Hon Pre-Calc Test Chapter 11 *2016 - 2017*

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1. Find the equation of the sphere (in center radius form) with endpoints of its diameter at (2, 3, -2) and (6, 5, 8)

Center = $(\frac{6+2}{2}, \frac{3+5}{2}, \frac{-2+8}{2}) = (4, 4, 3)$

Radius = $\frac{1}{2}$ x distance between endpoints = $\frac{1}{2}\sqrt{4^2+2^2+10^2} = \sqrt{30}$

$\therefore (x-4)^2 + (y-4)^2 + (z-3)^2 = 30$

2. Find the exact volume of the sphere given by the equation:

$$9x^2 + 9y^2 + 9z^2 - 18x - 6y - 72z + 73 = 0$$

$$9(x^2-2x) + 9\left(y^2 - \frac{2}{3}y\right) + 9(z^2 - 8z) + 73 = 0$$

$$9(x-1)^2 + 9(y-\frac{1}{3})^2 + 9(z-4)^2 = 81$$

$$(x-1)^2 + (y-\frac{1}{3})^2 + (z-4)^2 = 9 = r^2$$

$\therefore r = 3$

Sphere volume = $\frac{4}{3}\pi 3^3 = 36\pi$

3. Find vector $\vec{z}$, given $\vec{u}$ = <-1, 3, 2>, $\vec{v}$ = <1, -2, -2>, and $\vec{w}$ = <5, 0, -5> if $2\vec{z} - 4\vec{u} = \vec{w}$

$2\vec{z} - 4\vec{u} = \vec{w}$, ➔ $2\vec{z} = \vec{w} + 4\vec{u}$

$\therefore \vec{z} = \frac{1}{2}\vec{w} + 2\vec{u}$

$= \frac{1}{2}$<5, 0, -5> + 2 <-1, 3, 2>

$= <\frac{1}{2}, 6, \frac{3}{2}>$

4. Given: $\vec{u} = 8i + 3j - k$ and $\vec{v} = -3i + 5j + 10k$

a) Find a unit vector in the direction of $\vec{u}$

$$\frac{\vec{u}}{\|\vec{u}\|} = \frac{<8,3,-1>}{\sqrt{8^2+3^2+(-1)^2}} = \frac{<8,3,-1>}{\sqrt{74}} = \frac{\sqrt{74}<8,3,-1>}{74}$$

b) Find $\|\vec{v}\|$

$$\|\vec{v}\| = \sqrt{(-3)^2+5^2+10^2} = \sqrt{134}$$

c) Find $\vec{u} \cdot \vec{v}$

$$\vec{u} \cdot \vec{v} = 8 * (-3) + 3 * 5 + (-1) * 10 = -19$$

d) Find the angle between $\vec{u}$ and $\vec{v}$ in degrees rounded to nearest hundredth.

$$\cos\theta = \frac{|\vec{u} \cdot \vec{v}|}{\|\vec{u}\|\|\vec{v}\|} = \frac{|-19|}{\sqrt{74} * \sqrt{134}} = \frac{19}{\sqrt{9916}}$$

$\therefore \theta \simeq 79.00°$

5. The components of vector $\vec{v}$ and its new initial point are given. Find the new terminal point.

$\vec{v} = <4, \frac{3}{2}, -\frac{1}{4}>$ initial point = $(2, 1, -\frac{3}{2})$

terminal point = (x, y, z)

$\vec{v} = <4, \frac{3}{2}, -\frac{1}{4}>$ = terminal point – initial point

$= (x, y, z) - (2, 1, -\frac{3}{2})$

$\therefore (x, y, z) = (6, \frac{5}{2}, -\frac{7}{4})$

6. Determine if $\vec{u}$ and $\vec{v}$ are parallel, orthogonal, or neither.

a) $\vec{u} = <-1, 3, -1>$, $\quad \vec{v} = <2, -1, 5>$

$\vec{u} \cdot \vec{v} = -2 + (-3) + (-5) = -10$

$\therefore$ neither

b) $\vec{u} = <-2, 3, -1>$, $\quad \vec{v} = <2, 1, -1>$

$\vec{u} \cdot \vec{v} = (-4) + 3 + 1 = 0$

$\therefore$ orthogonal

7. Determine the value of c such that $\|c\vec{u}\| = 12$, where $\vec{u} = -2i + 2j - 4k$

$\vec{u} = < -2, 2, -4 >, \ \therefore c\,\vec{u} = < -2c, 2c, -4c >$

$\therefore \|c\vec{u}\| = \sqrt{4c^2 + 4c^2 + 16c^2} = \sqrt{24c^2} = 12$

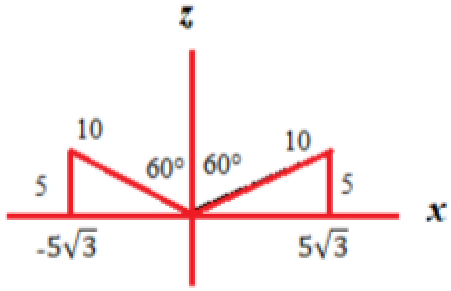
$\therefore 24c^2 = 144$

$c^2 = 6$, $c = \pm\sqrt{6}$

8. Write the **exact** component form of $\vec{v}$ if $\vec{v}$ lies in the *xz* plane, has magnitude 10, and makes an angle of 60° with the positive *z*-axis.

