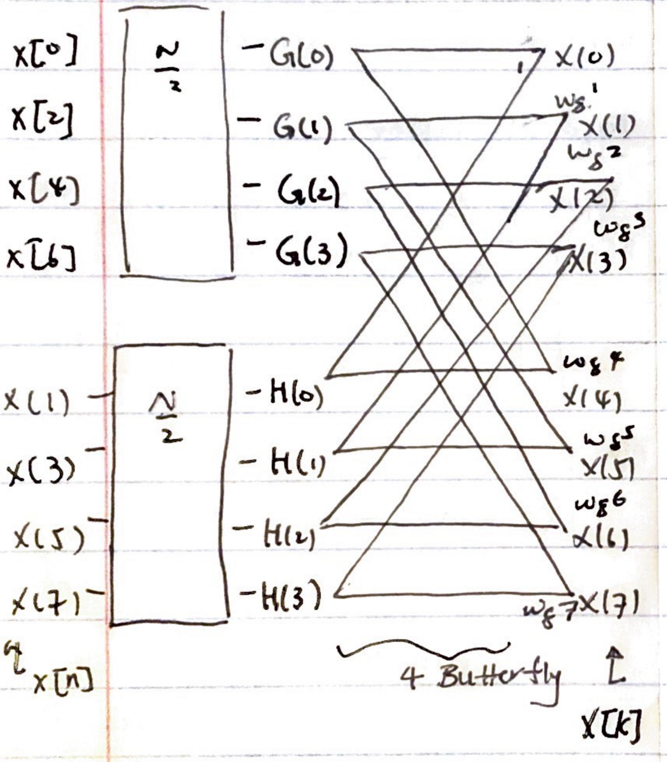
Z-transform

stable : Right sided.

$Z = e^{jw}$ (unit circle).

Transfer Function ↑ ↑ frequency Resp.

Type	ZERO
1	—
2	$\omega = \pi$
3	$\omega = 0, \pi$
4	$\omega = 0$



$-60\,dB = 20\log(A)$

$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{nk}$

$n = 2r$

$= \sum_{r=0}^{\frac{N}{2}-1} x[2r] W_N^{2rk} + \sum_{r=0}^{\frac{N}{2}-1} x[2r+1] W_N^{(2r+1)k}$

$= \sum_{r=0}^{\frac{N}{2}-1} x(2r)(W_{\frac{N}{2}})^{rk}$

$+ W_N^k \, x(2r+1)\,(W_{\frac{N}{2}})^{rk}$

$= \underbrace{G[k]}_{\frac{N}{2}\text{ long}} + \underbrace{W_N^k H[k]}$

down sampling - decimation

$x[n] \rightarrow \boxed{\downarrow M} \rightarrow x(nM) \longleftrightarrow \frac{1}{M}\sum_{i=0}^{M-1} X\left(\frac{\omega}{M} - \frac{2\pi i}{M}\right)$

If $M\omega_B < \pi$ → no need filter

$M\omega_B > \pi$ → need prefilter.

upsampling - interpolation.

$x[n] \rightarrow \boxed{\uparrow L} \rightarrow x\left(\frac{n}{L}\right) \longleftrightarrow X(L\omega)$

$x[n] \rightarrow \boxed{\downarrow M} \rightarrow \boxed{H(z)} \rightarrow y_a(n)$ } eq.

$x(n) \rightarrow \boxed{H(z^M)} \rightarrow \boxed{\downarrow M} \rightarrow y_b(n)$

$x[n] \rightarrow \boxed{H(z)} \rightarrow \boxed{\uparrow L} \rightarrow y_a(n)$ } eq.

$x[n] \rightarrow \boxed{\uparrow L} \rightarrow \boxed{H(z^L)} \rightarrow y_b(n)$

$x[n] \rightarrow \boxed{\uparrow L} \rightarrow H\left(\min\left\{\frac{\pi}{M}, \frac{\pi}{L}\right\}\right) \rightarrow \boxed{\downarrow M} \rightarrow x_f(n)$

$M > L$ — Net Reduction of S. Rate. (need prefilter)

$M < L$ — net inc of S.R. (perfect.).

