9/6/20.

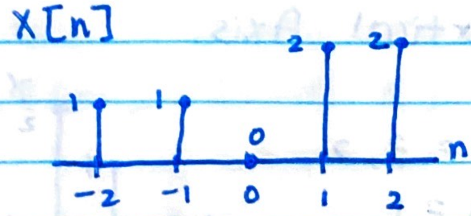
DSP. Analytical Problem HW #1 Aiden chen

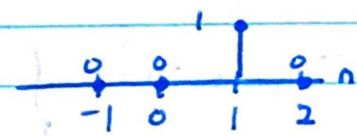
4). Consider the signal $x[n]$.

$$x[n] = \begin{cases} 1 & n = -2, -1 \\ 0 & n = 0 \\ 2 & n = 1, 2 \\ 0 & \text{else} \end{cases}$$

4a.) $y_1[n] = x[n] * \delta[n-1]$

$x[n]$ $\quad * \quad$ $\delta[n-1] = h[n]$





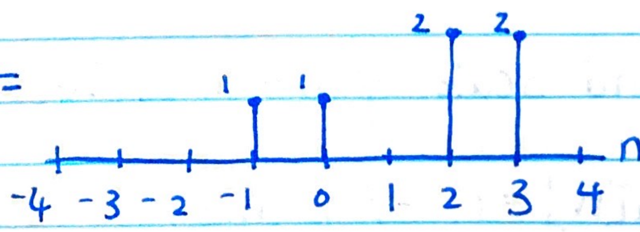
$$y_1[n] = x[n] * h[n]$$

$$= \sum_{k=-\infty}^{\infty} x[k] \cdot h[n-k]$$

$$= \sum_{k=-2}^{-1} 1 \cdot h[n-k] + \sum_{k=1}^{2} 2 \cdot h[n-k]$$

$$= h[n-(-2)] + h[n-(-1)] + 2h[n-1] + 2h[n-2]$$

=



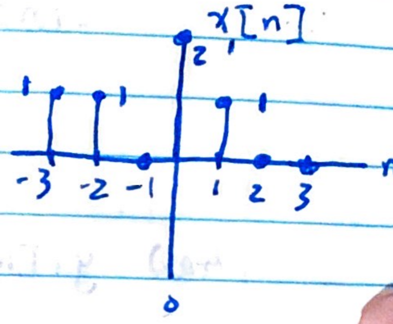
$$= \{1, \underset{\uparrow_0}{1}, 0, 2, 2\}$$

4b) $y_2[n] = -3x[-2n+1]$

steps. Shift $x[n]$ → Flip → Scale → Fold.

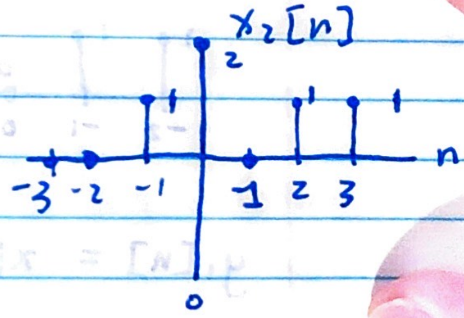
1. Shift $x[n]$ 1 to the left

$$x_1[n] = x[n+1] = \begin{cases} 1, & n=-3,-2 \\ 0, & n=-1 \\ 2, & n=0,1 \\ 0, & \text{else} \end{cases}$$



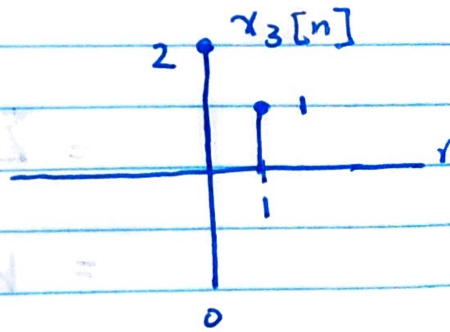
2.) Flip $x[n]$ across vertical Axis.

