Hon Pre-Calc
Test Chapter 3

Name _____

No Calculators!!! Leave Answers in Simplified Calculator Ready Form!!! Circle All Final Answers!!!

Short Answer

1. Given: $f(x) = -1\log_2(5-2x) - 2$. Use interval notation to write:

   a) Domain =

   b) Range =

   c) Write the equation(s) of any asymptotes

2. Find the exact rational value.

   $\log_9 27 + \log_{27} 81$

3. Solve for x:

   $$\frac{5}{4-3e^{4x}} = 25$$

4. Find the exact value of the logarithmic expression:

   a) $\log_2 \sqrt[4]{64}$

   b) $\log_3(-27)$

5. Solve for x:

   $\ln(x^2-2) - \ln x = 1$

6. Solve for x:

   $8e^{2x} - 2e^x - 15 = 0$

7. Solve: $(\ln x)^5 = \ln\left(x^9\right)$

8. Solve: $4\left(3^{x-1}\right) = 8\left(2^{2x-3}\right)$

9. Solve for x in terms of a:

$$\log_b x = 3 - a + \log_b\left(\frac{a^4 b^a}{b^3}\right)$$

10. Solve for x:

$$2x \ln\left(\frac{1}{x}\right) - x = 0$$

11. A bacteria culture starts with 3000 bacteria. After 3 hours there are 48,000 bacteria present. How long (IN MINUTES) did it take to double?

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12. A sample of a radioactive isotope decreases from 6 mg to 4 mg in 30 years. What is the half life period of the isotope? **(Use natural logs to solve)**

13. Suppose the number of bugs in your house is growing exponentially. If there were 54 bugs after 3 weeks and 486 bugs after 5 weeks, how many bugs were there initially?

14. The population of Florida and Michigan are growing exponentially. If Florida currently has 22 million people and Michigan currently has 10 million people and if the populations of the two states increase annually by 2% and 4%, respectively, when will their populations be the same? **(Use natural logs to solve)**

15. Solve for x:

$$\ln\sqrt{x+4} - \ln\sqrt{x-2} = \ln\sqrt{x+1}$$

16. Expand:

$$\ln\left[\frac{\sqrt[3]{x-1}\,(3x-2)^4}{(x+1)\sqrt{x-1}}\right]^2$$

17. $\frac{1}{2}\log_a(x+3)+\frac{1}{2}\log_a(x-3)=\frac{2}{3}\log_a 125$

18. Condense: $\ln\left(x^2+3x+9\right)-\ln\left(x^3-27\right)+\ln(x-3)+\ln(x-2)-\ln\left(x^2-4\right)$

Hon Pre-Calc
Test Chapter 3

Name _____ 71/90

No Calculators!!! Leave Answers in Simplified Calculator Ready Form!!! Circle All Final Answers!!!

Short Answer

1. Given: $f(x) = -1\log_2(5-2x) - 2$. Use interval notation to write:

   a) Domain = $(-\infty, \frac{5}{2})$

   b) Range = $(-\infty, -2)$

   c) Write the equation(s) of any asymptotes

   $y = -2$

   $x = \frac{5}{2}$

2. Find the exact rational value.

   $\log_9 27 - \log_{27} 81$

   $\log_{9^3} 3^3 - \log_{3^3} 3^4$

   $3 - \frac{4}{3} = \frac{9}{3} - \frac{4}{3} = \frac{5}{3}$

   $(3^3)^x = 3^4$

   $3x = 4$

   $x = \frac{4}{3}$

3. Solve for x:

   $\frac{5}{4-3e^{4x}} = 25$

   $5 = 25(4-3e^{4x})$

   $\frac{1}{5} = 4 - 3e^{4x}$

   $-\frac{19}{5} = -3e^{4x}$

   $\frac{19}{15} = e^{4x}$

   $\ln\left(\frac{19}{15}\right) = 4x$

   $x = \frac{\ln\left(\frac{19}{15}\right)}{4}$

4. Find the exact value of the logarithmic expression:

   a) $\log_2 \sqrt[4]{64}$

   $\log_2 \sqrt[4]{4^3} = \log_2 (4)^{\frac{3}{4}} = \frac{3}{4}\log_2 4 = \frac{3}{4}(2) = \frac{3}{2}$

