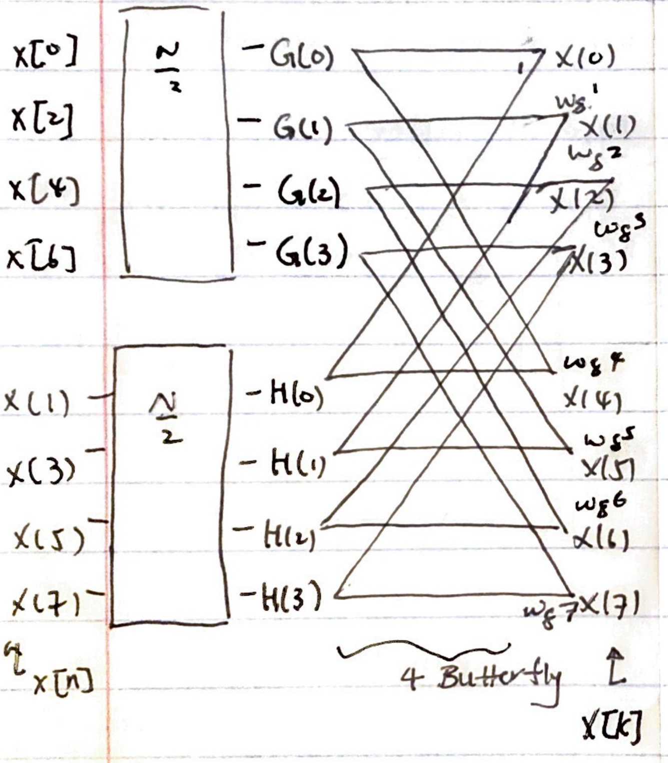
Z-Transform

stable : Right sided.

$Z = e^{jw}$ (unit circle).

Transfer Function ↑ ↑ frequency Resp.

Type	ZERo
1	—
2	$\omega = \pi$
3	$\omega = 0, \pi$
4	$\omega = 0$



$-60\,dB = 20\log(A)$

$X[K] = \sum_{n=0}^{N-1} x[n] W_N^{nk}$

$n = 2r$

$= \sum_{r=0}^{\frac{N}{2}-1} x[2r] W_N^{2rk} + \sum_{r=0}^{\frac{N}{2}-1} x[2r+1] W_N^{(2r+1)n}$

$= \sum_{r=0}^{\frac{N}{2}-1} x(2r)(W_{\frac{N}{2}})^{rk}$

$+ W_N^k x(2r+1) (W_{\frac{N}{2}})^{rk}$

$= \underbrace{G[k]}_{\frac{N}{2}\text{ long}} + \underbrace{W_N^k H[k]}$

down sampling - decimation

$x[n] \rightarrow \boxed{\downarrow M} \rightarrow x(nM) \longleftrightarrow \frac{1}{M}\sum_{i=0}^{M-1} X(\frac{\omega}{M} - \frac{\pi i}{M})$

If $MW_B < \pi$ → no need filter

$MW_B > \pi$ → need prefilter.

upsampling - interpolation.

$x[n] \rightarrow \boxed{\uparrow L} \rightarrow x(\frac{n}{L}) \longleftrightarrow X(L\omega)$

$x[n] \rightarrow \boxed{\downarrow M} \rightarrow \boxed{H(z)} \rightarrow y_a(n)$ } eq.

$x(n) \rightarrow \boxed{H(z^M)} \rightarrow \boxed{\downarrow M} \rightarrow y_b(n)$ }.

$x[n] \rightarrow \boxed{H(z)} \rightarrow \boxed{\uparrow L} \rightarrow y_a(n)$ } eq.

$x[n] \rightarrow \boxed{\uparrow L} \rightarrow \boxed{H(z^L)} \rightarrow y_b(n)$ }

$x[n] \rightarrow \boxed{\uparrow L} \rightarrow H(\min\{\frac{\pi}{M}, \frac{\pi}{L}\}) \rightarrow \boxed{\downarrow M} \rightarrow x_f(n)$

$M > L$) Net Reduction of S. Rate. (need prefilter)

$M < L$) net inc of S.R. (perfect.).

