12/14/20

Review

$$H(z) = \frac{b_0 + b_1 z^{-1} + \dots + b_M z^{-M}}{1 + a_1 z^{-1} + \dots + a_N z^{-N}} = \frac{B(z)}{A(z)}$$

N can be very large.

$$H_d(z) = \frac{B(z)}{A(z)}$$

$$\underbrace{H_d(z)}\,\underbrace{A(z)} = \underbrace{B(z)}$$

$$h_d[0] + h_d[1] z^{-1} + h_d[2] z^{-2} + \dots$$

$$(h_d[0] + h_d[1] z^{-1} + h_d[2] z^{-2} \dots)(1 + a_1 z^{-1} + a_2 z^{-2} \dots + a_N z^{-N})$$

$$= h_d[0] + h_d[0] a_1 z^{-1} + h_d[1] z^{-1} + h_d[2] z^{-2} + h_d[0] a_2 z^{-2} + h_d[1] a_1 z^{-2} + \dots$$

$$= b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_M z^{-M}$$

$$\underbrace{\begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ \vdots \\ b_M \end{bmatrix}}_{b} = \underbrace{\begin{bmatrix} h_d[0] & 0 & 0 & \dots & 0 \\ h_d[1] & h_d[0] & 0 & \dots & 0 \\ h_d[2] & h_d[1] & h_d[0] & 0 \dots & 0 \\ & & & & \\ h_d[M] & h_d[M-1] & \dots & & h_d[M-N] \end{bmatrix}}_{A \ (M+1)\times(N+1)} \underbrace{\begin{bmatrix} 1 \\ a_1 \\ a_2 \\ \vdots \\ a_N \end{bmatrix}}_{N+1 \text{ vector}}$$

$a = A^{-1} b$ if A is square matrix that is invertible

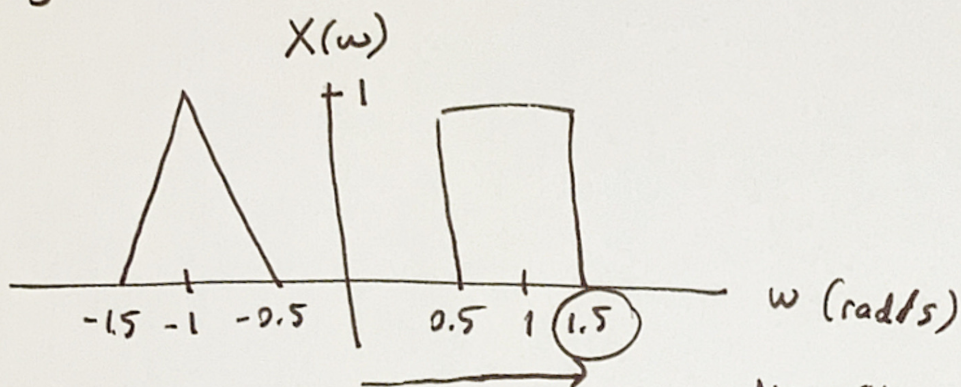
what if A is fat : $\begin{bmatrix} & A & \end{bmatrix}$

\# equations < \# unknowns

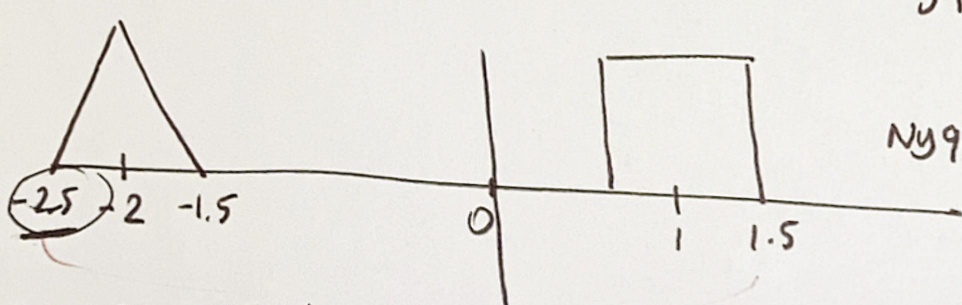
$\Rightarrow$ we need to zero pad the $\underline{b}$ vector