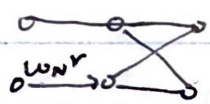
overlap when $\frac{\omega_1}{L} > \frac{2\pi - \omega_1}{L}$

* $W_N^{n(k+N)} = W_N^{nk}$ $\quad W_N^{(r+\frac{N}{2})} = -W_N^{r}$

$W_N^{kN} = 1$ $\quad W_N^{\frac{N}{2}(\text{ODD } k)} = -1$

FFT: $O(\underbrace{N}\,\underbrace{\log_2}N)$

\# of multip. Stage



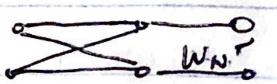
N = 8

1	1	1	1	1	1	1	1
1	W_8^1	W_8^2	W_8^3	W_8^3	-1	$-W_8^1$	$-W_8^2$	$-W_8^3$
1	$-j$	-1	j	j	1	$-j$	-1	j
1	W_8^3	j	W_8^1	W_8^1	-1	$-W_8^3$	$-j$	$-W_8$
1	-1	1	-1	-1	1	-1	1	-1
1	$-W_8^1$	W_8^2	$-W_8^3$	$-W_8^3$	-1	W_8^1	$-W_8^2$	W_8^3
1	j	-1	$-j$	$-j$	1	j	-1	$-j$
1	$-W_8^3$	j	$-W_8^1$	$-W_8^1$	-1	W_8^3	j	W_8^1

Even col: Top ½ = Bot ½ ODD col: Top ½ = −Bot ½

$\overset{\text{sum}}{\begin{bmatrix} I \\ I \end{bmatrix}} \overset{\text{diff}}{\begin{bmatrix} I \\ -I \end{bmatrix}} \underset{\text{Twiddle}}{\begin{bmatrix} I & \\ & \ddots \end{bmatrix}} \underset{\text{DFT.}}{\begin{bmatrix} F_4 & \\ & F_4 \end{bmatrix}} \begin{bmatrix} x_{even} \\ x_{odd} \end{bmatrix}$

FFT of freq:



$X[2r] = \sum_{n=0}^{\frac{N}{2}-1} \left[x[n] + x\left[n+\frac{N}{2}\right]\right] W_{\frac{N}{2}}^{nr}$

$X[2r+1] = \sum .. \left[x(n) - x\left(n+\frac{N}{2}\right]\right) W_N^{n} W_{\frac{N}{2}}^{nr}$

ZERO LOCATION

if $h(n) = h(N-1-n)$

$H(z) = z^{-(N-1)} H(\frac{1}{z})$

If z_0 is ZERO of Real Linear-phase filter $\{z_0^*, \frac{1}{z_0}, \frac{1}{z_0^*}\}$ also is ZERO.

General ZERO exist in group 4

Gen. ZERO at unit circle → 2

ZERO at axis = 2

1 and −1 count.

$\min E = \max_{\omega \in [0,\pi]} |A(\omega) - A_d(\omega)|$

Least Square Approximation of h[n]

Type I, III : M equations

Type II, IV : $\frac{N}{2} - 1$ equations

Ripple Reduce	Transition band (less sharp)
Ripple Inc	Transition band (more sharp)
more points	Transition band (more sharp)
less points	- - - band (less sharp)

$A(\omega) = \sum h \cdot \cos(\cdot) + h.$

$\underset{L \times 1}{[A]} = \underset{L \cdot M+1}{[\cos(\cdot)]} \underset{M+1}{[h]}$ Type I/III

$DFT\{\frac{1}{N} X[k]\} = x[-n]$

DFT:

Length N h[n] → ZERO PAD w/ M−1 ZEROS → $\boxed{N+M-1 \text{ DFT}}$ → ⊗ → $\boxed{DFT}$

Length M x[n] → ZERO PAD N−1 ZEROS → $\boxed{N+M-1 \text{ DFT}}$ → ⊗

x(n) → $\boxed{\downarrow M}$ → $E_0(z)^M$ → ⊕

↓ $\boxed{z}$ → $\boxed{\downarrow M}$ → $E_1(z^M)$ → ⊕

↓ $\boxed{z}$ → - - - -

Polyphase decimation $\frac{N}{M}$ tap

x(n) → $\boxed{\downarrow M}$ → $E_0(z)$ → ⊕

↓ $\boxed{z}$ → $\boxed{\downarrow M}$ → $E_1(z)$ → ⊕

↓ - - Same Rate - -

Polyphase Interpolation

x(n) → $\boxed{\uparrow L}$ → $E_0(z^L)$ → ⊕ →.

no ZERO → $\boxed{\uparrow L}$ → $E_1(z^L)$ → $\boxed{z^{-1}}$

$h(n) = -h(N-1-n)$

FIR Filter Design :

notch filter

Why we Desire them?

