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RESEARCH ARTICLE

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Normal forms and internal regularization of nonlinear differential-algebraic control systems

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Abstract

In this article, we propose two normal forms for nonlinear differential-algebraic control systems (DACSs) under external feedback equivalence, using a notion called maximal controlled invariant submanifold. The two normal forms simplify the system structures and facilitate understanding the various roles of variables for nonlinear DACSs. Moreover, we study when a given nonlinear DACS is internally regularizable, that is, when there exists a state feedback transforming the DACS into a differential-algebraic equation (DAE) with internal regularity, the latter notion is closely related to the existence and uniqueness of solutions of DAEs. We also revise a commonly used method in DAE solution theory, called the geometric reduction method. We apply this method to DACSs and formulate it as an algorithm, which is used to construct maximal controlled invariant submanifolds and to find internal regularization feedbacks. Two examples of mechanical systems are used to illustrate the proposed normal forms and to show how to internally regularize DACSs.

KEYWORDS

differential-algebraic equations, external feedback equivalence, internal regularization, mechanical systems, nonlinear control systems, normal forms

1 | INTRODUCTION

Consider a nonlinear differential-algebraic control system DACS of the form

$$\Xi^u : E(x)\dot{x} = F(x) + G(x)u, \tag{1}$$

where $x \in X$ is the generalized state, with X an n-dimensional differentiable manifold (or an open subset of $\mathbb{R}^n$) and $u \in \mathbb{R}^m$ is the control vector. For the differentiable manifold X, we denote by TX the tangent bundle of X and by T_xX the tangent space of X at $x \in X$. The maps $E : TX \to \mathbb{R}^l$, $F : X \to \mathbb{R}^l$ and $G : X \to \mathbb{R}^{l\times m}$ are smooth and the word "smooth" will always mean C^∞-smooth throughout the article. For each $x \in X$, we have $E(x) : T_xX \to \mathbb{R}^l$, which is the linear map $\dot{x} \mapsto E(x)\dot{x}$. In particular, if X is an open subset of $\mathbb{R}^n$, then for each $x \in X$, we have $E(x) : \mathbb{R}^n \to \mathbb{R}^l$, that is, $E(x) \in \mathbb{R}^{l\times n}$. A DACS of the form (1) will be denoted by $\Xi^u_{l,n,m} = (E, F, G)$ or, simply, Ξ^u. A particular case of (1) is a linear DACS of the form

$$\Delta^u : E\dot{x} = Hx + Lu, \tag{2}$$