Table 1: **Properties of the Discrete-Time Fourier Transform**

$$x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega})e^{j\omega n}d\omega$$

$$X(\omega) = \sum_{n=-\infty}^{+\infty} x[n]e^{-j\omega n}$$

Property	**Aperiodic Signal**	**DTFT**
	$x[n]$ $y[n]$	$\left.\begin{array}{l} X(\omega) \\ Y(\omega) \end{array}\right\}$ Periodic with period 2π
Linearity	$ax[n] + by[n]$	$aX(\omega) + bY(\omega)$
Time-Shifting	$x[n - n_0]$	$e^{-j\omega n_0}X(\omega)$
Frequency-Shifting	$e^{j\omega_0 n}x[n]$	$X(\omega - \omega_0)$
Conjugation	$x^*[n]$	$X^*(-\omega)$
Time Reversal	$x[-n]$	$X(-\omega)$
Time Expansions	$x_{(k)}[n] = \begin{cases} x[n/k], & \text{if } n = \text{multiple of } k \\ 0, & \text{if } n \neq \text{multiple of } k \end{cases}$	$X(k\omega)$
Convolution	$x[n] * y[n]$	$X(\omega)Y(\omega)$
Multiplication	$x[n]y[n]$	$\frac{1}{2\pi}\int_{2\pi} X(\theta)Y(\omega - \theta)d\theta$
Differencing in Time	$x[n] - x[n-1]$	$(1 - e^{-j\omega})X(\omega)$
Accumulation	$\sum_{k=-\infty}^{n} x[k]$	$\frac{1}{1 - e^{-j\omega}}X(\omega)$ $+\pi X(0)\sum_{k=-\infty}^{+\infty}\delta(\omega - 2\pi k)$
Differentiation in Frequency	$nx[n]$	$j\frac{dX(\omega)}{d\omega}$
Conjugate Symmetry for Real Signals	$x[n]$ real	$\begin{cases} X(\omega) = X^*(-\omega) \\ \Re e\{X(\omega)\} = \Re e\{X(-\omega)\} \\ \Im m\{X(\omega)\} = -\Im m\{X(-\omega)\} \\ \lvert X(\omega)\rvert = \lvert X(-\omega)\rvert \\ \sphericalangle X(\omega) = -\sphericalangle X(-\omega) \end{cases}$
Symmetry for Real, Even Signals	$x[n]$ real and even	$X(\omega)$ real and even
Symmetry for Real, Odd Signals	$x[n]$ real and odd	$X(\omega)$ purely imaginary and odd
Even-odd Decomposition of Real Signals	$x_e[n] = \mathcal{E}v\{x[n]\}$ $[x[n]$ real$]$ $x_o[n] = \mathcal{O}d\{x[n]\}$ $[x[n]$ real$]$	$\Re e\{X(\omega)\}$ $j\Im m\{X(\omega)\}$

Parseval's Relation for Aperiodic Signals

$$\sum_{n=-\infty}^{+\infty} \lvert x[n]\rvert^2 = \frac{1}{2\pi}\int_{2\pi} \lvert X(\omega)\rvert^2 d\omega$$

