**Rensselaer Polytechnic Institute**
**Department of Electrical, Computer, and Systems Engineering**
**ECSE 4530: Digital Signal Processing, Fall 2020**

Homework #4: due Thursday, Nov. $5^{\text{th}}$, at the beginning of class.

**MATLAB Grader Problems**

A full description of all the Grader problems is provided at the Grader link. You don't need to physically hand anything in for the Grader problems (but make sure that you hit Submit and see a green check mark to make sure your solution has been recorded). At this point you should be taking advantage of some of Matlab's vector and matrix-based syntax.

1. (10 points). Directly compute the DFT via multiplication by the Fourier matrix.
2. (20 points). Compute a DFT whose length is a power of 2 by recursively using the radix-2 FFT.

**Analytical Problems**: Clearly show your work and label your answers.

3. (20 points) **Discrete Fourier Transform (DFT).** Compute the $N$-point DFTs of the following signals:

   (a) $x[n] = \delta[n]$

   (b) $x[n] = \delta[n - n_0]$, where $0 < n_0 < N$

   (c) $x[n] = a^n$, where $0 \le n \le N-1$

   (d) $x[n] = \begin{cases} 1, & 0 \le n \le N/2-1, \\ 0, & N/2 \le n \le N-1 \end{cases}$, where $N$ is even

   (e) $x[n] = e^{j\frac{2\pi}{N}k_0 n}$, where $0 \le n \le N-1$

   (f) $x[n] = \cos\left(\frac{2\pi}{N}k_0 n\right)$, where $0 \le n \le N-1$

   (g) $x[n] = \sin\left(\frac{2\pi}{N}k_0 n\right)$, where $0 \le n \le N-1$

   (h) $x[n] = \begin{cases} 1, & n \text{ even}, \\ 0, & n \text{ odd} \end{cases}$, where $0 \le n \le N-1$

4. (10 points) **DFT Matrix.** Recall the definition of DFT which is given as

$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{kn},$$

   where $W_N^k$ for $k = 0, \ldots, N-1$ are called the $N^{\text{th}}$ roots of unity.

   (a) Plot the roots of unity for $N = 2$, $N = 4$, and $N = 8$ in the complex plane.

   (b) Write down the $N \times N$ complex DFT matrix $F$ for $N = 2$ and $N = 8$ in terms of $W_N$'s.

5. (20 points) **Fast Fourier Transform (FFT).** Recall (from Lecture 14) that the DFT summation can be split into sums over the odd and even indexes of the input signal. The splitting into sums over even and odd time indexes is called decimation in time.

   In this problem we consider a slightly different approach. Assume that you are given three 8-point FFT chips and asked to build a system that computes a 24-point DFT. Show explicitly how you should interconnect those chips to be able to compute a 24-point DFT.

6. (20 points) **Computation of the DFT using FFT. Write a FFT subroutine in Matlab to compute the following DFTs $X[k]$ and plot their magnitudes $|X[k]|$.**

   (a) The $N = 64$ point DFT of length 16 sequence $x[n] = \begin{cases} 1, & n = 0, 1, \ldots, 15, \\ 0, & \text{otherwise.} \end{cases}$

   (b) The $N = 128$ point DFT of length 16 sequence in the previous part,

   (c) The $N = 64$ point DFT of length 8 sequence $x[n] = \begin{cases} 1, & n = 0, 1, \ldots, 7, \\ 0, & \text{otherwise.} \end{cases}$

   (d) The $N = 64$ point DFT of length 64 sequence $x[n] = \begin{cases} 10e^{j\frac{\pi}{8}n}, & n = 0, 1, \ldots, 63, \\ 0, & \text{otherwise.} \end{cases}$

   Answer the following questions for each of the above parts.

   i What is the frequency interval between successive samples for the plots in parts (a)-(d)?

   ii What is the value of the spectrum at 0 frequency obtained from the plots in parts (a)-(d)?

   iii What is the frequency interval between successive nulls in the spectrum in parts (a)-(d)? What is the relationship between this interval (between the successive nulls) and the length of each signal?

   iv What is the difference between the plots obtained for (a) and (b)? Explain.

**Suggested reading material from textbook**: Section 7.1, 7.2 (DFT), Section 8.1.3 (FFT), along with the Examples and their solutions.

**Suggested practice problems from textbook**: 7.5, 7.11, 7.18, 7.20, 7.23, 8.1, 8.2, 8.13.