Class Notes for Digital Signal Processing

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Lecture 1: Introduction

Summary

DSP can involve linear and non-linear operation. The processing on signal is conducted in time-domain, frequency-domain and spatial-temporal domain wavelet.
Some taste of dsp tools: *Digital Filter, Z-plane, Signal Sampling, Discretization, Quantization.*

Snippet of DSP

Digital Filter The filter can be an linear and time-invarient (LTI) for an non-linear time-invarient filter. A digital filter is usually LTI. An LTI filter is a type of filter which exhibit the same effect on signal the same at all time. The output produced as a result of going through the filter is some linear transform of the input. And in the form of equation, we say:

$$\text{output} = \text{input} * \text{impulse response}$$

This is not a multiplication, this is done in a process called **convolution**, and it is a big deal in the DSP class.

Z-plane Z plane is a plane which plot the roots and zeros of the system and check on the stability of the signal, system, or filter.

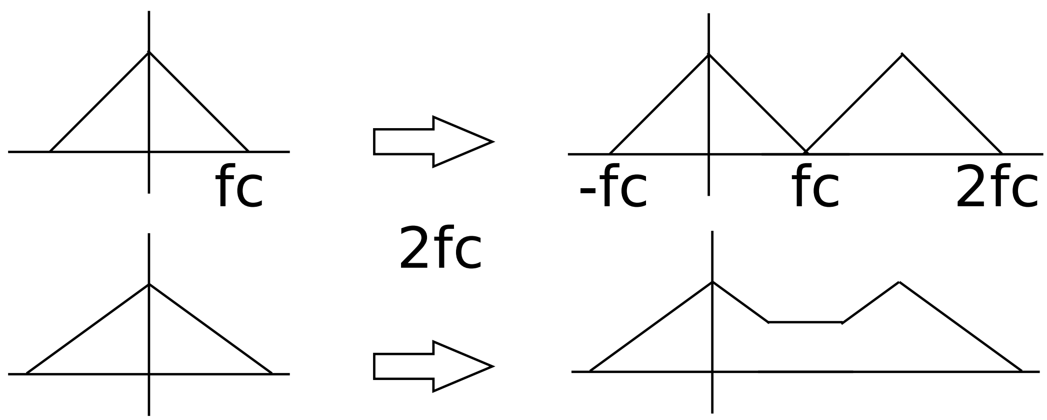
Signal Sampling The processings/algorithms cannot be applied on signal unless it is a digital signal. Samplers (ADC Converters) are used everyday to convert signals captured from the environment to digital signals that can be manipulated/extracted to obtain new information. This samplings are done in 2 stages: Discretization, Quantization.

Discretization is determining the value/amplitude of the signal during a finite time interval. Whereas *quantization* is a process which approximate the amplitude by a value from a finite set of values. Both stages involve complicated procedure because there's a desire to be accurate but also the demand of process efficient.

And now let's look at some theorem and properties of signals.

The Nyquist-Shannon Sampling Theorem

Definition: Reconstruct signal from its samples if and only if its sampling rate is greater than twice the highest frequency component in the signal.



On the figure above, the sampling theorem stated that when the sampling frequency is twice of the maximum frequency ($fs_1 = 2fc_1$) of the original signal (fc_1), then the signal sampled will not be distorted. However, if the same sampling frequency ($fs_1 = 2fc_1$) is used to sample original signal with maximum frequency, then the sampled signal is distorted because $fs_1 < 2fc_2$.

Discrete Time Signal Property

Time Shift $x(t-t_0)$: $x(t)$ shifted $t_0|t_0 > 0$ unit toward the right. Under this form, $t_0 < 0$ shift will be toward the left.

Time Reversal/Folding $x(-t)$ is the same graph as $x(t)$ except it's flipped about the y-axis.

Time Scaling $x(\alpha t)$ will be a new shape based on $x(t)$ where the x co-ordinates correspond to each of the y co-ordinates on the curve $x(t)$ will be divided by α. To translate that to math equations:

$$x_1 = x(t), x_2 = x(\alpha t) \leftrightarrow (\frac{x_1}{\alpha}, y) = (x_2, y)$$

Important: When you have a mix of time scaling and shift and reversal, the order of which you follow is **Shift** $\rightarrow$ **Flip** $\rightarrow$ **Scale**.

Even & Odd Signal Every signal can be broken down into an even and an odd part.

$$x(t) = Ev(x(t)) + Od(x(t))$$

And the part has some uniqueness in it to distinguish one part from the other by looking at the part's time reversal shape.

$$Ev(x(t)) : x(t) = x(-t)$$
$$Od(x(t)) : -x(t) = x(-t)$$

In otherwords, to determine if a signal is **strictly** odd or even, check its time reversal shape and see which statement will fit. If the signal is neither **strictly** odd or even, then you would need to broke down the signal into $Ev(x(t))$ and $Od(x(t))$ using the equation above.