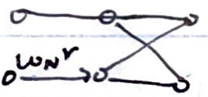
overlap when $\frac{\omega_1}{L} > \frac{2\pi - \omega_1}{L}$

* $W_N^{n(k+N)} = W_N^{nk}$ $\quad W_N^{(r+\frac{N}{2})} = -W_N^r$

$W_N^{KN} = 1$ $\quad W_N^{\frac{N}{2}(\text{ODD }K)} = -1$

FFT : $O(\underbrace{N}\ \underbrace{\log_2 N})$

\# of multip. Stage



N = 8

$$\begin{matrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & W_8^1 & W_8^2 & W_8^3 & W_8^3 & -1 & -W_8^1 & -W_8^2 & -W_8^3 \\
1 & -j & -1 & j & j & 1 & -j & -1 & j \\
1 & W_8^3 & j & W_8^1 & W_8^1 & -1 & -W_8^3 & -j & -W_8^1 \\
1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 \\
1 & -W_8^1 & W_8^2 & -W_8^3 & -W_8^3 & -1 & W_8^1 & -W_8^2 & W_8^3 \\
1 & j & -1 & -j & -j & 1 & j & -1 & -j \\
1 & -W_8^3 & j & -W_8^1 & -W_8^1 & -1 & W_8^3 & -j & W_8^1
\end{matrix}$$

Even col: Top ½ = BoT ½ ODD col: Top ½ = - Bot ½

$$\begin{bmatrix} \overset{\text{sum}}{\begin{pmatrix} I \\ I \end{pmatrix}} & \overset{\text{diff}}{\begin{pmatrix} I \\ -1 \end{pmatrix}} \end{bmatrix} \begin{bmatrix} I & \\ & \underbrace{[\ddots]}_{\text{Twiddle}} \end{bmatrix} \underbrace{\begin{bmatrix} F_4 & \\ & F_4 \end{bmatrix}}_{\text{DFT.}} \begin{bmatrix} x_{even} \\ x_{odd} \end{bmatrix}$$

FFT of freq:

$X[2r] = \sum_{n=0}^{\frac{N}{2}-1} [x[n] + x[n+\frac{N}{2}]] W_{\frac{N}{2}}^{nr}$

$X[2r+1] = \sum .. \ [x(n) - x(n+\frac{N}{2}]) W_N^n W_{\frac{N}{2}}^{nr}$

ZERO LOCation

if $h(n)=h(N-1-n)$

$H(z)=z^{-(N-1)}H(\frac{1}{z})$

·Genera ZERO exit in group 4

Gen. ZERO at unit circle → 2

ZERO at axis = 2

1 and -1 count.

If z_0 is ZERO of Real Linear-phase filter $\{z_0^*, \frac{1}{z_0}, \frac{1}{z_0^*}\}$ also is ZERO.

$\min E = \max_{w\in[0,\pi]} |A(w) - A_d(w)|$

Least Square Approximation of h[n]

Type I, III : M equations

Type II, IV : $\frac{N}{2}-1$ equations

Ripple Reduce	Transition band (less sharp)
Ripple Inc	Transition band (more sharp)
more points	Transition band (more sharp)
less points	- - - band (less sharp)

