Name: ____________________

Rensselaer Polytechnic Institute
Department of Electrical, Computer, and Systems Engineering
ECSE 4530: Digital Signal Processing, Fall 2017

Exam #2. Closed book, one one-sided page of notes.
November 16, 2017, 10:00-11:20 AM

Show all work for full credit.

- No calculators or other electronic devices are allowed.
- Tables are provided with the exam.
- Reduce expressions involving exponentials, sines, and cosines as much as possible.
- If the sinc function is needed, use the definition $\mathrm{sinc}(x) = \frac{\sin x}{x}$.
- When in doubt, show more work!

Crib sheet		**5**
1		**21**
2		**14**
3		**14**
4		**12**
5		**18**
6		**16**
Total		**100**

1. (21 points.) Indicate whether each of the following statements is true (T) or false (F). No explanation is needed.

____ a. Let $H(\omega)$ be the Discrete-time Fourier Transform (DTFT) of a 10-point impulse response $h[n]$ of a Linear Time-Invariant (LTI) system. Then $H(-\frac{\pi}{2}) = H(\frac{3\pi}{2})$ must hold.

____ b. Let $\hat{H}[k], k = 1, \ldots., 10$ denote the 10-point Discrete Fourier Transform (DFT) of $h[n]$ in (a). Then $\hat{H}[3] = H(\frac{2\pi}{5})$, with $H(\omega)$ defined in (a).

____ c. In order to use radix-2 Fast Fourier Transform (FFT) to compute the DFT of a length-600 signal, we need to zero pad the signal to a length of 1024.

____ d. The process of downsampling can never introduce aliasing.

____ e. The process of upsampling can never introduce aliasing.

____ f. The FIR filter $h[n] = [-5, 3, 0, 3, 5]$ has linear phase.

____ g. The downsampler $\boxed{\downarrow 2}$ is a linear time-invariant system.

2. (14 points.) Let $x[n] = 3\delta[n] + \delta[n-1] - 3\delta[n-2]$.

   (a) (8 points.) What is the DTFT $X(\omega)$ of $x[n]$? Sample the DTFT $X(\omega)$ at two frequencies $k\pi$ with $k = 0, 1$, what is the resulting signal $\hat{X}[k]$, $k = 0, 1$?

   (b) (6 points.) Take the 2-point Inverse DFT of $\hat{X}[k]$, $k = 0, 1$, what is the resulting signal $y[n]$?

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3. (14 points.) Let $x_1[n]$ and $x_2[n]$ be two N-point sequences. Let $X_1[k]$ and $X_2[k]$ ($k = 0, ..., N-1$) denote their N-point DFTs, respectively. $x[n] = x_1[n]x_2[n]$. Let $X[k]$ ($k = 0, ..., N-1$) be the N-point DFT of $x[n]$. Show that $X[k]$ equals the circular convolution of $X_1[k]$ and $X_2[k]$ divided by N, i.e.,

$$X[k] = \frac{1}{N} X_1[k] \circledast X_2[k] \tag{1}$$

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4. (12 points.) Describe the steps of computing a 15-point DFT using Cooley-Tukey FFT. Please also write down the matrix representation of the input, the matrix representation of the output, and the matrix representation of the twiddle factors.

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5. (18 points.)

The Continuous-time Fourier Transform $X_c(\omega)$ of a continuous-time signal $x_c(t)$ is shown in Figure 1. $x_c(t)$ sampled with a sampling period of $T_s = 1/300$ seconds. The resulting discrete-time signal $x_d[n]$ passes an expander $\boxed{\uparrow 2}$) that inserts a zero after each of the original sequence values in $x_d[n]$. For example, $[1,2,3,...]$ becomes $[1,0,2,0,3,0,...]$. The resulting signal $z_d[n]$ passes a discrete-system with frequency response $H(\omega)$. The output signal $y_d[n]$ is related to $z_d[n]$ through

$$y_d[n] = z_d[n-1] + z_d[n] + z_d[n+1].$$

$y_d[n]$ is converted to a continuous-time signal $y_c(t)$ with sampling period T_s. The system diagram is shown in Figure 2.

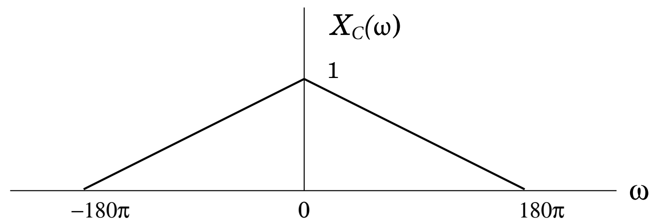

Figure 1: CTFT of $x_c(t)$

(a) (6 points.) Draw the frequency response of $x_d[n]$ and $z_d[n]$.

(b) (4 points.) What is $H(\omega)$?

(c) (8 points.) Let $Y_c(\omega)$ be the Continuous-time Fourier Transform of $y_c(t)$. What is the value of $Y_c(50\pi)$?

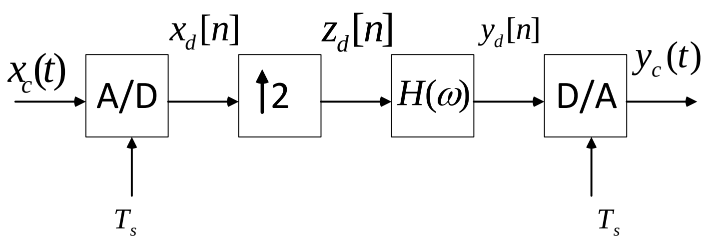

Figure 2: System diagram

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6. 16 Let $x_c(t)$ be a real-valued continuous-time signal with the highest frequency at 400π radians/second. Let $y_c(t)$ be a time delayed version of $x_c(t)$, $y_c(t) = x_c(t - \frac{1}{1000})$.

   (a) (3 points.) If $x[n] = x_c(\frac{n}{500})$. Is it theoretically possible to recover $x_c(t)$ from $x[n]$? Justify your answer.

   (b) (3 points.) If $y[n] = y_c(\frac{n}{500})$. Is it theoretically possible to recover $y_c(t)$ from $y[n]$? Justify your answer.

   (c) (10 points.) We would like to obtain $y[n]$ from $x[n]$. Is it possible to achieve that using the system in Figure 3? If so, find the frequency response $H(\omega)$. If not, explain why.

   Hint 1: First establish the relationship between the DTFTs of $x[n]$ and $y[n]$.

   Hint 2: Can $H(\omega)$ be a low-pass filter?

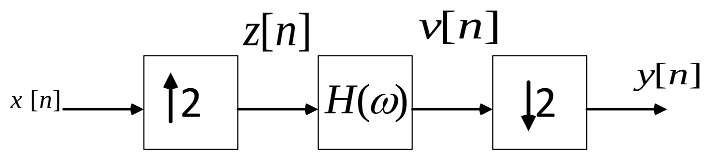

Figure 3: System diagram

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Feedback and suggestions for the class?