$$x_2[n] = x_1[-n] = \begin{cases} 1, & n=3,2 \\ 0, & n=1 \\ 2, & n=0,-1 \\ 0, & \text{else} \end{cases}$$



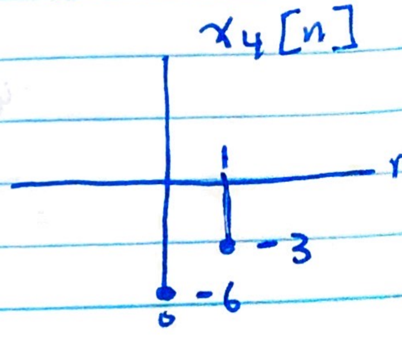
3.) Scale $x[n]$ by 2.

$$x_3[n] = x_2[2n] = \begin{cases} 1, & n=1 \\ 2, & n=0 \\ 0, & \text{else.} \end{cases}$$



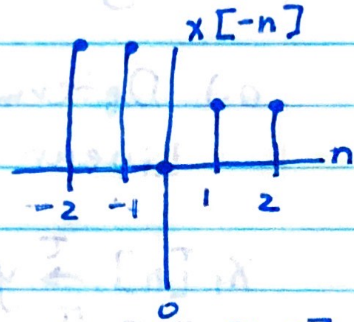
4.) Fold and Scale up Amplitude by -3.

$$x_4[n] = -3x_3[n] = \begin{cases} -3, & n=1 \\ -6, & n=0 \\ 0, & \text{else} \end{cases}$$

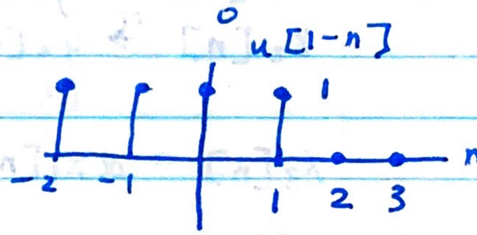


4c.) $y_3[n] = x[-n]\,u[1-n]$

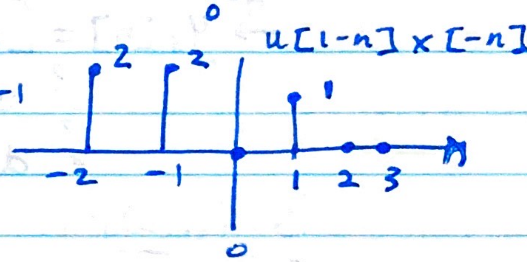
$$x[-n] = \begin{cases} 1, & n = 2, 1 \\ 0, & n = 0 \\ 2, & n = -1, -2 \\ 0, & \text{else} \end{cases}$$



$$u[1-n] = \begin{cases} 1, & n \leq 1 \\ 0, & \text{else} \end{cases}$$



$$x[-n]\,u[1-n] = \begin{cases} 2, & n = -2, -1 \\ 0, & n = 0 \\ 1, & n = 1 \\ 0, & \text{else.} \end{cases}$$

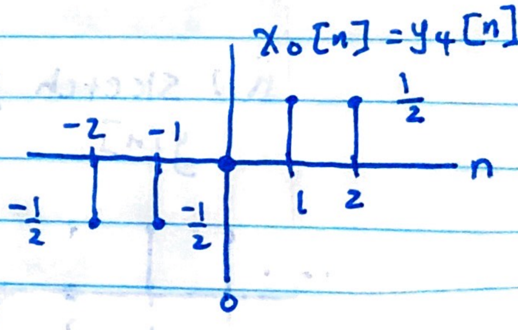


4d.) $y_4[n] = \mathrm{Odd}(x[n])$

$$x_o[n] = \mathrm{odd}(x[n]) = \tfrac{1}{2}(x[n] - x[-n])$$
$$= \tfrac{1}{2}x[n] - \tfrac{1}{2}x[-n]$$

