

Class Notes for Digital Signal Processing

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Lecture 1: Introduction

Summary

DSP can involve linear and non-linear operation. The processing on signal is conducted in time-domain, frequency-domain and spatial-temporal domain wavelet.

Some taste of dsp tools: *Digital Filter, Z-plane, Signal Sampling, Discretization, Quantization.*

Snippet of DSP

Digital Filter The filter can be an linear and time-invariant (LTI) for an non-linear time-invariant filter. A digital filter is usually LTI. An LTI filter is a type of filter which exhibit the same effect on signal the same at all time. The output produced as a result of going through the filter is some linear transform of the input. And in the form of equation, we say:

$$\text{output} = \text{input} * \text{impulse response}$$

This is not a multiplication, this is done in a process called **convolution**, and it is a big deal in the DSP class.

Z-plane Z plane is a plane which plot the roots and zeros of the system and check on the stability of the signal, system, or filter.

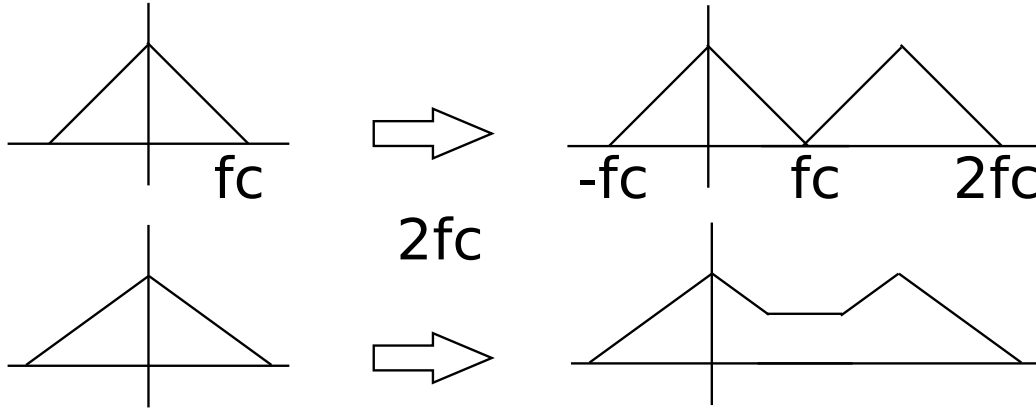
Signal Sampling The processings/algorithms cannot be applied on signal unless it is a digital signal. Samplers (ADC Converters) are used everyday to convert signals captured from the environment to digital signals that can be manipulated/extracted to obtain new information. This samplings are done in 2 stages: Discretization, Quantization.

Discretization is determining the value/amplitude of the signal during a finite time interval. Whereas *quantization* is a process which approximate the amplitude by a value from a finite set of values. Both stages involve complicated procedure because there's a desire to be accurate but also the demand of process efficient.

And now let's look at some theorem and properties of signals.

The Nyquist-Shannon Sampling Theorem

Definition: Reconstruct signal from its samples if and only if its sampling rate is greater than twice the highest frequency component in the signal.



On the figure above, the sampling theorem stated that when the sampling frequency is twice of the maximum frequency ($f_{s1} = 2f_{c1}$) of the original signal (f_{c1}), then the signal sampled will not be distorted. However, if the same sampling frequency ($f_{s1} = 2f_{c1}$) is used to sample original signal with maximum frequency, then the sampled signal is distorted because $f_{s1} < 2f_{c2}$.

Discrete Time Signal Property

Time Shift $x(t-t_0)$: $x(t)$ shifted $t_0|t_0 > 0$ unit toward the right. Under this form, $t_0 < 0$ shift will be toward the left.

Time Reversal/Folding $x(-t)$ is the same graph as $x(t)$ except it's flipped about the y-axis.

Time Scaling $x(\alpha t)$ will be a new shape based on $x(t)$ where the x co-ordinates correspond to each of the y co-ordinates on the curve $x(t)$ will be divided by α . To translate that to math equations:

$$x_1 = x(t), x_2 = x(\alpha t) \leftrightarrow \left(\frac{x_1}{\alpha}, y\right) = (x_2, y)$$

Important: When you have a mix of time scaling and shift and reversal, the order of which you follow is **Shift** \rightarrow **Flip** \rightarrow **Scale**.

Even & Odd Signal Every signal can be broken down into an even and an odd part.

$$x(t) = Ev(x(t)) + Od(x(t))$$

And the part has some uniqueness in it to distinguish one part from the other by looking at the part's time reversal shape.

$$Ev(x(t)) : x(t) = x(-t)$$

$$Od(x(t)) : -x(t) = x(-t)$$

In otherwords, to determine if a signal is **strictly** odd or even, check its time reversal shape and see which statement will fit. If the signal is neither **strictly** odd or even, then you would need to broke down the signal into $Ev(x(t))$ and $Od(x(t))$ using the equation above.

Periodicity $x(t+T) = x(t)$ means $y_0 = x(t_0)$ can be found again after T seconds for all t in x .