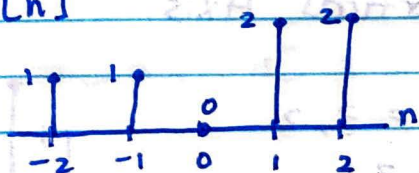DSP. Analytical Problem   HW #1   Aiden chen

4). Consider the signal $x[n]$.

$$x[n] = \begin{cases} 1 & n = -2, -1 \\ 0 & n = 0 \\ 2 & n = 1, 2 \\ 0 & else \end{cases}$$

4a.) $y_1[n] = x[n] * \delta[n-1]$

$X[n]$ $\delta[n-1] = h[n]$
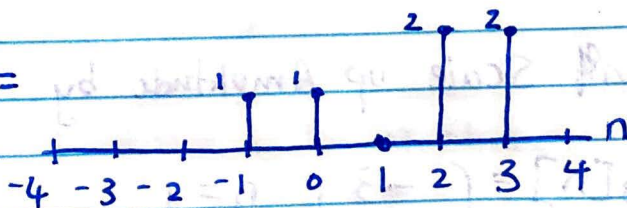
$y_1[n] = x[n] * h[n]$ using analytical approach is easier

$$= \sum_{k=-\infty}^{\infty} x[k] \cdot h[n-k]$$

find out k-Region where $x[k] \neq 0$

$$= \sum_{k=-2}^{-1} 1 \cdot h[n-k] + \sum_{k=1}^{2} 2 \cdot h[n-k]$$

$$= h[n-(-2)] + h[n-(-1)] + 2h[n-1] + 2h[n-2]$$

now sum up the h-term

$y_1[n] =$

$$= \{1, 1, 0, 2, 2\}$$
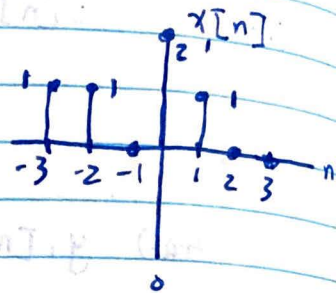↑
0

Represent in a set form.

4b) $y_2[n] = -3x[-2n+1]$

steps. Shift $x[n] \rightarrow$ Flip $\rightarrow$ Scale. $\rightarrow$ Fold. & Scale
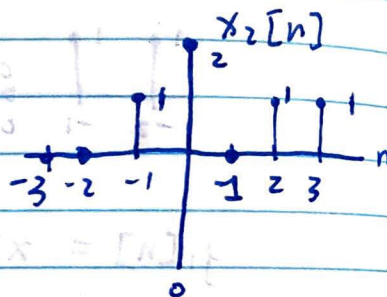time   Amplitude

1. Shift $x[n]$ 1 to the left

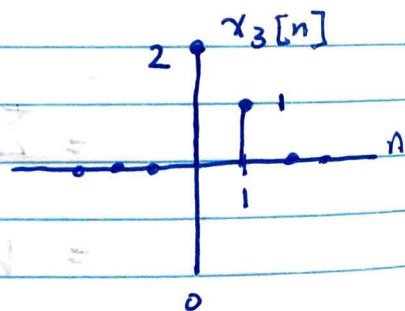$$x_1[n] = x[n+1] = \begin{cases} 1 & , n = -3, -2 \\ 0 & , n = -1 \\ 2 & , n = 0, 1 \\ 0 & , else \end{cases}$$

2.) Flip $x[n]$ across vertical Axis.

$$x_2[n] = x_1[-n] = \begin{cases} 1 & , n = 3, 2 \\ 0 & , n = 1 \\ 2 & , n = 0, -1 \\ 0 & , else \end{cases}$$
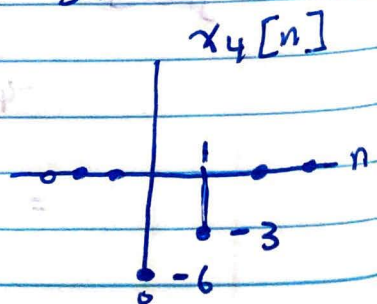

3.) Scale $x[n]$ by 2.

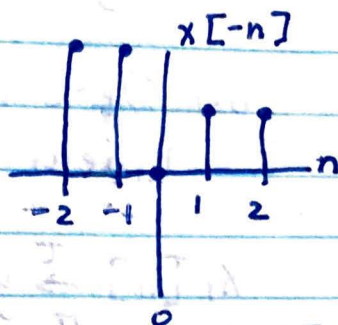$$x_3[n] = x_2[2n] = \begin{cases} 1 & , n = 1 \\ 2 & , n = 0 \\ 0 & , else. \end{cases}$$

4.) Fold down & Scale up Amplitude by 3.

