

Hon Pre-Calc Test Chapter 4 2018 - 2019

1. Use the value of the trigonometric function to evaluate the following: $-\sin(-t) = -\frac{3}{10}$

a) $\sin(t) =$

b) $\sin(\pi + t) =$

c) $-\cos\left(\frac{\pi}{2} + t\right) =$

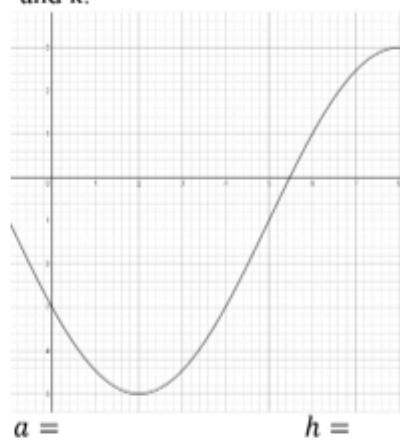
2. The point $(-2, 7)$ is on the terminal side of an angle θ whose reference angle is θ' . Find:

a) $\tan \theta'$

b) $\sin \theta$

c) $\sec \theta$

3. From the **sine** graph find a, b, h ($h > 0$), and k.



$b =$

$k =$

4. Determine if the function is odd, even, or neither.

a) $f(x) = x + \tan x$

b) $f(x) = x + \sec x$

c) $f(x) = x \csc x$

5. Given: $f(x) = \frac{1}{3} \sec\left(\frac{\pi x}{2} + \frac{\pi}{2}\right) + \frac{2}{3}$

State the domain and range of f

a) $\text{Domain} =$

b) $\text{Range} =$

6. Write an algebraic expression that is equivalent to the expression:

$$\csc\left(\arctan\left(\frac{|x|}{\sqrt{2}}\right)\right)$$

7. A 2-inch diameter pulley on an electric motor that runs at 17 rpm is connected by a belt to a 4-inch diameter pulley on a saw arbor.

a) Find the angular speed of each pulley.

b) Find the revolutions per minute of the saw.

8. Find the central angle in radians of a sector with an arc length of 2π inches and an area of 10π square inches.

9. Convert 16.62° to degrees, minutes, seconds

10. Given: $\sec(t) = \frac{5\sqrt{5}}{7}$ find:
a) $\cos(\pi - t)$

b) $\sec(\pi + t)$

11. Given: $\cot \alpha = \frac{5|x|}{3}$ and $\sin \alpha < 0$, Find:

a) $\sec \alpha$

b) $\sin \alpha$

12. What is the reference angle of 17 radians?

13. Given: $\sin(-x) = -\frac{1}{3}$, $\tan x = -\frac{\sqrt{2}}{4}$

a) $\cos x =$

b) $\csc x =$

14. What are the following exact values:

a) $\cos \frac{25\pi}{3}$

b) $\sec \frac{15\pi}{4}$

c) $\csc -\frac{13\pi}{3}$

d) $\cot -\frac{16\pi}{3}$

15. Find the exact value of the expressions:

a) $\sin\left(\cos^{-1}\frac{\sqrt{5}}{5}\right)$

b) $\cot\left(\sin^{-1}-\frac{\sqrt{2}}{2}\right)$

16. Write an algebraic expression that is equivalent to the expression:

$\tan\left(\arccos\frac{x}{3}\right)$

17. Evaluate the expression:

a) $\sin^{-1}\left(\cos\frac{5\pi}{6}\right)$

b) $\tan^{-1}\left(\tan-\frac{5\pi}{6}\right)$

c) $\csc^{-1}\left(\sec\frac{2\pi}{3}\right)$

18. Simplify completely to one single trigonometric function:

$$\frac{\cos \theta \cot \theta}{1 - \sin \theta} - 1$$

19. Consider: $g(x) = \frac{\sin x}{x}$ Determine:
 $\lim_{x \rightarrow 0^+} g(x)$

20. Given $f(x) = e^{-x} \cos x$

- a) Find y-intercept =
- b) Use limit notation to state the end behavior.