$h_d(n) = \delta[n-n_d] \leftrightarrow H_d(\omega) e^{-j\omega n_d}$

$H(\omega) = A(\omega) e^{-j(k_1 + k_2\omega)}$

A→D · Fs scale · Repeat

D→A · T scale · Remove $|F| > F_s/2$

Filter Design Process

1.) choose desired Freq. Resp.

2.) choose allow-able class of filter
Length-N FIR Filter

3.) choose measure of quality (How close)

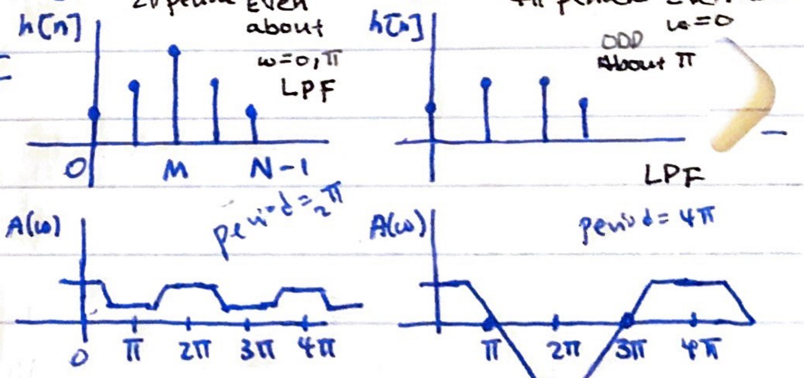
4.) Apply Algo to find best val.

5.) Choose best Realization of filter.

Type of Filter

Type I : N ODD

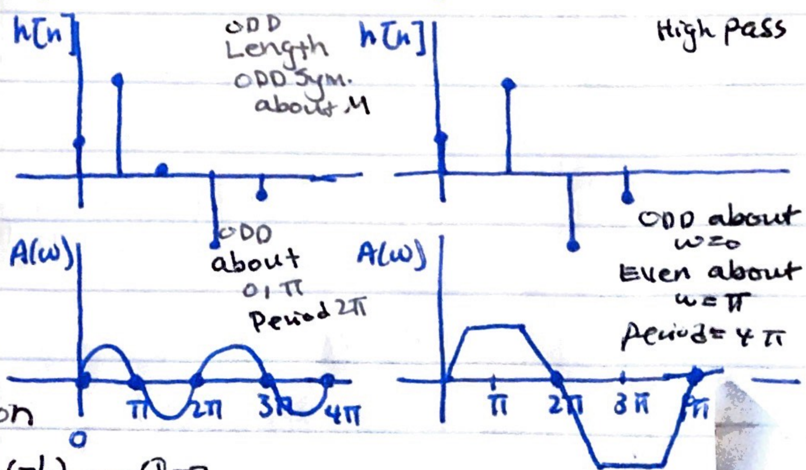
Type II : N Even



$h(n) = h(N-1-n)$

Type III : N ODD

Type IV : N Even



I'm aware of the Academic integrity. I affirm that I'll not give or receive help on this exam. and all work is on my own

Exam 2 Aiden Chen

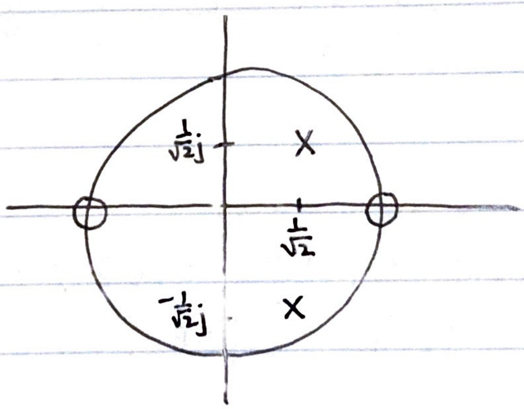
1

1. $H(z) = \dfrac{z^2-1}{z^2-z+0.5} = \dfrac{(z+1)(z-1)}{\left(z-\left(\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}}\right)\right)\left(z-\left(\frac{1}{\sqrt{2}}-j\frac{1}{\sqrt{2}}\right)\right)}$