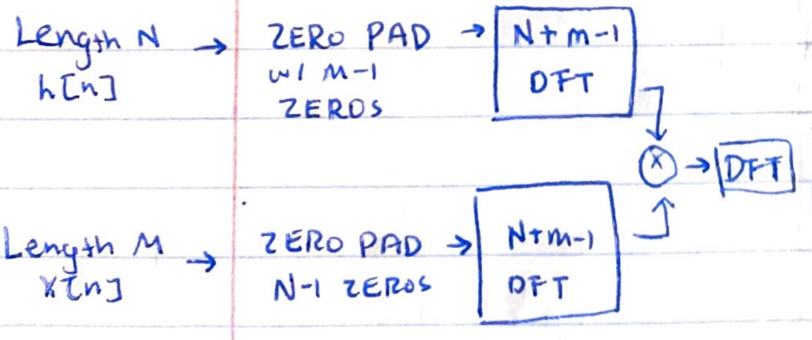
$A(w) = \sum h \cdot \cos(\cdot) + h.$

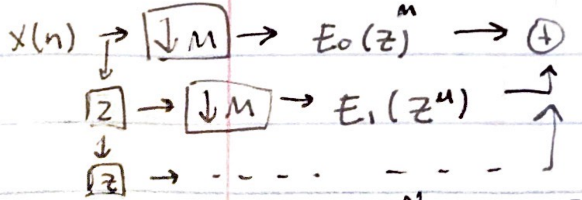
$[A] = [\cos(\cdot)]\,[h]$ Type I/III

$\underset{L\times 1}{[A]} = \underset{L\cdot M+1}{[\cos(\cdot)]}\,\underset{M+1}{[h]}$

$DFT\{\frac{1}{N}x[k]\} = x[-n]$

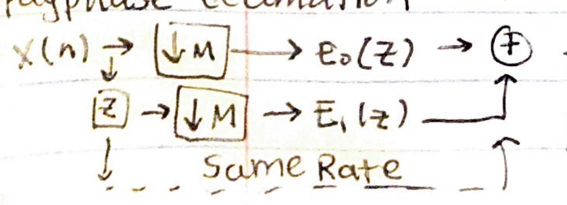
DFT:



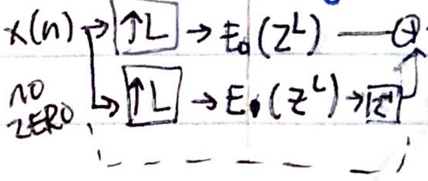


$\frac{N}{M}$ tap

Polyphase decimation



Polyphase Interpolation



FIR Filter Design :

notch filter

Why we Desire them?

$h_d(n) = \delta[n-n_d] \leftrightarrow H_d(w)e^{-jwn_d}$

$H(w) = A(w)e^{-j(K_1+K_2w)}$

A→D · Fs scale
· Repeat

D→A · T scale
· Remove $|F| > F_s/2$

Filter Design Process

1.) choose desired Freq. Resp.

2.) choose allow-able class of filter
Length-N FIR Filter

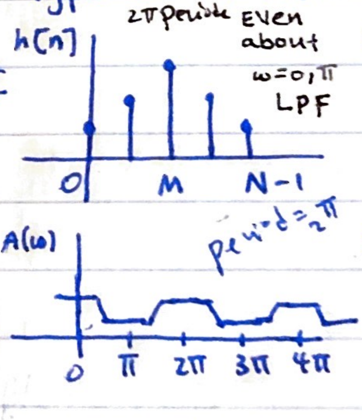
3.) choose measure of quality (How close)

4.) Apply Algo to find best val.

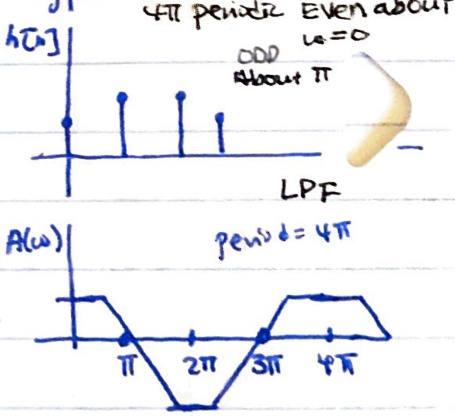
5.) Choose best Realization of filter.