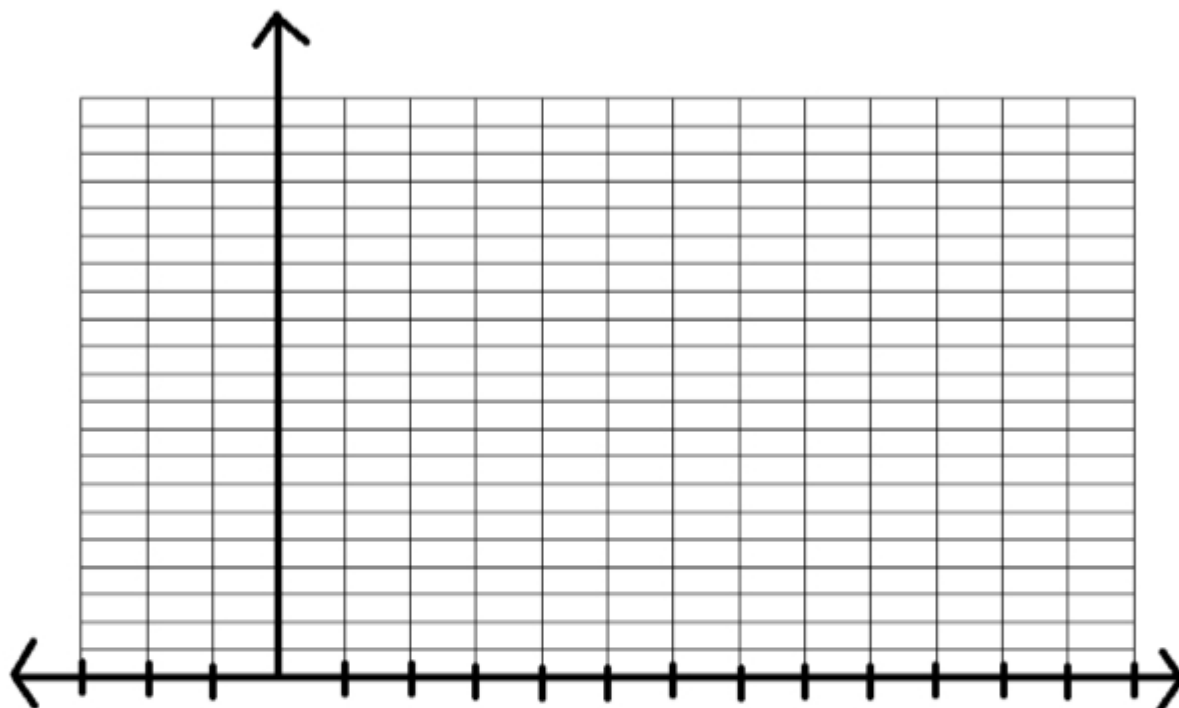
21. A ball that is bobbing up and down on the end of a string has a maximum displacement of 3 inches. Motion (in ideal conditions) is modeled by $y = \frac{1}{4} \cos 16t$ ($t > 0$). Where y is measured in feet and t is the time in seconds.

- a) What is the frequency of the oscillations?
- b) Determine the first time the weight passes the point of equilibrium ($y = 0$)

22. Two fire towers are 40 km apart, where tower A is due west of tower B. A fire is spotted from the towers, and the bearings from A to B are N 70° E and N 52° W, respectively. Find the distance d (**in simplified calculator ready form**) of the fire from the line segment AB.

23. **Ferris Wheel:** As you ride the Ferris wheel your distance from the ground varies sinusoidally with time. When the last seat is filled and the Ferris wheel starts ($t = 0$ sec), you notice it takes you 15 seconds to get to the top. The platform to load the chairs is 8 feet off the ground and the Ferris wheel towers at a height of 72 feet above the ground. You time the ride and you see that you made 6 revolutions in 8 min.

- a) Sketch only **ONE** cycle of the graph of the distances of your height from the ground. Scale your graph so it starts and ends perfectly on the graph provided.



