Rensselaer Polytechnic Institute
Department of Electrical, Computer, and Systems Engineering
ECSE 4530: Digital Signal Processing, Fall 2020

Exam #2.

November 16, 2020, 10:10-11:30 AM

Show all work for full credit.

- Closed book, closed notes.
- 1 two-sided (or 2 one-sided) crib sheet is allowed.
- Electronic devices (including calculators) are not permitted.
- Reduce expressions involving exponentials, sines, and cosines as much as possible.
- If the sinc function is needed, use the definition $\text{sinc}(x) = \frac{\sin x}{x}$.
- Geometric series formula: $\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}, \quad |a| < 1$.
- Finite sum formula: $\sum_{n=M}^{N-1} a^n = \frac{a^M - a^N}{1-a}, \quad a \neq 1$.
- When in doubt, show your work.

Good luck!

1		**25**
2		**30**
3		**15**
4		**30**
Total		**100**

Append the below phrase into your handwritten solutions and upload a single pdf file that includes your handwritten crib sheets to Gradescope.

I am aware of the Academic Integrity policy. I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

Name

Signature

1. (25 points) **Filter design using the z-transform.** We are given a transfer function for a linear and time invariant discrete-time system:

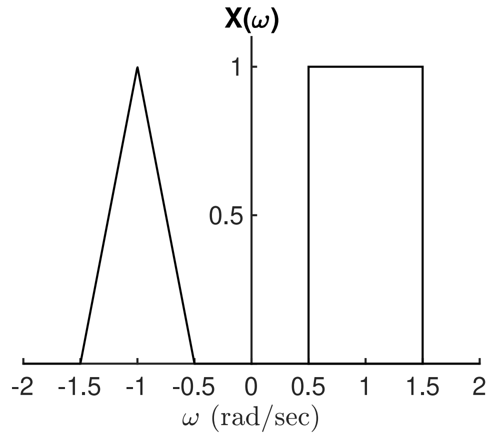
$$H(z) = \frac{z^2 - 1}{z^2 - z + 0.5}$$

   (a) (10 points.) Give the pole-zero diagram of $H(z)$.

   (b) (10 points.) Using the pole-zero diagram, plot the magnitude of the frequency response $|H(\omega)|$ for $\omega \in (-\pi, \pi)$. Indicate the values of $|H(0)|$, $|H\left(\frac{\pi}{2}\right)|$ and $|H\left(-\frac{\pi}{2}\right)|$.

   (c) (5 points.) What type of digital filter is $H(\omega)$? (Low pass, high pass, band pass) Explain your answer.

2. (30 points) **Sampling.** We consider a continuous time signal $x(t)$ with a Fourier transform $X(\omega)$ as shown below.



   (a) (2 points.) What is the Nyquist rate of $x(t)$?

   (b) (2 points.) Is $x(t)$ real or complex valued?

   (c) (6 points.) $x(t)$ is sampled with a sampling period of $T = \frac{2\pi}{4}$ seconds. Call the resulting signal $x[n]$. Plot the Fourier transform (DTFT) of $x[n]$ in the frequency range $\omega \in [-4\pi, 4\pi]$.

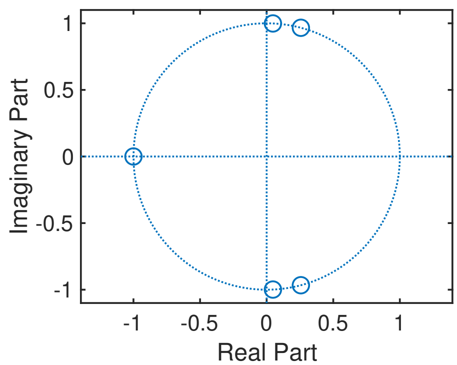
   (d) (10 points.) Now assume that we want to compute a length $N = 4$ DFT of $x[n]$, i.e., $X[k]$, by applying radix-2 decimation in time (DIT) FFT (Fast Fourier Transform) algorithm on where the input to the block diagram is the polyphase components of $x[n]$, i.e., $e_0[n]$ and $e_1[n]$. Sketch the block diagram of the DIT FFT implementation and indicate the branch gains, inputs and outputs.

   (e) (10 points.) Now assume that we upsample $x[n]$ by a factor of $L = 2$ and denote the resulting signal by $y[n]$. We convert the upsampled signal $y[n]$ from discrete time to continuous time $y(t)$ using a sampling period of $T_1 = \frac{\pi}{4}$. We then reconstruct a new continuous time signal $z(t)$ by interpolating $y(t)$ with an ideal low pass filter with cutoff $\omega_c = 2$ and gain $L = 2$. Sketch the block diagrams to indicate the relationship between $x[n]$ and $z(t)$. Plot the Fourier transforms of $y(t)$ and $z(t)$.

3. (15 points.) True/false and fill in the blank questions. You do not need to explain your reasoning. Read each question carefully.

   1. ______ Circular (or cyclic) convolution always produces the same result as linear convolution.
   2. ______ A length 8 Discrete Fourier Transform (DFT) computation requires 64 multiplications.
   3. ______ The twiddle factors (W_N^{nk}, complex roots of DFT) are uniformly distributed on the unit circle.
   4. ______ Prefiltering is always required in downsampling.

5. ______ We can compute the inverse DFT of a length 4 signal using 2 stages of butterflies where each butterfly computes a length 2 DFT.
6. ______ We can change the sampling rate by a non-integer factor by cascading interpolator and decimator operations.
7. ______ For a length N DFT, $W_N^{k\frac{N}{2}} = -1$ if k is odd.
8. ______ The complexity of a 8 point Decimation in Time (DIT) FFT algorithm is twice the complexity of a 4 point DIT FFT algorithm.
9. ______ The zeros of a real Finite Impulse Response (FIR) linear phase filters can look like below.



10. Equivalent systems of polyphase representations provide computational savings by filtering at ______________________ sampling rate.
11. Polyphase decomposition of a signal can be obtained via ______________________.
12. The purpose of sinc interpolation after upsampling is ______________________.
13. Linear interpolation is as good as sinc interpolation if ______________________.
14. If we have two length 8 signals $x[n]$ and $y[n]$ in the time domain, the length 15 Discrete Fourier Transform (DFT) of their linear convolution is the same as ______________________ ______________________ the zero padded signals $x[n]$ and $y[n]$ with ______ zeros.
15. Because of Covid-19 ______________________

____

______________________.

4. (30 points.) The parts of this problem are independent of each other. Read each question carefully. You can refer to the tables to verify your solutions.

   (a) (15 points.) **Downsampling and Upsampling.** Consider the two different ways of cascading a compressor $M = 2$ and an expander $L = 2$ as shown below. Show that $y_1[n]$ and $y_2[n]$ are different. Hint: You can give a counter example.

(b) (15 points.) **Discrete Fourier Transform (DFT) of a DFT.** Let $X[k]$ be the N-point DFT of the sequence $x[n]$, $0 \le n \le N-1$. What is the N-point DFT of the sequence $y[n] = X[n]$, $0 \le n \le N-1$?

(c) Make up exam question

(15 points.) **Discrete Fourier Transform (DFT).** Let $X[k]$ be the N-point DFT of the sequence $x[n]$, $0 \le n \le N-1$. We define a $2N$-point sequence $y[n]$ as

$$y[n] = \begin{cases} x\left[\frac{n}{2}\right], & \text{n even} \\ 0, & \text{n odd.} \end{cases}$$

Determine the $2N$-point DFT of $y[n]$ in terms of $X[k]$.