Type of Filter

Type I : N ODD

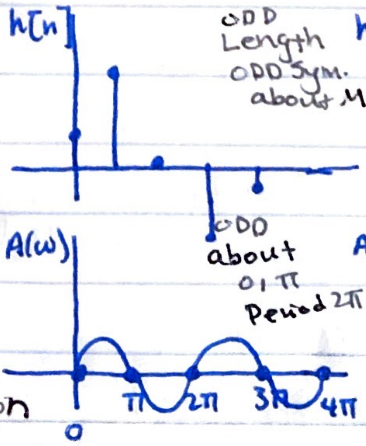


Type II : N Even

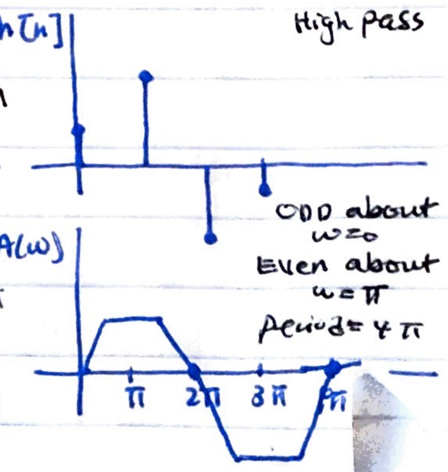


$h(n)=h(N-1-n)$

Type III : N ODD



Type IV : N Even



$h(n) = -h(N-1-n)$

I'm aware of the Academic integrity.
I affirm that I'll not give or receive
help on this exam. and all work is on my own ①

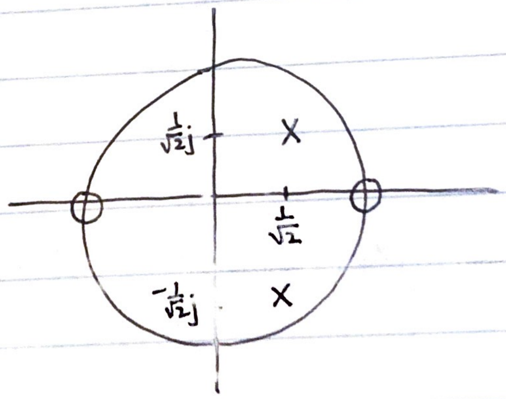
Exam 2 Aiden Chen

①. $H(z) = \frac{z^2-1}{z^2-z+0.5} = \frac{(z+1)(z-1)}{\left(z-\left(\frac{1}{\sqrt2}+j\frac{1}{\sqrt2}\right)\right)\left(z-\left(\frac{1}{\sqrt2}-j\frac{1}{\sqrt2}\right)\right)}$

$\sqrt{b^2-4ac} = \sqrt{1-4\frac{1}{2}} = \sqrt{1-2} = \sqrt{-1} = j$

$\frac{-b\pm\sqrt{\ }}{2a} = \frac{1}{\sqrt2}\pm\frac{j}{\sqrt2}$

a.) pole-ZERO



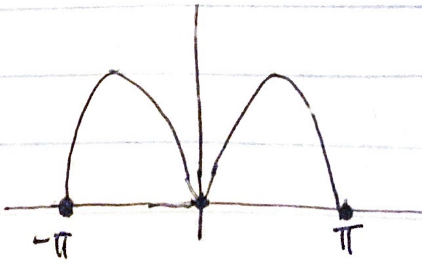
b.) $|H(0)| = \left|\frac{0}{\ }\right| = \frac{0}{\ } = \boxed{0}$. $e^{jw=0} = 1$

$|H(\frac{\pi}{2})| = \left|\frac{e^{j\frac{\pi}{2}}-1}{(e^{j\frac{\pi}{2}})^2-e^{j\frac{\pi}{2}}+0.5}\right| = \frac{\sqrt{1^2+1^2}}{\sqrt{1^2+(\frac{1}{2})^2}} = \frac{\sqrt{2}}{\sqrt{1+\frac{1}{4}}} = \sqrt{\frac{2}{\frac{5}{4}}}$

$= \sqrt{\frac{8}{5}}$

$|H(-\frac{\pi}{2})| = \sqrt{\frac{8}{5}}$

Bandpass because at $w=0,\pi$ it is Rejected and in between is passed.