- b) Write an equation of this sinusoidal using the **SINE** function.

- c) Predict your exact height above ground at $221\frac{2}{3}$ seconds.

Hon Pre-Calc Test Chapter 4

Name _____

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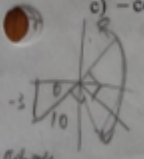
Circle All Final Answers!!! No Calculators!!!! Leave Answers In Simplified Calculator Ready Form (If Necessary)!!!

Short Answer

1. Use the value of the trigonometric function to evaluate the following: $-\sin(-t) = -\frac{3}{10}$

a) $\sin(t) = \boxed{-\frac{3}{10}}$

b) $\sin(\pi+t) = \boxed{\frac{3}{10}}$

c) $-\cos\left(\frac{\pi}{2}+t\right) \text{ find } \sin\theta$

 $\boxed{-\frac{3}{10}}$

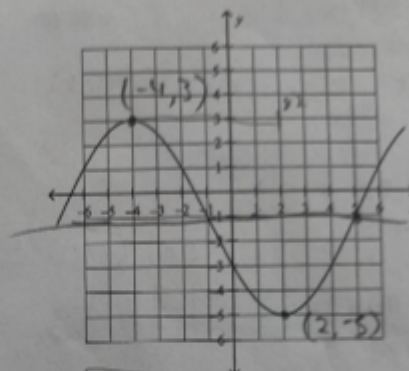
2. The point $(-2, 7)$ is on the terminal side of an angle θ whose reference angle is θ' . Find:

a) $\tan \theta' = \boxed{\frac{7}{2}}$

b) $\sin \theta = \boxed{\frac{7\sqrt{53}}{53}}$

c) $\sec \theta = \boxed{-\frac{\sqrt{53}}{2}}$

3. From the **sine** graph find a, b, h ($h > 0$), and k.



$k = \frac{3-5}{2} = -1$

$6 \div 2 = 12$

$a = \boxed{4}$

$h = \boxed{5}$

$b = \boxed{\frac{\pi}{6}}$

$k = \boxed{-1}$

$\frac{2\pi}{b} = 12$
 $b = \frac{\pi}{6}$

$y = 4\sin_{\frac{\pi}{6}}(x-5) - 1$

(2)

4. Determine if the function is odd, even, or neither.

a) $f(x) = x + \tan x$
 $f(-x) = -x - \tan x = -(x + \tan x)$

odd

b) $f(x) = x + \sec x$
 $f(-x) = -x + \sec x$

neither

c) $f(x) = x \csc x$
 $f(-x) = -x (\csc(-x)) = -x (-\csc x) = x \csc x = f(x)$

even

5. Given: $f(x) = \frac{1}{3} \sec\left(\frac{\pi x}{2} + \frac{\pi}{2}\right) + \frac{2}{3}$

State the domain and range of the f .

a) Domain = $(-\infty, \infty), x \neq 0 + 2n \text{ (n int)}$

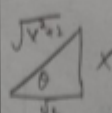
$\frac{2\pi}{\frac{\pi}{2}} = 2 \cdot \frac{2\pi}{\frac{\pi}{2}} = 4\pi$
 $\frac{2}{2} = 1$

b) Range = $\frac{1}{3} + \frac{2}{3} = 1$

$\left(-\infty, \frac{1}{3}\right] \cup \left[\frac{1}{3}, \infty\right)$

6. Write an algebraic expression that is equivalent to the expression:

$\csc\left(\arctan\left(\frac{|x|}{\sqrt{2}}\right)\right)$



$\csc \theta = \frac{\sqrt{x^2 + 2}}{|x|}$

7. A 2-inch diameter pulley on an electric motor that runs at 17 rpm is connected by a belt to a 4-inch diameter pulley on a saw arbor.

a) Find the angular speed of each pulley.

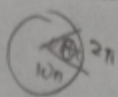
$v = \frac{d}{t}$
 $17 \text{ rpm} = 34\pi \text{ rad/min}$ (for 2-inch pulley)
 $17 \text{ rpm} = 17\pi \text{ rad/min}$ (for 4-inch pulley)

b) Find the revolutions per minute of the saw.

