9/6/20.

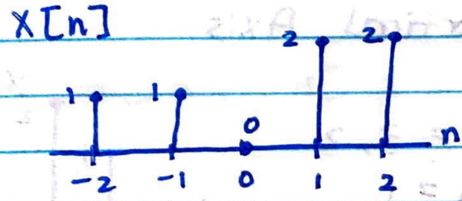
DSP. Analytical Problem HW #1 Aiden chen

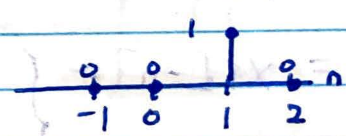
4). Consider the signal $x[n]$.

$$x[n] = \begin{cases} 1 & n = -2, -1 \\ 0 & n = 0 \\ 2 & n = 1, 2 \\ 0 & \text{else} \end{cases}$$

4a.) $y_1[n] = x[n] * \delta[n-1]$

$x[n]$ $*$ $\delta[n-1] = h[n]$





$y_1[n] = x[n] * h[n]$ using analytical approach is easier

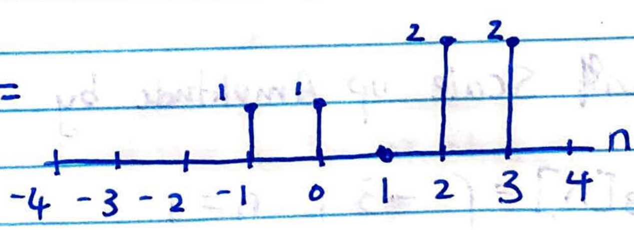
$$= \sum_{k=-\infty}^{\infty} \underbrace{x[k]}_{\text{find out } k\text{-Region where } x[k] \neq 0} \cdot h[n-k]$$

$$= \sum_{k=-2}^{-1} 1 \cdot h[n-k] + \sum_{k=1}^{2} 2 \cdot h[n-k]$$

$$= h[n-(-2)] + h[n-(-1)] + 2h[n-1] + 2h[n-2]$$

now sum up the h-term

$y_1[n] =$



$$= \{1, \underset{\substack{\uparrow \\ 0}}{1}, 0, 2, 2\}$$

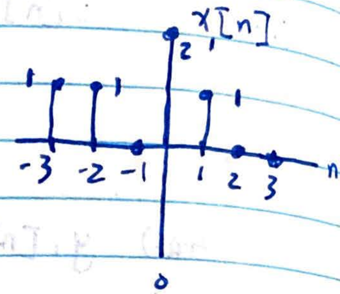
Represent in a set form.

4b) $y_2[n] = -3\,x[-2n+1]$

steps. Shift $x[n]$ → Flip → Scale time → Fold & Scale Amplitude

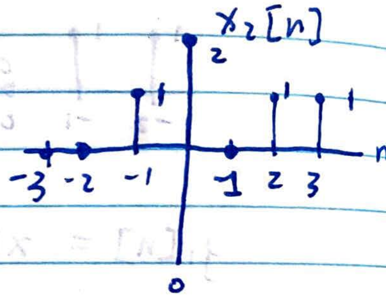
1. Shift $x[n]$ 1 to the left

$$x_1[n] = x[n+1] = \begin{cases} 1, & n=-3,-2 \\ 0, & n=-1 \\ 2, & n=0,1 \\ 0, & \text{else} \end{cases}$$



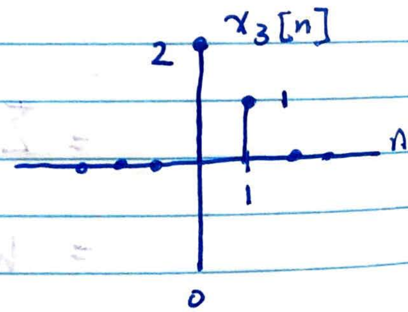
2.) Flip $x[n]$ across vertical Axis.

$$x_2[n] = x_1[-n] = \begin{cases} 1, & n=3,2 \\ 0, & n=1 \\ 2, & n=0,-1 \\ 0, & \text{else} \end{cases}$$



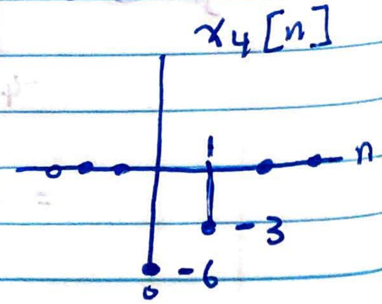
3.) Scale $x[n]$ by 2.