   b) $\log_3(-27)$

   undefined

5. Solve for x:

   $\ln\left(x^2-2\right) - \ln x = 1$

   $\ln\left(\frac{x^2-2}{x}\right) = 1$

   $e = \frac{x^2-2}{x}$

   $ex = x^2 - 2$

   $x^2 - ex - 2 = 0$

   $(x-2e)(x+e) = 0$

   $x = 2e, \quad x = e$

6. Solve for x:

   $8e^{2x} - 2e^x - 15 = 0$

   $0 = (e^x - 12)(e^x + 10)$

   $e^x = 12, \quad e^x \neq -10$

   $x = \ln 12$

$(e^{9\ln x})^5 = x$

$F = ie^{tb}$

7. Solve: $(\ln x)^5 = \ln(x^9)$

$\ln x \ln x \ln x \ln x \ln x = 9 \ln x$

$(\ln x)^4 = 9$

$\ln x = \sqrt[4]{9} = \sqrt[4]{3^2} = \sqrt{3}$

$\sqrt[4]{3^2} = 3^{2/4} = 3^{1/2} = \sqrt{3}$

$x = e^{\sqrt{3}}$

8. Solve: $4(3^{x-1}) = 8(2^{2x-3})$

$4 = 8(2^{-1}) = \frac{1}{2}(16) = 4$

$3^{x-1} = 2(2^{2x-3})$

$\ln 3^{x-1} = \ln 2 + \ln 2^{2x-3}$

$(x-1)\ln 3 = \ln 2 + (2x-3)\ln 2$

$x\ln 3 - \ln 3 = \ln 2 + 2x\ln 2 - 3\ln 2$

$x\ln 3 - \ln 3 = \ln 2 + x\ln 4 - \ln 8$

$x\ln 3 - x\ln 4 = \ln 2 + \ln 3 - \ln 8$

$x(\ln(\frac{3}{4})) = \ln(\frac{6}{8}) = \ln(\frac{3}{4})$

$x = 1$

9. Solve for x in terms of a:

$$\log_b x = 3 - a + \log_b\left(\frac{a^4 b^a}{b^3}\right)$$

$\log_b x - \log_b(\frac{a^4 b^a}{b^3}) = 3 - a$

$\log_b(\frac{x}{\frac{a^4 b^a}{b^3}}) = \log_b(\frac{xb^3}{a^4 b^a}) = 3 - a$

$b^{3-a} = \frac{xb^3}{a^4 b^a}$

$a^4 b^a b^{3-a} = xb^3$

$a^4 b^{(a+3-a)} = xb^3$

$a^4 b^3 = xb^3$

$x = a^4$

10. Solve for x:

$$2x\ln\left(\frac{1}{x}\right) - x = 0$$

$x(2\ln(\frac{1}{x}) - 1) = 0$

$x = 0$, $2\ln(\frac{1}{x}) - 1 = 0$

$2\ln(\frac{1}{x}) = 1$

$\ln(\frac{1}{x}) = \frac{1}{2}$

$e^{\frac{1}{2}} = \frac{1}{x}$

$\sqrt{e} = \frac{1}{x}$

$x = \frac{1}{\sqrt{e}}$

11. A bacteria culture starts with 3000 bacteria. After 3 hours there are 48,000 bacteria present. How long (IN MINUTES) did it take to double?

$48000 = 3000(2)^t$

$16 = 2^t$ -3

$t = \log_2 16 = 4$

$t = 4$ hours $\times 60 = 240$ minutes

$48000 = 3000e^{3b}$

$16 = e^{3b}$

$\ln 16 = 3b$

$b = \frac{\ln 16}{3}$

$t = \frac{3\ln 2}{\ln 16}$

$= \frac{3(\log_{16} 2)}{1} = 3(\frac{1}{4})$

$= 0.75 = 45$ mins

45 mins

$\frac{3\ln 2}{\ln 16}$

$3\ln_{16} 16^{1/4}$

3ln

-4

$3(\frac{1}{4})$

(0.75)

