Name: ______________________________

Rensselaer Polytechnic Institute
Department of Electrical, Computer, and Systems Engineering
ECSE 4530: Digital Signal Processing, Fall 2019

Exam #2. Closed book, closed notes.
November 21, 2019, 10:00-11:20 AM

Show all work for full credit.

- Electronic devices are not permitted.
- Reduce expressions involving exponentials, sines, and cosines as much as possible.
- If the sinc function is needed, use the definition $\text{sinc}(x) = \frac{\sin x}{x}$.
- Useful ratio to dB conversion formula: $10\log_{10}(2) = 3$ dB.

Good luck!

1		**25**
2		**25**
3		**25**
4		**25**
Total		**100**

1. (25 points.) True/false and fill in the blank questions. You do not need to explain your reasoning. Read each question carefully.

   1. ______ A fast Fourier transform (FFT) is an algorithm that computes the DFT of a sequence, or its inverse (IDFT).

   2. ______ Cyclic convolution of two discrete time signals in time domain corresponds to multiplication of their discrete Fourier transforms (DFTs) in frequency domain.

   3. ______ Parseval's relation states that the sum (or integral) of the square of a function is equal to the sum (or integral) of the square of its transform.

   4. ______ We cannot compute the DFT for aperiodic signals.

   5. ______ Frequency response of a discrete time signal is 2π periodic.

   6. ______ A linear-phase FIR filter is always even symmetric or odd symmetric about the middle tap.

   7. ______ A length $N = 8$ FFT diagram can be implemented in 3 stages.

   8. ______ A length 1000 DFT operation involves $10,000$ multiplications.

   9. ______ Linear phase FIR filters are not stable.

   10. ______ The downsampler $\boxed{\downarrow 3}$ is not linear.

   11. ______ The upsampler $\boxed{\uparrow 2}$ is not time invariant.

   12. ______ If the original signal is sampled at M times just above the Nyquist rate, we do not need to prefilter it before we downsample it by a factor of M.

   13. ______ The output of the cascaded system of a downsampler and an upsampler as shown below satisfies $y[n] = x[n]$.

   $$x[n] \longrightarrow \boxed{\downarrow 2} \longrightarrow \boxed{\uparrow 2} \longrightarrow y[n]$$

   14. ______ The Remez exchange algorithm tries to maximize the number of extremal frequencies in designing Chebyshev FIR filters.

   15. ______ We may have aliasing if the sampling rate is above the Nyquist rate.

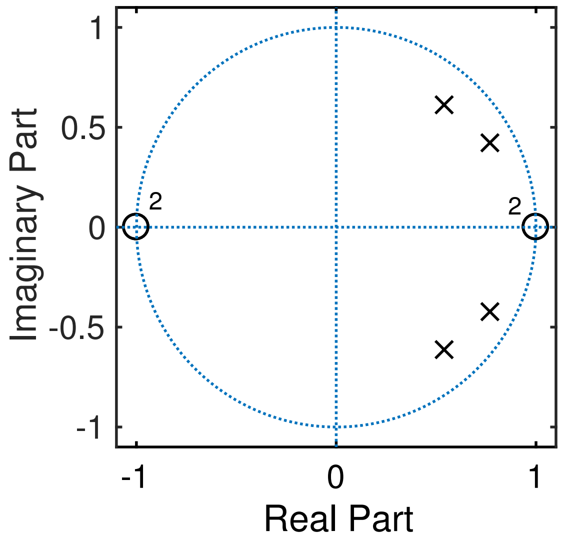
   16. ______ Linear interpolation is good when the adjacent signal samples are very close to each other.

   17. ______ We cannot do discrete time processing of continuous time signals.

18. ______________________ is twice the bandwidth of a bandlimited signal.

19. The fundamental element of a radix-2 FFT is colloquially known as a ______________________

20. Given a transfer function $H(z)$ with pole-zero diagram as below, $H(\omega)$ is a ______________________ filter.



21. Based on the pole-zero diagram as above, given that the above system is ______________________, then the filter is stable.