$v_1 = \frac{2}{t} = \frac{r \cdot \theta}{t} = \frac{1.2 \pi \cdot 17}{t} \text{ in/min}$
 $v_2 = \frac{4}{t} \text{ in/min}$

$\frac{17}{2} = 8.5 \text{ rpm}$

8. Find the central angle in radians of a sector with an arc length of 2π inches and an area of 10π square inches.



$$\frac{1}{2}r^2\theta = 10\pi$$

$$r\theta = 2\pi$$

$$\frac{1}{2}r = 5$$

$$r = 10$$

$$10(\theta) = 2\pi$$

$$\theta = \frac{\pi}{5} \text{ radians}$$

9. Convert 16.62° to degrees, minutes, seconds.

$$16 + \frac{62}{100} \cdot \frac{60}{60} = \frac{186}{5}$$

$$\begin{array}{r} 38 - 4 \\ 5 \overline{) 186} \\ 15 \\ \hline 36 \\ 5 \end{array}$$

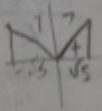
$$16^\circ, 38', 24''$$

$$38 + \frac{4}{60} \cdot 60 = 38.666666666666664$$

$$16^\circ 38' 24''$$

$$16^\circ 37' 12''$$

10. Given: $\sec t = \frac{5\sqrt{5}}{7\sqrt{5}}$ Evaluate:



a) $\cos(\pi - t)$

$$\frac{-\sqrt{5}}{7} = -\frac{\sqrt{5}}{7}$$

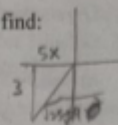
b) $\sec(\pi + t)$

$$\frac{-5\sqrt{5}}{7}$$

11. Given: $\cot \alpha = \frac{5|x|}{3}$ and $\sin \alpha < 0$ find:

a) $\sec \alpha$

$$\frac{\sqrt{25x^2 + 9}}{5|x|}$$



b) $\sin \alpha = \frac{-3}{\sqrt{25x^2 + 9}} = \frac{-3\sqrt{25x^2 + 9}}{25x^2 + 9}$

12. What is the reference angle of 17 radians?

$$17 - 6.28 = 10.72$$

$$\begin{array}{r} 17 \\ 12.56 \\ 4.24 \\ 3.14 (\pi) \\ \hline 1.30 \end{array}$$

$$17 - 5\pi \text{ radians}$$

$$-4$$

13. Given: $\sin(-x) = -\frac{1}{3}$, $\tan x = -\frac{\sqrt{2}}{4}$

$\sin x = \frac{1}{3}$
 $\cos x = -\frac{\sqrt{2}}{4}$
 $\tan x = -\frac{\sqrt{2}}{4}$
 $\sec x = -\frac{4}{\sqrt{2}} = -2\sqrt{2}$

a) $\cos x = ?$ $\boxed{-\frac{\sqrt{2}}{4}}$

b) $\csc x = ?$ $\boxed{3}$

14. What are the following exact values:

a) $\cos \frac{25\pi}{3} \left(\frac{\pi}{3} \right) = \boxed{\frac{1}{2}}$

b) $\sec \frac{15\pi}{4} \left(\frac{\pi}{4} \right) = \boxed{\sqrt{2}}$

c) $\csc \frac{-13\pi}{3} \left(\frac{\pi}{3} \right) = \boxed{-\frac{2\sqrt{3}}{3}}$

$\sin(-\frac{\pi}{3}) = -\frac{\sqrt{3}}{2}$

d) $\cot \frac{-16\pi}{3} \left(\frac{2\pi}{3} \right) = \boxed{-\frac{\sqrt{3}}{3}}$