$\ln 2 = t(\frac{\ln 16}{3})$

$6000 = 3000e^{t(\frac{\ln 16}{3})}$

$2 = e^{t(\frac{\ln 16}{3})}$

7. Solve: $(\ln x)^3 = \ln(x^9)$

$\ln x \ln x \ln x = 9 \ln x$

$(\ln x)^4 = 9$

$\ln x = \sqrt[4]{9} = \sqrt[4]{3^2} = \sqrt{3}$

$\boxed{x = e^{\sqrt{3}}}$

8. Solve: $4(3^{x-1}) = 8(2^{2x-3})$

$3^{x-1} = 2(2^{2x-3})$

$\ln 3^{x-1} = \ln 2 + \ln 2^{2x-3}$

$(x-1)\ln 3 = \ln 2 + (2x-3)\ln 2$

$x\ln 3 - \ln 3 = \ln 2 + 2x\ln 2 - 3\ln 2$

$x\ln 3 - \ln 3 = \ln 2 + x\ln 4 - \ln 8$

$x\ln 3 - x\ln 4 = \ln 2 + \ln 3 - \ln 8$

$x(\ln(\frac{3}{4})) = \ln(\frac{6}{8}) = \ln(\frac{3}{4})$

$\boxed{x = 1}$

9. Solve for x in terms of a:

$$\log_b x = 3 - a + \log_b\left(\frac{a^4 b^a}{b^3}\right)$$

$\log_b x - \log_b(\frac{a^4 b^a}{b^3}) = 3 - a$

$\log_b(\frac{x}{\frac{a^4 b^a}{b^3}}) = \log_b(\frac{xb^3}{a^4 b^a}) = 3 - a$

$b^{3-a} = \frac{xb^3}{a^4 b^a}$

$a^4 b^a b^{3-a} = xb^3$

$a^4 b^{(a+3-a)} = xb^3$

$a^4 b^3 = xb^3$

$\boxed{x = a^4}$

10. Solve for x:

$$2x\ln\left(\frac{1}{x}\right) - x = 0$$

$x(2\ln(\frac{1}{x}) - 1) = 0$

$x = 0$, $2\ln(\frac{1}{x}) - 1 = 0$

$2\ln(\frac{1}{x}) = 1$

$\ln(\frac{1}{x}) = \frac{1}{2}$

$e^{\frac{1}{2}} = \frac{1}{x}$

$\sqrt{e} = \frac{1}{x}$

$\boxed{x = \frac{1}{\sqrt{e}}}$

11. A bacteria culture starts with 3000 bacteria. After 3 hours there are 48,000 bacteria present. How long (IN MINUTES) did it take to double?

$48000 = 3000(2)^t$

$16 = 2^t$

$t = \log_2 16 = 4$

$t = 4$ hours $\times 60 = \boxed{240 \text{ minutes}}$

$48000 = 3000e^{3b}$

$16 = e^{3b}$

$\ln 16 = 3b$

$b = \frac{\ln 16}{3}$

$3\ln$

$6000 = 3000e^{t(\frac{\ln 16}{3})}$

$2 = e^{t(\frac{\ln 16}{3})}$

45 mins

$\frac{3\ln 2}{\ln 16}$

$3\ln_{16} 16^{1/4}$

$3(\frac{1}{4})$

(0.75)

$t = \frac{3\ln 2}{\ln 16}$

$= 3(\log_{16} 2) = 3(\frac{1}{4})$

$= 0.75 = 45$ mins

$\ln 2 = t(\frac{\ln 16}{3})$

P=6
A=4

$\frac{8^3}{3^2}=27$ $\frac{2^3}{2^2}=\frac{4}{4}=2$

12. A sample of a radioactive isotope decreases from 6 mg to 4 mg in 30 years. What is the half life period of the isotope? **(Use natural logs to solve)**

$4=6\left(\frac{1}{2}\right)^{30/t}$ $A=Pe^{rt}$

$4=6e^{(\frac{1}{2})t}$ $3=6e^{\frac{1}{2}t}$

$\frac{2}{3}=e^{\frac{1}{2}t}$ $4=6e^{30r}$

$\ln\left(\frac{2}{3}\right)=\frac{1}{2}t$ $\frac{1}{2}=e^{(\frac{1}{2})t}$

$2\ln\left(\frac{2}{3}\right)=t$ $\ln\left(\frac{1}{2}\right)=\frac{1}{2}t$ $\ln\left(\frac{2}{3}\right)=30\ln\frac{1}{2}$

$t=2\ln\left(\frac{1}{2}\right)$

$4=6e^{(30)(2\ln\frac{1}{2})}$ $\frac{2}{3}=e^{30\ln\frac{1}{2}}$

$4=6e^{30\ln(\frac{1}{2})}$

13. Suppose the number of bugs in your house is growing exponentially. If there were 54 bugs after 3 weeks and 486 bugs after 5 weeks, how many bugs were there initially?