22. A ______________________, in FFT algorithms, is any of the trigonometric constant coefficients that are multiplied by the data in the course of the algorithm.

23. A bandlimited signal cannot be also ______________________.

24. Ideal low pass filtering of a signal after upsampling is equivalent to ______________________.

25. DSP is ______________________________________.

2. (25 points.) Transfer function.

   We are given a transfer function for a linear and time invariant discrete-time system:

$$H(z) = \frac{(z-j)(z+j)}{z(z-0.5)(z+0.5)}$$

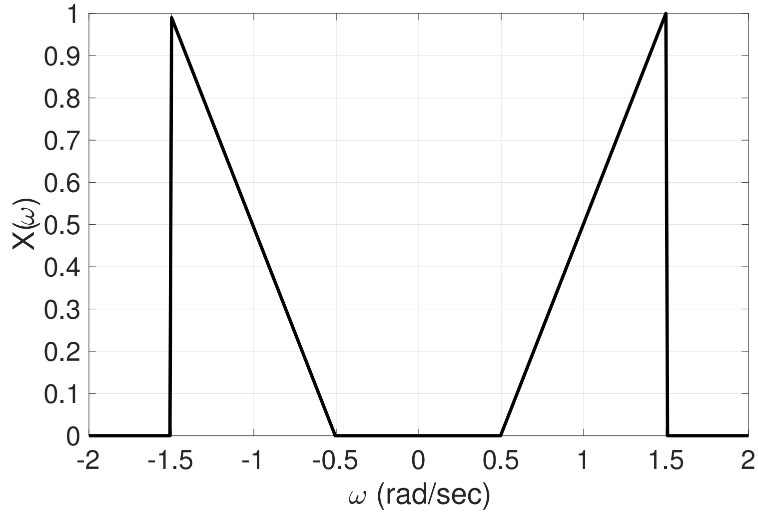
   (a) (6 points.) Determine the difference equation corresponding to this system.

   (b) (6 points.) Give the pole-zero diagram of $H(z)$.

(c) (8 points.) Using the pole-zero diagram, plot the frequency response of $H(\omega)$ for $\omega \in (-\pi, \pi)$. Your plot does not need to be very precise, only indicate the values of $H(0)$, $H\left(\frac{\pi}{2}\right)$ and $H\left(-\frac{\pi}{2}\right)$.

(d) (5 points.) Determine the value of $|H(\omega)|$ at $\omega = \pi/4$.

3. (25 points.) Sampling Theorem. We consider a real signal $x(t)$ with a Fourier transform $X(\omega)$ as shown below.

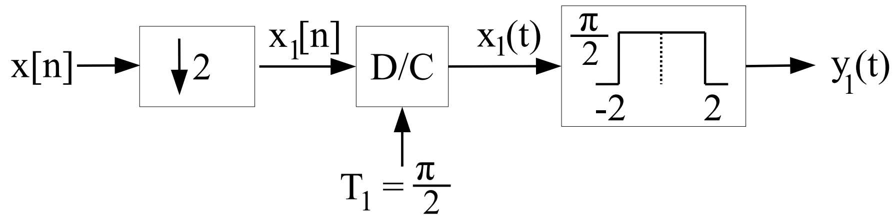


(a) (2 points.) What is the Nyquist rate of $x(t)$?

(b) (3 points.) We have a complex filter $h(t)$, which has a flat passband extending only from 0.5 to 1.5 rad/sec. The passband has a 3 dB gain in the range $0.5 < \omega < 1.5$, but otherwise has a value of 0 ($-\infty$ in dB), including for all negative values of ω. Plot the Fourier transform (continuous time) of $h(t)$ in the frequency range $[-2, 2]$.

(c) (5 points.) $x(t)$ is bandpass filtered with the complex filter $h(t)$. Call the resulting signal $y(t)$. Plot the Fourier transform (continuous time) of $y(t)$ in the frequency range $[-6, 6]$.