<u>Exam 2</u>

2. Sampling

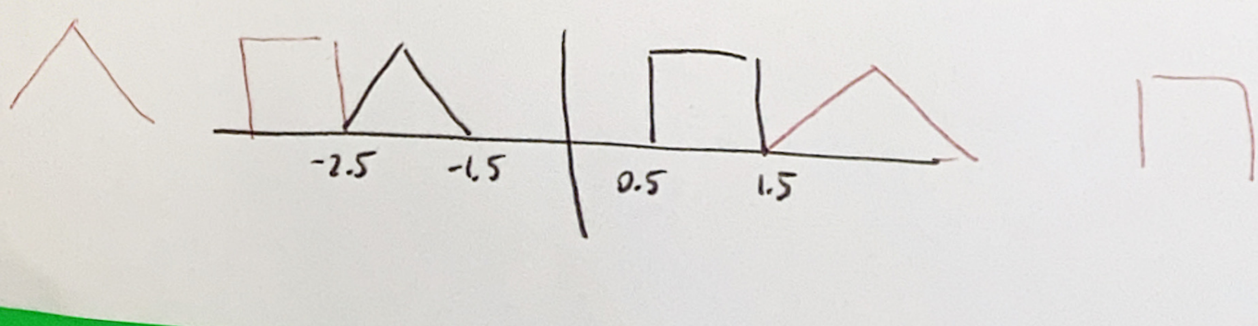


Nyquist rate = 3 rad/



Nyquist rate $\stackrel{?}{=}$ 5 rad/s

Nyquist rate = 4



$$X(\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt$$ CTFT

If we have a real signal $\boxed{x(t) = x^*(t)}$

$$X^*(\omega) = \int_{-\infty}^{+\infty} \underbrace{x^*(t)}_{x(t)}\, e^{j\omega t}\, dt = X(-\omega)$$

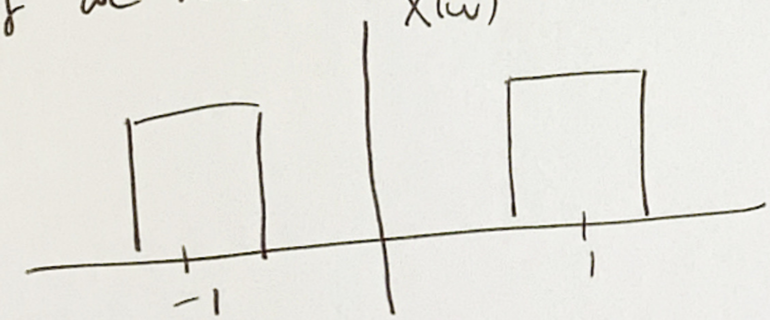
$$X^*(-\omega) = \int x(t)\, e^{-j\omega t}\, dt = X(\omega)$$

$$\boxed{X^*(-\omega) = X(\omega)}$$ → not satisfied for Problem 2.

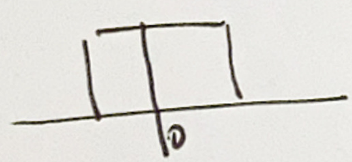
$X(\omega)$ is real in Exam 2.

