HW #3. 10/13/20.

4.) Find H(z)

4a.) $$H(z) = \frac{(z+1)(z-1-j)(z-1+j)}{(z-0.5+0.5j)(z-0.5-0.5j)(z-0.75)}$$

$$= \frac{(z+1)(z^2-2z+ [illegible])}{(z^2 - z + [illegible])}$$

$$= \frac{z^3 - 2z^2 + \cancel{2z} + z^2 - \cancel{2z} + 2}{z^3 - 2z^2 + 0.5z + 0.75z + \frac{1}{2}z - \frac{1}{2}\frac{3}{4}}$$

$$= \frac{z^3 - z^2 + 2}{z^3 - 1.75z^2 + 1.25z - 3/8}$$

$$= \frac{1 - z^{-1} + 2z^{-3}}{1 - 1.75z^{-1} + 1.25z^{-2} - 3/8 z^{-3}}$$

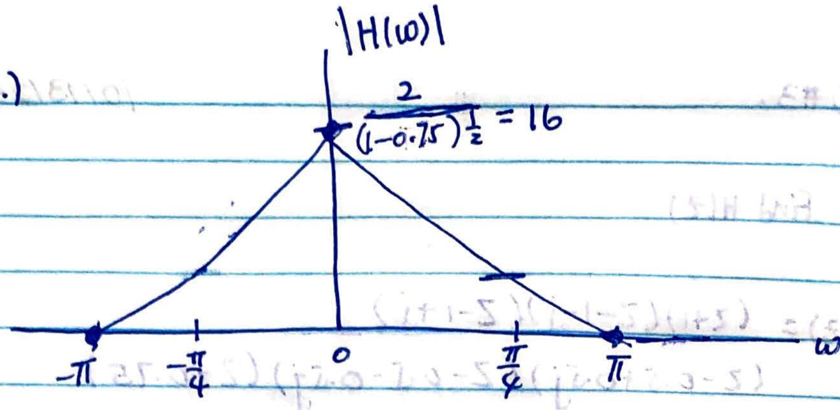
4c.) Find the difference eq.

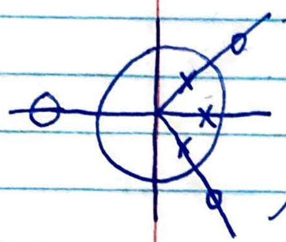
$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - z^{-1} + 2z^{-3}}{1 - 1.75z^{-1} + 1.25z^{-2} - 3/8 z^{-3}}$$

$$Y(z)(1 - 1.75z^{-1} + 1.25z^{-2} - 3/8 z^{-3}) = X(z)(1 - z^{-1} + 2z^{-3})$$

$$y(n) - 1.75y(n-1) + 1.25y(n-2) - \frac{3}{8}y(n-3) =$$
$$x(n) - x(n-1) + 2x(n-3)$$

4b.)

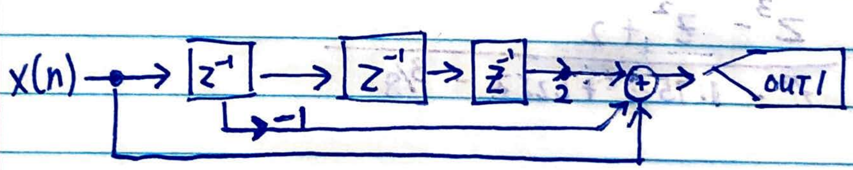


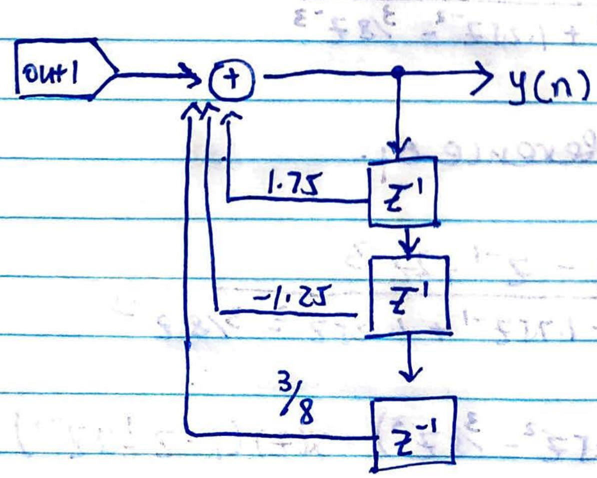


These pole and ZERO will cancel out because they are aligned to each other. at same angles

$$H(z) = \frac{z+1}{(z-0.75)\frac{1}{2}} \Rightarrow H(e^{j\omega}) = \frac{e^{j\omega}+1}{(e^{j\omega}-0.75)\frac{1}{2}}$$

4d.)





