(w_0)x(w_0)$

In DTFT, w in the X(w) is a real value.

In Z-transform, w in the X(w) is a complex value.

An LTI system only exist if $Y(w)$ is defined at region where X(w) is defined. In other words, if Y(w) is found to be non-zero at w_0 where $X(w_0)$ is zero, then it is not a valid LTI System. → For $H(w_0) \ne 0$, $Y(w_0)$ must $= 0$.

For difference eq, use Z then e^{jw}.

Exponential to Sinusoidal

$cos(x)$	$\frac{e^{jx}+e^{-jx}}{2}$
$sin(x)$	$\frac{e^{jx}-e^{-jx}}{2j}$
e^{jx}	$cos(x)+jsin(x)$

z-transform.

$x(n) = -u(-1-n)$

↑ LEFT signal.

Pole:

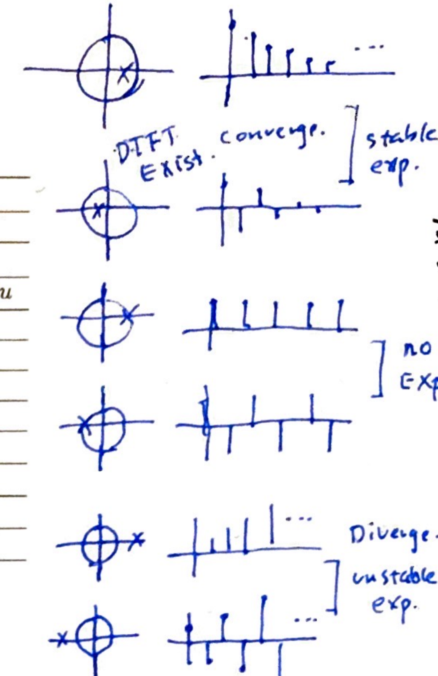
$\alpha \in \mathbb{R}$ → pure exp.

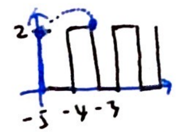
$|\alpha| > 1$ → explode

$|\alpha| < 1$ → converge to 0.

$\alpha \in \mathbb{C}$ → osillatory sig.

$|\alpha| = 1$ → $|a^n| = 1$





$T = 10^{-6}s$ (t_{on})

$N = 2T = 2\cdot10^{-6}s$ {total p}

$a_K = \frac{1}{2T}\int_T 2\cdot e^{-j\frac{2\pi}{2T}Kt}dt = \frac{1}{j\pi K}(1-e^{-j\pi K})$ $K \ne 0$.

$= \frac{1}{2T}X(\omega)\Big|_{\omega=\frac{\pi K}{T}}$

$a_0 = \frac{1}{2T}\int_0^{T} 2\,dt = 1$

$\boxed{b_k = a_k H\left(\frac{\pi k}{T}\right)}$.

a_k need both condition!

$h(n) = \text{Sinc}^2(t/T) = \mathcal{F}(\text{sinc}) * \mathcal{F}\{\text{sinc}\}$ $\frac{1}{2\pi}$ $w_c = \frac{1}{T}$

$\text{sinc}\, t/T = \frac{\sin(t/T)}{t/T} \cdot \frac{\pi}{\pi} = \underbrace{\frac{\sin(t/T)}{\pi t}}_{\text{good Form}} \underbrace{T\pi}_{\text{scalar}} \leftrightarrow \begin{cases} T\pi, & |w| \le w_c \\ 0, & \text{else} \end{cases}$

The height of Triangle is $T\pi \cdot T\pi \cdot \frac{1}{2\pi} \cdot \frac{1}{T}$.

$|z|$ must be $\{\mathbb{C}\} - \{0\} - \{\pm\infty\}$.

for finite ~~Single~~ value.

{ . . . . }

← Both → Sided signal

I am aware of the Academic Integrity Policy. I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

Aiden Chen

Exam 1

10/8/20.

