Rensselaer Polytechnic Institute
Department of Electrical, Computer, and Systems Engineering
ECSE 4530: Digital Signal Processing, Fall 2020

Exam #1.
October 8, 2020, 10:10-11:30 AM

<u>Show all work for full credit.</u>

- Closed book, closed notes.
- 1 one-sided crib sheet is allowed.
- Electronic devices (including calculators) are not permitted.
- Reduce expressions involving exponentials, sines, and cosines as much as possible.
- If the sinc function is needed, use the definition $\text{sinc}(x) = \frac{\sin x}{x}$.
- Geometric series formula: $\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}, \quad |a| < 1$.
- Finite sum formula: $\sum_{n=M}^{N-1} a^n = \frac{a^M - a^N}{1-a}, \quad a \neq 1$.
- When in doubt, show your work.

Good luck!

1		**20**
2		**35**
3		**45**
Total		**100**

<u>Append the below phrase into your handwritten solutions and upload a single pdf file that includes your handwritten crib sheets to Gradescope.</u>

I am aware of the Academic Integrity policy. I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

Name

Signature

1. (20 points.) **Discrete-time system properties.** Consider the system given by the input-output relationship

$$y[n] = x[n-1]\cos^2\left(\frac{\pi}{8}(n+1) + \frac{\pi}{4}\right)$$

You need to prove if each of the below statements (a)-(c) is true, and otherwise give a counter example.

Determine if the given system is

   (a) (5 points.) linear.
   (b) (5 points.) time-invariant.
   (c) (5 points.) causal.
   (d) (5 points.) Determine the output $y[n]$ if the input is $x[n] = \cos\left(\frac{\pi}{8}n\right)$.

2. (35 points.) **Z-transform.** Consider the system which has the following transfer function:

$$H(z) = \frac{1-3z^{-1}+6z^{-2}-4z^{-3}}{(1-z^{-1})(1-0.4z^{-1})(1-0.8z^{-1})}, \quad \text{ROC: } a < |z| < b$$

   (a) (10 points.) Plot the pole-zero diagram for the given system. Indicate the ROC.
   (b) (5 points.) Determine the finite constants a and b.
   (c) (3 points.) Is this system stable? Explain your reasoning.
   (d) (2 points.) Is this system causal? Explain.
   (e) (10 points.) Determine the impulse response $h[n]$ of the system.
   (f) (5 points.) Discuss whether $h[n]$ is even or not.

3. (45 points.) **Mixed bag.** The parts of this problem are independent of each other. The idea here is to use your knowledge of the Linear time-invariant (LTI) systems, discrete-time signals, Discrete Time Fourier Transform (DTFT) properties, such as oddness, evenness, Parseval's relation, $\sum_{n=-\infty}^{\infty} x[n] = X(0)$ and $x[0] = \frac{1}{2\pi}\int_{2\pi} X(\omega)\mathrm{d}\omega$, stability, causality, etc. You can refer to the tables to verify your solutions.

   (a) (7 points.) True/false and fill in the blank questions. You do not need to explain your reasoning. Read each question carefully.

      ______ LTI systems can be completely characterized by its impulse response.

      ______ $x[n]\delta[n-1] = x[1]$

      ______ $x[n] * \delta[n+1] = x[n+1]$ where $*$ denotes convolution.

      ______ If $x[n]$ is an odd signal, then $x[0] = 0$.

      ______ For stable systems, the ROC is towards outwards.

      ______ If $x[n]$ is real and even, then its DTFT $X(\omega)$ is also even.

      ______ The DTFT of a rectangular pulse is a sinc waveform.

   (b) (10 points.) The pole-zero diagram of the causal signal $x[n]$ has two poles at -2 and 3. Plot the ROC for the time reversed signal $x[-n]$. Indicate the ROC.
   (c) (13 points.) Derive the discrete-time signal $x[n]$ that has DTFT $X(\omega) = \frac{1}{(1-0.5e^{-j\omega})^2}$ using DTFT properties. You can refer to the tables to verify your solutions.
   (d) (15 points.) Compute and plot the DTFTs $X_1(\omega)$, $X_2(\omega)$, $X_3(\omega)$ of

$$x_1[n] = \{1,1,\underline{1},1,1\}, \qquad x_2[n] = \{1,0,1,0,\underline{1},0,1,0,1\}$$

$$x_3[n] = \{1,0,0,1,0,0,\underline{1},0,0,1,0,0,1\}.$$

   Determine the relation between $X_1(\omega)$, $X_2(\omega)$, $X_3(\omega)$.