Nonlinear Analysis: Hybrid Systems 46 (2022) 101238

Contents lists available at ScienceDirect

Nonlinear Analysis: Hybrid Systems

journal homepage: www.elsevier.com/locate/nahs

Impulse-free jump solutions of nonlinear differential–algebraic equations

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ARTICLE INFO

Article history:
Received 12 May 2021
Received in revised form 11 March 2022
Accepted 6 June 2022
Available online 8 July 2022

Keywords:
Nonlinear differential–algebraic equations
Discontinues solutions
Inconsistent initial values
Impulse-free jumps
Geometric methods
Singular perturbed systems

ABSTRACT

In this paper, we propose a novel notion called impulse-free jump solution for nonlinear differential–algebraic equations (DAEs) of the form $E(x)\dot{x} = F(x)$ with inconsistent initial values. The term "impulse-free" means that there are no Dirac impulses caused by jumps from inconsistent initial values, i.e., the directions of the jumps stay in $\ker E(x)$. We show that our proposed impulse-free jump rule is a coordinate-free concept, meaning that the calculation of the impulse-free jump does not depend on the coordinates of the DAE, which is a main advantage compared to some existing jump rules for nonlinear DAEs. We find that the existence and uniqueness of impulse-free jumps are closely related to the notion of geometric index-1 and the involutivity of the distribution defined by $\ker E(x)$. Moreover, a singular perturbed system approximation is proposed for nonlinear DAEs; we show that solutions of the perturbed system approximate both impulse-free jump solutions and $\mathcal{C}^1$-solutions of nonlinear DAEs. Finally, we show by some examples that our results of impulse-free jumps are useful for the problems like consistent initialization of nonlinear DAEs and transient behavior simulations of electric circuits.

1. Introduction

Consider a nonlinear differential–algebraic equation (DAE) in quasi-linear form

$$\Xi : \quad E(x)\dot{x} = F(x), \tag{1}$$

where $x \in X$ is a vector of the generalized states and $(x, \dot{x}) \in TX$, where TX is the tangent bundle of the open subset X in $\mathbb{R}^n$ (or an n-dimensional smooth manifold). The maps $E : TX \to \mathbb{R}^l$ (attaching $(x, \dot{x}) \mapsto E(x)\dot{x}$) and $F : X \to \mathbb{R}^l$ are $\mathcal{C}^\infty$-smooth, and for each $x \in X$, we have that $E(x) : \mathbb{R}^n \to \mathbb{R}^l$ is a linear map. We will denote a DAE of the form (1) by $\Xi_{l,n} = (E, F)$ or, simply, Ξ. A linear DAE of the form

$$\Delta : \quad E\dot{x} = Hx \tag{2}$$

will be denoted by $\Delta_{l,n} = (E, H)$ or, simply, Δ, where $E \in \mathbb{R}^{l\times n}$ and $H \in \mathbb{R}^{l\times n}$. A linear DAE is called *regular* if $l = n$ and $\det(sE - H) \in \mathbb{R}[s] \setminus \{0\}$.

Definition 1.1 ($\mathcal{C}^1$*-solutions and Consistency Space*)**.** The trajectory $x : \mathcal{I} \to X$ for some open interval $\mathcal{I} \subseteq \mathbb{R}$ is called a $\mathcal{C}^1$-solution of the DAE $\Xi_{l,n} = (E, F)$ if x is continuously differentiable and satisfies $E(x(t))\dot{x}(t) = F(x(t))$ for all $t \in \mathcal{I}$.

A point $x_c \in X$ is called *consistent* (or *admissible* [1]) if there exists a $\mathcal{C}^1$-solution $x : \mathcal{I} \to X$ and $t_c \in \mathcal{I}$ such that $x(t_c) = x_c$. The *consistency space* $S_c \subseteq X$ is the set of all consistent points.

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https://doi.org/10.1016/j.nahs.2022.101238