$\sqrt{b^2-4ac} = \sqrt{1-4\frac{1}{2}} = \sqrt{1-2} = \sqrt{-1} = j$

$\dfrac{-b\pm\sqrt{\ }}{2a} = \dfrac{1}{\sqrt{2}} \pm \dfrac{j}{\sqrt{2}}$

a.) pole-ZERO

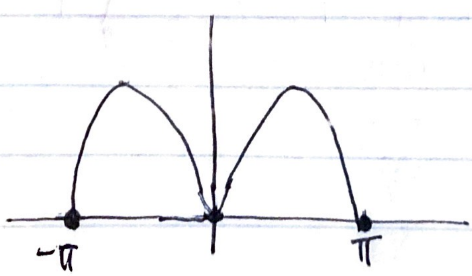


b.) $|H(0)| = \left|\dfrac{0}{\ }\right| = \dfrac{0}{\ }$ $\boxed{0}$. $e^{jw=0} = 1$

$|H(\frac{\pi}{2})| = \left|\dfrac{e^{j\frac{\pi}{2}}-1}{(e^{j\frac{\pi}{2}})^2 - e^{j\frac{\pi}{2}}+0.5}\right| = \dfrac{\sqrt{1^2+1^2}}{\sqrt{1^2+(\frac{1}{2})^2}} = \dfrac{\sqrt{2}}{\sqrt{1+\frac{1}{4}}} = \sqrt{\dfrac{2}{\frac{5}{4}}}$

$H(-\frac{\pi}{2}) = \sqrt{\frac{4}{5}}$ $= \sqrt{\frac{8}{5}}$

Bandpass because at $w=0,\pi$ it is Rejected and in between is passed.



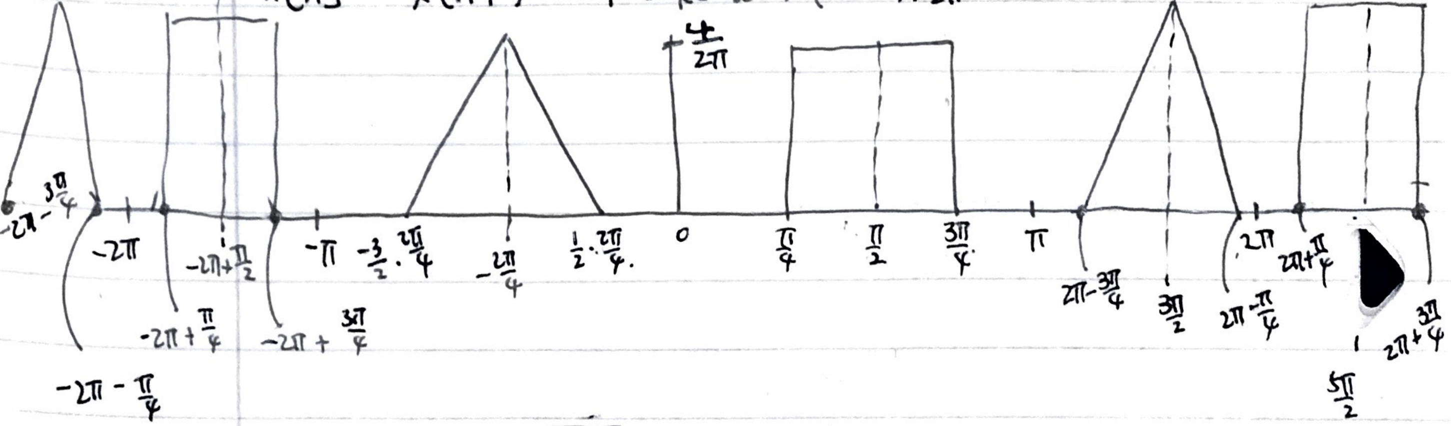
2a.) Nyquist Rate is $2 \cdot 1.5 = 3 \frac{Rad}{s}$

2b.) $x(t)$ is real.