Periodicity $x(t+T) = x(t)$ means $y_0 = x(t_0)$ can be found again after T seconds for all t in x.

Lecture 2: Introduction Day 2

Summary

A family of signals that are the building blocks of complex signal. Important functions and the relationships between those simple special signals.
Review of special signal: *Delta function, step function, sinusoid, exponential, sampling property, Euler's formula, discrete time convolution.*

Last Lecture

Digital Time Signal To process ANY signals using any toolkit being discussed in this course, the signals need to be in the digital format (in other words, must be sampled and not be continuous).

Important Functions

$$\delta[n] = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases}$$

Unit Impulse Function In continuous time, unit impulse is only defined at t=0, with weight being **unbounded**. In discrete time, unit impulse is only defined at n=0, with weight being of **1**.

$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

Unit Step Function In continuous time, unit step is a function with weight being 1 for t $\geq$ 0 and 0 otherwise. The relationships between $u(t)$ and $\delta(t)$ is:

$$u(t) = \int_t \delta(t)\, dt \tag{1}$$

$$\delta(t) = \frac{d}{dt} u(t) \tag{2}$$

This is similar to the discrete time, except in discrete time, you cannot use intergration or differentiation, so this relationships can be shown as:

$$
\begin{aligned}
u[n] &= \delta[n] + \delta[n-1] + \cdots \\
&= \sum_{k=0}^{\infty} \delta[n-k] \\
\delta[n] &= u[n] - u[n-1]
\end{aligned}
$$

Sampling Property

Can you answer this: $x[n]\delta[n-n_0]$? If you draw the two plots and multiply them together, you notice that it will evaluate to:

$$
x[n]\delta[n-n_0] = \begin{cases} x[n_0], & n = n_0 \\ 0, & else \end{cases}
$$

To put more things into perspective, when you have a spectrum of x[n] which has different height for different n. You can imagine that as a summation of different time shifted version of impulse with a weight being multiplied to the impulse. To put this in terms of equation:

$$
\begin{aligned}
x[n] &= \sum_{-\infty}^{\infty} x[k]\delta[n-k] \\
&= x[n] * \delta[n]
\end{aligned}
$$

Complex Number

$$
\begin{aligned}
z &= x + jy, \quad \text{(Rectangular)} \\
z &= re^{j\theta}, \qquad \text{(Polar)} \\
r &= |z| = \sqrt{x^2 + y^2} \\
\theta &= \angle z = \arctan(\frac{y}{x})
\end{aligned}
$$

Euler's Formula

$$e^{j\theta} = cos(\theta) + jsin(\theta)$$
$$cos(\theta) = \frac{1}{2}(e^{j\theta} + e^{-j\theta})$$
$$sin(\theta) = \frac{1}{2j}(e^{j\theta} - e^{-j\theta})$$

Sinusoid

The general function of a sinusoid is a function of sin or cosine.

$$x(t) = Asin(w_0 t + \phi)$$

This function above is periodic with $T = \frac{w_0}{2\pi} = \frac{1}{f}$.

Exponential

The general function of an exponential is:

$$x(t) = \mathbf{C}e^{\alpha t}$$

If $\mathbf{C}, \alpha \subset \mathbb{R}$:

$$\alpha > 0 \rightarrow x(t) \text{ is an exponential grow function.}$$

$$\alpha < 0 \rightarrow x(t) \text{ is an exponential decay function.}$$

If $\mathbf{C}, \alpha \subset \mathbb{C}$:

$$\mathbf{C} = re^{j\theta}, \alpha = b + jy$$
$$x(t) = \mathbf{C}e^{\alpha t} = re^{j\theta}e^{(b+jy)t} = re^{bt}e^{j(\theta+yt)}$$
$$= re^{bt}[cos(\theta + yt) + jsin(\theta + yt)]$$

Real Envelope

The real and the non-sinusoidal constant part of x(t), re^{bt}, is called the real envelope in that it is an imaginary curve that set the boundary within which the signal is contained.

The detail discussed above is for continuous time, for the discrete time, the only difference here is this:

$$
\begin{aligned}
\alpha &= e^{\beta} \subset \mathbb{R}, \mathbf{C} = \phi + jw_0 \subset \mathbb{C} \\
x[n] &= \mathbf{C}e^{\beta n}, n \subset \mathbb{Z} \\
&= \mathbf{C}\alpha^n \\
&= |\mathbf{C}||\alpha|^n[cos(w_0 n + \phi) + jsin(w_0 n + \phi)] \\
e^{jw_0 n} &\equiv e^{j(w_0+2\pi)n}, e^{j(w_0+2\pi)n} \leq 1 = e^{2\pi n}
\end{aligned}
$$

In other words, in discrete time, there's no infinitely high frequency. Only a 2π-wide range of frequency.

Convolution (LTI)

You choose 1 function to time-reverse. Start n at negative inifnity and slide it to the right to positive infinity. It's better to watch youtube video to explain on this to save typesetting everything