1.) DTFT.

$$y(n) = x(n-1)\cos^2\left(\frac{\pi}{8}(n+1)+\frac{\pi}{4}\right).$$

a.) Linear? Yes.

b.) TI? No.

c.) Causal? Yes.

d.) find $y(n)$ if $x(n) = \cos(\frac{\pi}{8}n)$ $y(n) = \cos(\frac{\pi}{8}(n-1))\cos^2(\frac{\pi}{8}(n+1)+\frac{\pi}{4})$

a.) $x_1(n) \rightarrow y_1(n) = x_1(n-1)\cos^2(\frac{\pi}{8}(n+1)+\frac{\pi}{4})$

$x_2(n) \rightarrow y_2(n) = x_2(n-1)\cos^2(\frac{\pi}{8}(n+1)+\frac{\pi}{4})$

$x_3 = x_1 + x_2 \rightarrow y_3(n) = x_3(n-1)\cos^2(\frac{\pi}{8}(n+1)+\frac{\pi}{4})$

$= [x_1(n-1) + x_2(n-1)]\cos^2(\frac{\pi}{8}(n+1)+\frac{\pi}{4})$

= Linear!

b.) $x_1(n) \xrightarrow{\text{shift}} x_1(n-n_0) \xrightarrow{T} y(n) = x(n-n_0-1)\cos^2(\frac{\pi}{8}(n+1)+\frac{\pi}{4})$

$x_1(n) \xrightarrow{T} x(n-1)\cos^2(\frac{\pi}{8}(n+1)+\frac{\pi}{4}) \rightarrow x(n-1-n_0)\cos^2(\frac{\pi}{8}(n-n_0+1)+\frac{\pi}{4})$.

c.) $y(1) = x(1-1) = x(0)$

$y(0) = x(-1)$

$y(-1) = x(-2)$.

2.) Z - transform.

$$H(z) = \frac{1-3z^{-1}+6z^{-2}-4z^{-3}}{\cancel{(1-z^{-1})}(1-0.4z^{-1})(1-0.8z^{-1})}$$ Roc: $\alpha < |z| < b$.

a.) Plot the Pole ZERO Diagram., Include ROC.

Poles:

$$\boxed{\begin{array}{l} z = 0.4 \\ z = 0.8 \end{array}}$$

$1 = 0.4z^{-1}$ $1 = 0.8z^{-1}$

$z = .4$ $z = 0.8$

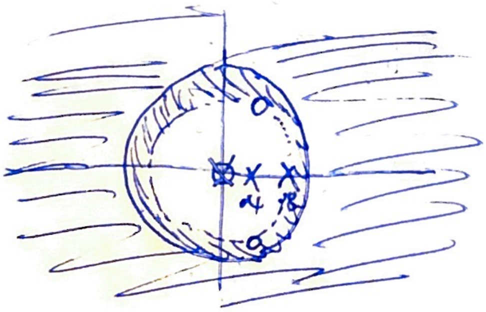
$\mathrm{Roc}_{①}: |z| > 0.8$

ZEROS: $1 - 3z^{-1} + 6z^{-2} - 4z^{-3} = 0$

$(1 - z^{-1})(1 + -2z^{-1} + 4z^{-2}) = 0$

$\cancel{(1-z^{-1})}(z^{-1} - (\frac{1}{4} \pm j\frac{\sqrt{12}}{4})) = 0$

1	1	-3	6	-4
	↓	1	-2	4
	1	-2	4	0



Roc:

$|z| > 0$.

$1 - 2x + 4x^2$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\frac{\cancel{2} \pm j\sqrt{12}}{\cancel{2}(4)}$

$$\boxed{\text{ZERO: } \frac{1}{4} \pm j\sqrt{12}/4}$$

$p_0 = \frac{1}{4} + j\sqrt{12}/4.$

$p_0^* = \frac{1}{4} - j\frac{1}{4}\sqrt{12}$

$= \sqrt{\frac{1}{4}^2 + (\frac{1}{4}\sqrt{12})^2}$

$= \frac{1+12}{16} = \frac{13}{16}.$ >

2.) Z - transform.

$$H(z) = \frac{1-3z^{-1}+6z^{-2}-4z^{-3}}{(\cancel{1-z^{-1}})(1-0.4z^{-1})(1-0.8z^{-1})} \qquad \text{Roc: } a < |z| < b.$$

a.) Plot the Pole zero Diagram, Include ROC.