$$x_3[n] = x_2[2n] = \begin{cases} 1, & n=1 \\ 2, & n=0 \\ 0, & \text{else.} \end{cases}$$



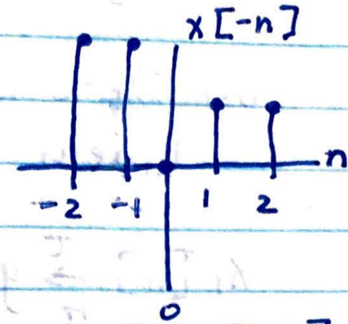
4.) Fold down & Scale up Amplitude by -3.

$$x_4[n] = -3x_3[n] = \begin{cases} -3, & n=1 \\ -6, & n=0 \\ 0, & \text{else} \end{cases}$$

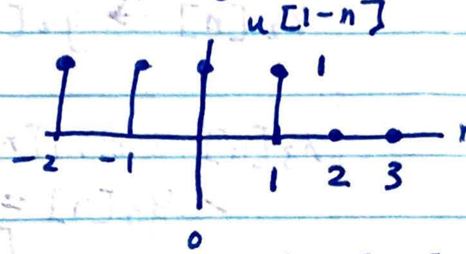


4c.) $y_3[n] = x[-n]\,u[1-n]$

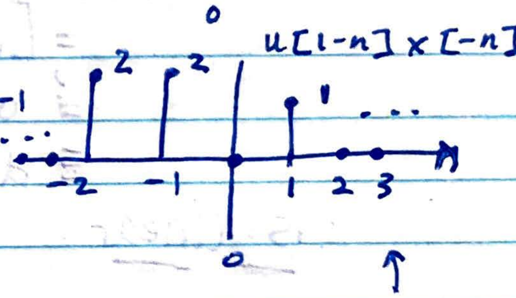
$$x[-n] = \begin{cases} 1, & n = 2, 1 \\ 0, & n = 0 \\ 2, & n = -1, -2 \\ 0, & \text{else} \end{cases}$$



$$u[1-n] = \begin{cases} 1, & n \leq 1 \\ 0, & n > 1 \end{cases}$$



$$x[-n]\,u[1-n] = \begin{cases} 2, & n = -2, -1 \\ 0, & n = 0 \\ 1, & n = 1 \\ 0, & \text{else.} \end{cases}$$



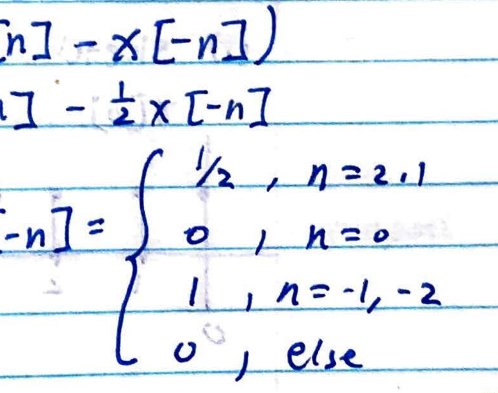
multiply point-by-point

4d.) $y_4[n] = \text{Odd}(x[n])$

$$x_o[n] = \text{odd}(x[n]) = \tfrac{1}{2}(x[n] - x[-n])$$
$$= \tfrac{1}{2}x[n] - \tfrac{1}{2}x[-n]$$

$$\tfrac{1}{2}x[n] = \begin{cases} \tfrac{1}{2}, & n = -2, -1 \\ 0, & n = 0 \\ 1, & n = 1, 2 \\ 0, & \text{else} \end{cases} \qquad \tfrac{1}{2}x[-n] = \begin{cases} 1/2, & n = 2, 1 \\ 0, & n = 0 \\ 1, & n = -1, -2 \\ 0, & \text{else} \end{cases}$$

$$x_o[n] = \begin{cases} \tfrac{1}{2} - 1 = -\tfrac{1}{2}, & n = -2, -1 \\ 0 - 0, & n = 0 \\ 1 - \tfrac{1}{2} = \tfrac{1}{2}, & n = 1, 2 \\ 0, & \text{else.} \end{cases}$$



5.) $y[n] = x[n^2]$

a.) Determine whether System is ____ Linear and Time-invariant.

