Fourier series (FS)

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \qquad a_k = \frac{1}{T}\int_T x(t)e^{-jk\omega_0 t}dt$$

$\omega_0 = \frac{2\pi}{T}$

Property/signal	Time domain	Transform domain
Linearity	$Ax(t) + By(t)$	$Aa_k + Bb_k$
Time shifting	$x(t-\tau)$	$e^{-jk\omega_0\tau}a_k$
Time reversal	$x(-t)$	a_{-k}
Time scaling	$x(at), a>0$ (periodic $\frac{T}{a}$)	a_k
Conjugation	$x^*(t)$	a^*_{-k}
Symmetry	$x(t)$ real	$a_k = a^*_{-k}$
Differentiation	$\frac{d}{dt}x(t)$	$jk\omega_0 a_k$
Integration	$\int_{-\infty}^{t} x(t)dt$, $a_0 = 0$	$\frac{a_k}{jk\omega_0}$
Convolution	$\int_T h(\tau) * x(t-\tau)d\tau$	$Ta_k b_k$
Multiplication	$x(t)y(t)$	$\sum_{m=-\infty}^{\infty} a_m b_{k-m}$
Cosine	$2A\cos(\omega_0 t + B)$	$a_1 = Ae^{jB}, a_{-1} = Ae^{-jB}$
Parseval	$\frac{1}{T}\int_T \lvert x(t)\rvert^2 dt$ $=$	$\sum_{k=-\infty}^{\infty} \lvert a_k\rvert^2$

Fourier transform (FT)

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega \qquad X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

Property/signal	Time domain	Transform domain
Linearity	$ax(t) + by(t)$	$aX(j\omega) + bY(j\omega)$
Time shifting	$x(t-\tau)$	$e^{-j\omega\tau}X(j\omega)$
Time scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X(j\omega/a)$
Conjugation	$x^*(t)$	$X^*(-j\omega)$
Symmetry	$x(t)$ real	$X(j\omega) = X^*(-j\omega)$
Differentiation	$\frac{d}{dt}x(t)$	$j\omega X(j\omega)$
Integration	$\int_{-\infty}^{t} x(\tau)d\tau$	$\frac{1}{j\omega}X(j\omega) + \pi X(0)\delta(\omega)$
Convolution	$\int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$	$H(j\omega)X(j\omega)$
Multiplication	$x(t)y(t)$	$\frac{1}{2\pi}\int_{-\infty}^{\infty} X(ju)Y(j\omega - ju)du$
Delta	$\delta(t)$	1
One	1	$2\pi\delta(\omega)$
Exponent	$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$
Cosine	$\cos(\omega_0 t)$	$\pi[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)]$
Sine	$\sin(\omega_0 t)$	$\frac{\pi}{j}[\delta(\omega-\omega_0) - \delta(\omega+\omega_0)]$
Unit step	$u(t)$	$\frac{1}{j\omega} + \pi\delta(\omega)$
Decaying step	$u(t)e^{-at}$, $a>0$	$\frac{1}{a+j\omega}$
Rectangular pulse	$\Pi(\frac{t}{2T})$	$\frac{2\sin(\omega T)}{\omega}$
Sinc (normalized)	$\frac{\sin(Wt)}{\pi t}$	$\Pi(\frac{\omega}{2W})$
Parseval	$\int_{-\infty}^{\infty} \lvert x(t)\rvert^2 dt$ $=$	$\frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(j\omega)\rvert^2 d\omega$

Discrete-time Fourier transform (DTFT)

$$x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega})e^{j\omega n}d\omega \qquad X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

Property/signal	Time domain		Transform domain
Linearity	$ax[n]+by[n]$		$aX(e^{j\omega})+bY(e^{j\omega})$
Time shifting	$x[n-n_0]$		$e^{-j\omega n_0}X(e^{j\omega})$
Time reversal	$x[-n]$		$X(e^{-j\omega})$
Conjugation	$x^*[n]$		$X^*(e^{-j\omega})$
Symmetry	$x[n]$ real		$X(e^{j\omega}) = X^*(e^{-j\omega})$
Convolution	$\sum_{m=-\infty}^{\infty} x[m]y[n-m]$		$X(e^{j\omega})Y(e^{j\omega})$
Multiplication	$x[n]y[n]$		$\frac{1}{2\pi}\int_{2\pi} X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta$
Delta	$\delta[n]$		1
One	1		$2\pi\sum_{m=-\infty}^{\infty}\delta(\omega-2\pi m)$
Exponent	$e^{j\omega_0 n}$		$2\pi\sum_{m=-\infty}^{\infty}\delta(\omega-\omega_0-2\pi m)$
Cosine	$\cos[w_0 n]$		$\pi\sum_{m=-\infty}^{\infty}[\delta(\omega-\omega_0-2\pi m)+\delta(\omega+\omega_0-2\pi m)]$
Sine	$\sin[w_0 t]$		$\frac{\pi}{j}\sum_{m=-\infty}^{\infty}[\delta(\omega-\omega_0-2\pi m)-\delta(\omega+\omega_0-2\pi m)]$
Decaying step	$u[n]a^n$, $\|a\|<1$		$\frac{1}{1-ae^{-j\omega}}$
Rectangular pulse	$\Pi_N[n]$		$\frac{\sin[\omega(N+\frac{1}{2})]}{\sin(\omega/2)}$
Sinc (normalized)	$\frac{\sin[Wn]}{\pi n}$		$\sum_{m=-\infty}^{\infty}\Pi(\frac{\omega-2\pi m}{2W})$
Parseval	$\sum_{n=-\infty}^{\infty}\|x[n]\|^2$	$=$	$\frac{1}{2\pi}\int_{2\pi}\|X(e^{j\omega})\|^2$ dw
	$e^{an}u[n]$, $\alpha<0$		$\frac{1}{1-e^{-(a+j\omega)}}$