Table 2: **Basic Discrete-Time Fourier Transform Pairs**

Signal $x[n]$	**DTFT** $X(\omega)$
$a^n u[n], \quad \lvert a\rvert < 1$	$\dfrac{1}{1 - ae^{-j\omega}}$
$x[n] = \begin{cases} 1, & \lvert n\rvert \le N_1 \\ 0, & \lvert n\rvert > N_1 \end{cases}$	$\dfrac{\sin[\omega(N_1 + \frac{1}{2})]}{\sin(\omega/2)}$
$\frac{\sin Wn}{\pi n} = \frac{W}{\pi} \operatorname{sinc}\left(\frac{Wn}{\pi}\right)$ $0 < W < \pi$	$X(\omega) = \begin{cases} 1, & 0 \le \lvert\omega\rvert \le W \\ 0, & W < \lvert\omega\rvert \le \pi \end{cases}$ $X(\omega)$ periodic with period 2π
$\delta[n]$	1
$u[n]$	$\dfrac{1}{1 - e^{-j\omega}} + \displaystyle\sum_{k=-\infty}^{+\infty} \pi\delta(\omega - 2\pi k)$
$\delta[n - n_0]$	$e^{-j\omega n_0}$
$(n+1)a^n u[n], \quad \lvert a\rvert < 1$	$\dfrac{1}{(1 - ae^{-j\omega})^2}$
$\dfrac{(n+r-1)!}{n!(r-1)!} a^n u[n], \quad \lvert a\rvert < 1$	$\dfrac{1}{(1 - ae^{-j\omega})^r}$
$\displaystyle\sum_{k=\langle N\rangle} a_k e^{jk(2\pi/N)n}$	$2\pi \displaystyle\sum_{k=-\infty}^{+\infty} a_k \delta\left(\omega - \frac{2\pi k}{N}\right)$
$e^{j\omega_0 n}$	$2\pi \displaystyle\sum_{l=-\infty}^{+\infty} \delta(\omega - \omega_0 - 2\pi l)$
$\cos \omega_0 n$	$\pi \displaystyle\sum_{l=-\infty}^{+\infty} \{\delta(\omega - \omega_0 - 2\pi l) + \delta(\omega + \omega_0 - 2\pi l)\}$
$\sin \omega_0 n$	$\dfrac{\pi}{j} \displaystyle\sum_{l=-\infty}^{+\infty} \{\delta(\omega - \omega_0 - 2\pi l) - \delta(\omega + \omega_0 - 2\pi l)\}$
$x[n] = 1$	$2\pi \displaystyle\sum_{l=-\infty}^{+\infty} \delta(\omega - 2\pi l)$
Periodic square wave $x[n] = \begin{cases} 1, & \lvert n\rvert \le N_1 \\ 0, & N_1 < \lvert n\rvert \le N/2 \end{cases}$ and $x[n+N] = x[n]$	$2\pi \displaystyle\sum_{k=-\infty}^{+\infty} a_k \delta\left(\omega - \frac{2\pi k}{N}\right)$
$\displaystyle\sum_{k=-\infty}^{+\infty} \delta[n - kN]$	$\dfrac{2\pi}{N} \displaystyle\sum_{k=-\infty}^{+\infty} \delta\left(\omega - \frac{2\pi k}{N}\right)$

Table 3: **Properties of the z-Transform**

Property	Sequence	Transform	ROC
	$x[n]$ $x_1[n]$ $x_2[n]$	$X(z)$ $X_1(z)$ $X_2(z)$	R R_1 R_2
Linearity	$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$	At least the intersection of R_1 and R_2
Time shifting	$x[n - n_0]$	$z^{-n_0}X(z)$	R except for the possible addition or deletion of the origin
Scaling in the z-Domain	$e^{j\omega_0 n}x[n]$ $z_0^n x[n]$ $a^n x[n]$	$X(e^{-j\omega_0}z)$ $X\left(\frac{z}{z_0}\right)$ $X(a^{-1}z)$	R $z_0 R$ Scaled version of R (i.e., $\|a\|R =$ the set of points $\{\|a\|z\}$ for z in R)
Time reversal	$x[-n]$	$X(z^{-1})$	Inverted R (i.e., R^{-1} = the set of points z^{-1} where z is in R)
Time expansion	$x_{(k)}[n] = \begin{cases} x[r], & n = rk \\ 0, & n \neq rk \end{cases}$ for some integer r	$X(z^k)$	$R^{1/k}$ (i.e., the set of points $z^{1/k}$ where z is in R)
Conjugation	$x^*[n]$	$X^*(z^*)$	R
Convolution	$x_1[n] * x_2[n]$	$X_1(z)X_2(z)$	At least the intersection of R_1 and R_2
First difference	$x[n] - x[n-1]$	$(1 - z^{-1})X(z)$	At least the intersection of R and $\|z\| > 0$
Accumulation	$\sum_{k=-\infty}^{n} x[k]$	$\frac{1}{1-z^{-1}}X(z)$	At least the intersection of R and $\|z\| > 1$
Differentiation in the z-Domain	$nx[n]$	$-z\frac{dX(z)}{dz}$	R

Initial Value Theorem
If $x[n] = 0$ for $n < 0$, then
$x[0] = \lim_{z\to\infty} X(z)$

