Verifications of solutions for linear discontinuous DAEs

Team: Hycomes team, INRIA center at Rennes University
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About the Internship

Differential-algebraic equations (DAEs) arise naturally when modeling dynamical systems from first principles. In many cases, physical laws are expressed as combinations of differential and algebraic equations. This modeling approach is common in constrained mechanics, chemical and biological processes, power systems, and especially analog circuit design—where idealized components (e.g., resistors, capacitors, inductors) and Kirchhoff's laws define the system dynamics. When these systems experience abrupt changes—such as switching in electrical circuits, mechanical contacts, or discontinuous control inputs—*discontinuous DAEs* emerge. However, there is currently no comprehensive theoretical foundation for studying such systems.

Discontinuous DAEs are relevant to many research areas, including systems and control, hybrid systems, and computer-aided simulation. A notable example is *switched DAEs*. While time-dependent switching has been extensively studied [1-2], progress on *state-dependent switching*, a subclass of discontinuous DAEs, remains limited. Challenges include:

- Their hybrid trajectories caused by switching and algebraic constraints;
- The diverse behaviors on the boundaries of active regions;
- The well-posedness of solutions.

In a recent publication [3], we focus on *piecewise-affine DAEs (PWA-DAEs)*:

$$\Delta_i : \; E_i\dot{x} = A_i x + b_i, \quad x \in \mathcal{X}_i \tag{1}$$

where $\{\mathcal{X}_i\}$ is a polyhedral partition of $\mathbb{R}^n$, and $E_i, A_i \in \mathbb{R}^{n\times n}$, $b_i \in \mathbb{R}^n$. We introduced a notion of a **jump-flow solution** for PWA-DAEs, inspired by the concept of **Filippov solutions** for differential inclusions [4]. Based on this solution framework, different boundary behaviors are studied. For example, it is shown that when a jump (transient) direction and the classical ODE flow both point toward a boundary, a **jump-flow sliding behavior** occurs.

However, the validation of the proposed solution concept has not yet been carried out. It is known that the solution of a continuous DAE can be approximated by a singularly perturbed ODE system [5], and classical sliding behavior in switching ODEs can be validated via hysteresis switching. The objective of this internship is to verify the proposed solution concept in [6] using **singular perturbation methods** and **hysteresis-based switching**.

[1] D. Liberzon and S. Trenn. “On stability of linear switched differential algebraic equations”. In: *IEEE CDC* 2009, pp. 2156–2161.
[2] Y. Chen and S. Trenn. “On impulse-free solutions and stability of switched nonlinear differential–algebraic equations”. In: *Automatica* 156 (2023), p. 111208
[3] Y. Chen and S. Trenn. “Solution concepts for linear piecewise affine differential-algebraic equations”. In: *IEEE CDC* 2024, pp. 7804–7809.
[4] J. Cortes. “Discontinuous dynamical systems”. In: *IEEE Control Systems Magazine* 28.3 (2008), pp. 36–73.
[5] H.K. Khalil, *Nonlinear Systems*, third ed., Prentice-Hall, Upper Saddle River, NJ, 2001.

Responsibilities

- Propose and develop validation methods for solutions of PWA-DAEs.
- Conduct numerical simulations of the proposed methods using MATLAB/Simulink or Modelica.
- Prepare technical reports and documentation of research findings.
- Participate actively in team meetings and brainstorming sessions.

Requirements

- Master’s student in systems and control, applied mathematics, computer science, or a related field.
- Proficiency in academic English writing and fluency in spoken English.
- Strong mathematical reasoning and problem-solving skills.
- Familiarity with one or more of the following topics is an advantage:
  DAEs, switched systems, complementarity systems, sliding-mode control, hybrid systems simulation, or Modelica.