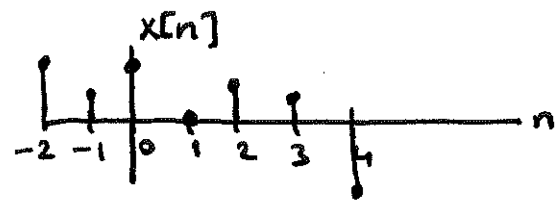
## Announcements

- Lecture 1, page 3 (sampling figures are corrected).
- Next week will be in class

  Monday    Last name between "A..." "Le..."
  Thursday    "Li..." "Zh..."

## Last lecture

DT signal $x[n]$, $n \in$ integers




$x[n]$ is periodic if $x[n] = x[n+N]$, $N \in$ "+" integer, for all $n$.

## Today's lecture

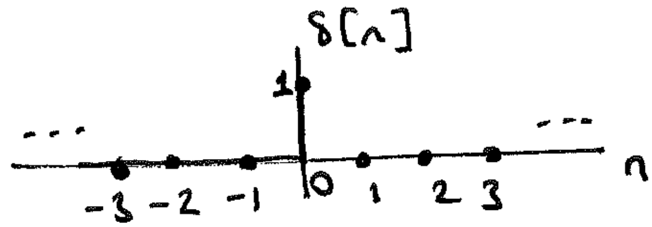
- Unit step function, unit impulse (delta) function, properties
- Complex signals and periodicity (MATLAB grader example)
- Systems and their properties

HW0

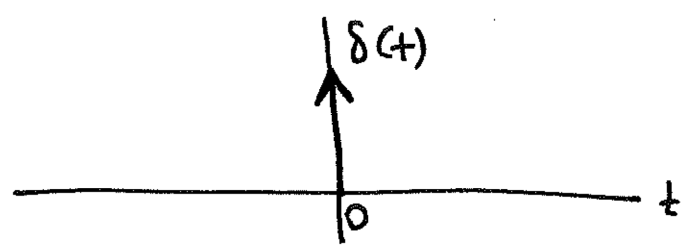
## Some Important Functions

Unit impulse function (delta function)

$$\delta[n] = \begin{cases} 0 & n \neq 0 \\ 1 & n = 0 \end{cases}$$

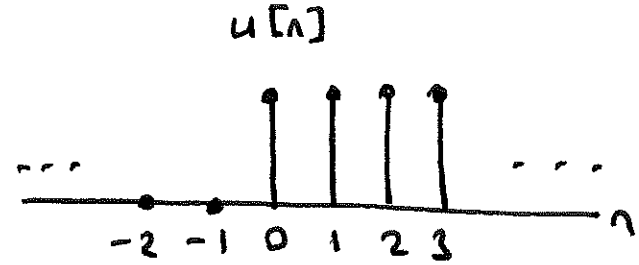



Continuous time $\delta(t)$



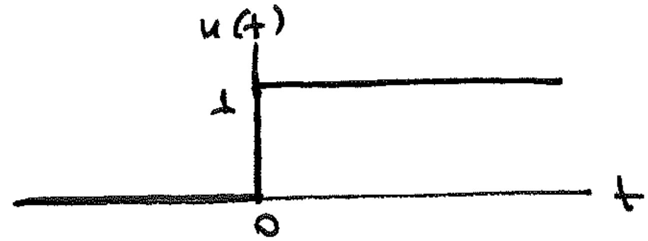
Unit step function

$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$




Continuous time

$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

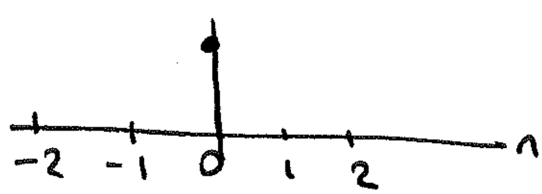



In CT, $\delta(t) = \dfrac{du(t)}{dt}$

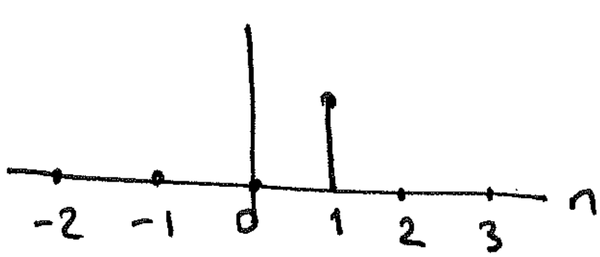
In DT, $\underbrace{\delta[n] + \delta[n-1] + \delta[n-2] + \cdots}_{} = u[n]$