lies in the *xz* plane → $y = 0$

→ only draw *x*, *z* axis



$\therefore \vec{v} = (\pm 5\sqrt{3}, 0, 5)$

9. Find the a unit vector that is orthogonal to both $\vec{u}$ and $\vec{v}$ if $\vec{u} = \boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}$ and $\vec{v} = \boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$

$\vec{u} \times \vec{v} = \begin{vmatrix} i & j & k \\ 1 & 1 & -1 \\ 1 & 1 & 1 \end{vmatrix} = <2, -2, 0>$

unit vector $= \frac{<2,-2,0>}{\sqrt{2^2+(-2)^2}} = <\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, 0>$

10. Find the area of the parallelogram ABCD with the following vertices: A(3, 2, -1), B(-2, 2, -3), C(3, 5, -2), D(-2, 5, -4)

$\overrightarrow{AB} = (-2, 2, -3) - (3, 2, -1) = <-5, 0, -2>$

$\overrightarrow{AC} = (3, 5, -2) - (3, 2, -1) = <0, 3, -1>$

$\|\overrightarrow{AB} \times \overrightarrow{AC}\| = \left\| \begin{vmatrix} i & j & k \\ -5 & 0 & -2 \\ 0 & 3 & -1 \end{vmatrix} \right\|$

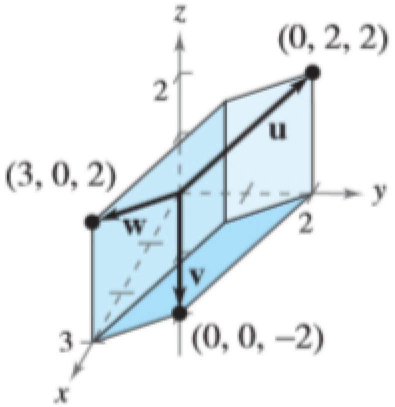
$= \|< 0 - (-6), \ -(5 - 0), \ (-15) - 0 >\|$

$= \|< 6, -5, -15 >\|$

$= \sqrt{6^2 + (-5)^2 + (-15)^2}$

$= \sqrt{286}$

11. Find the volume of the given parallelepiped.



$\vec{u} = < 0, 2, 2 >$
$\vec{v} = < 0, 0, -2 >$
$\vec{w} = < 3, 0, 2 >$
Volume = $|\vec{u} \cdot (\vec{v} \times \vec{w})|$
$= \left| < 0, 2, 2 > \cdot \begin{vmatrix} i & j & k \\ 0 & 0 & -2 \\ 3 & 0 & 2 \end{vmatrix} \right|$
$= |< 0, 2, 2 > \cdot < 0, -6, 0 >| = |-12| = 12$

12. Find a set of parametric equations of the line that…

a) passes through (-3, 8, 15) and (1, -2, 16)

$\vec{a}$ = (1, -2, 16) – (-3, 8, 15) = <4, -10, 1>
$\therefore \begin{cases} x = -3 + 4t \\ y = 8 - 10t \\ z = 15 + t \end{cases}$

b) passes through (2, -3, 5) and is parallel to
$\begin{cases} x = 5 + 2t \\ y = 7 - 3t \\ z = -7 + t \end{cases}$

$\vec{a}$ = <2, -3, 1>
$\therefore \begin{cases} x = 2 + 2t \\ y = -3 - 3t \\ z = 5 + t \end{cases}$

13. Find the general form of the equation of the plane passing through (5, -1, 4), (1, -1, 2), (2, 1, -3)
A B C

$\overrightarrow{AB}$ = <-4, 0, -2>
$\overrightarrow{AC}$ = <-3, 2, -7>
$\overrightarrow{AB} \times \overrightarrow{AC}$ = <0-(-4), -(28-6), -8+0> = <4, -22, -8>

general form of plane equation (use point C)
$= 4(x-2) - 22(y-1) - 8(z+3) = 0$
$\rightarrow 4x - 22y - 8z - 10 = 0$
$\rightarrow 2x - 11y - 4z - 5 = 0$

14. Given: $x + y - z = 0$
$2x - 5y - z = 1$

a) Find the angle in degrees between the following two planes with the given equations:

$\overrightarrow{n_1} = < 1, 1, -1 >, \overrightarrow{n_2} = < 2, -5, -1 >$

$$\cos\theta = \frac{|\overrightarrow{n_1} \cdot \overrightarrow{n_2}|}{\|\overrightarrow{n_1}\|\|\overrightarrow{n_2}\|} = \frac{|2 - 5 + 1|}{\sqrt{3} * \sqrt{30}} = \frac{2}{\sqrt{90}}$$
$\therefore \theta \simeq 77.83°$

b) Find the line of intersection in parametric form. **(NO fractions and solve in terms of Z)**

$\begin{cases} 2x + 2y - 2z = 0 & - ① \\ 2x - 5y - z = 1 & - ② \end{cases}$
① – ②: $7y - z = -1 \rightarrow y = \frac{1}{7}(z - 1)$
sub into $x + y - z = 0$
$\rightarrow x + \frac{1}{7}(z-1) - z = 0$
$\rightarrow x = \frac{1}{7}(6z + 1)$
let $y = t$, $\therefore t = \frac{1}{7}(z - 1) \rightarrow z = 7t + 1$
$x = \frac{1}{7}(6z + 1) = \frac{1}{7}[6(7t + 1) + 1] = 6t + 1$

$\begin{cases} x = 1 + 6t \\ y = t \\ z = 7t + 1 \end{cases}$

15. Find the exact distance between the given point and plane.

(-1, 2, 5) and $2x - 3y + z = 6$
P

$\vec{n}$ = <2, -3, 1>
set Q(0, 0, 6) is on the plane $2x - 3y + z = 6$
$\overrightarrow{PQ}$ = <1, -2, 1>

$$D = \frac{|PQ \cdot n|}{\|n\|} = \frac{|2+6+1|}{\sqrt{1^2+(-2)^2+1^2}} = \frac{9}{\sqrt{14}}$$

$$= \frac{9\sqrt{14}}{14}$$

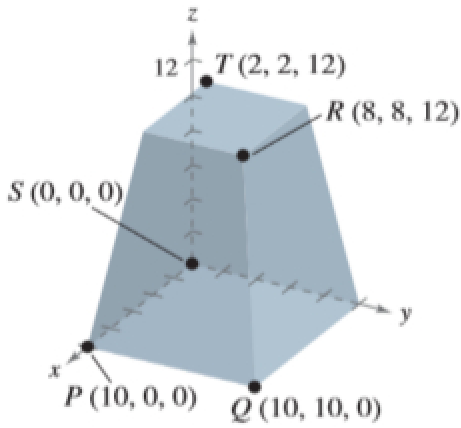
16. Find the general form for the equation of the plane that passes through (2, 2, 1) and (-1, 1, -1) and is perpendicular to the plane with equation $2x - 3y + z = 3$

$\vec{n} = < 2, -3, 1 >$
P(2,2,1), Q(-1,1,-1) → $\overrightarrow{PQ}$ = <-3, -1, -2>

$$\vec{n} \times \overrightarrow{PQ} = \begin{vmatrix} i & j & k \\ 2 & -3 & 1 \\ -3 & -1 & -2 \end{vmatrix}$$
$= (6-(-1))i - (-4-(-3))j + (-2-9)k$
= <7, 1, -11>

use point (2, 2, 1)
$\rightarrow 7(x-2) + 1(y-2) - 11(z-1) = 0$
$\rightarrow 7x + y - 11z - 5 = 0$

17. A tractor fuel tank has the shape and dimensions shown in the figure. In fabricating the tank, it is necessary to know the angle between two adjacent sides. Find this angle in degrees.



$\overrightarrow{ST}$ = <2, 2, 12>
$\overrightarrow{SP}$ = <10, 0, 0>

$$\overrightarrow{n_{TSP}} = \overrightarrow{ST} \times \overrightarrow{SP} = \begin{vmatrix} i & j & k \\ 2 & 2 & 12 \\ 10 & 0 & 0 \end{vmatrix} = <0, 120, -20>$$

$\overrightarrow{QR}$ = <-2, -2, 12>
$\overrightarrow{QP}$ = <0, -10, 0>

$$\overrightarrow{n_{PQR}} = \overrightarrow{QR} \times \overrightarrow{QP} = \begin{vmatrix} i & j & k \\ -2 & -2 & 12 \\ 0 & -10 & 0 \end{vmatrix} = <120, 0, 20>$$

$$\cos\theta = \frac{|<0,120,-20> \cdot <120,0,20>|}{\sqrt{0^2+120^2+(-20)^2}\sqrt{120^2+0^2+20^2}}$$
$$= \frac{|0+0-400|}{14800} = \frac{4}{148} = \frac{1}{37}$$

$\theta \simeq 88.45°$