# Honors Pre-Calc Test Chapter 7 (2017)

**Show ALL work for full credit!!! Circle ALL final answers!!! Leave any general solutions in terms of Z!!!**

**Short answer**

1. Describe $m$ and $n$ so the system has no solutions.

$$\begin{cases} mx + 3y = n \\ 4x - 5y = 9 \end{cases} \Rightarrow \begin{cases} 5mx + 15y = 5n \\ 12x - 15y = 27 \end{cases}$$

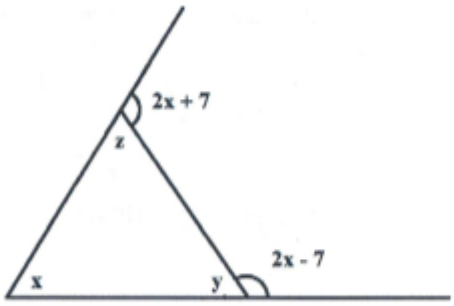
$5mx + 12x = 5n + 27$

$5x(m + \frac{12}{5}) = 5(n + \frac{27}{5})$

$m = -12/5$

$n \neq -27/5$

2. Find the values of $x$, $y$, and $z$ in the figure. (angle measurements are in degrees)




$x + y + z = 180°$

$z = 180° - (2x + 7)°$

$z = 173° - 2x$

$y = 180° - (2x - 7)°$

$y = 187° - 2x$

$x + 173° - 2x + 187° - 2x = 180°$

$-3x = -180°$

$x = 60°$

$y = 187° - 120°$

$y = 67°$

$z = 173° - 120°$

$z = 53°$

$(60°, 67°, 53°)$

3. Solve the following system:

$$\begin{cases} 2x + y - z = 7 \\ x - 2y + 2z = -9 \\ 3x - y + z = 5 \end{cases}$$

$$+\begin{cases} 2x + y - z = 7 \\ 3x - y + z = 5 \end{cases}$$

$5x = 12$

$x = 12/5$

$$+\begin{cases} 4x + 2y - 2z = 14 \\ x - 2y + 2z = -9 \end{cases}$$

$5x = 5$

$x = 1$

$1 \neq 12/5$

No solution

4. Solve the following system:

$$\begin{cases} x^2 + y^2 = 169 \\ x^2 - 8y = 104 \end{cases}$$

$x^2 = 104 + 8y$

$104 + 8y + y^2 = 169$

$y^2 + 8y - 65 = 0$

$(y - 5)(y + 13) = 0$

$y = 5, -13$

$x^2 + 5^2 = 169$

$x^2 = 144$

$x = \pm 12$

$x^2 + (-13)^2 = 169$

$x^2 = 0$

$x = 0$

$(12, 5)$

$(-12, 5)$

$(0, -13)$

5. Solve the following system:

$$\begin{cases} x - 3y + 2z = 18 \\ 5x - 13y + 12z = 80 \end{cases}$$

$$\begin{cases} 5x-15y+10z=90 \\ -(5x-13y+12z=80) \end{cases}$$

$-2y-2z=10$

$-2y=2z+10$

$y=-z-5$

$x-3y+2z=18$

$x=18+3y-2z$

$x=18+3(-z-5)-2z$

$x=3-5z$

$(3-5z, -5-z, z)$

6. What are the dimensions of a rectangular piece of land if its perimeter is 44 kilometers and its area is 120 sq. kilometers?