$$X(\omega) = X^*(\omega)$$

If we had $X(\omega)$



$Y(\omega)$ → $X(\omega)$

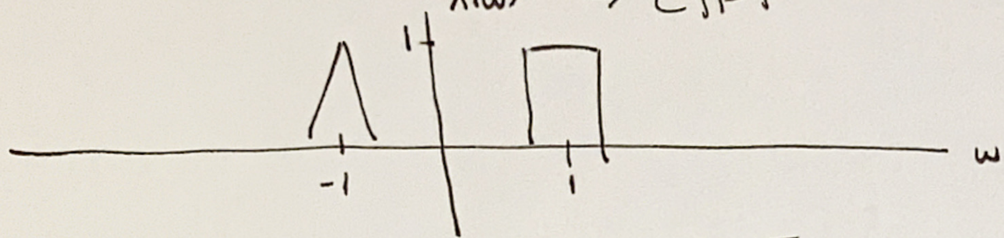


$y(t)\cos(t) = x(t)$

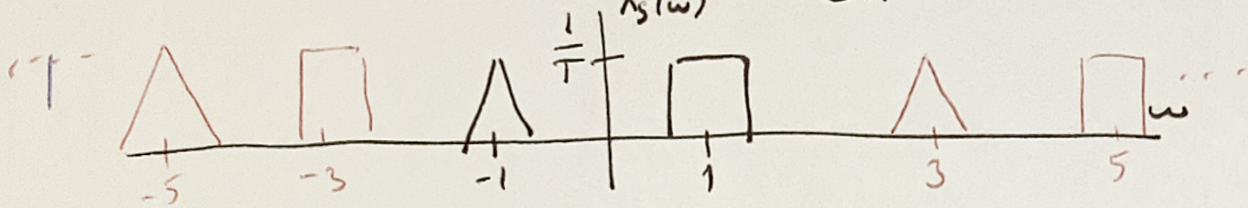
$x(t) \rightarrow x_s(t)$

$T = \frac{2\pi}{4}$, $w_s = \frac{2\pi}{T} = 4$

$X(\omega) \rightarrow$ CTFT

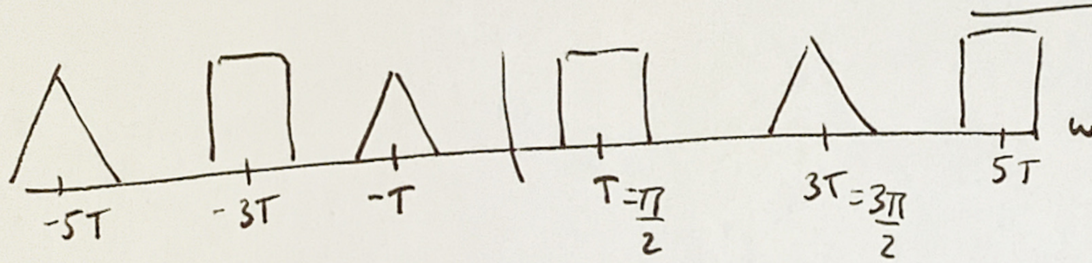


$X_s(\omega) \rightarrow$ CTFT



$X(\omega)$ DTFT

$\boxed{X(\omega) = X_s\left(\frac{\omega}{T}\right)}$

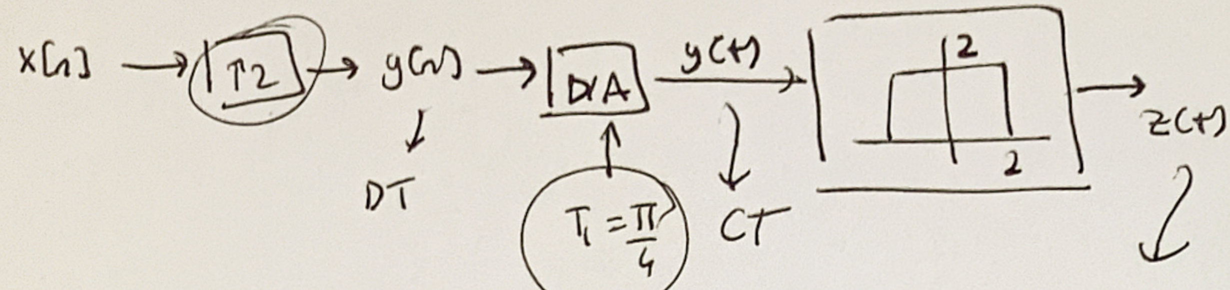


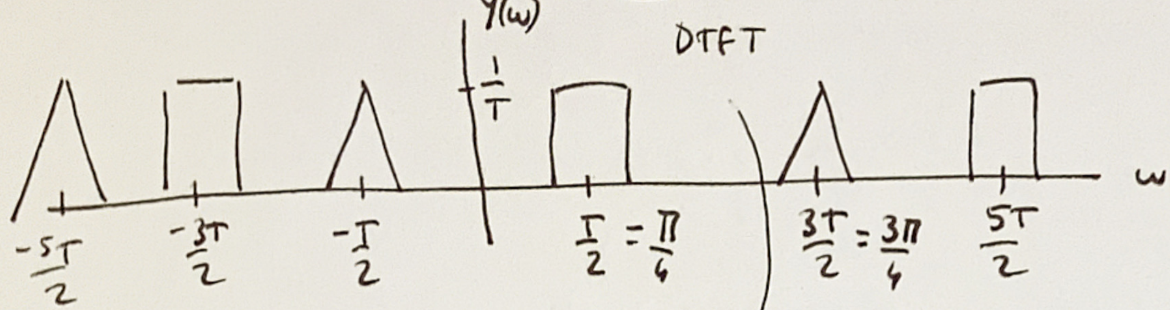
d) '4 point DFT' $X[0], X[1], X[2], X[3]$

$x[0], x[1], x[2], x[3]$

$x[n] \rightarrow \boxed{\downarrow 2} \rightarrow e_0[n]$

e)



