Rensselaer Polytechnic Institute
Department of Electrical, Computer, and Systems Engineering
ECSE 4530: Digital Signal Processing, Fall 2020

Homework #5: due Thursday, Nov. 19$^{\text{th}}$, at the beginning of class.

MATLAB Grader Problems

A full description of all the Grader problems is provided at the Grader link. You don't need to physically hand anything in for the Grader problems (but make sure that you hit Submit and see a green check mark to make sure your solution has been recorded).

1. (5 points). Make a more-finely-spaced time vector.

2. (5 points). Reconstruct intermediate signal values using linear interpolation.

3. (10 points). Reconstruct intermediate signal values using ideal (sinc) interpolation.

4. (10 points). Resample a signal by a rational factor.

Analytical Problems: Be sure to show intermediate steps that explain your reasoning and do not forget to provide the labelings.

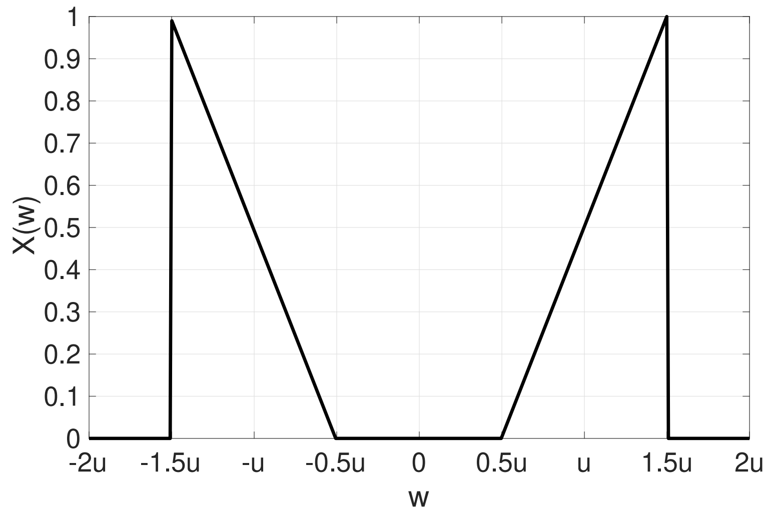
5. (10 points) **Nyquist rate.** Consider a bandlimited continuous-time signal $x(t)$ such that $X(\omega) = 0$ for $\omega > \omega_B$. Determine the Nyquist rate for the following continuous time signals:
   (a) $y(t) = x(t) + x(t-2)$
   (b) $y(t) = x(2t)$
   (c) $y(t) = (x(t))^2$
   (d) $y(t) = x(t)\cos(\omega_0 t)$
   (e) $y(t) = x(t) * \frac{1}{t}\sin(\omega_c t)$ (Recall that $*$ means regular convolution.)

6. (10 points) **Aliasing.** Consider a signal $x(t)$ whose continuous-time Fourier transform is

$$X(\omega) = \begin{cases} 1 & 20 < |\omega| < 40 \\ 0 & \text{otherwise} \end{cases}$$

   (a) What is the Nyquist rate of $x(t)$?
   (b) Will there be aliasing if $x[n]$ is created by sampling $x(t)$ every $T = \dfrac{2\pi}{20}$ seconds? (Hint: sketch what happens in the frequency domain.)

7. (10 points) **Filtering and Sampling.** Let $x(t)$ be a real signal with Fourier transform $X(\omega)$ as given below, where $u > 0$ is a constant. We apply two operations on $x(t)$:
   - First, $x(t)$ is bandpass filtered with a complex filter $h(t)$, which has a flat passband extending only from $0.5u$ to $1.5u$ rad/sec. The passband has a 3 dB gain in the range $0.5u < \omega < 1.5u$, but otherwise has a value of 0 ($-\infty$ in dB), including for all negative values of ω. Call the resulting signal $y(t)$.
   - The signal is sampled with a sampling period of $T = \frac{\pi}{2u}$ seconds. Call the resulting signal $y[n]$.