$\tan(\frac{2\pi}{3}) = \sqrt{3}$

15. Find the exact value of the expressions:

a) $\sin \left(\cos^{-1} \left(\frac{\sqrt{5}}{5} \right) \right)$

$\cos \theta = \frac{\sqrt{5}}{5}$

$\sin \theta = \frac{\sqrt{20}}{5} = \boxed{\frac{2\sqrt{5}}{5}}$

b) $\cot \left(\sin^{-1} \left(-\frac{\sqrt{2}}{2} \right) \right)$

$\sin \theta = -\frac{\sqrt{2}}{2}$

$\cot \theta = \frac{\cos \theta}{\sin \theta} = \frac{0}{-1} = \boxed{0}$

16. Write an algebraic expression that is equivalent to the expression:

$\tan \left(\arccos \frac{x}{3} \right)$

$\tan \theta = \frac{\sqrt{9-x^2}}{x}$

$\pm \frac{\sqrt{9-x^2}}{x}$

17. Evaluate the expression:

a) $\sin^{-1} \left(\cos \frac{5\pi}{6} \right)$

$\cos \frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$

$\sin^{-1} \left(-\frac{\sqrt{3}}{2} \right) = -\frac{\pi}{3}$

b) $\tan^{-1} \left(\tan \frac{-5\pi}{6} \right)$

$\tan \frac{-5\pi}{6} = -\frac{1}{\sqrt{3}}$

$\tan^{-1} \left(-\frac{1}{\sqrt{3}} \right) = -\frac{\pi}{6}$

c) $\csc^{-1} \left(\sec \frac{2\pi}{3} \right)$

$\sec \frac{2\pi}{3} = -2$

$\csc^{-1}(-2) = -\frac{\pi}{6}$

18. Simplify completely to one single trigonometric function:

$$\begin{aligned} \frac{\cos \theta \cot \theta}{1 - \sin \theta} - 1 &= \frac{\cos \theta \cdot \frac{\cos \theta}{\sin \theta}}{1 - \sin \theta} - 1 = \frac{\frac{\cos^2 \theta}{\sin \theta}}{1 - \sin \theta} - 1 \\ &= \frac{\cos^2 \theta}{\sin \theta (1 - \sin \theta)} - \frac{1 - \sin \theta}{1 - \sin \theta} = \frac{\cos^2 \theta - (1 - \sin \theta) \sin \theta}{\sin \theta (1 - \sin \theta)} \\ &= \frac{\cos^2 \theta - \sin \theta + \sin^2 \theta}{\sin \theta (1 - \sin \theta)} = \frac{1 - \sin \theta}{\sin \theta (1 - \sin \theta)} = \frac{1}{\sin \theta} \\ &= \csc \theta \end{aligned}$$

19. Consider: $g(x) = \frac{\sin x}{x}$ Determine: $\lim_{x \rightarrow 0^+} g(x)$

$$\lim_{x \rightarrow 0^+} g(x) = 1$$

20. Given: $f(x) = e^{-x} \cos x$ $x \rightarrow 0$

a) Find y-intercept = $(0, 1)$

$$e^{-0} (\cos 0) = 1 \cdot 1 = 1$$

- b) Use limit notation to state the end behavior.

$$\begin{aligned} \lim_{x \rightarrow \infty} f(x) &= 0 \\ \lim_{x \rightarrow -\infty} f(x) &= 0 \end{aligned}$$

21. A ball that is bobbing up and down on the end of a string has a maximum displacement of 3 inches. Its motion (in ideal conditions) is modeled by

$$y = \frac{1}{4} \cos 16t \quad (t > 0). \text{ Where } y \text{ is measured in feet and } t \text{ is the time in seconds.}$$

- a) What is the frequency of the oscillations?

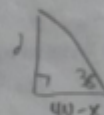
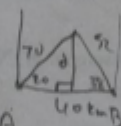
$$f = \frac{1}{T} = \frac{8}{\pi} \text{ cycles per sec.}$$

- b) Determine the first time the weight passes the point of equilibrium ($y = 0$).

$$\begin{aligned} \cos 16t &= 0 \\ 16t &= \frac{\pi}{2} \\ t &= \frac{\pi}{32} \end{aligned}$$

$$\frac{\pi}{32} \text{ sec.}$$

22. Two fire towers are 40 km apart, where tower A is due west of tower B. A fire is spotted from the towers, and the bearings from A and B to the fire are $N 70^\circ E$ and $N 52^\circ W$, respectively. Find the distance d (in simplified calculator ready form) of the fire from the line segment AB.