Discrete Fourier transform (DFT)

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X(k)e^{j\frac{2\pi}{N}nk} \qquad X(k) = \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}nk}$$

Property/signal	Time domain		Transform domain
Linearity	$ax[n]+by[n]$		$aX(k)+bY(k)$
Time shifting	$x[n-n_0]_{modN}$		$e^{-j(\frac{2\pi}{N}n_0 k)}X(k)$
Time reversal	$x^*[-n]_{modN}$		$X^*(k)$
Conjugation	$x^*[n]$		$X^*(-k)_{modN}$
Symmetry	$x[n]$ real		$X(k) = X^*(-k)_{modN}$
Convolution	$\sum_{m=0}^{N-1} x[m]_{modN}y[n-m]_{modN}$		$X(k)Y(k)$
Multiplication	$x[n]y[n]$		$\frac{1}{N}\sum_{l=0}^{N-1} X(l)_{modN}Y(k-l)_{modN}$
Parseval	$\sum_{n=0}^{N-1}\|x[n]\|^2$	$=$	$\frac{1}{N}\sum_{k=0}^{N-1}\|X(k)\|^2$

Laplace transform

$$x(t) = \frac{1}{2\pi j}\int_{\sigma - j\infty}^{\sigma + j\infty} X(s)e^{st}ds \qquad X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt$$

Property/signal	Time domain	Transform domain
Linearity	$ax(t) + by(t)$	$aX(s) + bY(s)$
Time shifting	$x(t-\tau)$	$e^{-s\tau}X(s)$
time scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X(s/a)$
Conjugation	$x^*(t)$	$X^*(s^*)$
Differentiation	$\frac{d}{dt}x(t)$	$sX(s)$
Integration	$\int_{-\infty}^{t} x(\tau)d\tau$	$\frac{1}{s}X(s)$
Convolution	$\int_{-\infty}^{\infty} x(\tau)y(t-\tau)d\tau$	$X(s)Y(s)$
Delta	$\delta(t)$	1
Unit step	$u(t)$	$\frac{1}{s}$ $(Re\{s\} > 0)$
Decaying step	$e^{-at}u(t)$	$\frac{1}{s+a}$ $(Re\{s\} > -a)$
Decaying step	$-e^{-at}u(-t)$	$\frac{1}{s+a}$ $(Re\{s\} < -a)$
Causal Cosine	$\cos(\omega_0 t)u(t)$	$\frac{s}{s^2+\omega_0^2}$ $(Re\{s\} > 0)$
Causal Sine	$\sin(\omega_0 t)u(t)$	$\frac{\omega_0}{s^2+\omega_0^2}$ $(Re\{s\} > 0)$

Z transform

$$x[n] = \frac{1}{2\pi j}\oint X(z)z^{n-1}dz \qquad X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

Property/signal	Time domain	Transform domain
Linearity	$ax[n] + by[n]$	$aX(z) + bY(z)$
Time shifting	$x[n-n_0]$	$z^{-n_0}X(z)$
time reversal	$x[-n]$	$X(z^{-1})$
Conjugation	$x^*[n]$	$X^*(z^*)$
Convolution	$\sum_{m=-\infty}^{\infty} x[m]y[n-m]$	$X(z)Y(z)$
Delta	$\delta[n]$	1
Unit step	$u[n]$	$\frac{1}{1-z^{-1}}$ $(\lvert z\rvert > 1)$
Decaying step	$a^n u[n]$	$\frac{1}{1-az^{-1}}$ $(\lvert z\rvert > a)$
Decaying step	$-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$ $(\lvert z\rvert < a)$

General

Description	Equation
Rectangular pulse in continuous-time	$\Pi(x) = \begin{cases} 1 & \lvert x\rvert < \frac{1}{2} \\ \frac{1}{2} & \lvert x\rvert = \frac{1}{2} \\ 0 & elsewhere \end{cases}$
Rectangular pulse in discrete-time	$\Pi_N[n] = \begin{cases} 1 & \lvert n\rvert \leq N \\ 0 & elsewhere \end{cases}$
Unit step in continuous-time	$u(x) = \begin{cases} 1 & x > 0 \\ \frac{1}{2} & x = 0 \\ 0 & elsewhere \end{cases}$
Unit step in discrete-time	$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & elsewhere \end{cases}$
Sinc in continuous-time	$\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$
Cosine of sum of angles	$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$
Sine of sum of angles	$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$

$$\cos x = \frac{e^{ix} + e^{-ix}}{2}$$

$$\sin x = \frac{e^{ix} - e^{-ix}}{2j}$$

$$e^{ix} = \cos x + j\sin x$$

$$e^{-ix} = \cos x - j\sin x$$

$$\frac{\omega}{\pi}\,\mathrm{sinc}\left[\frac{\omega n}{\pi}\right] = \frac{\sin[\omega n]}{\pi n} \overset{F}{\underset{DTFT}{\longleftrightarrow}} X(\omega) = \begin{cases} 1, & 0 \leq \omega \leq W_c \\ 0, & W_c < \lvert\omega\rvert < \pi \end{cases}$$

$$\sum_{K=0}^{n-1} (a)^K = \frac{1-a^n}{1-a}$$

$$\downarrow$$

$$\sum_{K=0}^{6} (a)^K = \frac{1-a^7}{1-a}.$$

$$\sum_{K=1}^{2} a^K \Rightarrow \sum_{K=0}^{1} a^{K+1}$$