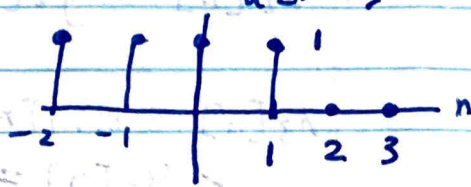$$x_4[n] = -3x_3[n] = \begin{cases} -3 & , n = 1 \\ -6 & , n = 0 \\ 0 & , else \end{cases}$$
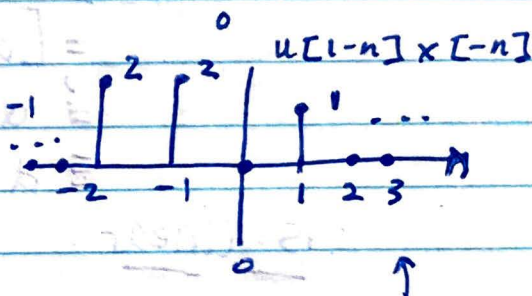
4c.) $y_3[n] = x[-n] u[1-n]$

$$x[-n] = \begin{cases} 1, & n = 2, 1 \\ 0, & n = 0 \\ 2, & n = -1, -2 \\ 0, & else \end{cases}$$

$x[-n]$

$$u[1-n] = \begin{cases} 1, & n \leq 1 \\ 0, & n > 1 \end{cases}$$

$u[1-n]$

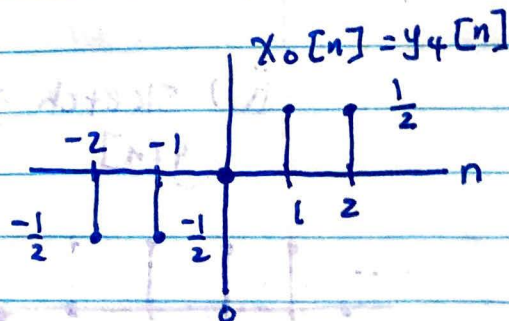$$x[-n] u[1-n] = \begin{cases} 2, & n = -2, -1 \\ 0, & n = 0 \\ 1, & n = 1 \\ 0, & else \end{cases}$$

$u[1-n] x[-n]$

multiply point-by-point

4d.) $y_4[n] = Odd(x[n])$

$$X_o[n] = odd(x[n]) = \tfrac{1}{2}(x[n] - x[-n])$$
$$= \tfrac{1}{2} x[n] - \tfrac{1}{2} x[-n]$$

$$\tfrac{1}{2} x[n] = \begin{cases} \tfrac{1}{2}, & n = -2, -1 \\ 0, & n = 0 \\ 1, & n = 1, 2 \\ 0, & else \end{cases} \qquad \tfrac{1}{2} x[-n] = \begin{cases} \tfrac{1}{2}, & n = 2, 1 \\ 0, & n = 0 \\ 1, & n = -1, -2 \\ 0, & else \end{cases}$$

$$X_o[n] = \begin{cases} \tfrac{1}{2} - 1, & n = -2, -1 \quad = -\tfrac{1}{2} \\ 0 - 0, & n = 0 \\ 1 - \tfrac{1}{2} = \tfrac{1}{2}, & n = 1, 2 \\ 0, & else \end{cases}$$

$X_o[n] = y_4[n]$

5) $y[n] = x[n^2]$

a.) Determine whether System is
Linear and Time - invariant.

$$x_1[n] \xrightarrow{T} y_1[n] = x_1[n^2]$$
$$x_2[n] \xrightarrow{T} y_2[n] = x_2[n^2]$$

$$x_3[n] = \alpha x_1[n] + b x_2[n]$$
$$\rightarrow y_3[n] = x_3[n^2]$$
$$= [\alpha x_1[n^2] + b x_2[n^2]]$$
$$\overset{?}{=} \alpha y_1[n] + b y_2[n]$$
$$= \alpha x_1[n^2] + b y_2[n^2]$$

∴ is linear

Time Invarian

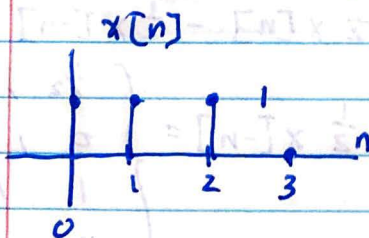$$x[n] \rightarrow y[n] = x[n$$
$$z[n] = x[n-n_0]$$
$$z[n] \rightarrow z[n^2]$$
$$= x[n^2-n_0]$$
$$y[n-n_0] \rightarrow x[(n-n_0)$$