Poles:

$$\boxed{\begin{aligned} z &= 0.4 \\ z &= 0.8 \end{aligned}}$$

$1 = 0.4z^{-1}$ $\quad$ $1 = 0.8z^{-1}$

$z = .4$ $\quad$ $z = 0.8$

$Roc_{①}: |z| > 0.8$

ZEROS:

$$1 - 3z^{-1} + 6z^{-2} - 4z^{-3} = 0$$

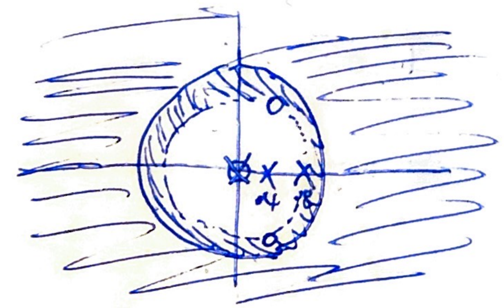
$$(1 - z^{-1})(1 + -2z^{-1} + 4z^{-2}) = 0$$

$$(\cancel{1-z^{-1}})(z^{-1} - (\tfrac{1}{4} \pm j\sqrt{12}/4)) = 0$$

1	1	-3	6	-4
	↓	1	-2	4
	1	-2	4	0

Roc:

$|z| > 0$.



$1 - 2x + 4x^2$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{\cancel{2} \pm j\sqrt{12}}{\cancel{2}(4)}$$

Total Roc: $\frac{8}{10} < |z| < \infty$.

$$\boxed{\text{ZERO: } \tfrac{1}{4} \pm j\sqrt{12}/4}$$

$p_0 = \frac{1}{4} + j\sqrt{12}/4$.

$p_0^* = \frac{1}{4} - j\frac{1}{4}\sqrt{12}$

$= \sqrt{\frac{1}{4}^2 + (\frac{1}{4}\sqrt{12})^2}$

$= \frac{1+12}{16} = \frac{13}{16}$. >

b.) find a and b.

$\alpha < |z| < b.$

$b = \infty$

$a = 0.8$

c.) System is stable b/c ROC contain unit circle.

d.) System is causal because ROC is outside of All the poles, there's no left sided signal.

e). find h(n).

$$\frac{B}{1-.4z^{-1}} + \frac{C}{1-0.8z^{-1}}$$

$$\Rightarrow B(1-z^{-1})(1-0.8z^{-1}) + C(1-z^{-1})(1-0.4z^{-1})$$

$$= (1-z^{-1})\left(z^{-1} - \left(\tfrac{1}{4} \pm j\tfrac{\sqrt{12}}{4}\right)\right)$$

$\Rightarrow z = .4 \Rightarrow$ $0 + B(1-4)(1-0.8)(4) + 0 = (1-4)(4 - (\tfrac{1}{4} \pm j\sqrt{12}/4)$

$$B = \frac{-3(4 - (\tfrac{1}{4} \pm j\sqrt{12}/4)}{-3(0.2)}$$

$\Rightarrow z = .8 \Rightarrow$ $0 + 0 + C(1-8)(1-0.4(8)) = (1-8)(8 - (\tfrac{1}{4} \pm j\sqrt{12}/4)$

$$C = \frac{-7(8 - (\tfrac{1}{4} \pm j\sqrt{12}/4)}{-7(1-(.4)(8))}$$

$$h(n) = 2\left(4 - \left(\frac{1}{4} + j\frac{\sqrt{12}}{4}\right)\right)\left(4 - \left(\frac{1}{4} - j\frac{\sqrt{12}}{4}\right)\right)(.4)^n u(n)$$

$$+ \frac{1}{(1-0.4(8))}\left(8 - \left(\frac{1}{4} + j\frac{\sqrt{12}}{4}\right)\right)\left(8 - \left(\frac{1}{4} - j\frac{\sqrt{12}}{4}\right)\right)(0.8)^n u(n)$$

$n > 0.$

f.) $h(n)$ is is not even because

$$h(-n) \neq h(n)$$

3a.) T/F.

T — LTI can be complety characterize by Impulse Response.

F — $X(n)\,\delta(n-1) = X(1)$

T. — $X(n) * \delta(n+1) = X(n+1)$.

T — $X(n)$ odd

T. — ROC point outward.

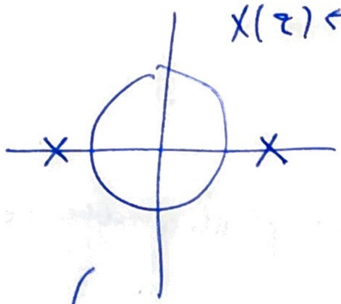
T — $X(n)$ Real, even, $X(\omega)$ is also even

F — DTFT of Rect $\rightarrow$ 

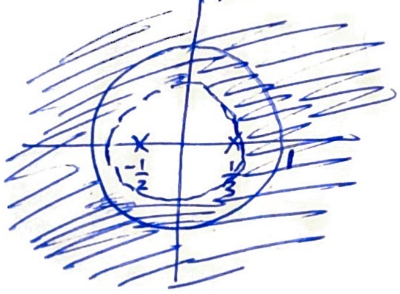
not a sinc waveform

3b.)

