Rensselaer Polytechnic Institute
Department of Electrical, Computer, and Systems Engineering
ECSE 4530: Digital Signal Processing, Fall 2020

Homework #2: due Thursday, Oct. 1^{st}, at the beginning of class.

The homeworks in this class will be a mixture of paper-and-pencil problems to turn in using Gradescope for electronic submission, and MATLAB problems to submit online. Note that it may not always be easy to figure out why your code isn't passing the tests from the online interface. it might be better to use your local copy of MATLAB to debug/solve the problems, then upload the solution to the website once you've got it working. Use Piazza to ask questions (but please remember not to post code that will give the problem away to other students).

MATLAB Grader Problems
A full description of all the Grader problems is provided at the Grader link. You don't need to physically hand anything in for the Grader problems (but make sure that you hit Submit and see a green check mark to make sure your solution has been recorded).

1. (10 points). Compute the Discrete-Time Fourier Transform using numerical integration.

2. (15 points). Determine the response of a real LTI system to a pure cosine input.

3. (10 points). Estimate the time-varying dominant frequency of a signal (like a whistle or pure instrument note) using the spectrogram.

4. (15 points). Recreate a synthetic signal from its estimated dominant frequencies. This can be combined with the previous problem to make a "chiptune" version of a real input signal, and could be further enhanced by using harmonics and/or a signal envelope as explored in Homework #1.

Analytical Problems: Be sure to show intermediate steps that explain your reasoning and do not forget to provide the labelings.

5. (10 points) **Fourier Series and Fourier Transform.**

   (a) Consider the signal $x[n] = \{-1, 0, \underline{1}, 2, 4\}$ with Fourier transform $X(\omega) = X_R(\omega) + jX_I(\omega)$. Determine the signal $y[n]$ with the Fourier transform $Y(\omega) = X_I(\omega) + X_R(\omega)e^{j2\omega}$.

   (b) Assume that $x[n]$ is aperiodic and with Fourier transform $X(\omega)$. Now we construct a periodic signal
   $$y[n] = \sum_{k=-\infty}^{\infty} x[n - kN].$$

   i. What is the period of $y[n]$?
   ii. Determine the Fourier series coefficients of $y[n]$ using $X(\omega)$.

6. (20 points) **Discrete-Time Fourier Transform (DTFT).**

   (a) Compute the discrete-time Fourier transform (DTFT) $X(\omega)$ of $x[n] = \text{sinc(n)} \cdot \text{sinc(n)}$.

   (b) Compute $\sum_{n=-\infty}^{\infty} x^2[n]$ for $x[n]$ as given in Part (a).

(c) Compute and plot the DTFT $X(\omega)$ of the signal $x[n] = x[n+10]$ given as follows

$$x[n] = \begin{cases} 3 - |n+2|, & n = -5, \ldots, 1 \\ 0, & n = -9, \ldots, -6. \end{cases}$$

(d) Determine the range of values of α and β for which the LTI system with input $x[n] = \alpha^n u[n]$ and impulse response $h[n] = \beta^n (u[n] - u[n-3])$ is stable.

(e) Consider a linear time-invariant (LTI) system with impulse response $h[n] = \frac{1}{2}e^{-n}u[n] + \frac{1}{2}e^{-3n}u[n]$. Let $y[n]$ be the output for the input $x[n] = e^{-n}u[n]$.

i. Compute the frequency response $H(\omega)$ of the system.

ii. Compute the discrete-time Fourier transform of $y[n]$, i.e. $Y(\omega)$.

iii. Compute $y[n]$ using $Y(\omega)$.

7. (10 points) **Frequency response of LTI systems.**

(a) Consider an LTI system with impulse response $h[n] = \left(\frac{1}{4}\right)^n u[n]$.

i. Determine and sketch the magnitude response, i.e., $|H(\omega)|$.

ii. Determine and sketch the magnitude response of the LTI system for $x[n] = \cos\left(\frac{3\pi n}{10}\right)$.

(b) An FIR filter is described by the relation

$$2y[n] = x[n] + x[n-1].$$

i. Is this system LTI?

ii. Let $x_1[n] = \delta[n]$, $x_2[n] = u[n]$ and $x_3[n] = e^{j\omega n}$ be the respective inputs to the above FIR filter. Determine which of these inputs are eigenfunctions of the system.

8. (10 points) **z-transform.** Determine the z-transform of the following signals. Do not forget to specify the region of convergence (ROC) for each part.

(a) $x[n] = \{3, 0, 0, 0, 0, \underline{6}, 1, 4\}$

(b) $x[n] = (0.1^n + 0.1^{-n})u[n]$

Readings from textbook: 2.4, 3.1-3.3, 4.1-4.4, 5.1-5.2.
Suggested practice problems from textbook: 2.45, 2.48, 3.14, 3.18, 3.40, 3.42, 4.4, 4.5, 4.9, 4.10, 5.4.