$$= \underbrace{\sum_{k=0}^{\infty} \delta[n-k]}_{\text{summation}}$$

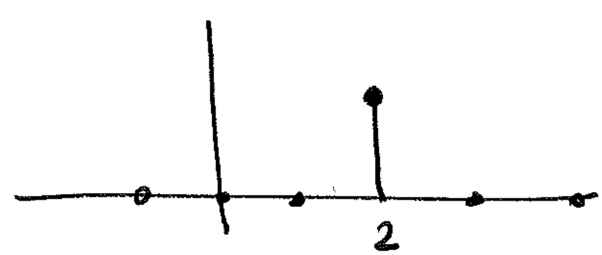
$\delta[n]$



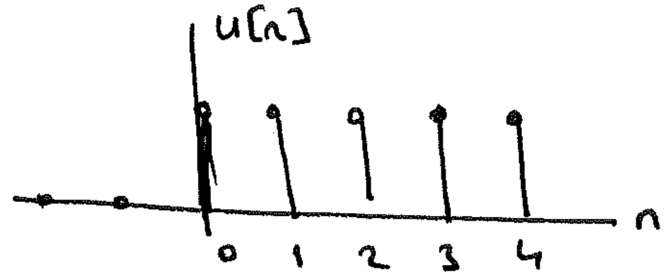

$\delta[n-1]$



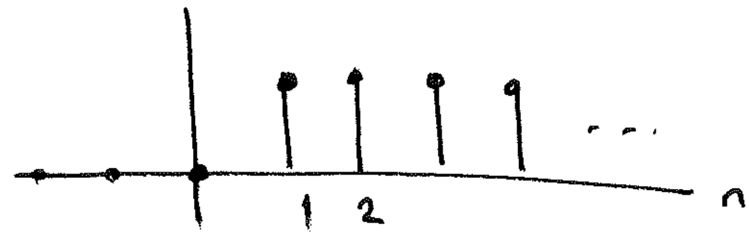

$\delta[n-2]$



$$\delta[n] = \underbrace{u[n] - u[n-1]}_{\text{difference equation.}}$$



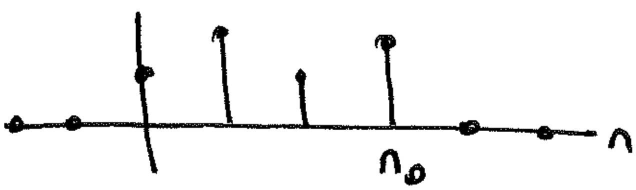

u[n-1]



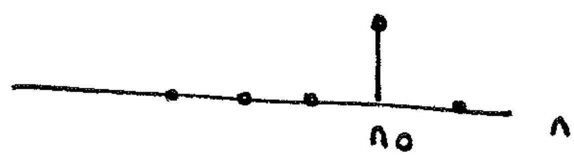

## Sampling property

$x[n]\delta[n-n_0]$ ?  $n_0 \in \underbrace{\mathbb{Z}}_{\text{integer}}$

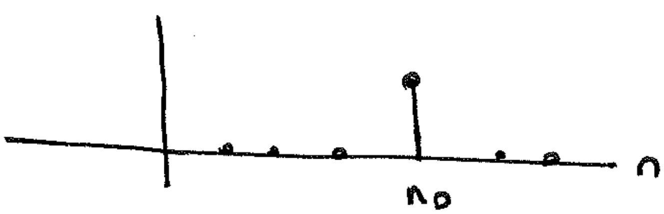
x[n]




