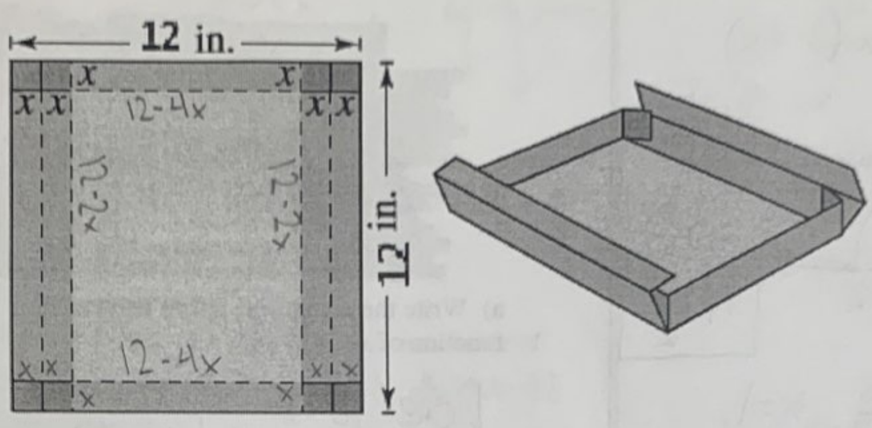
Hon Pre Calculus

Quiz 2.1 - 2.5 Name [redacted]

Show All Work!!! Circle All Final Answers!!! NO Calculators!!

1. An open box with locking tabs is to be made from a square piece of material 12 inches on a side. This is to be done by cutting equal squares from the corners and folding along the dashed lines shown in the figure.



$\frac{46}{50}$

a) Write the function $V(x)$ that represents the volume of the box.

$$\boxed{V(x) = x(12-4x)(12-2x)}$$

b) Determine the domain of the function.

$12-4x>0 \quad 12-2x>0 \quad x>0$

$\frac{-4x}{-4} > \frac{-12}{-4} \quad \frac{-2x}{-2} > \frac{-12}{-2} \quad (0,3)$

$x<3 \quad \cancel{x<6}$

$\boxed{\text{Domain: } (0,3)}$

c) Find the value of x that will maximize the volume.

$V(x) = x(144-48x-24x+8x^2)$

$V(x) = x(144-72x+8x^2)$

$V(x) = 8x^3-72x^2+144x$

$V'(x) = 24x^2-144x+144$

$= 24(x^2-6x+6)$

$x = \frac{6 \pm \sqrt{(-6)^2-4(1)(6)}}{2}$

$x = \frac{6 \pm \sqrt{12}}{2}$

$x = \frac{6 \pm 2\sqrt{3}}{2}$

$x = 3 \pm \sqrt{3}$

$\cancel{3+\sqrt{3}} < 3$

$x = \boxed{3-\sqrt{3} \text{ inches}}$

P= maximum product

X= first #   y = second #

2. Find two positive real numbers with a maximum product such that sum of three times the first and twice the second is 6.

$P = xy \longrightarrow P = x\left(\frac{6-3x}{2}\right)$

$3x + 2y = 6$

$2y = 6 - 3x$

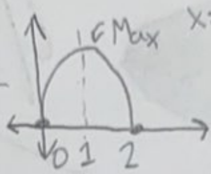
$y = \frac{6-3x}{2}$

$P = x\left(3 - \frac{3}{2}x\right)$

$x = 0 \quad x = \frac{3}{1} \cdot \frac{2}{3}$

$x = 2$

Max



$x = 1$

$3(1) + 2y = 6$

$\cancel{2}y = \frac{3}{\cancel{2}}$

$y = \frac{3}{2} \quad x = 1$

$\boxed{1, \frac{3}{2}}$

3. Find the quotient: $\dfrac{x^{3n} + 9x^{2n} + 27x^n + 27}{x^n + 3}$

$$
\begin{array}{r}
x^{2n} + 6x^n + 9 \\
x^n + 3 \overline{\big) \cancel{x^{3n}} + 9x^{2n} + 27x^n + 27} \\
-\cancel{x^{3n}} \mp 3x^{2n} \qquad\qquad \\
\hline
\cancel{6x^{2n}} + 27x^n \qquad \\
-\cancel{6x^{2n}} \mp 18x^n \qquad \\
\hline
\cancel{9x^n} + \cancel{27} \\
-\cancel{9x^n} \mp \cancel{27} \\
\hline
0
\end{array}
$$

$= \boxed{x^{2n} + 6x^n + 9}$

4. A Norman window is constructed by adjoining a semicircle to the top of an ordinary rectangular window (see - figure). The perimeter of the entire window is 24 feet.

a) Write the area of the entire window as a function of x.