$$\tfrac{1}{2}x[n] = \begin{cases} \tfrac{1}{2}, & n = -2, -1 \\ 0, & n = 0 \\ 1, & n = 1, 2 \\ 0, & \text{else} \end{cases} \qquad \tfrac{1}{2}x[-n] = \begin{cases} 1/2, & n = 2, 1 \\ 0, & n = 0 \\ 1, & n = -1, -2 \\ 0, & \text{else} \end{cases}$$

$$x_o[n] = \begin{cases} \tfrac{1}{2} - 1 = -\tfrac{1}{2}, & n = -2, -1 \\ 0 - 0, & n = 0 \\ 1 - \tfrac{1}{2} = \tfrac{1}{2}, & n = 1, 2 \\ 0, & \text{else.} \end{cases}$$



5) $y[n] = x[n^2]$

a.) Determine whether System is ______ Linear and Time-invariant.

$$x_1[n] \xrightarrow{T} y_1[n] = x_1[n^2]$$
$$x_2[n] \xrightarrow{T} y_2[n] = x_2[n^2]$$

$$x_3[n] = \alpha x_1[n] + b x_2[n]$$

$$\xrightarrow{T} y_3[n] = [\alpha x_1[n^2] + b x_2[n^2]]$$
$$\stackrel{?}{=} \alpha y_1[n] + b y_2[n]$$
$$\stackrel{?}{=} \alpha x_1[n^2] + b y_2[n^2]$$

is linear

Time-Invariant

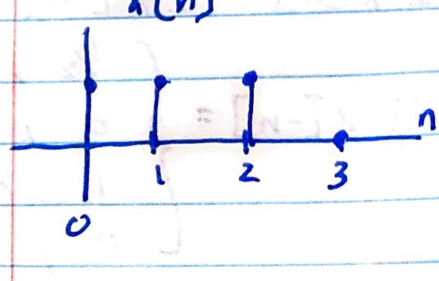
$$x_1[n] \to y_1[n] = x_1[n^2]$$
$$\to y_2[n] = x_1[(n-n_0)^2]$$

$$x_2[n] \to y_1[n] = x_2[n-n_0]$$
$$\to y_2[n] = x_2[n^2-n_0]$$
$$\neq x_1[(n-n_0)^2]$$

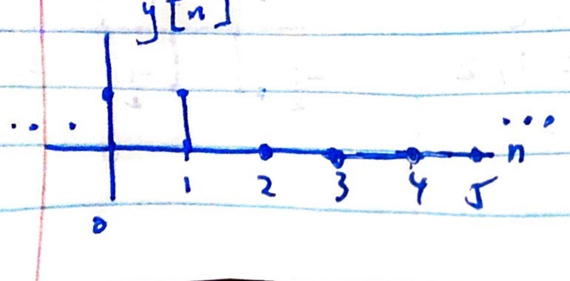
TI fail!

5 b.). $x[n] = \begin{cases} 1 & 0 \leq n \leq 2 \\ 0 & \text{else.} \end{cases}$