$\rightarrow$




$\delta[n-n_0]$



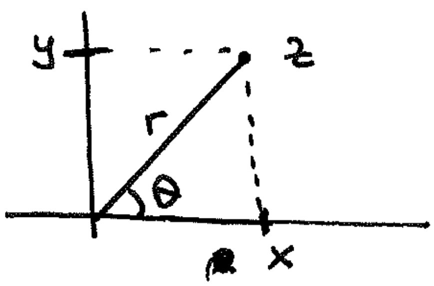

$$x[n] = \sum_{k=-\infty}^{+\infty} x[k]\delta[n-k] = x[n] * \delta[n]$$

## Complex Numbers

$$z = \underbrace{x}_{Re(z)} + j\underbrace{y}_{Im(z)} \qquad \text{(Cartesian)}$$

$$z = \underbrace{r}_{|z| \,:\, \text{magnitude}}\, e^{j\theta} \qquad \text{(Polar)}$$

$e^{j\theta} \rightarrow \angle z$ angle/phase.




$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$x = r\cos\theta, \qquad y = r\sin\theta$$

**Euler's formula:** $e^{j\theta} = \cos\theta + j\sin\theta$

$$\boxed{j = e^{j\frac{\pi}{2}}}$$

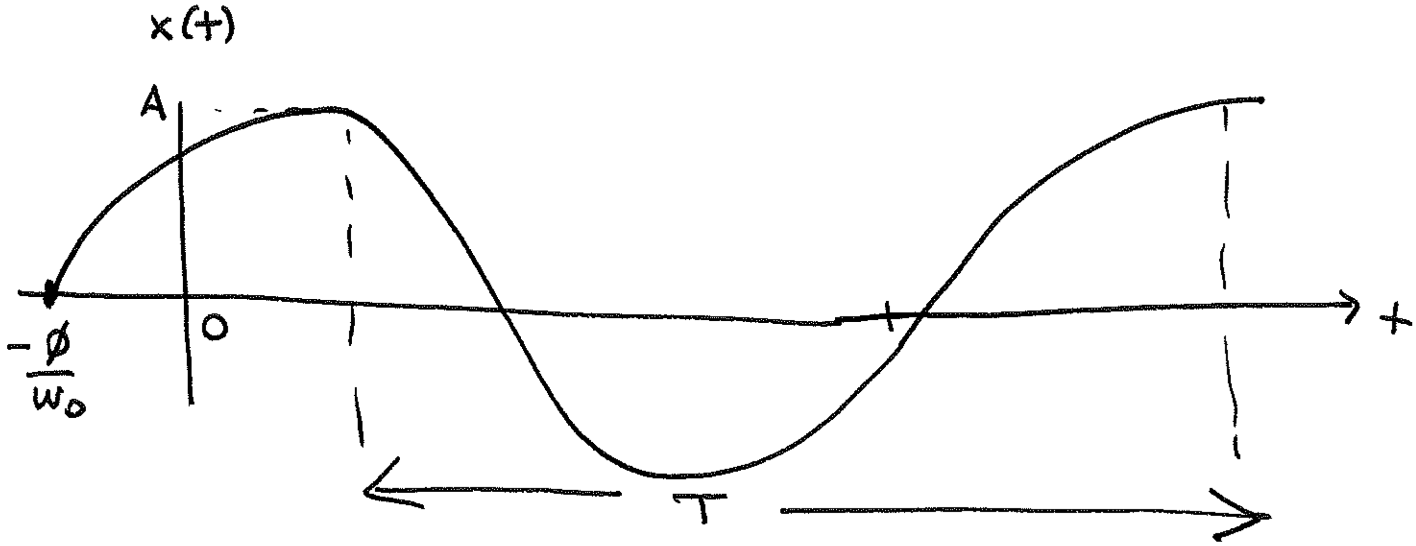
$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