$X(z) \leftrightarrow x(n)$



$x(-n) \leftrightarrow X(z^{-1})$



$$\frac{1}{(z+2)(z-3)} = X(z) \leftrightarrow x(n)$$

$$\frac{1}{(z^{-1}+2)(z^{-1}-3)} \qquad X(z^{-1}) \leftrightarrow x(-n)$$

$z = -\frac{1}{2}$ $\qquad$ $z = \frac{1}{3}$

$|z| > -2$ $\qquad$ $|z| > 3.$

$$\frac{1}{2(z^{-1}\cdot\frac{1}{2}+1)\,3(\frac{1}{3}z^{-1}-1)}$$

$|z| > |-\frac{1}{2}|$ $\qquad$ $|z| > |\frac{1}{3}|.$

$$\boxed{ROC: |z| > \frac{1}{3}}$$

3c.) find $x(n)$ where $X(w) = \dfrac{1}{(1-\frac{1}{2}e^{-jw})^2}$

$$X(w) = \frac{1}{1 - e^{-jw} + \frac{1}{4}e^{-jw2}} \qquad z = e^{jw}$$

$$= \frac{1}{1 - z^{-1} + \frac{1}{4}z^{-2}}$$

$$X(z) = \frac{1 - \frac{1}{2}z^{-1}}{1 - z^{-1} + \frac{1}{4}z^{-2}}$$

① $r = \frac{1}{2}$

② $2r\cos w_0 = 1$

$\cos w_0 = 1$

$w_0 = 0.$

$\cos(0) = \overset{\sin}{\cancel{\cos}}(90+0)$

③ $\cancel{r\sin w_0 = 1}$

$\cancel{\frac{1}{2}\sin(0) = 1}$

$\boxed{1 - r\cos(w_0)z^{-1} + A = 1}$

$\therefore A = +r\sin(\frac{\pi}{2}+w_0)z^{-1}$

$$X(z) = \frac{1}{1 - z^{-1} + \frac{1}{4}z^{-2}} + \frac{\frac{1}{2}z^{-1}}{1 - z^{-1} + \frac{1}{4}z^{-2}}$$

$$X(z) \longleftrightarrow x(n) = (\tfrac{1}{2})^n\cos(0)u(n) + (\tfrac{1}{2})^n\sin(\tfrac{\pi}{2}n)u(n).$$

3d).

$$x_1(n) = \{1,1,\underline{1},1,1\}$$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x_1(n) e^{-j\omega n}$$

$$= e^{+j2\omega} + e^{+j\omega} + e^{0} + e^{-j\omega} + e^{-2j\omega}$$

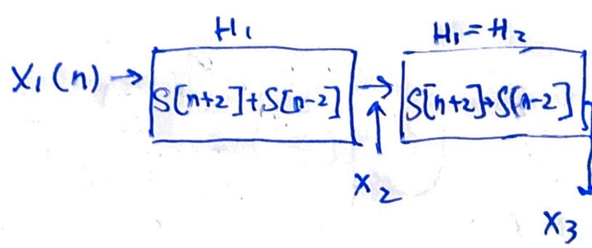
$$X(\omega) = 2\cos(2\omega) + 2\cos(\omega) + 1$$

$$x_2(n) = \{1,0,1,0,\underline{1},0,1,0,1\}$$

$$X_2(\omega) = e^{+j4\omega} + e^{+j2\omega} + 1 + e^{-j2\omega} + e^{-j4\omega}$$

$$= 2\cos(4\omega) + 2\cos(2\omega) + 1$$

$$X_3(\omega) = \{1,0,0,1,0,0,\underline{1},0,0,1,0,0,1\}$$



$$= 2\cos(6\omega) + 2\cos(3\omega) + 1$$

$$X_2(\omega) = X_1(\omega)\left[S[\omega-2] + S[\omega+2]\right]$$

$$X_3(\omega) = X_2(\omega)\left[S[\omega-2] + S[\omega+2]\right]$$

$$X_3(\omega) = X_1(\omega)\left[S[\omega-4] + S[\omega+4]\right]$$ 10