(a) Plot the Fourier transform (continuous time) of $y(t)$ in the frequency range $[-6u, 6u]$.

(b) Plot the Fourier transform (DTFT) of $y[n]$ in the corresponding frequency range, namely $\omega \in [-3\pi, 3\pi]$.

(c) Describe by a block diagram or equations how can we get back the input $x(t)$ from $y[n]$.

(d) Find the energy of the input $x(t)$ and the energy of the filtered signal $y(t)$.

Note: The decibel (dB) is a unit of measurement used to express the power or amplitude ratio on a logarithmic scale. The amplitude ratio in decibels (dB) is 20 times base 10 logarithm of the ratio. The power ratio in decibels (dB) is 10 times base 10 logarithm of the ratio.

8. (10 points) We know that for a discrete time signal $x[n]$, it's easy to create $x[n-k]$ when k is an integer. We can also think about the result when k is not an integer; for example, if $k = 0.5$, the effect would be the same as if we determined the continuous-time signal corresponding to the original samples, and took new samples exactly in between the original ones (a.k.a. a "half-sample delay"). However, we can accomplish this process directly using a digital filter $h_k[n]$. Determine the ideal frequency response of this filter, $H_k(\omega)$.

9. (10 points) **Downsampling and Upsampling.** Consider the two different ways of cascading a compressor $M = 2$ and an expander $L = 2$ as shown in Figure 1. Show that $y_1[n]$ and $y_2[n]$ are different. Hence, the two systems are not identical.

Figure 1: Cascaded systems.

Now consider the system in Figure 2 with $H(z)$ being the transfer function of an LTI system. Determine the transfer function of the whole system with input $x[n]$ and output $y[n]$.

10. (10 points) **Polyphase signal processing.** Consider a digital filter with transfer function $H(z)$.

(a) Determine the transfer function $H_e(z)$ of even numbered samples $h_e[n] = h[2n]$.

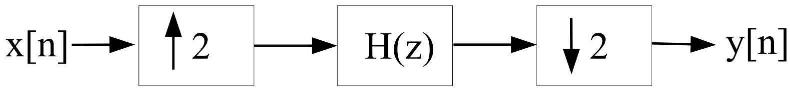


Figure 2: Cascaded system with a digital filter in the middle.

(b) Determine the transfer function $H_o(z)$ odd numbered samples $h_o[n] = h[2n+1]$.

(c) Express $H(z)$ in terms of $H_e(z)$ and $H_o(z)$.

(d) Now assume that we want to decompose $h[n]$ into M components: $h_1[n] = h[Mn]$, $h_2[n] = h[Mn+1], \ldots, h_M[n] = h[Mn+M-1]$. Show that $H(z)$ can be decomposed into a $M-$component polyphase filter structure with transfer function $H(z)$ that can be expressed in terms of the transfer functions of $h_k[n]$'s.

11. (10 points) **FIR filter design.** Design an FIR digital filter with linear phase that approximates the ideal frequency response

$$H_d(\omega) = \begin{cases} 1, & \text{for} \quad |\omega| \le \frac{\pi}{3} \\ 0, & \text{for} \quad \frac{\pi}{3} < |\omega| \le \pi. \end{cases}$$

(a) Determine the coefficients of a 15-tap filter based on the IDFT method.

(b) Determine and plot the magnitude and phase response of the filter.

Suggested reading material from textbook: Sections 6.1 (Sampling Theorem), 11.1-11.4 (downsampling and upsampling), 11.5, 11.10 (polyphase/multirate signal processing), 5.4.1-5.4.2, 10.1-10.2 (FIR filter design), along with the Examples and their solutions.

Suggested practice problems from textbook: 6.1, 6.9, 6.10, 6.11, 6.15, 11.1, 11.4, 11.5, 11.10, 11.11, 5.4, 5.7, 5.30, 5.32, 10.1, 10.6.