$54=P(R)^3$ $54=P\left(\frac{\sqrt{243}}{\sqrt{27}}\right)^3$

$486=P(R)^5$

$P=\frac{54}{\left(\frac{\sqrt{243}}{\sqrt{27}}\right)^3}$

$\frac{54}{R^3}=\frac{486}{R^5}$

$54R^5=486R^3$

$\frac{54}{486}R^2=1$

$R^2=\frac{486}{54}=\frac{243}{27}$

$R=\sqrt{\frac{243}{27}}$

-3

root

14. The population of Florida and Michigan are growing exponentially. If Florida currently has 22 million people and Michigan currently has 10 million people and if the populations of the two states increase annually by 2% and 4%, respectively, when will their populations be the same? **(Use natural logs to solve)**

$A=22(1.02)^t$ $A=22e^{1.027t}$

$A=10(1.04)^t$ ~~$A=10e^{1.047t}$~~

$22e^{1.027t}=10e^{1.047t}$

$2.2e^{1.027t}=e^{1.047t}$

$2.2=\frac{e^{1.047t}}{e^{1.027t}}$

$2.2=e^{0.02t}$

$\ln 2.2=0.02t$

$t=\frac{\ln 2.2}{0.02}$

15. Solve for x:

$$\ln\sqrt{x+4}-\ln\sqrt{x-2}=\ln\sqrt{x+1}$$

$\ln\left(\frac{\sqrt{x+4}}{\sqrt{x-2}}\right)=\ln\sqrt{x+1}$

$\left(\frac{\sqrt{x+4}}{\sqrt{x-2}}\right)^2=\left(\sqrt{x+1}\right)^2$

$\frac{x+4}{x-2}=x+1$

$x+4=(x-2)(x+1)$

$x+4=x^2-2x+x-2$

$x+4=x^2-x-2$

$x^2-2x-6=0$

$x=\frac{2\pm\sqrt{(-2)^2-4(1)(-6)}}{2(1)}=\frac{2\pm\sqrt{4+24}}{2}$

$x=\frac{2\pm\sqrt{28}}{2}=\frac{2\pm2\sqrt{7}}{2}=1\pm\sqrt{7}$

$x=1+\sqrt{7}$

-5

16. Expand:

$$\ln\left[\frac{\sqrt[3]{x-1}\,(3x-2)^4}{(x+1)\sqrt{x-1}}\right]^2$$

$2\ln\sqrt[3]{x-1} + 2\ln(3x-2)^4 - 2\ln(x+1) - 2\ln\sqrt{x-1}$

$2\ln(x-1)^{\frac{1}{3}} + 2\ln(3x-2)^{\frac{1}{4}} - 2\ln(x+1) - 2\ln(x-1)^{\frac{1}{2}}$

$\frac{2}{3}\ln(x-1) + \frac{1}{2}\ln(3x-2) - 2\ln(x+1) - \ln(x-1)$

$-\frac{1}{3}\ln(x-1) + \frac{1}{2}\ln(3x-2) - 2\ln(x+1)$

-2

17. $\frac{1}{2}\log_a(x+3) + \frac{1}{2}\log_a(x-3) = \frac{2}{3}\log_a 125$

$\frac{1}{2}\log_a(x+3) + \frac{1}{2}\log_a(x-3) - \frac{2}{3}\log_a 5 = 0$

$\log_a\left(\frac{\sqrt{x+3}\sqrt{x-3}}{(\sqrt[3]{125})^2}\right) = 0$

$1 = \frac{\sqrt{x+3}\sqrt{x-3}}{25}$

$25 = \sqrt{x+3}\sqrt{x-3}$

$625 = (x+3)(x-3)$

$625 = x^2 - 9$

$x^2 - 634 = 0$

$x = \pm\sqrt{634}$

-1

18. Condense: $\ln\left(x^2+3x+9\right) - \ln\left(x^3-27\right) + \ln(x-3) + \ln(x-2) - \ln\left(x^2-4\right)$

$\ln(x^2+3x+9) - (\ln(x-3) + \ln(x^2+3x+9)) + \ln(x-3) + \ln(x-2) - \ln(x-2) - \ln(x+2)$

$\ln\left(\frac{(x^2+3x+9)(x-3)(x-2)}{(x-3)(x^2+3x+9)(x-2)(x+2)}\right) = \ln\left(\frac{1}{x+2}\right)$

-3