$\begin{cases} xy=120 \\ 2x+2y=44 \Rightarrow y=22-x \end{cases}$

$x(22-x)=120$

$22x-x^2-120=0$

$x^2-22x+120=0$

$(x-12)(x-10)=0$

$x=10, 12$

$10y=120$ $\quad$ $12y=120$

$y=12$ $\quad$ $y=10$

12 km by 10 km
10 km by 12 km

7. Find the equation of the parabola: $y = ax^2 + bx + c$ that passes through (2, 0), (3, -1) and (4, 0).

$\begin{cases} 0=4a+2b+c \\ -1=9a+3b+c \\ 0=16a+4b+c \end{cases}$

$\begin{cases} 0=4a+2b+c \\ -(0=16a+4b+c) \end{cases}$

$0=-12a-2b$

$b=-6a$

$0=4a+2b+c$

$0=4-12+c$

$c=8$

$\begin{cases} 0=4a+2b+c \\ -(-1=9a+3b+c) \end{cases}$

$1=-5a-b$

$1=-5a+6a$

$a=1$

$b=-6$

$y=x^2-6x+8$

8. A chemist needs 10 liters of a 25% acid solution. The solution is to be mixed from three solutions whose concentrations are 10%, 20%, and 50%. How many liters of each solution will be used if the chemist wishes to use the least amount of the 50% solution as possible?

$x+y+z=10$ $\quad$ $x=10-y-z$

$0.1x+0.2y+0.5z=10(0.25)$ $\quad$ $x=25-2y-5z$

$10-y-z=25-2y-5z$ $\quad$ $x=10-(15-4z)-z$

$y=15-4z$ $\quad$ $x=3z-5$

$(3z-5, 15-4z, z)$

$z \geq 5/3$ $\quad$ $z \leq 15/4$

$(0, \underset{10\%}{25/3}, \underset{20\%}{5/3}, \underset{50\%}{})$

9. An airplane flying into a headwind travels the 1800-mile flying distance between Pittsburgh and Phoenix in 3 hours and 36 minutes. On the return flight, the distance is traveled in 3 hours.

a) Find the airspeed of the plane.

$r = d/t$

$r_1 = \frac{1800}{3.6}$ $r_2 = \frac{1800}{3}$

$r_1 = 500$ $r_2 = 600$

$+\begin{cases} 500 = s - w \\ 600 = s + w \end{cases}$

$1100 = 2s$

$\boxed{s = 550}$

b) Find the wind speed.

