Hon Pre-Calc
Test Chapter 2

Show All Work!!! Circle All Final Answers!! No Calculators!!!

Short Answer

1. A driver averaged 60 mph on the round trip between Rochester Hills, Michigan, and Gaylord, Michigan, 200 miles away. The average speeds for going and returning were x and y miles per hour respectively. Find an equation solved for y in terms of x.

$$60 = \frac{400}{\frac{200}{x} + \frac{200}{y}}$$

$$60 = \frac{400}{\frac{200y + 200x}{xy}}$$

$$3 = \frac{20xy}{200y + 200x}$$

$$3 = \frac{xy}{10y + 10x}$$

$$30y + 30x = xy$$

$$30y - xy = -30x$$

$$y(30 - x) = -30x$$

$$y = \frac{30x}{x - 30} \text{ m}$$

2. Find the remainder: $\dfrac{x^4}{(x-1)^3}$

$$\begin{array}{r} x+3 \\ x^3-3x^2+3x-1 \overline{)\, x^4+0+0+0+0} \\ -x^4+3x^3-3x^2+x \\ \hline 3x^3-3x^2+x \\ -3x^3+9x^2-9x+3 \\ \hline 6x^2-8x+3 \end{array}$$

$6x^2 - 8x + 3$

$(x-1)(x-1)$

$(x^2-2x+1)(x-1)$

$x^3 - x^2 - 2x^2 + 2x + x - 1$

$x^3 - 3x^2 + 3x - 1$

3. Consider: $f(x) = \dfrac{3x^2 + 10x - 8}{x^2 - 16}$

$$\frac{(3x-2)(x+4)}{(x+4)(x-4)}$$

a) Use interval notation to write the domain and range

Domain = $(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$

Range = $(-\infty, \frac{7}{4}) \cup (\frac{7}{4}, 3) \cup (3, \infty)$

b) Use limit notation to describe the end behavior.

$\lim_{x \to \infty} f(x) = 3$

$\lim_{x \to -\infty} f(x) = 3$

c) Identify the x and y location of any holes (write answer as an ordered pair).

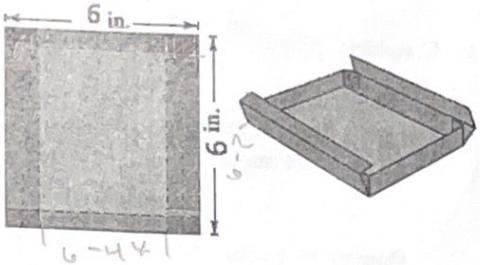
$(-4, \frac{7}{4})$

d) Use limit notation to describe the behavior around any vertical asymptotes.

$\lim_{x \to 4^+} f(x) = \infty$

$\lim_{x \to 4^-} f(x) = -\infty$

4. An open box with locking tabs is to be made from a square piece of material 6 inches on a side. This is to be done by cutting equal squares from the corners and folding along the dashed lines as shown.



a) Write a function $v(x)$ that represents the volume.

$V(x) = x(6-2x)(6-4x)$

$V(x) = x(36-24x-12x+8x^2)$

$V(x) = 8x^3 - 36x^2 + 36x$ X

b) What is the domain of the volume function.

$(0, \frac{3}{2})$

c) What is the value of x that will maximize the volume?

$24x^2 - 72x + 36$

$2x^2 - 6x + 3$

$\frac{6 \pm \sqrt{36-4(3)(2)}}{4} \rightarrow \frac{6 \pm \sqrt{36-24}}{4}$

$\frac{6 \pm \sqrt{12}}{4} \rightarrow \frac{6 \pm 2\sqrt{3}}{4} \rightarrow \frac{3-\sqrt{3}}{2}$ in

5. Find a polynomial with integer coefficients of least degree with 0 (mult 3) and $1+i\sqrt{2}$ as zeros that also

x^3

has the following end behavior: $\begin{cases} \lim_{x \to \infty} f(x) = -\infty \\ \lim_{x \to -\infty} f(x) = +\infty \end{cases}$

$-(x^3)(x-(1+i\sqrt{2}))(x-(1-i\sqrt{2}))$

$-(x^3)((x-1)-i\sqrt{2})((x-1)+i\sqrt{2})$

$-(x^3)((x-1)^2-(+1)(2))$

$-(x^3)(x^2-2x+1+2)$

$-(x^3)(x^2-2x+3) \rightarrow -x^3(x^2-2x+3)$

$-(x^5-2x^4+3x^3)$

$-x^5+2x^4-3x^3$

$\frac{2 \pm \sqrt{4-4(3)}}{2} \rightarrow \frac{2 \pm \sqrt{-8}}{2} \rightarrow \frac{2 \pm 2i\sqrt{2}}{2}$

$1 \pm i\sqrt{2}$

6. Given: $f(x) = 2x^3 - 3x^2 - 36x + 12$

$6x^2 - 6x - 36$ $(x-3)(x+2)$

$x^2 - x - 6$ $\rightarrow$

a) Find the location (x coordinate) of the relative minimum of the function:

$x = 3$

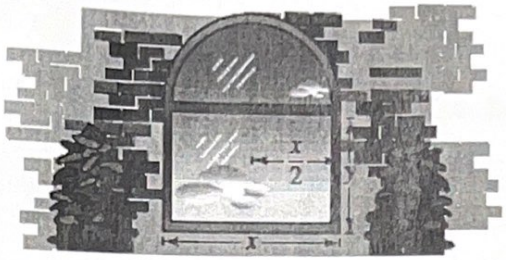
b) Use interval notation to state where the function in increasing

$(-\infty, -2) \cup (3, \infty)$

c) Use interval notation to state where the function is decreasing.