4e.) $H(z) = \dfrac{1 - z^{-1} + 2z^{-3}}{1 - 1.75z^{-1} + 1.25z^{-2} - \frac{3}{8}z^{-3}}$

4ei) $x(n) = 3^n$

if you have an impulse Response, $H(z)$ then any output with input $y(n)$:

$$y(n) = H(e^{j\omega})\, x(e^{j\omega})$$

Here: $e^{j\omega} = 3 \Rightarrow y(n) = H(3)x(n)$

$$= x(n)\, 1.3675$$

$$= x(n)\, \frac{160}{117}$$

4eii) $x(n) = 3^{n-10}$

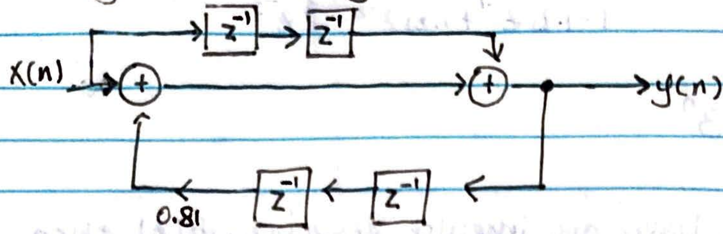
$$y(n) = 3^{-10} \cdot 3^n \cdot H(3) = \frac{160}{117} 3^{-10} 3^n$$

4eiii) $x(n) = 3^{10-n} = 3^{10} \cdot \left(\frac{1}{3}\right)^n$

$$y(n) = 3^{10} \cdot \left(\tfrac{1}{3}\right)^n \underbrace{\left[\frac{1 - \left(\frac{1}{3}\right)^{-1} + 2\left(\frac{1}{3}\right)^{-3}}{1 - 1.75\left(\frac{1}{3}\right)^{-1} + 1.25\left(\frac{1}{3}\right)^{-2} - \frac{3}{8}\left(\frac{1}{3}\right)^{-3}}\right]}_{H(1/3)}$$

$$= 3^{10} \cdot \left(\tfrac{1}{3}\right)^n \cdot (-16.64) \ldots$$

5.) Filter Design.



5a.)

$x(n) + [0.81\, y(n-2)] + x(n-2) = y(n)$

$H(z) = \frac{Y(z)}{H(z)}$

$X(z) + 0.81 z^{-2} Y(z) + z^{2} X(z) = Y(z)$

$X(z)(1 + z^{-2}) = (1 - 0.81 z^{-2}) Y(z)$

5b.) $= \frac{Y(z)}{X(z)} = \frac{1 + z^{-2}}{1 - 0.81 z^{-2}}$

ZERO: $z = \pm j$

poles

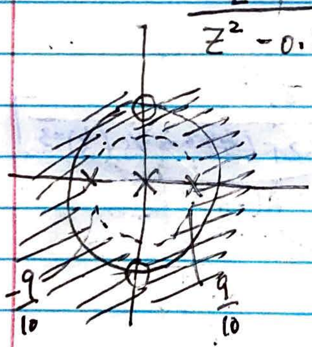
$z = \pm 9/10.$

$= \frac{z^2 + 1}{z^2 - 0.81}$

ROC: $|z| >$

$z = \pm \frac{1}{9}.$

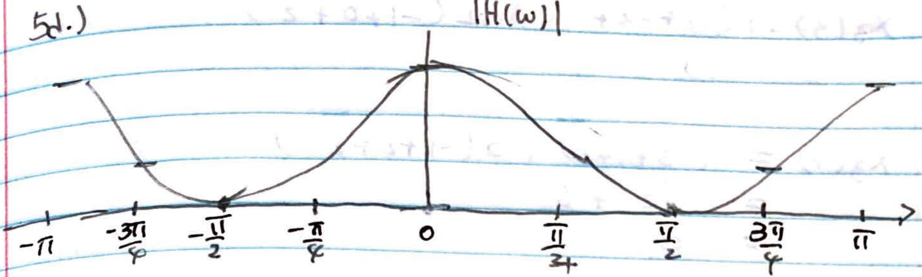
5c.)



STABLE. Roc contain the unit Circle.

$|z| \ldots |9.|$

5d.)



BandStop: filter; since it poss most frequeny exceep those in a specific Range.

Hw #3.

Problem 6a.)

$x(n) = \{ \underline{1}, 1, 1, 2, 2, 2 \}$

$h(n) = \{ \underline{-1}, 2, -3, 1, 0, 2 \}$

$N = 6$

$N-1 = 5$

$W_6 = e^{j\frac{\pi}{3}}$

$$x(n) \circledast h(n) = y(n)$$

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \\ y(4) \\ y(5) \end{bmatrix} = \begin{bmatrix} h(0) & h(5) & h(4) & h(3) & h(2) & h(1) \\ h(1) & h(0) & h(5) & h(4) & h(3) & h(2) \\ h(2) & h(1) & h(0) & h(5) & h(4) & h(3) \\ h(3) & h(2) & h(1) & h(0) & h(5) & h(4) \\ h(4) & h(3) & h(2) & h(1) & h(0) & h(5) \\ h(5) & h(4) & h(3) & h(2) & h(1) & h(0) \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \\ x(4) \\ x(5) \end{bmatrix}$$