2c.) $x(t) \rightarrow \boxed{} \rightarrow x[n]$

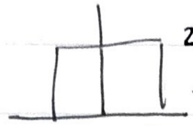
$\uparrow$ $T = \frac{2\pi}{4}$

$x[n] = x(nT) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X\left(\omega - k\frac{4}{2\pi}\right)$



2d.) $X(n) \rightarrow \boxed{\downarrow 2} \rightarrow E_0(z^2) \rightarrow (+) \rightarrow X[k].$

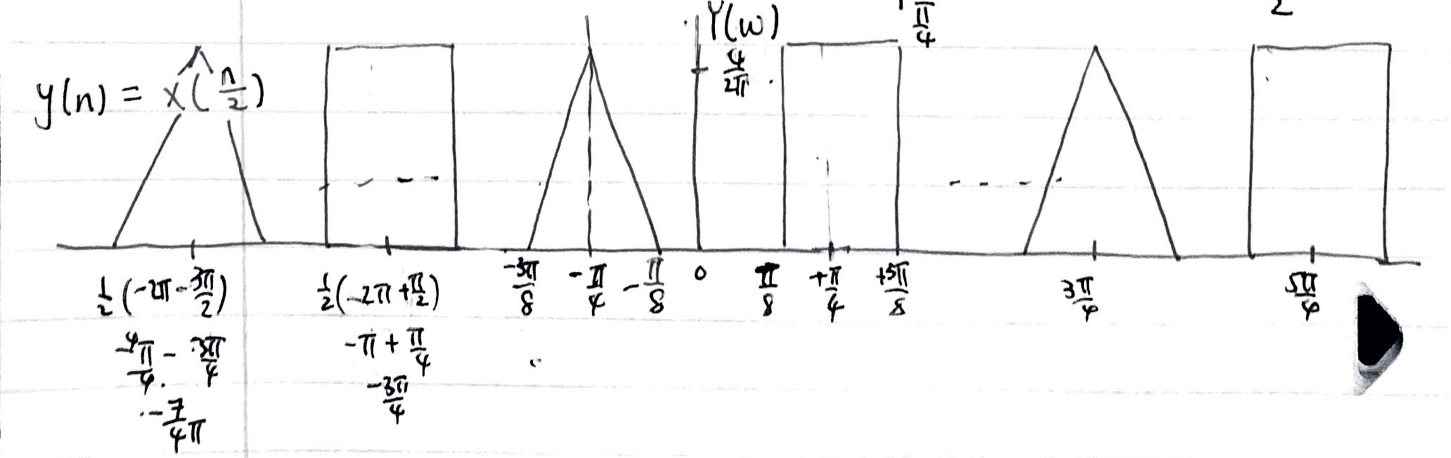
$\boxed{z^{+1}} \rightarrow \boxed{\downarrow 2} \rightarrow E_1(z^2) \rightarrow$

2e.) $x[n] \rightarrow \boxed{\uparrow 2} \rightarrow y(n) \rightarrow \boxed{D/C} \rightarrow y(t) \rightarrow$  $\rightarrow z(t)$