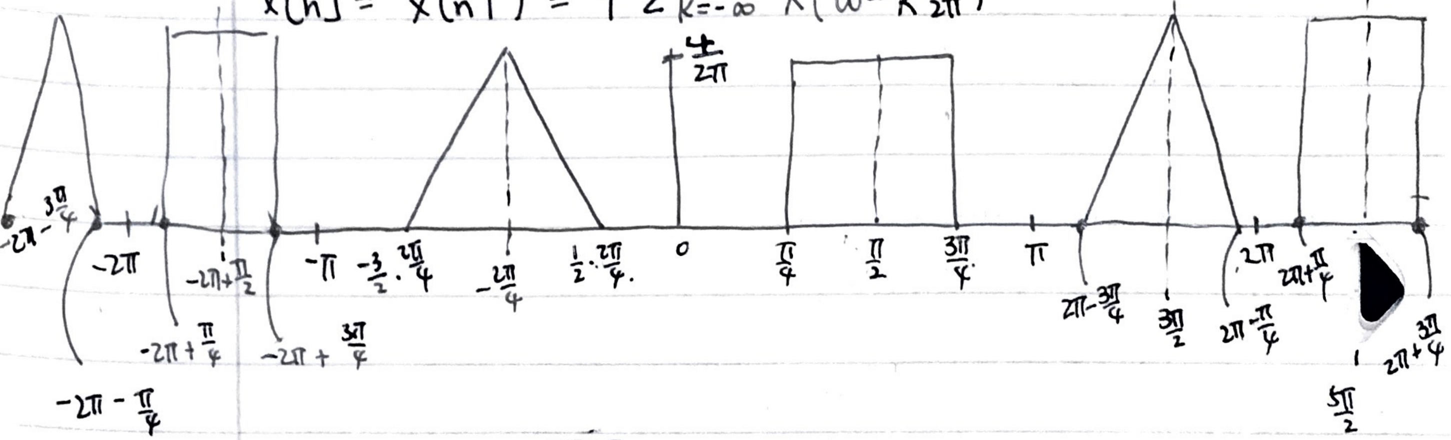
2a.) Nyquist Rate is $2 \cdot 1.5 = 3 \frac{Rad}{s}$

2b.) $x(t)$ is real.

2c.) $x(t) \rightarrow \boxed{} \rightarrow x[n]$

$\uparrow$ $T = \frac{2\pi}{4}$

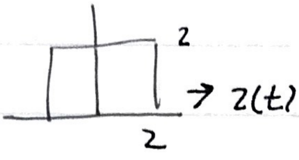
$x[n] = x(nT) = \frac{1}{T} \Sigma_{k=-\infty}^{\infty} X(\omega - k\frac{4}{2\pi})$



2d.) $x(n) \rightarrow \boxed{\downarrow 2} \rightarrow E_0(z^2) \rightarrow \oplus \rightarrow x[k]$.

$\boxed{z^{+1}} \rightarrow \boxed{\downarrow 2} \rightarrow E_1(z^2) \rightarrow$

2e.) $x[n] \rightarrow \boxed{\uparrow 2} \rightarrow y(n) \rightarrow \boxed{D/C} \rightarrow y(t) \rightarrow$

