Rensselaer Polytechnic Institute
Department of Electrical, Computer, and Systems Engineering
ECSE 4530: Digital Signal Processing, Fall 2020

Homework #1 Solution: due Monday, Sep. 14th, at the beginning of class.

Analytical Problems:

4. (10 points) **Discrete-time signals.** Consider the discrete-time signal $x[n]$ given by

$$x[n] = \begin{cases} 1, & n = -2, -1, \\ 0, & n = 0, \\ 2, & n = 1, 2 \\ 0, & \text{elsewhere.} \end{cases}$$

Sketch each of the following signals. Be sure to show intermediate steps that explain your reasoning and do not forget to provide the labelings.

(a) $y_1[n] = x[n] * \delta[n-1] = x[n-1] = \{\ldots, 1, \underline{1}, 0, 2, 2, 0, \ldots\}$ (* denotes the convolution operator)

(b) $y_2[n] = -3x[-2n+1]$
Remember the order: Shift, Flip, Scale.
Shift $z[n] = x[n+1] = \{\ldots, 1, 1, 0, \underline{2}, 2, 0, \ldots\}$
Flip $w[n] = z[-n] = \{\ldots, 0, 0, 2, \underline{2}, 0, 1, 1, 0, \ldots\}$
Scale $y_2[n] = -3w[2n] = -3x[-2n+1] = \{\ldots, 0, 0, 0, \underline{-6}, -3, 0, 0, \ldots\}$

(c) $y_3[n] = x[-n]u[1-n]$

$$u[1-n] = \begin{cases} 1, & n \le 1, \\ 0, & \text{elsewhere.} \end{cases} = \{\ldots 1, 1, \underline{1}, 1, 0, 0, \ldots\}$$

$$x[-n] = \{\ldots 0, 2, 2, \underline{0}, 1, 1, 0, \ldots\}$$

$$y_3[n] = \{\ldots, 0, 2, 2, \underline{0}, 1, 0, 0, \ldots\}$$

(d)

$$y_4[n] = \text{Odd}(\text{x[n]}) = \frac{\text{x[n]} - \text{x[}-\text{n]}}{2} = \frac{\{\ldots, 1, 1, \underline{0}, 2, 2, 0, \ldots\} - \{\ldots 0, 2, 2, \underline{0}, 1, 1, 0, \ldots\}}{2}$$
$$= \{\ldots, 0, -0.5, -0.5, \underline{0}, 0.5, 0.5, 0, \ldots\}$$

5. (25 points) **System properties.** Consider the system $y[n] = x[n^2]$.

(a) The system is linear. The long proof:

$$x_1[n] \to y_1[n] = x_1[n^2], \quad x_2[n] \to y_2[n] = x_2[n^2]$$
$$x[n] = ax_1[n] + bx_2[n] \to y[n] = x[n^2] = ax_1[n^2] + bx_2[n^2] = ax_1[n]^2 + bx_2[n]^2$$

Or you can give a counterexample: Let $x[n] = 1$, then $y[n] = 1$. When $x[n] = 2$, $y[n] = 4 \neq 2$.
The system is time variant. Why:

$$x[n] \to y[n] = x[n^2]$$
$$x[n-k] \to y_1[n] = x[(n-k)^2]$$
$$= x[n^2 + k^2 - 2nk] \neq y[n-k]$$

(b) Assume that the following signal is applied to the system:

$$x[n] = \begin{cases} 1, & 0 \le n \le 2 \\ 0, & \text{else.} \end{cases}$$

i. $x[n] = \{0, \underline{1}, 1, 1, 0, 0, \ldots\}$
ii. $y[n] = x[n^2] = \{\ldots, 0, 0, 1, \underline{1}, 1, 0, 0, 0, \ldots\}$.
iii. $z_1[n] = y[n-3] = \{\ldots, \underline{0}, 0, 1, 1, 1, 0, 0, \ldots\}$.
iv. $x[n-3] = \{0, \underline{0}, 0, 0, 1, 1, 1, 0, \ldots\}$.
v. $z_2[n] = \{0, 1, 0, \underline{0}, 0, 1, 0, 0, 0, 0, \ldots\}$
vi. $z_1[n] \neq z_2[n]$. The system is time variant.
vii. $y[n]$ is not periodic. This is clear from part (b) ii.

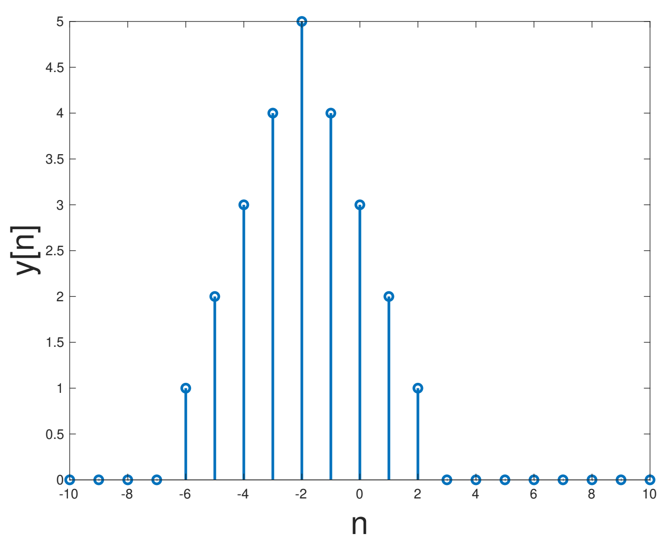
6. (15 points) **Convolution.** Compute and sketch the convolution $y[n] = x[n] * h[n]$ for the following signal and impulse response pairs.

(a) $x[n] = \begin{cases} 1, & n = -2, -1, 0, 1, 2 \\ 0, & \text{else,} \end{cases}$, $h[n] = x[n+2]$.

$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=-\infty}^{\infty} x[k]x[n+2-k] = \sum_{k=-2}^{2} x[n+2-k] = \sum_{l=0}^{4} x[n+l]$. Then,

$$y[n] = \begin{cases} 7+n, & -6 \le n \le -2 \\ 3-n, & -2 < n \le 2 \\ 0, & n < -6, \quad n > 2 \end{cases}$$

which is shown below.



(b) $x[n] = u[n], \quad h[n] = (1/4)^n u[n-2]$.

$$
\begin{aligned}
y[n] &= \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=-\infty}^{\infty} u[k](1/4)^{n-k}u[n-k-2] \\
&= \sum_{k=0}^{\infty} (1/4)^{n-k}u[n-k-2] = \sum_{k=0}^{n-2} (1/4)^{n-k} \\
&= \sum_{m=2}^{n} (1/4)^{m} = \frac{1-(1/4)^{n+1}}{1-(1/4)} - (1/4)^0 - (1/4)^1 = 1/12 - 1/3(1/4)^n
\end{aligned}
$$

for $n \geq 2$ (Note that we did a change of variables $n-k=m$ in the last sum). Hence,

$$y[n] = (1/12 - 1/3(1/4)^n)u[n-2].$$