Done on Seperate Sheet

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$\boxed{A = -\frac{1}{2}x^2 - \frac{1}{4}\pi x^2 + \frac{1}{8}\pi x + 12x}$

b) What should x be to maximize area of the window?

Done on Separate Sheet

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$\boxed{x = \frac{-96-\pi}{-8-4\pi}}$

-4

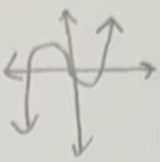
Use limit notation to describe the end behavior of $f(x) = -\left(\frac{1}{3} - \frac{1}{2}x^2 - 4x^5\right)$

Odd

Positive

$f(x) = -\frac{1}{3} + \frac{1}{2}x^2 + 4x^5$

$= 4x^5 + \frac{1}{2}x^2 - \frac{1}{3}$



$$\boxed{\lim_{x \to \infty} f(x) = \infty \qquad \lim_{x \to -\infty} f(x) = -\infty}$$

6. Find the value of k such that $x - 3$ is a factor of $x^3 - kx^2 + 2kx - 12$.

$$\begin{array}{r|rrrr} 3 & 1 & -k & 2k & -12 \\ & \downarrow & 3 & -3k+9 & -3k+27 \\ \hline & 1 & -k+3 & -k+9 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 3 & 1 & -5 & 10 & -12 \\ & \downarrow & 3 & -6 & 12 \\ \hline & 1 & -2 & 4 & 0 \end{array}$$

$-12 + (-3k+27) = 0$

$-3k + 27 = 12$

$-3k = -15$

$k = 5$

$\boxed{k=5}$

7. Consider: $f(x) = x^3 - \frac{1}{4}x^2 - x + \frac{1}{4}$. Find all zeros.

$f(x) = x^2\left(x - \frac{1}{4}\right) - 1\left(x - \frac{1}{4}\right)$

$f(x) = (x^2 - 1)\left(x - \frac{1}{4}\right)$

$f(x) = (x+1)(x-1)\left(x - \frac{1}{4}\right)$

$\boxed{x = -1,\ x = 1,\ x = \frac{1}{4}}$

8. Perform the operation and write the result in $a + bi$ form:

a) $\frac{2i}{2+i} + \frac{5}{2-i}$

$= \frac{5(2+i) + 2i(2-i)}{(2+i)(2-i)}$

$= \frac{10 + 5i + 4i - 2i^2}{4 - i^2}$

$= \frac{9i + 12}{5}$

$\boxed{= \frac{12}{5} + \frac{9}{5}i}$

b) $\sqrt{-6} \bullet \sqrt{-2}$

$\sqrt{-1} \cdot \sqrt{6} \cdot \sqrt{-1} \cdot \sqrt{2}$

$\sqrt{-1} \cdot \sqrt{-1} \cdot \sqrt{12}$

$-1 \cdot \sqrt{12}$

$-1 \cdot \sqrt{4} \cdot \sqrt{3}$

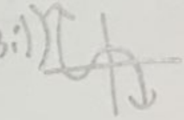
$\boxed{-2\sqrt{3}}$

9. Find a polynomial with real coefficients of least degree with -2 and $1+3i$ as zeros that also has the following behavior: $\begin{cases} \lim_{x \to \infty} f(x) = -\infty \\ \lim_{x \to -\infty} f(x) = +\infty \end{cases}$