**Sinusoids:** $x(t) = A\sin(\omega_0 t + \phi)$

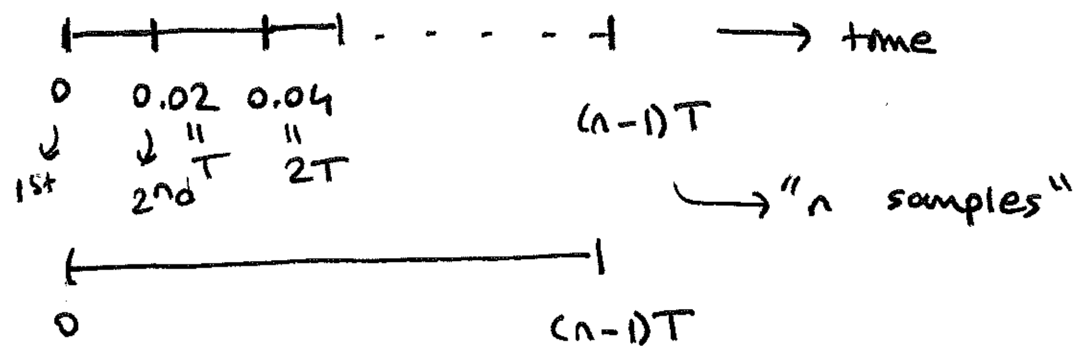
$A$ → amplitude, $\omega_0$ → angular frequency, $t$ → time, $\phi$ → phase

**Period:** $T = \dfrac{2\pi}{\omega_0} = \dfrac{1}{f}$



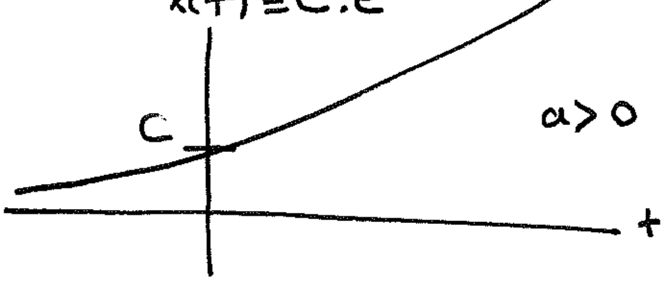

Matlab example with $n=1000$ samples

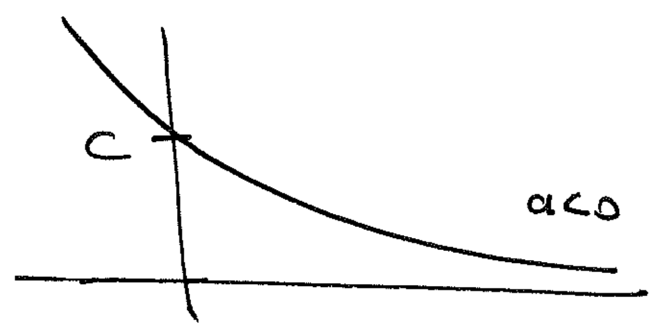
sample separation : $0.02 = T$




<u>Exponentials</u>

$x(t) = C.e^{at}$




both C, a are <u>real valued</u>

What about when $C, \alpha$ complex

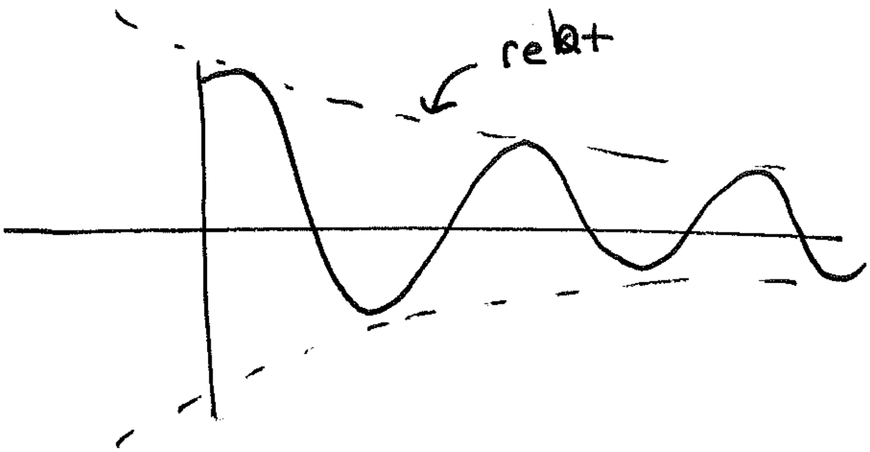
$C = r \cdot e^{j\theta}$ (polar)