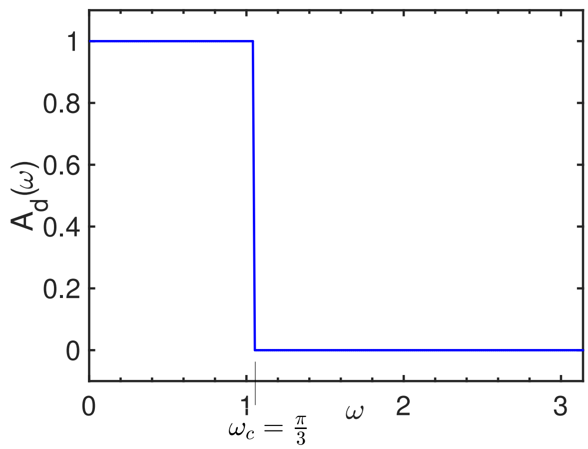
(d) (5 points.) The continuous time signal $y(t)$ is sampled with a sampling period of $T = \frac{\pi}{2}$ seconds. Call the resulting signal $y[n]$. Plot the Fourier transform (DTFT) of $y[n]$ in the corresponding frequency range, namely $\omega \in [-3\pi, 3\pi]$.



(e) (5 points.) Now assume that $x[n]$ is obtained from the original signal $x(t)$ by sampling it with a sampling period of $T = \frac{\pi}{4}$. Next, $x[n]$ is downsampled by a factor of 2 as shown above. Call the resulting signal $x_1[n]$. Plot the Fourier transform (DTFT) of $x_1[n]$ in the frequency range $\omega \in [-3\pi, 3\pi]$.

(f) (5 points.) The downsampled signal $x_1[n]$ is then converted from discrete sequence to continuous with $T_1 = \frac{\pi}{2}$, and a new signal is reconstructed with an ideal low-pass filter with cutoff frequency 2 and gain $\frac{\pi}{2}$, as illustrated above. What is the reconstructed signal $y_1(t)$? Plot the continuous time frequency responses of $X_1(\omega)$ and $Y_1(\omega)$.

4. (25 points.) Linear-phase FIR filter design. We want to design a Type-I, length $N = 15$ FIR digital filter with linear phase that approximates the ideal amplitude response as illustrated below:



$$A_d(\omega) = \begin{cases} 1, & \text{for} \quad |\omega| \le \frac{\pi}{3} \\ 0, & \text{for} \quad \frac{\pi}{3} < |\omega| \le \pi. \end{cases}$$

(a) (5 points.) Design the filter using the frequency-sampling method by taking $N = 15$ samples equally-spaced by $\frac{2\pi}{15}$ starting at $\omega = 0$. What is the vector A_d of desired amplitude response samples?

(b) (5 points.) What will the value of the frequency response of the designed filter be at $\omega = \frac{4}{5}\pi$? Determine both the magnitude and the phase response.

(c) (5 points.) Roughly sketch the amplitude response $A(\omega)$. You do not need to be very precise here. Is the response symmetric about $\omega = 0$? Is it symmetric about $\omega = \pi$? Explain your reasoning.

(d) (10 points.) Recall that a Type-I linear-phase filter satisfies the equation

$$h[n] = \frac{1}{N}\left[A[0] + \sum_{k=1}^{M} 2A[k]\cos\left(\frac{2\pi(n-M)k}{N}\right)\right], \quad \text{where } M = \frac{N-1}{2},$$

where $A[k]$'s are samples of amplitude response of the filter $h[n]$.

1. (3 points.) Determine the filter coefficients $h[5]$, $h[7]$, and $h[9]$.

2. (4 points.) How many equations you need to simultaneously solve in order to determine $h[n]$? Why?

3. (3 points.) Is $h[n]$ real or complex valued? Does it have any symmetry properties? Explain.

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