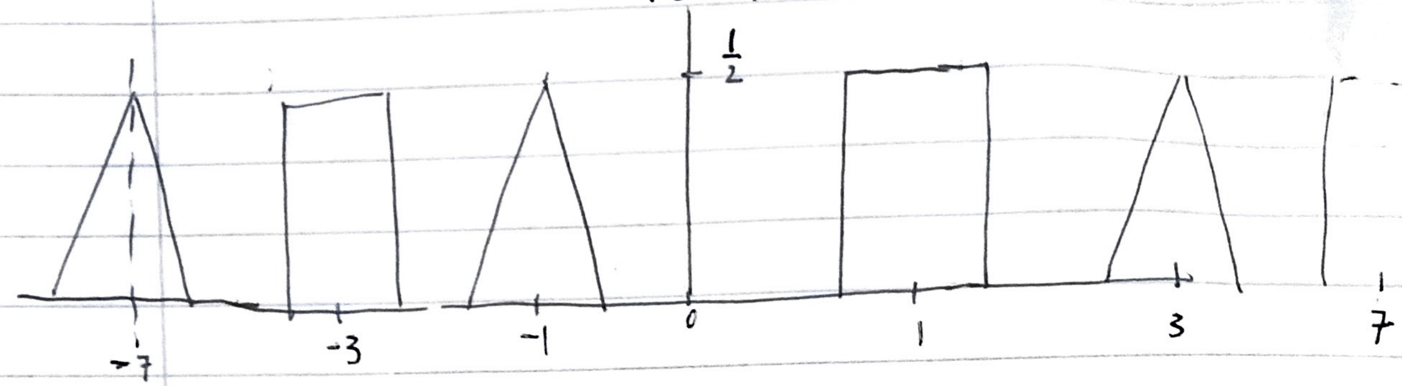
$\uparrow \frac{\pi}{4}$

$Y(\Omega) = y(t)$

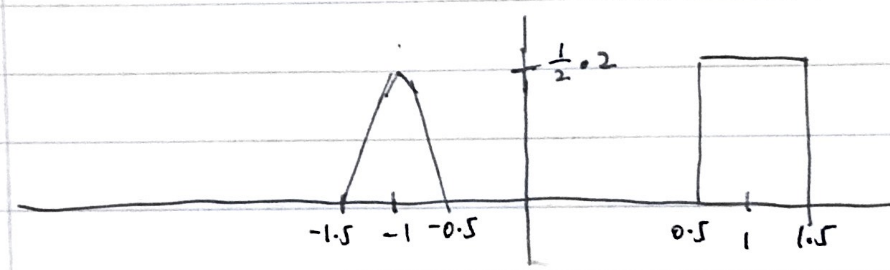
$y(n) = x\left(\frac{n}{2}\right)$



$Y(\Omega)$.



$Z(\omega)$



2

3.) 3.1 F

3.2 T

3.3. T.

3.4. F

3.5. T.

3.6. T.

3.7. T.

3.8. F.

3.9 T.

3.10 N/M.

3.11 decimation and interpolation

3.12. Remove ZERo.

3.13. Points are close together.

3.14. Cyclic Convolution , 7.

3.15. covid, I didn't get to do a lot of fun things.

$$\underbrace{4 \cdot \log_2}_{\text{stage.}}(4) = 8.$$

$$8 \underbrace{\log_2}_{3.}(8) = 24.$$

4.) ③

$x(n) \to \boxed{\uparrow 2} \to x_1(n) \to \boxed{\downarrow 2} \to y_1(n)$

$x_1(n) = x(\frac{n}{2}) \leftrightarrow X_1(\omega) = X(2\omega)$

$y_1(n) = x_1(2n) \leftrightarrow Y_1(\omega) = \frac{1}{2}\sum_{n=0}^{2-1} X_1(\frac{\omega}{2} - 2\pi i/2)$

$= \frac{1}{2}\sum_{n=0}^{1} X(2(\frac{\omega}{2} - \pi i))$

$= \frac{1}{2}(X(\omega) + X(\omega - 2\pi))$

$x(n) \to \boxed{\downarrow 2} \to x_2(n) \to \boxed{\uparrow 2} \to y_2(n)$

$x_2(n) = x(2n) \leftrightarrow X_2(\omega) = \frac{1}{2}\sum_{n=0}^{2-1} X(\frac{\omega}{2} - \frac{2\pi}{2} i)$.

$y_2(n) = x_2(\frac{n}{2}) \leftrightarrow Y_2(\omega) = X_2(2\omega) = \frac{1}{2}\sum_{n=0}^{1} X(\frac{2\omega}{2} - \pi i)$.

$= \frac{1}{2}(X(\omega) + X(\omega - \pi))$

So. $Y_1(\omega) \neq Y_2(\omega)$.

$y_1(n) \neq y_2(n)$

4b.) $X[k]$ be N-Point DFT. of $x[n]$, $0 \leq n \leq N-1$

what is $Y[k]$ when

$y[n] = X[n]$, $0 \leq n \leq N-1$.

$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{nk}$

$Y[k] = \sum_{n=0}^{N-1} y[n] W_N^{nk}$

$= \sum_{n=0}^{N-1} X[n] W_N^{nk}$

$m = l + n$.

$= \sum_{n=0}^{N-1} \left[\sum_{l=0}^{N-1} x[l] W_N^{kl}\right] W_N^{nk}$

$0 \leq l \leq N-1$

$0 \leq n \leq N-1$

$= \sum_{l=0}^{N-1} x[l] \sum_{n=0}^{N-1} W_N^{k(l+n)}$