$N \times 1$ $\qquad$ $N \times N$ $\qquad$ $N \times 1$

$$y(0) = h(0)x(0) + h(5)x(1) + h(4)x(2) + h(3)x(3) + h(2)x(4) + h(1)x(5)$$

$$= -1 + \cancel{2} + 0 + \cancel{2} + -\cancel{6} + \cancel{4} +$$

$$= 1$$

$$y(1) = h(1)x(0) + h(0)x(1) + h(5)x(2) + h(4)x(3) + h(3)x(4) + h(2)x(5)$$

$$= -1$$

$$y(2) = h(2)x(0) + h(1)x(1) + h(0)x(2) + h(5)x(3) + h(4)x(4) + h(3)x(5)$$

$$= 4$$

$$y(3) = h(3)x(0) + h(2)x(1) + h(1)x(2) + h(0)x(3) + h(5)x(4) + h(4)x(5)$$

$$= 2$$

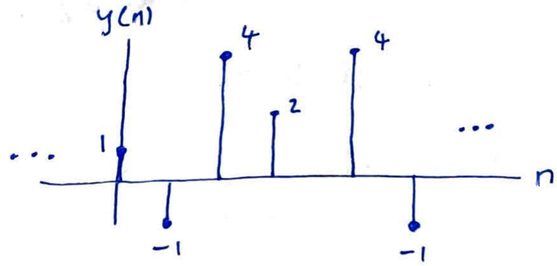
$$y(4) = h(4)x(0) + h(3)x(1) + h(2)x(2) + h(1)x(3) + h(0)x(4) + h(5)x(5)$$

$$= 4$$

$$y(5) = h(5)x(0) + h(4)x(1) + h(3)x(2) + h(2)x(3) + h(1)x(4) + h(0)x(5)$$

$$= -1$$

$$y(n) = \{\underline{1}, -1, 4, 2, 4, -1\}$$



Problem 6b:

$X(n) = \{-1, \underline{1}, -2, -1\}$ $\qquad$ $X(n-1) \circledast h(n-1) = y(n-2)$

$h(n) = \{-1, \underline{1}, 2\}$

Length - 6 Circular Convolution.

$N = 6$

$N-1 = 5$.

We can calculate $x(n) \circledast h(n)$ then shift final Result by 2 left.

$x_1(n) = \{\underline{-1}, 1, -2, -1, 0, 0\}$ ← Pad with 3-1 ZEROS.

$h(n) = \{\underline{-1}, 1, 2, 0, 0, 0\}$ ← Pad with 4-1 ZEROS.

① $y_1(n) = x_1(n) \circledast h_1(n)$

② $y(n) = y_1(n-2)$.

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① Find $y_1(n)$.

— PAD ZEROS.
— Non-ZEROS.

$$\begin{bmatrix} y_1(0) \\ y_1(1) \\ y_1(2) \\ y_1(3) \\ y_1(4) \\ y_1(5) \end{bmatrix} = \begin{bmatrix} h(0) & h(5) & h(4) & h(3) & h(2) & h(1) \\ h(1) & h(0) & h(5) & h(4) & h(3) & h(2) \\ h(2) & h(1) & h(0) & h(5) & h(4) & h(3) \\ h(3) & h(2) & h(1) & h(0) & h(5) & h(4) \\ h(4) & h(3) & h(2) & h(1) & h(0) & h(5) \\ h(5) & h(4) & h(3) & h(2) & h(1) & h(0) \end{bmatrix} \begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \end{bmatrix}$$

$y_1(0) = 1 + 0 + 0 + 0 + 0 + 0 = 1$

$y_1(1) = -1 + -1 = -2$

$y_1(2) = -2 + 1 + 2 = 1$

$y_1(3) = 0 + 2 + -2 + 1 = 1$

$y_1(4) = -4 + -1 = -5$

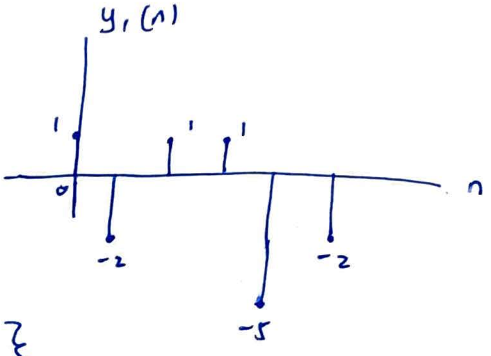
$y_1(5) = -2$

$y_1(n) = \{\underline{1}, -2, 1, 1, -5, -2\}$

(2) Find $y(n)$.

$y(n) = y_1(n-2)$

$y_1 = \{\underline{1}, -2, 1, 1, -5, -2\}$.



$y(n) = \{1, -2, \underline{1}, 1, -5, -2\}$