Rensselaer Polytechnic Institute
Department of Electrical, Computer, and Systems Engineering
ECSE 4530: Digital Signal Processing, Fall 2020

Homework #6: due Monday, Dec. 14^{th}, at the beginning of class.

MATLAB Grader Problems (No points, but you are encouraged to give it a shot!)

Analytical Problems: Be sure to show intermediate steps that explain your reasoning and do not forget to provide the labelings.

1. (10 points) Use the bilinear transformation with $T = 0.1$ to convert the analog filter with transfer function (in the Laplace domain)

$$H(s) = \frac{s+0.1}{(s+0.1)^2+9}$$

into a digital IIR filter. Compare the locations of the zeros in $H(z)$ with the locations of the zeros obtained by applying the impulse invariance method in the conversion of $H(s)$.

2. (30 points) Consider an autoregressive process that satisfies the difference equation

$$x[n] = 0.6x[n-1] + v[n]$$

where $v[n]$ is a white noise process with variance $\sigma_v^2 = 0.64$. Assume further that we observe the signal $y[n]$ which is given by

$$y[n] = x[n] + w[n]$$

where $w[n]$ is a white noise process with variance $\sigma_w^2 = 1$.

(a) Compute the correlation coefficients $r[0]$ and $r[1]$ for $\{x[n]\}$ by using the Yule-Walker equations. Then, obtain the value of $r[0]$ by using the equation for the variance of the white noise process in terms of the $r[j]$.
(b) Design a Wiener filter with length $M = 2$ to estimate $\{x[n]\}$.
(c) Determine the minimum mean square error (MMSE) for $M = 2$.

For Problem 3, in addition to the answers to the questions please turn in the MATLAB codes you wrote using Gradescope. Please use comments in your programs to clarify your statements.

To facilitate grading, please also turn in all MATLAB solutions in .pdf form using the MATLAB publish feature. Combine the listings with the scanned written homework problems.

```
To publish .pdf listings of MATLAB code:
 1. Go to the main script of your work.
 2. Switch to the "Publish" tab in MATLAB.
 3. Under "Publish", select "Edit Publishing Options".
 4. Under "output settings", select "pdf" for the option "Output file format"
 5. Click Publish.
```

In addition, please compress the executable code and email it to malakd@rpi.edu (You might want to cc your TA as well: guoh11@rpi.edu). Once your routine is working, zip the folder that contains all your code and everything else that is needed for the main files to run and email the zipped file to us. Please write "MATLAB Code for HW6" in the subject line.

3. (60 points) The LMS (least-mean-square) and the RLS (recursive-least-square) algorithms.

   Consider the setup in Figure 1 where

   - Input signal x[n] is a zero mean and independent and identically distributed (i.i.d.) wide sense stationary (WSS) process with variance 1.
   - Noise v[n] is a zero mean and i.i.d. WSS process with variance $\sigma_v^2 = 10^{-4}$.
   - Observation signal y[n] is the output of the system with frequency response H(ω) plus noise v[n].
   - W(ω) is an FIR filter with coefficients w[0], ..., w[$N-1$] that produces a delayed estimate of the input x[n], i.e. $\hat{x}[n-n_0]$ where n_0 is the delay.

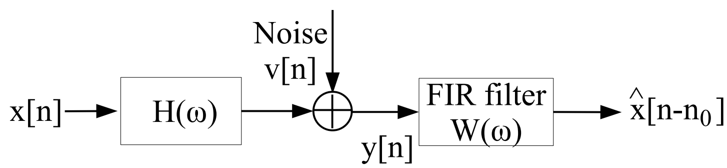

Figure 1: A linear system to recover $x[n]$ (also known as an equalizer).

   (a) Write a MATLAB function to calculate MMSE FIR filter coefficients. Below is the prototype for the function:

```
function [w, mmse]=findmmsefireq(h,Var,N,no)
% MATLAB function that calculates the MMSE FIR filter
% h : Impulse response of the linear system
% Var : Variance of the iid noise
% N : Length of the FIR filter
% no : Output delay
% w : FIR coefficients as output
% mmse: Minimum Mean Square Error as output
% This function assumes that the input signal x[n] has variance 1.
```

   (b) Given $h = [1, 1.5]$ and the FIR filter $W(\omega)$ with length $N = 10$, calculate MMSE for $n_0 = 0$ to 8 and plot it as a function of delay.

   (c) For the same frequency response H(ω),

   - Generate i.i.d x[n]'s with values -1 and 1.
   - Filter it through H(ω).
   - Add v[n] to the output to obtain the observation signal $y[n]$. For $n_0 = 8$ and $\mu = 0.03$ perform LMS for 500 samples and obtain square error convergence curve as a function of time. Repeat this for 1000 times to average the square error curves.

(d) Repeat part (c) for $\mu = 0.02$. Plot mmse convergence curves for part (c) and part (d) on the same graph (plot $10\log_{10}(mmse)$). On the same plot draw a line indicating the optimal mmse level. What are the excess MMSE and convergence times in each case.

(e) Find the convergence curve for $\mu = 0.5$. What is your observation? Explain the reason for this behavior based on the statistics of the observation signal $\mathrm{y}[n]$. Test to find the μ value which is the border value for convergence.

(f) Apply the RLS algorithm to obtain adaptive filter coefficients. Choose λ (forgetting factor) very close to 1.

Suggested reading material from textbook: Sections 10.3 (IIR filter design), 12.1-12.2 (adaptive filtering; moving average and autoregressive processes), 12.7 (Wiener and Kalman filters), 13.2-13.3 (adaptive filters: LMS, LS and RLS algorithms), 6.3.2-6.3.3 (quantizers) along with the Examples and their solutions.

Suggested practice problems from textbook: 10.10, 10.12, 10.17, 10.19, 10.23, 10.29, 10.30, 12.32, 12.36, 13.1, 13.8, 13.12, 6.19.