A

$$\tan 20 = \frac{20}{x} \quad \tan 38 = \frac{38}{40 - x}$$

$$\cot 20 = \frac{x}{20} \quad \cot 38 = \frac{40 - x}{38}$$

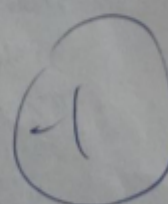
$$x = 20 \cot 20$$

$$\begin{aligned} d \cot 38 &= 40 - x \\ x &= 40 - d \cot 38 \end{aligned}$$

$$d \cot 20 = 40 - d \cot 38$$

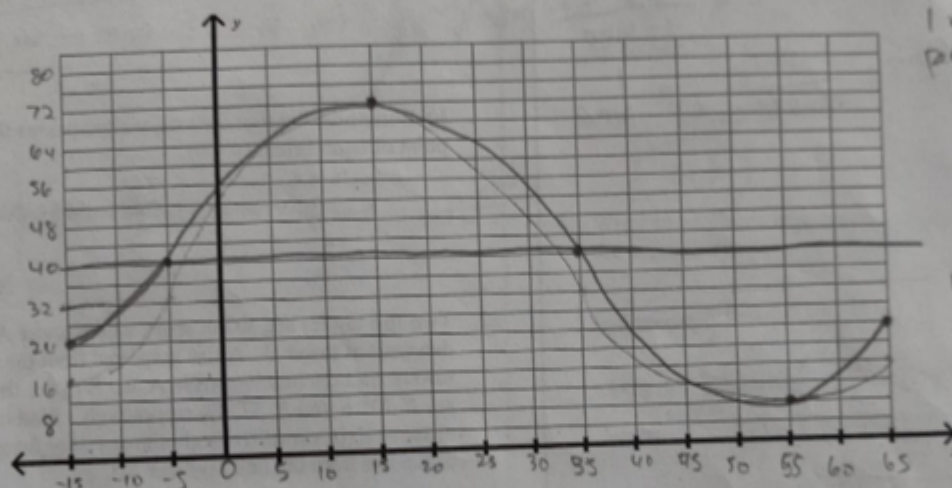
$$d \cot 20 + d \cot 38 = 40$$

$$d = \frac{40}{\cot 20 + \cot 38} \text{ km}$$



23. **Ferris Wheel:** As you ride the Ferris wheel your distance from the ground varies sinusoidally with time. When the last seat is filled and the Ferris wheel starts ($t=0$ sec), you notice it takes you 15 seconds to get to the top. The platform to load the chairs is 8 feet off the ground and the Ferris wheel towers at a height of 72 feet above the ground. You time the ride and you see that you made 6 revolutions in 8 min.

a) Sketch only **ONE** cycle of the graph of distances of your height from the ground. Scale your graph so it starts and ends perfectly on the graph provided.



b) Write an equation of this sinusoidal using the **SINE** function.

16 sec = 15
 $k = 40$ $h = 5$ $a = 16$ $2\pi/b = 80$ $b = \pi/40$

$$y = 32 \sin \frac{\pi}{40}(x+5) + 40$$

c) Predict your exact height above ground at $221\frac{2}{3}$ seconds

$665 + 15 = 680$

$$y = 32 \sin \frac{\pi}{40} \left(\frac{665}{3} + 5 \right) + 40$$

$$= 32 \sin \frac{\pi}{40} \left(\frac{670}{3} \right) + 40$$

$$= 32 \sin \frac{17\pi}{3} + 40$$

$$= 32 \sin \left(-\frac{\pi}{3} \right) + 40$$

$$= 32 \left(-\frac{\sqrt{3}}{2} \right) + 40$$

$$= 40 - 16\sqrt{3} \text{ ft}$$