i.) sketch $x[n]$



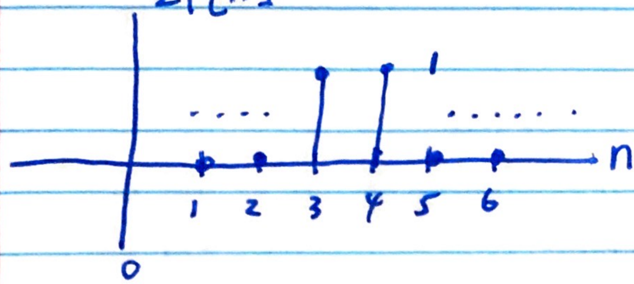
ii.) Sketch $y[n] = x[n^2]$



$$x[n^2] = \begin{cases} 1 & 0 \leq n \leq 1 \\ 0 & \text{else} \end{cases}$$

iii Sketch $z_1[n] = y[n-3]$

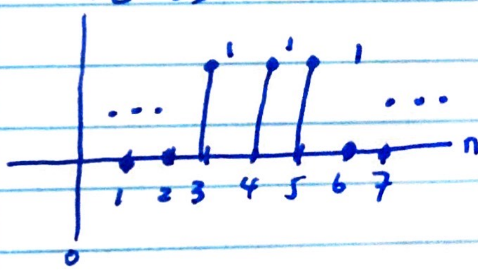
$z_1[n]$



iv.) Determine and sketch $x[n-3]$.

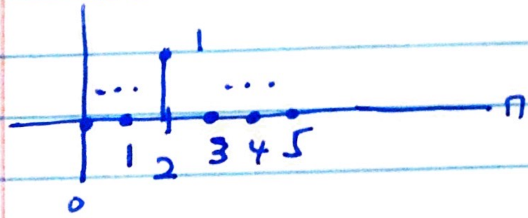
$$x[n-3] = \begin{cases} 1 & 3 \le n \le 5 \\ 0 & \text{else.} \end{cases}$$

$x[n-3]$



v.). $x[n-3] \xrightarrow{\tau} z_2[n] = \begin{cases} 1, & n = 2 \\ 0, & \text{else} \end{cases}$

$z_2[n]$



vi.) The system is not time-invariant. At different time, input and output are not linear.

vii). $y[n]$ is not periodic

6.) Sketch and compute convolution.

a.) $x[n] = \begin{cases} 1, & n = -2, -1, 0, 1, 2. \\ 0, & \text{else} \end{cases}$ $\quad h[n] = x[n+2]$

$x[n+2] = \begin{cases} 1; & n = -4, -3, -2, -1, 0 \\ 0, & \text{else.} \end{cases}$

$\uparrow$

n..

↳ $=1$ for $n < -2$.

$y[n] = x[n] * h[n]$

$= \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$

$= \sum_{k=-2}^{2} 1 \cdot h[n-k]$

$= h[n-(-2)] + h[n-(-1)] + h[n-(0)] + h[n-1] + h[n-2]$

=	-6	-5	-4	-3	-2	-1	0	1	2
1	1	1	1	1	1				
2		1	1	1	1	1			
3			1	1	1	1	1		
4				1	1	1	1	1	
5					1	1	1	1	1
+	1	2	3	4	5	4	3	2	1

$y[n] = \{1, 2, 3, 4, 5, 4, \underset{\uparrow 0}{\underline{3}}, 2, 1\}$

6b.)

$x[n] = u[n]$, $h[n] = (\frac{1}{4})^n u[n-2]$

$\cancel{y[n] = x[n] * h[n]}$

$\cancel{= \sum_{k=-\infty}^{\infty} h[k] x[n-k]}$

$\cancel{= \sum_{k=-\infty}^{\infty} (\frac{1}{4})^k u[k-2] u[n-k]}$

$u[k-2]=0$

$k<2$

$\cancel{y[n] = \sum_{k=2}^{n} (\frac{1}{4})^k}$

$h[n-k]=0$

$n-k<0.$

$n<k$

$y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k]$

$= \sum_{k=-\infty}^{\infty} u[k] (\frac{1}{4})^{n-k} u[n-2-k]$

$= \sum_{k=0}^{n-2} (\frac{1}{4})^{n-k}$

$= (\frac{1}{4})^n \sum_{k=0}^{n-2} 4^k$ $\quad \forall n \in \{\mathbb{Z}\} - \{n > 2\}$

$= (\frac{1}{4})^n \sum_{k=2}^{n} 4^k - 4^0 - 4^1$

$= (\frac{1}{4})^n \left[\sum_{k=2}^{n} 4^k - 1 - 4\right]$

$= (\frac{1}{4})^n \sum_{k=2}^{n} 4^k - (\frac{1}{4})^n 5$