

# Hon Pre-Calc

## Quiz 9.1 - 9.3

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45/50

Show All Work!!! Circle All Final Answers!!!

### Short Answer

1. Write an expression for the apparent  $n^{\text{th}}$  term of the sequence:

a)  $\frac{1}{1}, \frac{3}{2}, \frac{9}{6}, \frac{27}{24}, \frac{81}{120}, \dots$

$$\frac{3^{n-1}}{(n-1)!}$$

b)  $1, 7, 1, 7, 1, 7, 1, 7, \dots$

$$4 + 3(-1)^n$$

2. Simplify:  $\frac{1000!}{998!6!}$

$$= \frac{1000 \cdot 999 \cdot 998!}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 998!}$$

$$= \boxed{1387.5}$$

3. Use sigma notation to write the sum.

$$\frac{-1}{2} + \frac{1}{3} + \frac{3}{4} + \dots + \frac{27}{16}$$

$$\begin{aligned} -1 + 2n-2 \\ 2n-3 \end{aligned}$$

$$\sum_{i=1}^{15} \frac{2n-3}{i+1}$$

4. Find the  $a_{12}$  term in terms of  $x$  and  $y$  if  $a$  is arithmetic and,  $a_6 = x$ , and  $a_8 = y$

$$\begin{array}{ccccccc} x & & y & & & & \\ a_6 & a_7 & a_8 & a_9 & a_{10} & a_{11} & a_{12} \end{array}$$

$$a_{12} = x + 6d$$

$$a_{12} = y + 4d$$

$$a_{12} = 2x + 12d$$

$$a_{12} = 3y + 12d$$

$$\boxed{a_{12} = 3y - 2x}$$

5. Find the sum of the multiples of 5 from -2025 to 5015.

$$-2025, -2020, \dots, 5015$$

$$5015 = -2025 + (n-1)5$$

$$n = 1409$$

$$S_n = \frac{1409}{2} (-2025 + 5015)$$

$$S_n = \boxed{2,106,455}$$

6. Evaluate:  $\sum_{n=0}^{100} (100 - 2n)$

$$S_n = \frac{101}{2} (100 + (-100))$$

$$S_n = \boxed{0}$$

7. Given the following recursive formula.

$$\begin{cases} a_1 = 384n \\ a_{k+1} = -\frac{1}{2} a_k \end{cases}$$

384n, -192n, 96n, -48n, 24n...

Find  $a_{20}$  in simplest fraction form.

$$a_k = 384n \left(\frac{-1}{2}\right)^{k-1}$$

$$a_{20} = 384n \left(\frac{-1}{2}\right)^{19}$$

$$= \frac{-384n}{524288}$$

$$= \boxed{\frac{-3n}{4096}}$$

8. Find all infinite sums for the geometric sequence if  $g_4 = 4$  and  $g_8 = \frac{1}{4}$ .

4, 2, 1,  $\frac{1}{2}$ ,  $\frac{1}{4}$

$\times 0.5$   $\times 0.5$   $\times 0.5$   $\times 0.5$

$$\sum_{i=1}^{\infty} 4 \left(\frac{1}{2}\right)^{i-1}$$

$$= \frac{4 \cancel{1}}{1 - \frac{1}{2}}$$

$$= \frac{4}{\frac{1}{2}}$$

$$= \boxed{8}$$

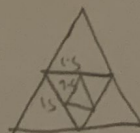
9. Use summation notation to write the sum:

$$\frac{2}{1} - \frac{1}{2} + \frac{1}{8} - \dots + \frac{1}{2048}$$

$$\frac{2}{1} - \frac{1}{2} + \frac{1}{8} - \frac{1}{32} + \frac{1}{128} - \frac{1}{512} + \frac{1}{2048}$$

$$\sum_{i=1}^7 2 \left(-\frac{1}{4}\right)^{i-1}$$

10. A side of an equilateral triangle is 30 inches long. A second equilateral triangle is inscribed in it by joining the midpoints of the sides of the first triangle. The process is continued. Find the perimeter of the 13th inscribed triangle in reduced fraction form.



$$G_n = \left(15 \left(\frac{1}{2}\right)^{n-1}\right) 3$$

$$G_n = \boxed{\frac{45}{4096} \text{ inches}}$$

11. Filled to capacity, a tank contains 20 gallons of pure antifreeze. Two gallons of liquid is drawn out and the tank is filled with water. If this operation is repeated several times, after how many operations will there be less than 0.5 gallons of pure antifreeze left in the tank?

$$20(0.9)^n < 0.5$$

$$\log_{0.9} 20 < \frac{1}{40}$$

$$n > \log_{0.9} \frac{1}{40} + 1$$

$$n > 36.011$$

37 operations



12. Evaluate:  $\sum_{i=0}^{\infty} \left(\frac{1}{10}\right)^i$

$$= \frac{1}{1 - \frac{1}{10}} = \frac{1}{\frac{9}{10}} = \boxed{\frac{10}{9}}$$

13. Alex collected 14 aluminum cans the first day, 20 cans the second day, 26 the third day, and so on, until he had collected a total of 3030 cans.

- a) How many cans did Alex collect in just his second week of collecting?

$\sum_{i=8}^{14} 6i + 8$

$$\Rightarrow S_n = \frac{7}{2}(56 + 92)$$

$$S_n = \boxed{518 \text{ cans}}$$

- b) How many days did he spend total collecting cans?

$$3030 = 14 + (n-1)6$$

$$6n - 6 = 3016$$

$$6n = 3022$$

$$n = 503.6$$

$$\boxed{504 \text{ days}}$$