$\alpha = b + jy$ (cartesian)

$b, y \in$ real.

$$x(t) = C e^{\alpha t}$$
$$= r e^{j\theta} e^{(b+jy)t}$$
$$= r \cdot e^{bt} \cdot e^{j(\theta + yt)}$$
$$= \underbrace{r \cdot e^{bt}}_{\text{real envelope}} \left[\cos(yt + \theta) + j\sin(yt + \theta)\right]$$

$$\mathrm{Re}(x(t)) = r \cdot e^{bt} \cdot \cos(yt + \theta) \quad , \quad b < 0$$




Periodic?

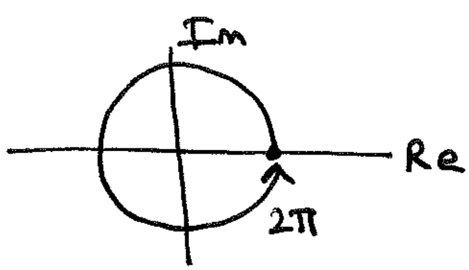
## Discrete Time Signals

$$x[n] = C e^{\beta n} \quad , \quad n \in \mathbb{Z}$$

$\alpha = e^{\beta}$

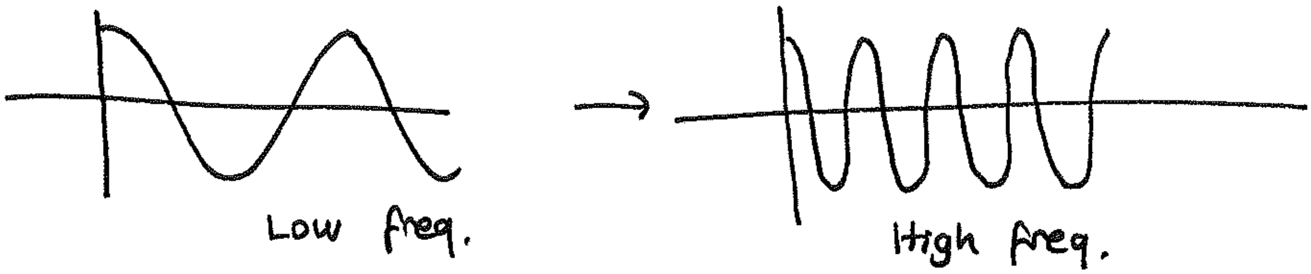
$$x[n] = C \alpha^n = |C| \, |\alpha|^n \left[\cos(\omega_0 n + \phi) + j \sin(\omega_0 n + \phi)\right]$$

$$e^{j(\omega_0 + 2\pi)n} = e^{j\omega_0 n} \cdot \underbrace{e^{j2\pi n}}_{1 \text{ for all } n \in \mathbb{Z}}$$




$$e^{j2\pi n} = \underbrace{\cos 2\pi n}_{1} + j \underbrace{\sin 2\pi n}_{0} = 1$$

