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RESEARCH ARTICLE

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Feedback linearization of nonlinear differential-algebraic control systems

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Abstract

In this article, we study feedback linearization problems for nonlinear differential-algebraic control systems (DACSs). We consider two kinds of feedback equivalences, namely, external feedback equivalence, which is defined (locally) on the whole generalized state space, and internal feedback equivalence, which is defined on the locally maximal controlled invariant submanifold (i.e., on the set where solutions exist). We define a notion called explicitation with driving variables, which is a class of ordinary differential equation control systems (ODECSs) attaching to a given DACS. Then we give necessary and sufficient conditions for both internal and external feedback linearization problems of the DACS. We show that the feedback linearizability of the DACS is closely related to the involutivity of the linearizability distributions of the explicitation systems. Finally, we illustrate the results of the by an academic example and a constrained mechanical system.

KEYWORDS

controlled invariant submanifolds, constrained mechanical system, differential-algebraic control systems, explicitation, external and internal feedback equivalence, feedback linearization

1 | INTRODUCTION

Consider a nonlinear differential-algebraic control system (DACS) of the form

$$\Xi^u : E(x)\dot{x} = F(x) + G(x)u, \tag{1}$$

where $x \in X$ is called the generalized state and $(x, \dot{x}) \in TX$, where TX is the tangent bundle of an open subset X in $\mathbb{R}^n$ (or, more general, of an n-dimensional smooth manifold X), and $u \in \mathbb{R}^m$ is the vector of inputs, and where $E : TX \to \mathbb{R}^l$, $F : X \to \mathbb{R}^l$ and $G : X \to \mathbb{R}^{l\times m}$ are smooth maps. The word "smooth" will always mean C^∞-smooth throughout the article. We denote a DACS of the form (1) by $\Xi^u_{l,n,m} = (E, F, G)$ or, simply, Ξ^u. A linear DACS is of the form

$$\Delta^u : E\dot{x} = Hx + Lu, \tag{2}$$

where $E, H \in \mathbb{R}^{l\times n}$ and $L \in \mathbb{R}^{l\times m}$. Denote a linear DACS by $\Delta^u_{l,n,m} = (E, H, L)$ or, simply, Δ^u. Linear DACSs have been studied for decades, there is a rich literature devoted to them (see, e.g., the surveys[1,2] and textbook[3]).