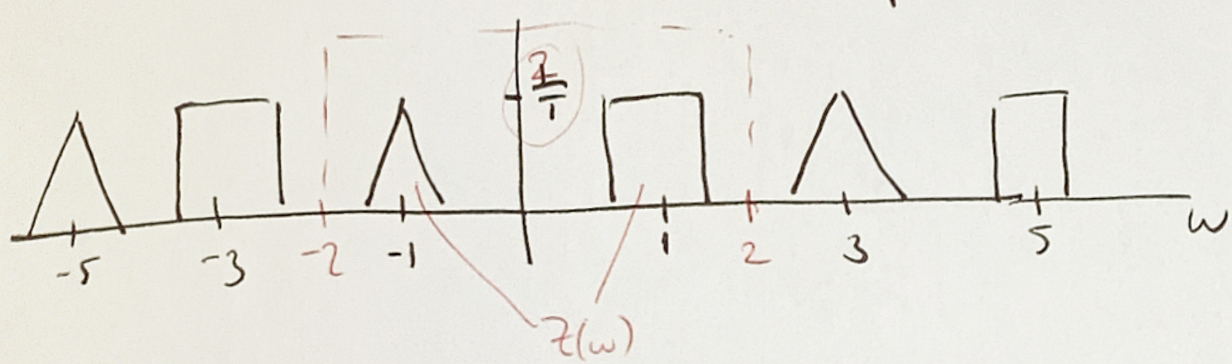


$\omega_{s_1} = \frac{2\pi}{T_1} = 8$, $\boxed{T_1 = \frac{\pi}{4}}$

DTFT

$Y_s(\omega)$ CTFT $Y(\omega) = Y_s\left(\frac{\omega}{T_1}\right)$

(divide freq. axis by T_1)



$Y_s(\omega) \rightarrow Y(\omega)$

Last year's final

deterministic

6/ e1. $\hat{h} = R_y^{-1} p$ (deterministic)

$$\text{MMSE} = E\left[\left(d[n] - \sum_{k=0}^{2} \hat{h}[k]\, y(n-k)\right)^2\right]$$

$$= E[d^2[n]] - \overbrace{2\sum_{k=0}^{2} \hat{h}[k]\, E[d[n]\, y(n-k)]}^{p^T \hat{h}}$$

$$+ \underbrace{\sum_{k=0}^{2} \sum_{l=0}^{2} \hat{h}[k]\, \hat{h}[l]\, E[y(n-k)\, y(n-l)]}_{\hat{h}^T R_y \hat{h}}$$

$$= E[d^2[n]] - 2p^T \underbrace{\hat{h}}_{R_y^{-1}p} + \underbrace{\hat{h}^T}_{(R_y^{-1}p)^T} R_y \underbrace{\hat{h}}_{R_y^{-1}p}$$

$$= E[d^2[n]] - 2p^T R_y^{-1} p + p^T R_y^{-1} \underbrace{R_y R_y^{-1}}_{I} p$$

$$= E[d^2[n]] - 2p^T R_y^{-1} p + p^T R_y^{-1} p$$

$$= E[d^2[n]] - p^T R_y^{-1} p = E[d^2[n]] - p^T (R_x + \sigma_w^2 I)^{-1}$$

* r[0] & r(0) are the same.

5 part d (Final last year)

$$x[n] = \underbrace{x_1[n]}_{\text{real}} + j\,\underbrace{x_2[n]}_{\text{real}}, \qquad n = 0, \ldots, 7$$

determine $X_1[k]$, $X_2[k]$ using DFT properties.

$$x[n] = x_1[n] + j\,x_2[n] \xrightarrow[\text{(linearity)}]{\text{DFT}} \boxed{\underbrace{X_1[k]}_{= X_1^*[-k]} + j\,\underbrace{X_2[k]}_{= X_2^*[-k]} = X[k]}$$

$$\boxed{X[-k] = X_1[-k] + j\,X_2[-k]}$$

$$= X_1^*[k] + j\,X_2^*[k]$$

- $\underline{X[k] + X[-k]} = \underbrace{(X_1[k] + X_1^*[k])}_{\text{real}} + j\,\underbrace{(X_2[k] + X_2^*[k])}_{\text{real}}$
- $X[k] - X[-k] = \underbrace{(X_1[k] - X_1^*[k])}_{\text{imaginary}} + j\,\underbrace{(X_2[k] - X_2^*[k])}_{\text{imaginary}}$

$$X_1[k] = a + jb \qquad X_1[k] + X_1^*[k] = 2a$$

$$X_1^*[k] = a - jb \qquad X_1[k] - X_1^*[k] = 2jb.$$

$$\text{Real}\{X[k] + X[-k]\} + \text{Imag}\{X[k] - X[-k]\} = 2X_1[k] ✓$$

$$\text{Real}\{X[k] - X[-k]\} + \text{Imag}\{X[k] + X[-k]\} = 2X_2[k]$$