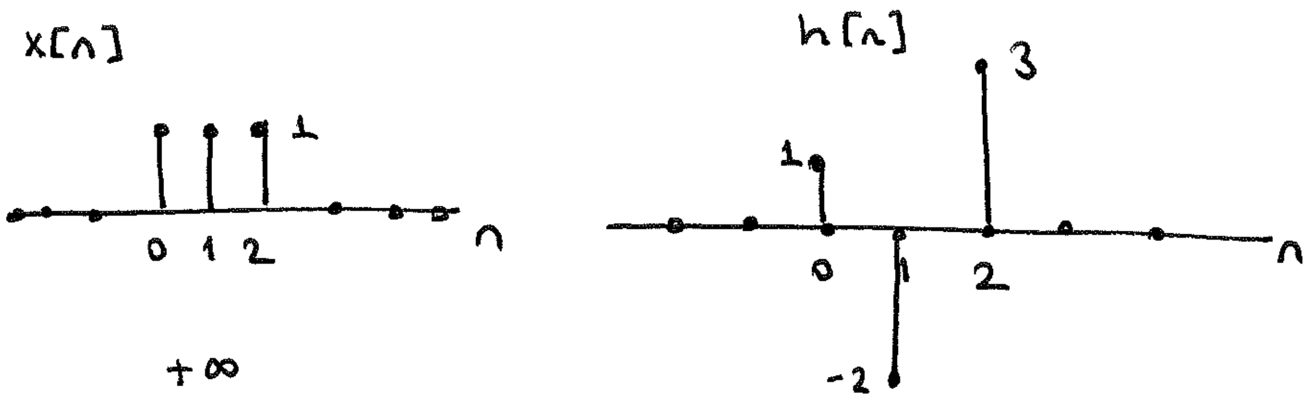
In CT




In DT, there is no ~~infinite~~ infinitely high frequency.

only a "2π-wide range" of frequencies in DT.

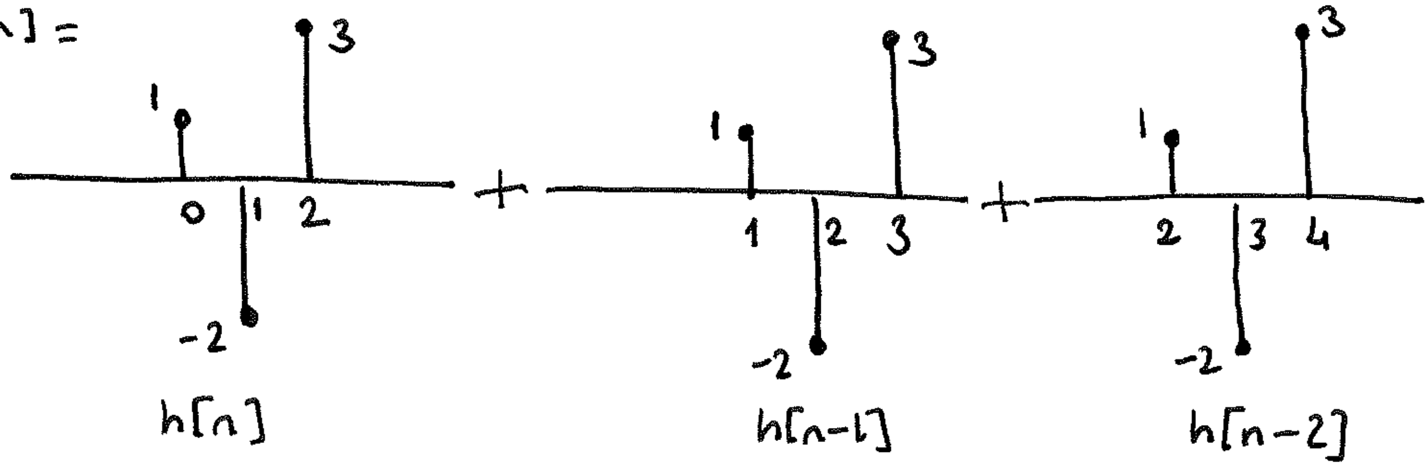
## Convolution (LTI)