$(-2, 3)$

7. A Norman window is constructed by adjoining a semicircle to the top of an ordinary rectangular window (see - figure). The perimeter of the entire window is 14 feet.



a) Write the area of the entire window as a function of x.

$14 = x + 2y + \frac{\pi x}{2}$ $\quad A = xy + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$

$14 - x - \frac{\pi x}{2} = 2y$ $\quad A = x\left(7 - x\left(\frac{1}{2} + \frac{\pi}{4}\right)\right) + \frac{\pi x^2}{8}$

$7 - \frac{x}{2} - \frac{\pi x}{4} = y$ $\quad A = 7x - \frac{x^2}{2} - \frac{2\pi x^2}{8} + \frac{\pi x^2}{8}$

$7 - x\left(\frac{1}{2} + \frac{\pi}{4}\right) = y$ $\quad A = 7x - \left(\frac{1}{2} + \frac{\pi}{8}\right)x^2$

b) What should x be to maximize area of the window?

$\frac{-7}{2\left(-\frac{1}{2} + \frac{\pi}{8}\right)}$ $\quad \frac{7}{1 + \frac{\pi}{4}} \rightarrow \frac{28}{4+\pi}$ ft

8. Simplify completely and put in standard $a+bi$ form.

$$\frac{1+i}{i} - \frac{3}{2+i}$$

$\frac{i-1}{-1} - \frac{3(2-i)}{4-i^2}$

$\frac{-1+i}{-1} \rightarrow \frac{1-i}{1} - \frac{(6-3i)}{5}$

$\frac{5-5i-6+3i}{5} \rightarrow \frac{-1-2i}{5}$

$-\frac{1}{5} - \frac{2}{5}i$

$x^2(x-2) - 1(x-2)$

9. Consider: $\frac{x^3 - 2x^2 - x + 2}{x^2 - 4}$

$\frac{(x+1)(x-1)(x-2)}{(x+2)(x-2)}$

a) Identify any vertical asymptotes

$x = -2$

b) Identify any horizontal asymptotes

none

c) Identify any slant asymptotes

$$\begin{array}{r} x - 2 \\ x^2+0x-4 \overline{)\, x^3 - 2x^2 - x + 2} \\ -x^3 + 0x^2 + 4x \\ \hline -2x^2 + 3x + 2 \\ +2x^2 - 4x + 2 \\ \hline -x + 2 \end{array}$$

$y = x - 2$

d) Identify the location of any holes

$\left(2, \frac{3}{4}\right)$

$\frac{(2+1)(2-1)}{(2+2)}$

$\frac{(3)(1)}{4} \rightarrow \frac{3}{4}$

$2x^4 - 5x^3 + 4x^2 - 5x + 2$

10. Consider: $f(x) = 2x^4 + 5x^3 + 4x^2 + 5x + 2$

a) Complete a P,N,I chart

P	N	I
0	4	0
0	2	2
0	0	4

b) List all possible rational zeros

$\frac{P}{q} \rightarrow \frac{-2, -1}{2, 1} \rightarrow \frac{P}{q} = -2, -1, -\frac{1}{2}$

c) Solve Completely

-2 | 2 | 5 | 4 | 5 | 2
↓ | | -4 | -2 | -4 | -2
| 2 | 1 | 2 | 1 | 0

$(2x^3 + x^2)(2x + 1)$

$x^2(2x+1)(2x+1)$

$(x^2+1)(2x+1)$

$x = \pm i, -\frac{1}{2}, -2$

11. Write the solution to the inequality using interval notation:

$$\frac{3x}{x-1} \leq \frac{x}{x+4} + 3$$

$\frac{3x}{x-1} \leq \frac{x+3x+12}{x+4}$

$\frac{3x}{x-1} - \frac{4x+12}{x+4} \leq 0$

$\frac{(4x+12)(x-1) - 3x(x+4)}{(x+4)(x-1)} \leq 0$

$\frac{4x^2 - 4x + 12x - 12 - 3x^2 - 12x}{(x+4)(x-1)} \leq 0$

$\frac{x^2 - 4x - 12}{(x+4)(x-1)} \leq 0 \rightarrow \frac{(x-6)(x+2)}{(x+4)(x-1)} \leq 0$

$-5: \frac{(-)(-)}{(-)(-)} +$ $\quad -3: \frac{(-)(-)}{+} -$ $\quad 0: +, \quad 2: -, \quad 7: +$

$(-\infty, -4) \cup [-2, 1) \cup [6, \infty)$

12. Find the domain of x.

$$\sqrt{\frac{x-1}{2x^2+3x-2}}$$

$(2x^2 + 4x)(-x - 2)$

$2x(x+2) - 1(x+2)$

$(2x-1)(x+2)$

$0 \leq \frac{x-1}{(2x-1)(x+2)}$

$(-2, 1] \cup (\frac{1}{2}, \infty)$

$(-2, \frac{1}{2}) \cup [1, \infty)$

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