$x_1[n] \xrightarrow{T} y_1[n] = x_1[n^2]$

$x_2[n] \xrightarrow{T} y_2[n] = x_2[n^2]$

$x_3[n] = \alpha x_1[n] + b x_2[n]$

$\rightarrow y_3[n] = x_3[n^2]$

$= [\alpha x_1[n^2] + b x_2[n^2]]$

$\stackrel{?}{=} \alpha y_1[n] + b y_2[n]$

$\equiv \alpha x_1[n^2] + b y_2[n^2]$

$\therefore$ is linear

Time Invariant

$x_1[n] \rightarrow y_1[n] = x_1[n$

$z[n] = x[n-n_0]$

$z[n] \rightarrow z[n^2]$

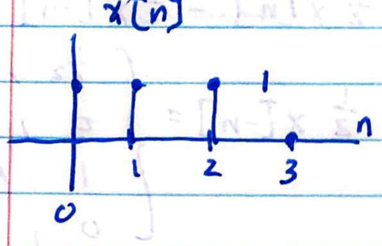
$= x[n^2-n_0]$

$y_1[n-n_0] \rightarrow x_1[(n-n_0)$

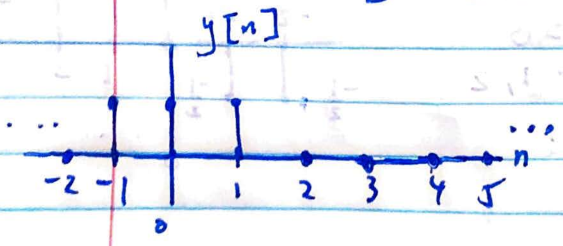
not time invariant

5 b.). $x[n] = \begin{cases} 1 & 0 \leq n \leq 2 \\ 0 & \text{else.} \end{cases}$

i.) sketch $x[n]$

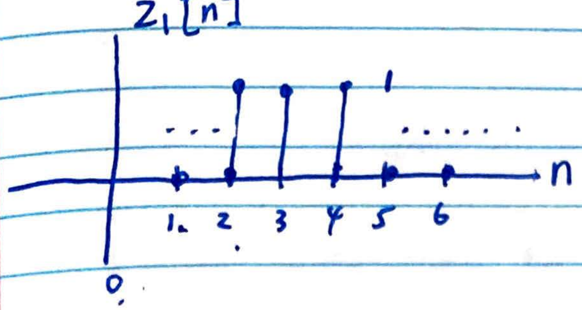


ii.) sketch $y[n] = x[n^2]$



$y[n] = \{\cdots, 0, 1, \underset{\uparrow}{1}, 1, 0, \cdots\}$

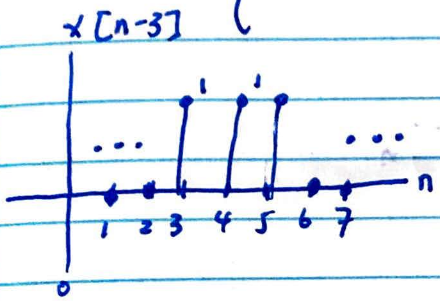
iii Sketch $Z_1[n] = y[n-3] = x[(n-3)^2]$



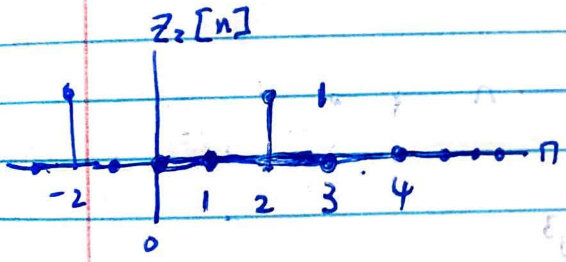
$\{\ldots, \underline{0}, 0, 1, 1, 1, 0, 0, 0 \ldots\}$

iv.) Determine and sketch $x[n-3]$.

$$x[n-3] = \begin{cases} 1 & 3 \le n \le 5 \\ 0 & \text{else.} \end{cases}$$



v.). $x[n-3] \xrightarrow{T} Z_2[n] = x[n^2-3]$.