$600 = 550 + w$

$\boxed{w = 50}$

10. Write the partial fraction decomposition of the rational expression:

$\frac{3}{x^4 + x}$ $\frac{3}{x(x^3+1)}$ $\frac{3}{x(x+1)(x^2-x+1)}$

$\frac{3}{x^4+x} = \frac{A}{x} + \frac{B}{x+1} + \frac{Cx+D}{x^2-x+1}$

$3 = A(x^3+1) + Bx(x^2-x+1) + (Cx+Dx)(x+1)$

$x = -1;\ 3 = A(0) - B(1+1+1) + (C-D)(0)$

$3 = -3B$ $B = -1$

$x = 0;\ 3 = A(1) + B(0)(1) + (0+0)(1)$

$A = 3$

$3 = 3(x^3+1) - x(x^2-x+1) + (Cx^2+Dx)(x+1)$

$3 = 3x^3 + x^2 - x + 3 + Cx^3 + Cx^2 + Dx^2 + Dx$

$0 = x(2x^2 + x - 1) + Cx^2(x+1) + Dx(x+1)$

$0 = x(2x-1)(x+1) + (Cx^2+Dx)(x+1)$

$-x(2x-1)(x+1) = x(Cx+D)(x+1)$

$-2x + 1 = Cx + D$

$C = -2$ $D = 1$

$\frac{3}{x} - \frac{1}{x+1} - \frac{2x-1}{x^2-x+1}$

11. Write the partial fraction decomposition of the improper rational expression:

$\frac{x^4}{(x-1)^3}$

$x^3 - 3x^2 + 3x - 1 \overline{)\, x^4 + 0x^3 + 0x^2 + 0x + 0}$

$x + 3$

$x^4 - 3x^3 + 3x^2 - x$

$3x^3 - 3x^2 + x + 0$

$3x^3 - 9x^2 + 9x - 3$

$6x^2 - 8x + 3$

$(x+3) + \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3}$

$6x^2 - 8x + 3 = A(x-1)^2 + B(x-1) + C$

$x = 1;\ 6(1) - 8(1) + 3 = A(0) + B(0) + C$ $C = 1$

$6x^2 - 8x + 2 = A(x^2 - 2x + 1) + B(x-1)$

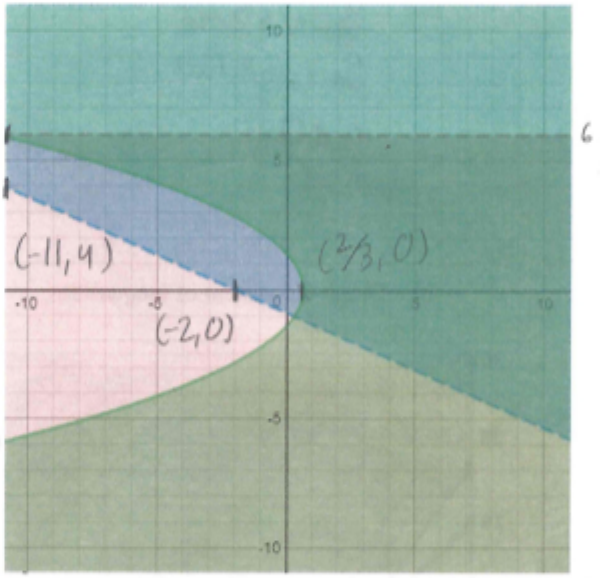
$A = 6;\ 6x^2 - 8x + 2 = 6x^2 - 12x + 6 + B(x-1)$

$B(x-1) = 4x - 4$

$B = 4$

$\boxed{x + 3 + \frac{6}{x-1} + \frac{4}{(x-1)^2} + \frac{1}{(x-1)^3}}$

12. Write the system of inequalities that has the following solution:




$\begin{cases} 0 = -2m + b \\ 4 = -11m + b \end{cases}$

$-4 = 9m$

$m = -4/9$

$0 = 8/9 + b$

$b = -8/9$

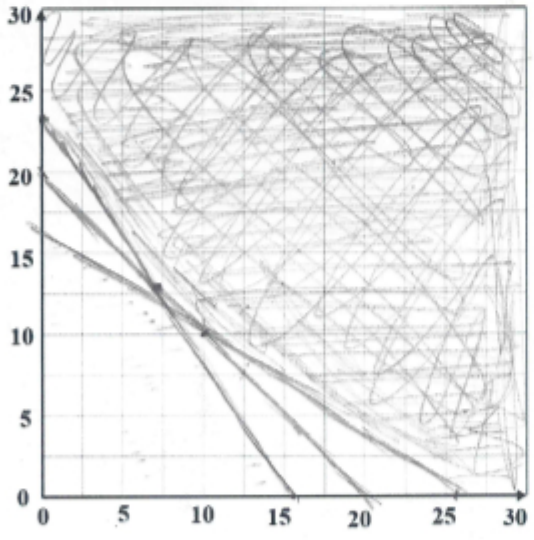
$\boxed{\begin{aligned} y &< 6 \\ y &> -\tfrac{4}{9}x - \tfrac{8}{9} \\ y^2 &\geq -3x + 2 \end{aligned}}$

13. A humanitarian agency can use two models of vehicles for a refugee rescue mission. Each Model-A vehicle costs $1000 and each Model-B vehicle costs $1500. Mission strategies and objectives indicate the following constraints:
    - A total of at least 20 vehicles must be used.
    - A Model-A vehicle can hold 45 supply boxes. A Model-B can hold 30. The agency must deliver at least 690 boxes to the refugee camp.
    - A Model-A vehicle can hold 20 refugees. A Model-B can hold 32. The agency must rescue at least 520 refugees.

    a) Write a system of inequalities modeling the constraints

$a+b \geq 20$
$3a+2b \geq 46$
$5a+8b \geq 130$
$a \geq 0$
$b \geq 0$

    b) Graph the system of inequalities




    c) Write the objective function

$C = 1000a + 1500b$

    d) List all the vertices of the feasible region

(0, 23)
(26, 0)
(6, 14)
(10, 10)

$b = 20 - a$
$3a + 2(20-a) = 46$
$a = 6 \quad b = 14$
$5a + 8(20-a) = 130$
$-3a = -30$
$a = 10 \quad b = 10$

    e) What is the optimal number of each vehicle?

(0, 23) => 34500
(26, 0) => 26000
(6, 14) => 27000
(10, 10) => 25000

(10, 10)

    f) What is the optimal cost?

$25000