Table 4: **Some Common z-Transform Pairs**

Signal	Transform	ROC
1. $\delta[n]$	1	All z
2. $u[n]$	$\frac{1}{1-z^{-1}}$	$\|z\| > 1$
3. $u[-n-1]$	$\frac{1}{1-z^{-1}}$	$\|z\| < 1$
4. $\delta[n-m]$	z^{-m}	All z except 0 (if $m > 0$) or ∞ (if $m < 0$)
5. $\alpha^n u[n]$	$\frac{1}{1-\alpha z^{-1}}$	$\|z\| > \|\alpha\|$
6. $-\alpha^n u[-n-1]$	$\frac{1}{1-\alpha z^{-1}}$	$\|z\| < \|\alpha\|$
7. $n\alpha^n u[n]$	$\frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$	$\|z\| > \|\alpha\|$
8. $-n\alpha^n u[-n-1]$	$\frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$	$\|z\| < \|\alpha\|$
9. $[\cos \omega_0 n]u[n]$	$\frac{1-[\cos \omega_0]z^{-1}}{1-[2\cos \omega_0]z^{-1}+z^{-2}}$	$\|z\| > 1$
10. $[\sin \omega_0 n]u[n]$	$\frac{[\sin \omega_0]z^{-1}}{1-[2\cos \omega_0]z^{-1}+z^{-2}}$	$\|z\| > 1$
11. $[r^n \cos \omega_0 n]u[n]$	$\frac{1-[r\cos \omega_0]z^{-1}}{1-[2r\cos \omega_0]z^{-1}+r^2z^{-2}}$	$\|z\| > r$
12. $[r^n \sin \omega_0 n]u[n]$	$\frac{[r\sin \omega_0]z^{-1}}{1-[2r\cos \omega_0]z^{-1}+r^2z^{-2}}$	$\|z\| > r$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

Table 5: **Properties of the Discrete Fourier Transform**

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{jk\omega_0 n} = \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{jk(2\pi/N)n}$$

$$X[k] = \sum_{n=0}^{N-1} x[n]e^{-jk\omega_0 n} = \sum_{n=0}^{N-1} x[n]e^{-jk(2\pi/N)n}$$

Property	**Periodic signal**	**DFT coefficients**
	$\left.\begin{matrix} x[n] \\ y[n] \end{matrix}\right\}$ Periodic with period N and fundamental frequency $\omega_0 = 2\pi/N$	$\left.\begin{matrix} X[k] \\ Y[k] \end{matrix}\right\}$ Periodic with period N
Linearity	$Ax[n] + By[n]$	$AX[k] + BY[k]$
Time shift	$x[n - n_0]$	$X[k]e^{-jk(2\pi/N)n_0}$
Frequency Shift	$e^{jM(2\pi/N)n}x[n]$	$X[k - M]$
Conjugation	$x^*[n]$	$X^*[-k]$
Time Reversal	$x[-n]$	$X[-k]$
Time Scaling	$x_{(m)}[n] = \begin{cases} x[n/m] & \text{if } n \text{ is a multiple of } m \\ 0 & \text{if } n \text{ is not a multiple of } m \end{cases}$ (periodic with period mN)	$\frac{1}{m}X[k]\left(\begin{matrix}\text{viewed as} \\ \text{periodic with} \\ \text{period } mN\end{matrix}\right)$
Periodic Convolution	$\sum_{r=\langle N\rangle} x[r]y[n-r]$	$X[k]Y[k]$
Multiplication	$x[n]y[n]$	$\frac{1}{N}\sum_{l=\langle N\rangle} X[l]Y[k-l]$
First Difference	$x[n] - x[n-1]$	$(1 - e^{-jk(2\pi/N)})X[k]$
Running Sum	$\sum_{k=-\infty}^{n} x[k]\left(\begin{matrix}\text{finite-valued and} \\ \text{periodic only if } X[0]=0\end{matrix}\right)$	$\left(\frac{1}{(1 - e^{-jk(2\pi/N)})}\right)X[k]$
Conjugate Symmetry for Real Signals	$x[n]$ real	$\begin{cases} X[k] = X^*[-k] \\ \Re e\{X[k]\} = \Re e\{X[-k]\} \\ \Im m\{X[k]\} = -\Im m\{X[-k]\} \\ \lvert X[k]\rvert = \lvert X[-k]\rvert \\ \sphericalangle X[k] = -\sphericalangle X[-k] \end{cases}$
Real and Even Signals	$x[n]$ real and even	$X[k]$ real and even
Real and Odd Signals	$x[n]$ real and odd	$X[k]$ purely imaginary and odd
Even-Odd Decomposition of Real Signals	$x_e[n] = \mathcal{E}v\{x[n]\}$ $[x[n]$ real$]$ $x_o[n] = \mathcal{O}d\{x[n]\}$ $[x[n]$ real$]$	$\Re e\{X[k]\}$ $j\Im m\{X[k]\}$

Parseval's Relation for Periodic Signals

$$\sum_{n=0}^{N-1} |x[n]|^2 = \frac{1}{N}\sum_{k=0}^{N-1} |X[k]|^2$$