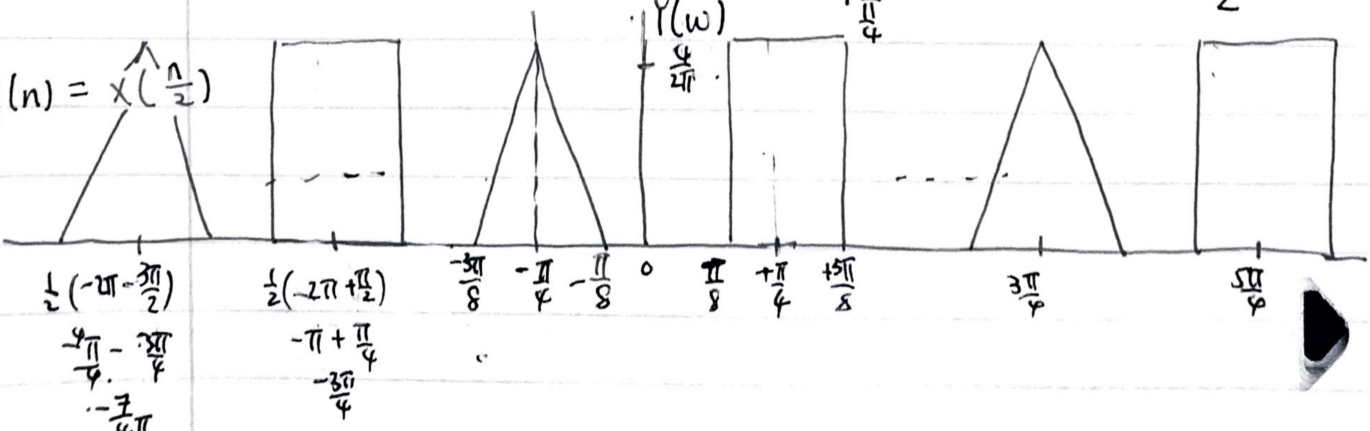


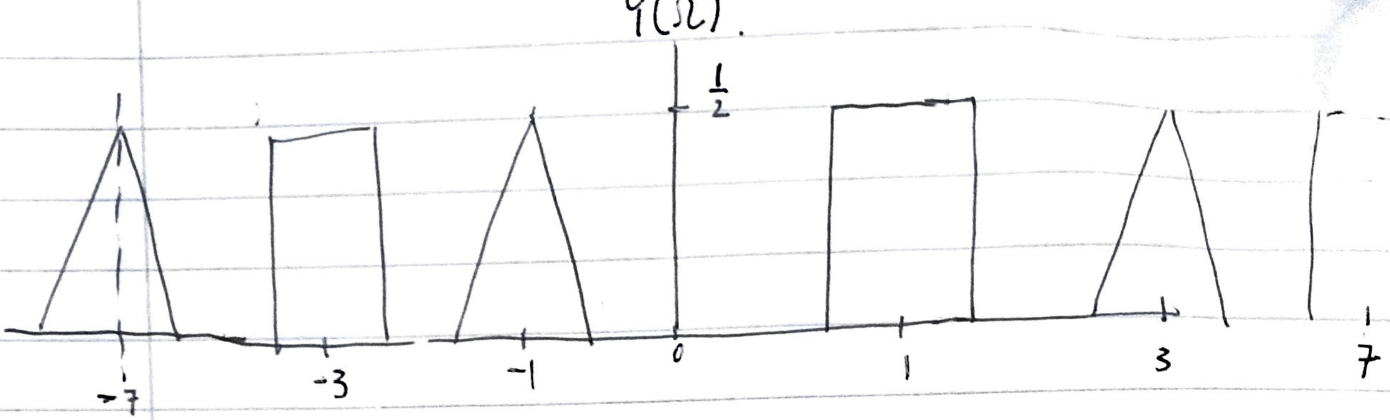
$\rightarrow z(t)$

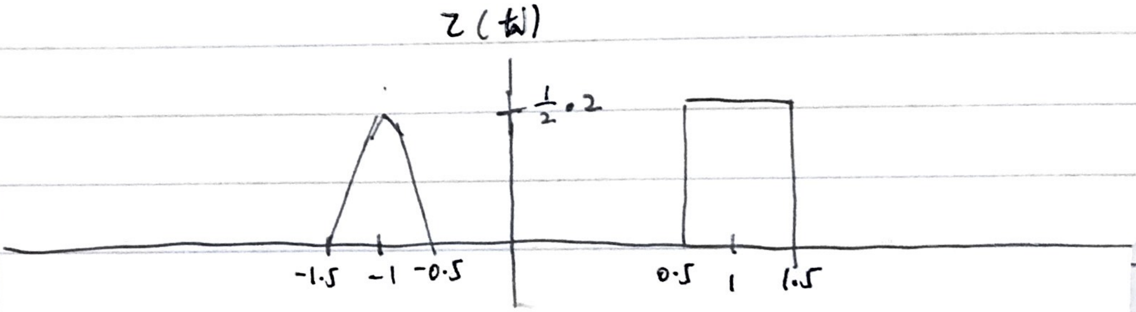
$\uparrow \frac{\pi}{4}$

$Y(\Omega)$ $\parallel$

$y(n) = x(\frac{n}{2})$




Y(Ω)
1/2
-7
-3
-1
0
1
3
7


Z(jω)
1/2 · 2
-1.5
-1
-0.5
0.5
1
1.5

②

3.) 3.1 F

3.2 T

3.3. T.

3.4. F

3.5. T.

3.6. T.

3.7. T.

3.8. F.

3.9 T.

3.10 N/M.

3.11 decimation and interpolation

3.12. Remove ZERo.

3.13. Points are close together.

3.14. Cyclic Convolution , 7.

3.15. covi9, I didn't get to do a lot of fun things.

4. $\underbrace{\log_2}_{\text{stage.}}(4) = 8. \downarrow$

$$8 \underbrace{\log_2(8)}_{3.} = 24.$$

4.)

$$x(n) \rightarrow \boxed{\uparrow 2} \rightarrow x_1(n) \rightarrow \boxed{\downarrow 2} \rightarrow y_1(n)$$

$$x_1(n) = x\left(\frac{n}{2}\right) \leftrightarrow X_1(\omega) = X(2\omega)$$

$$y_1(n) = x_1(2n) \leftrightarrow Y_1(\omega) = \frac{1}{2}\sum_{n=0}^{2-1} X_1\left(\frac{\omega}{2} - 2\pi i/2\right)$$

$$= \frac{1}{2}\sum_{n=0}^{1} X\left(2\left(\frac{\omega}{2} - \pi i\right)\right)$$