odd

negative

$f(x) = -(x+2)(x-(1+3i))(x-(1-3i))$



$= -(x+2)((x-1)-3i)((x-1)+3i)$

$= -(x+2)(x^2 - 2x + 10)$

$= -(x^3 \cancel{-2x^2} + 10x \cancel{+2x^2} - 4x + 20)$

$= -(x^3 + 6x + 20)$

$= \boxed{-x^3 - 6x - 20}$

10. Sketch a possible graph of the following function (Don't worry about y-scale):

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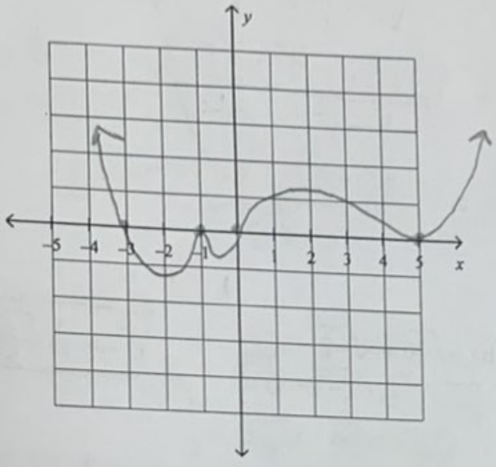
$f(x) = x^7 (x+3)^3 (x+1)^2 (x-5)^4$ even; positive

0 -3 -1 5

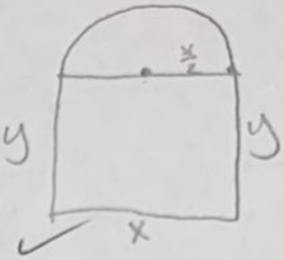
odd odd even even



$P = x + 2y + \frac{1}{2} \cdot \frac{2}{1} \cdot \frac{\pi}{1} \cdot \frac{x}{2}$

$P = x + 2y + \frac{\pi x}{2}$

$A = xy + \frac{1}{2} \cdot \frac{\pi}{1} \cdot \frac{x}{2} \cdot \frac{x}{2}$



$A = xy + \frac{\pi x^2}{8}$

$24 = x + 2y + \frac{\pi x}{2}$

$\frac{48}{2} - \frac{2x}{2} - \frac{\pi x}{2} = 2y$

$\frac{\cancel{2}y}{\cancel{2}} = \frac{48 - 2x - \pi x}{2} \cdot \frac{1}{2}$

$y = \frac{48 - 2x - \pi x}{4}$

$A = \frac{x}{1}\left(\frac{48 - 2x - \pi x}{4}\right) + \frac{\pi x}{8}$

$A = \frac{48x - 2x^2 - \pi x^2}{4} + \frac{\pi x}{8}$

$A = \frac{96x - 4x^2 - 2\pi x^2 + \pi x}{8}$

$A = 12x - \frac{1}{2}x^2 - \frac{1}{4}\pi x^2 + \frac{\pi x}{8}$

$\boxed{A = -\frac{1}{2}x^2 - \frac{1}{4}\pi x^2 + \frac{1}{8}\pi x + 12x}$

b)

$A = -\frac{1}{2}x^2 - \frac{1}{4}\pi x^2 + \frac{1}{8}\pi x + 12x$

$A = -\frac{1}{2}x^2 - \frac{\pi x^2}{4} + \frac{\pi x}{8} + 12x$

$A' = -x - \frac{2\pi x}{4} + \frac{\pi}{8} + 12$

$A' = -x - \frac{1}{2}\pi x + \frac{\pi}{8} + 12$

$-12 - \frac{\pi}{8} = -x\left(1 - \frac{1}{2}\pi\right)$

$x\left(-\frac{2}{2} - \frac{\pi}{2}\right) = -\frac{96}{8} - \frac{\pi}{8}$

$x\left(\frac{-2-\pi}{2}\right) = \frac{-96-\pi}{48} \cdot \frac{2}{-2-\pi}$

$x = \frac{-96+\pi}{+8-4\pi}$

$x = \frac{96+\pi}{8+4\pi}$

$x = \frac{4(24 + \frac{1}{4}\pi)}{4(2+\pi)}$

$x = \frac{\frac{96+\pi}{4}}{2+\pi}$

$x = \frac{96+\pi}{8+4\pi}$

$A' = 12 - x - \frac{1}{2}\pi x + \frac{\pi}{8}$

$\frac{-12 - \frac{\pi}{8}}{-1-\frac{1}{2}\pi} = \frac{x\left(-1-\frac{1}{2}\pi\right)}{-1-\frac{1}{2}\pi}$

$x = \frac{\frac{-96-\pi}{8}}{\frac{-2-\pi}{2}}$ $\quad \frac{-96-\pi}{48} \cdot \frac{2}{-2-\pi}$

$x = \frac{-96-\pi}{-8-4\pi}$