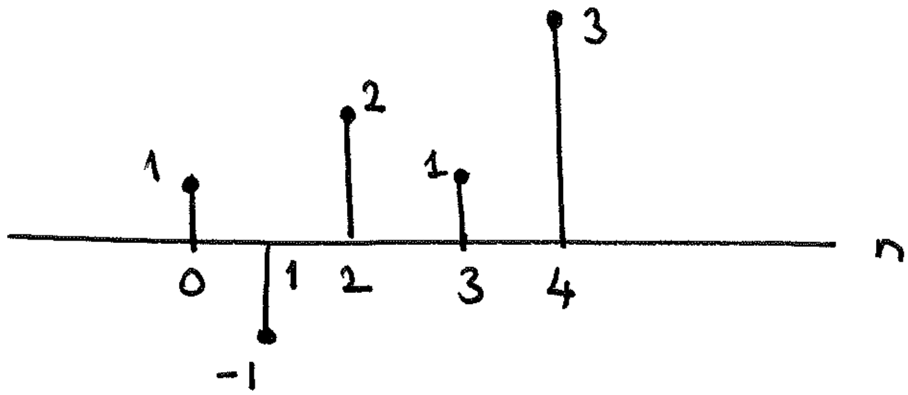

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]\,h[n-k] = x[n] * h[n]$$

$$= \sum_{k=0}^{2} h[n-k] = h[n] + h[n-1] + h[n-2]$$

y[n] =

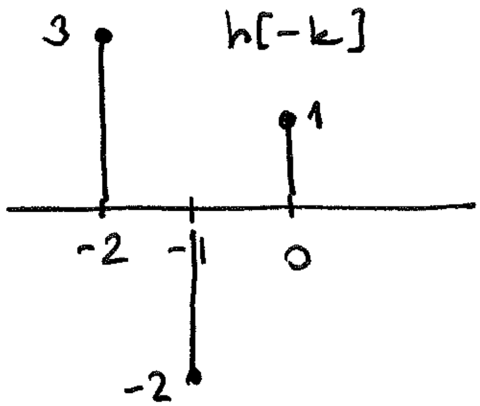


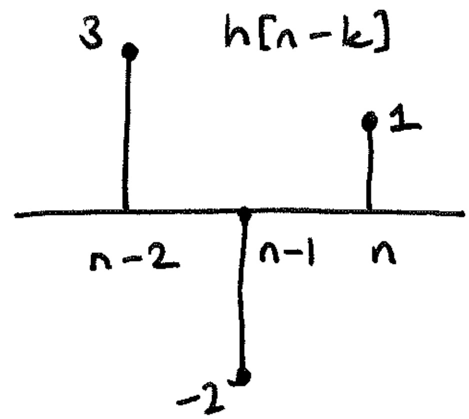
$=$




<u>Another way:</u>

flip $h[k] \longrightarrow h[-k]$ shift by "$n$" $\longrightarrow h[n-k]$



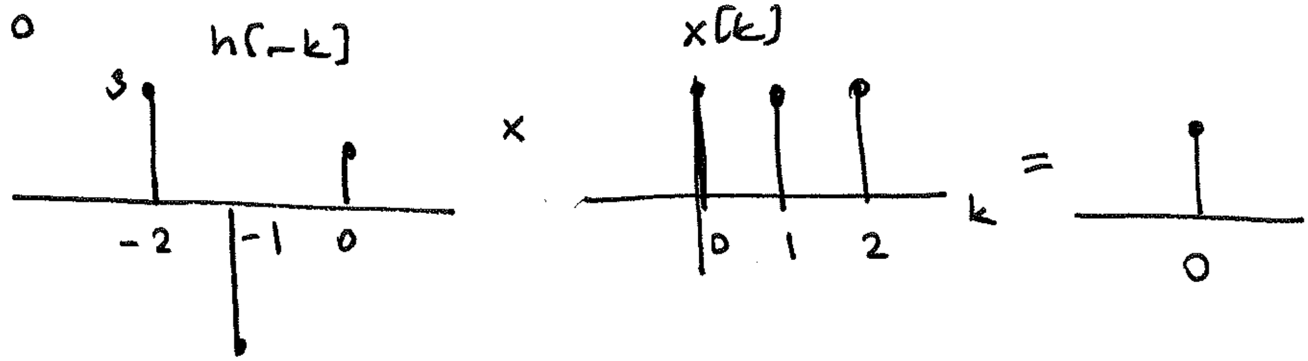

Explanation:

1. Flip $h[k]$ to obtain $h[-k]$
   Time offset $n$: $h[n-k]$
2. We start $n$ at $-\infty$ and slide it all the way to $+\infty$
3. Wherever 2 functions intersect, find the integral (sum) of the product

for $n<0 \rightarrow y[n]=0$

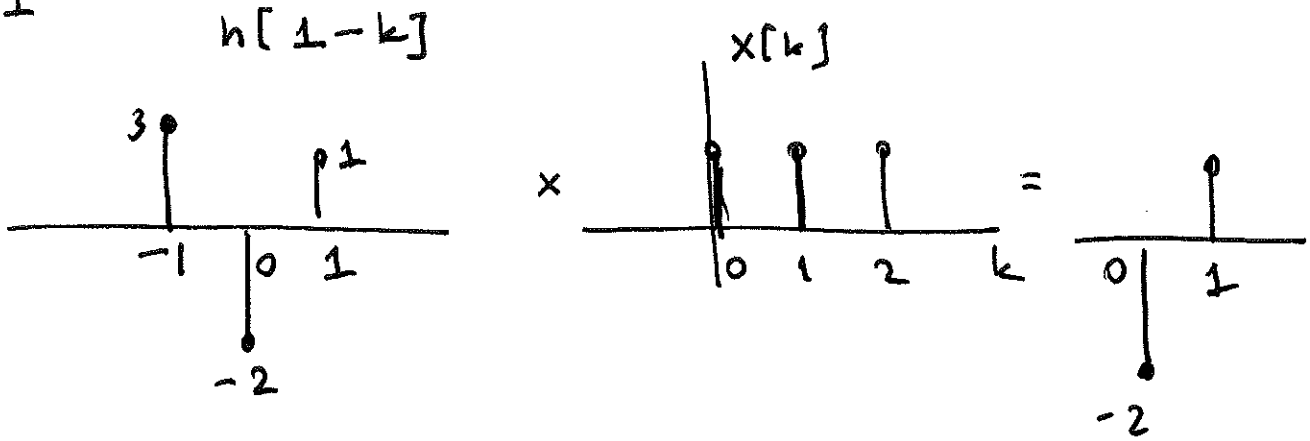
$n=0 \rightarrow y[0]=1$

$n=0$



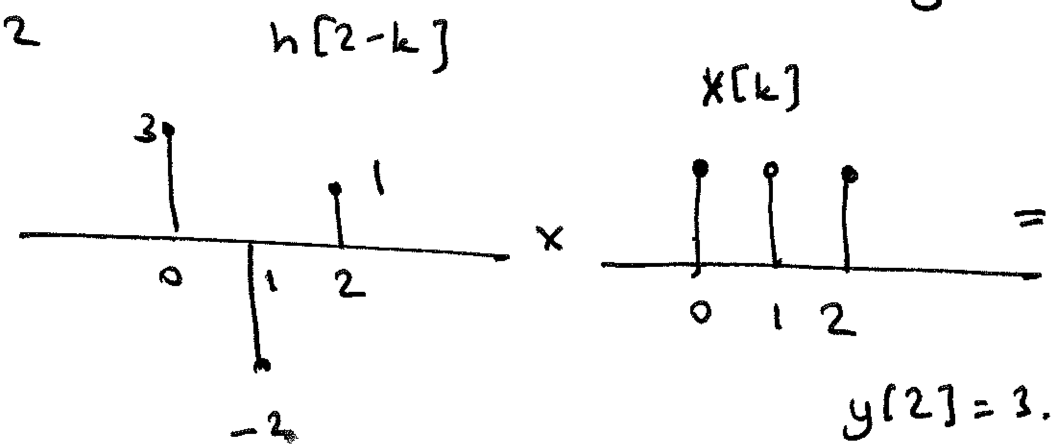

$y[0] = 1$

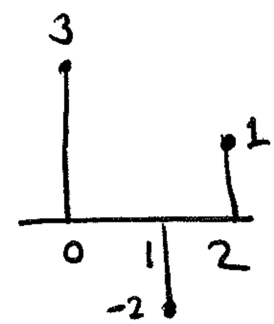
$n=1$




$y[1] = -2 + 1 = -1$

$n=2$




$y[2] = 3.1 - 2.1 + 1.1$

$= 2$

$\vdots$

$n > 2$ . .