$$= \frac{1}{2}\left(X(\omega) + X(\omega - 2\pi)\right)$$

$$x(n) \rightarrow \boxed{\downarrow 2} \rightarrow x_2(n) \rightarrow \boxed{\uparrow 2} \rightarrow y_2(n)$$

$$x_2(n) = x(2n) \leftrightarrow X_2(\omega) = \frac{1}{2}\sum_{n=0}^{2-1} X\left(\frac{\omega}{2} - \frac{2\pi}{2} i\right).$$

$$y_2(n) = x_2\left(\frac{n}{2}\right) \leftrightarrow Y_2(\omega) = X_2(2\omega) = \frac{1}{2}\sum_{n=0}^{1} X\left(\frac{2\omega}{2} - \pi i\right).$$

$$= \frac{1}{2}\left(X(\omega) + X(\omega - \pi)\right)$$

So. $Y_1(\omega) \neq Y_2(\omega)$.

$y_1(n) \neq y_2(n)$

4b.) $X[k]$ be N - Point DFT. of $x[n]$, $0 \leq n \leq N-1$

What is $Y[k]$ when

$$y[n] = X[n], \quad 0 \leq n \leq N-1.$$

$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{nk}$$

$$Y[k] = \sum_{n=0}^{N-1} y[n] W_N^{nk}$$

$$= \sum_{n=0}^{N-1} X[n] W_N^{nk}$$

$m = l + n$.

$$= \sum_{n=0}^{N-1}\left[\sum_{l=0}^{N-1} x[l] W_N^{kl}\right] W_N^{nk}$$

$0 \leq l \leq N-1$

$0 \leq n \leq N-1$

$$= \sum_{l=0}^{N-1} x[l] \sum_{n=0}^{N-1} W_N^{k(l+n)}$$