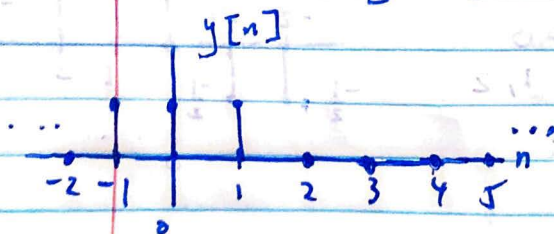not time invariant

5 b.) $x[n] = \begin{cases} 1 & 0 \le n \le 2 \\ 0 & \text{else} \end{cases}$
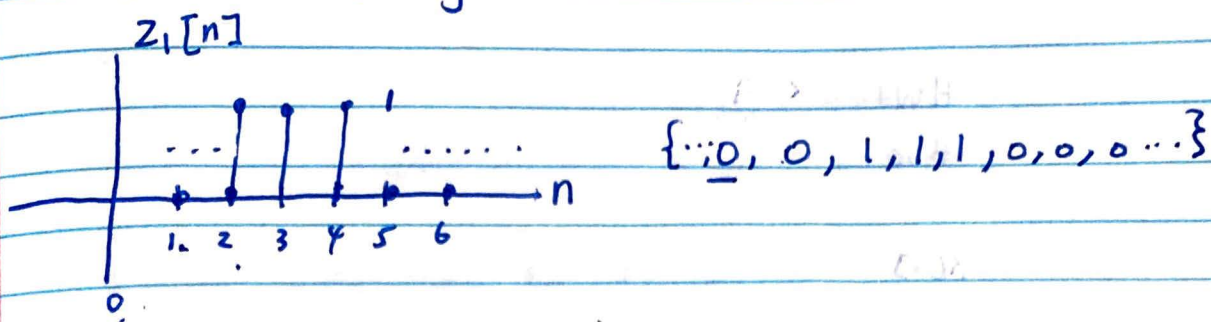
(i.) sketch $x[n]$

$x[n]$

(ii.) Sketch $y[n] = x[n^2]$

$y[n]$

$y[n] = \{ \cdots, 0, 1, 1, 1, 1, 0, \cdots \}$

iii Sketch $Z_1[n] = y[n-3] = X[(n-3)^2]$

$Z_1[n]$

$\{...0, 0, 1, 1, 1, 0, 0, 0...\}$

(iv.) Determine and sketch $x[n-3]$.

$$x[n-3] = \begin{cases} 1 & 3 \leq n \leq 5 \\ 0 & \text{else.} \end{cases}$$

$x[n-3]$

v.). $x[n-3] \xrightarrow{\pi} Z_2[n] = X[n^2 - 3.]$.

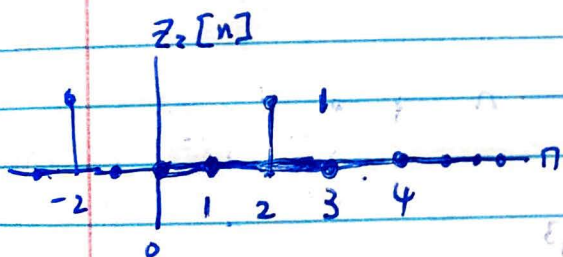$Z_2[n]$
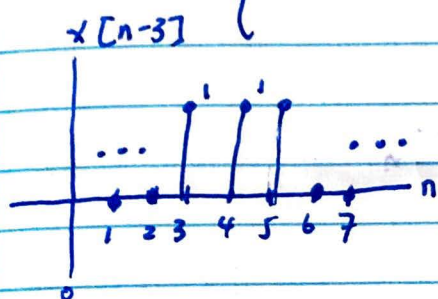
$Z_1[n] \neq Z_2[n]$.

not Time-Invariant

(vi.) The system is not time-invariant. At different time, input and output are different.
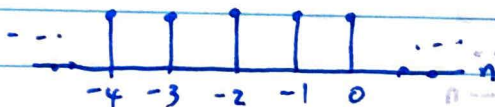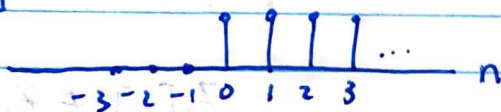
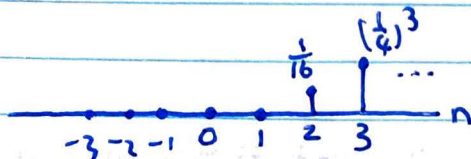vii). $y[n]$ is not periodic

HW1.