$Z_1[n] \neq Z_2[n]$.

not Time-Invariant

vi.) The system is not time-invariant. At different time, input and output are different.

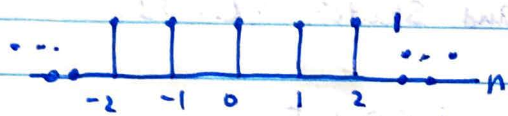
vii). $y[n]$ is not periodic

HW 01.

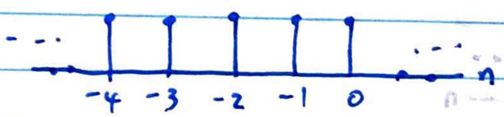
HW1.

#6a.

x[n]

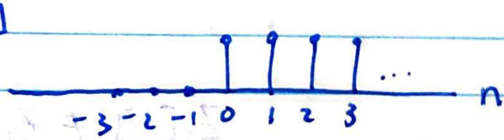


$h[n] = x[n+2]$

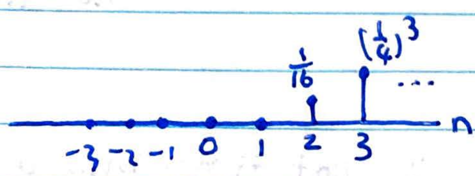


#6b.

x[n]



h[n]



6.) Sketch and compute convolution.

a.) $x[n] = \begin{cases} 1, & n = -2, -1, 0, 1, 2. \\ 0, & \text{else} \end{cases}$ $\quad h[n] = x[n+2]$

$x[n+2] = \begin{cases} 1; & n = -4, -3, -2, -1, 0 \\ 0, & \text{else.} \end{cases}$

↑
h[n].
↳ = 1 for n < -2.

$y[n] = x[n] * h[n]$

used analytical approach.

$= \sum_{k=-\infty}^{\infty} \underbrace{x[k]}_{\neq 0 \text{ if } -2<k<2} h[n-k]$

$= \sum_{k=-2}^{2} 1 \cdot h[n-k]$

$= \underset{①}{h[n-(-2)]} + \underset{②}{h[n-(-1)]} + \underset{③}{h[n-(0)]} + \underset{④}{h[n-1]} + \underset{⑤}{h[n-2]}$

used table to sum

= n	-6	-5	-4	-3	-2	-1	0	1	2
①	1	1	1	1	1				
②		1	1	1	1	1			
③			1	1	1	1	1		
④				1	1	1	1	1	
⑤					1	1	1	1	1
+	1	2	3	4	5	4	3	2	1

$y[n] = \{1, 2, 3, 4, 5, 4, \underline{3}, 2, 1\}$

↑
0.

$$y[n] = \sum_{k=-\infty}^{\infty} h[k] \cdot x[n-k]$$

$$= \sum_{k=-\infty}^{\infty} \left(\tfrac{1}{4}\right)^k u[k-2] \cdot x[n-k]$$

$$= \sum_{k=2}^{n} \left(\tfrac{1}{4}\right)^k u[n-k].$$

$\hookrightarrow n > k > 0$

$$= \sum_{k=2}^{n} \left(\tfrac{1}{4}\right)^k$$

$$= \sum_{k=0}^{n-2} \left(\tfrac{1}{4}\right)^{k+2}.$$

$$= \sum_{k=0}^{n-2} \left(\tfrac{1}{4}\right)^k \cdot \left(\tfrac{1}{4}\right)^2$$

$$= \left(\frac{\left(\frac{1}{4}\right)^0 - \left(\frac{1}{4}\right)^{n-2+1}}{1 - \frac{1}{4}}\right)\left(\tfrac{1}{4}\right)^2$$

$$= \frac{\left(1 - \left(\frac{1}{4}\right)^{n-1}\right)4}{4 - 1}\left(\tfrac{1}{4}\right)^2$$

$$= \frac{4 - \left(\frac{1}{4}\right)^n (4)(4)}{3}\left(\tfrac{1}{4}\right)^2$$

$$= \frac{\frac{4}{4^2} - \frac{\left(\frac{1}{4}\right)^n (4)^2}{4^2}}{3} = \frac{\frac{1}{4} - \left(\frac{1}{4}\right)^n}{3} = \frac{1}{12} - \frac{1}{3}\left(\tfrac{1}{4}\right)^n$$

$n \geq 0.$