#6a.

x[n]

h[n] = x[n+2]

#6b.

x[n]

h[n]

6.) sketch and compute convolution.

a.) $x[n] = \begin{cases} 1, & n=-2,-1,0,1,2. \\ 0, & else \end{cases}$ $h[n] = x[n+2]$

$x[n+2] = \begin{cases} 1; & n=-4,-3,-2,-1,0 \\ 0, & else. \end{cases}$

$\uparrow$
$h[n]$.
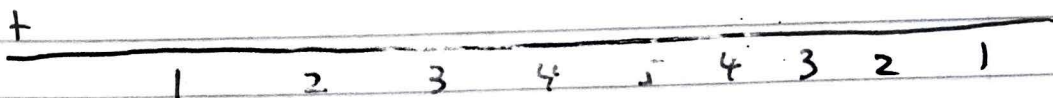$\quad\hookrightarrow =1$ for $n<-2$.

$y[n] = x[n] * h[n]$

used analytical approach.

$= \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$

$\hookrightarrow \neq 0$ if $-2<k<2$

$= \sum_{k=-2}^{2} 1\cdot h[n-k]$

$= h[n-(-2)] + h[n-(-1)] + h[n-(0)] + h[n-1] + h[n-2] -$
$\qquad\quad \textcircled{1} \qquad\qquad \textcircled{2} \qquad\qquad \textcircled{3} \qquad\qquad \textcircled{4} \qquad \textcircled{5}$

used
table
to
sum

n	-6	-5	-4	-3	-2	-1	0	1	2
①	1	1	1	1	1				
②		1	1	1	1	1			
③					1	1	1		
④			1		1	1	1	1	
⑤				1		1	1	1	1

$+$
___

| | 1 | 2 | 3 | 4 | | 4 | 3 | 2 | 1 |

$y[n] = \{ 1, 2, 3, 4, 5, 4, \underline{3}, 2, 1 \}$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad \uparrow$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad 0.$

$y[n]$

66.)
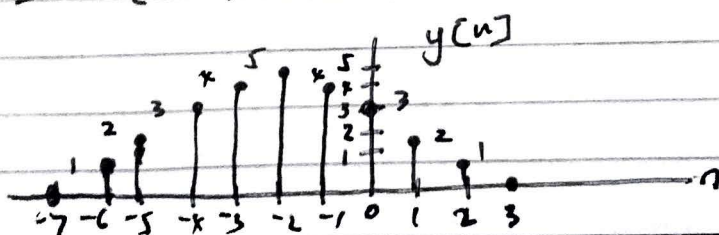
$$y[n] = \sum_{k=-\infty}^{\infty} h[k] \cdot x[n-k]$$

$$= \sum_{k=-\infty}^{\infty} \left(\tfrac{1}{4}\right)^k u[k-2] \cdot x[n-k]$$

$$= \sum_{k=2}^{n} \left(\tfrac{1}{4}\right)^k u[n-k] .$$
$$\hookrightarrow n > k > 0$$

$$= \sum_{k=2}^{n} \left(\tfrac{1}{4}\right)^k$$

$$= \sum_{k=0}^{n-2} \left(\tfrac{1}{4}\right)^{k+2} .$$

$$= \sum_{k=0}^{n-2} \left(\tfrac{1}{4}\right)^k \cdot \left(\tfrac{1}{4}\right)^2$$

$$= \left(\frac{\left(\tfrac{1}{4}\right)^0 - \left(\tfrac{1}{4}\right)^{n-2+1}}{1 - \tfrac{1}{4}} \right)\left(\tfrac{1}{4}\right)^2$$

$$= \frac{\left(1 - \left(\tfrac{1}{4}\right)^{n-1}\right) 4}{4 - 1} \left(\tfrac{1}{4}\right)^2$$

$$= \frac{4 - \left(\tfrac{1}{4}\right)^n (4)(4)}{3} \left(\tfrac{1}{4}\right)^2$$

$$= \frac{\frac{4}{4^2} - \frac{\left(\tfrac{1}{4}\right)^n (4)^2}{4^2}}{3} = \frac{\tfrac{1}{4} - \left(\tfrac{1}{4}\right)^n}{3} = \frac{1}{12} - \frac{1}{3}\left(\tfrac{1}{4}\right)^n$$

$$n \geq 0 .$$

$y[n]$ stem plot: values ... $.082$, $.078$ near $n=3,4$; $\tfrac{1}{8}$ at $n=2$; $-.25$ below axis; points at $n=0,1,2,3,4$