Hon Pre-Calc Test Chapter 12 *2017 - 2018*

Show All Work For FULL Credit!!! Circle All Final Answers!!!!

1. Evaluate the following limit **EXACTLY** (when possible)s:

a) $\lim_{x \to 2} \frac{x-2}{x^2-4}$

b) $\lim_{x \to 0} \frac{\sin \pi x}{x}$

c) $\lim_{x \to 4} \frac{2-\sqrt{x}}{x-4}$

d) $\lim_{x \to 2} \frac{x^4-16}{x+2}$

e) $\lim_{x \to \frac{\pi}{2}} [(1 - \sin x) \sec x]$

f) $\lim_{x \to 0} (1 + 4x)^{\frac{1}{x}}$

g) $\lim_{x \to 1} f(x)$ where $f(x) = \begin{cases} 4 - x^2, x \leq 0 \\ x + 4, x > 0 \end{cases}$

h) $\lim_{x \to 5^+} \frac{5-x}{25-x^2}$

i) $\lim_{x \to 0} \frac{4x}{\sin 4x}$

j) $\lim_{x \to 0} \frac{\frac{1}{x+4} - \frac{1}{4}}{x}$

2. You are given wire and are asked to form a right triangle with a hypotenuse of $\sqrt{22}$ inches whose area is as large as possible.

a) Write a function for the area in terms of *x, the length of a side of the triangle.*

b) What should x be in order to maximize the area?

c) What is the maximum area?

3. Given: $f(x) = \frac{3}{3-x}$ *and* $g(x) = \sin \pi x$ Find

a) $\lim_{x \to 2}[f(x)g(x)]$

b) $\lim_{x \to 2}[g(x) - f(x)]$

4. Find the derivative of the function $f(x) = \sqrt{3x+1}$ using the limit process.

5. Find the equation of all tangent line(s) that are tangent to the graph of $f(x) = 2x^3 - 6x^2$ perpendicular to the line with the equation $6y - x = 14$

6. Use the function and its derivative to determine any **EXACT** points on the graph of f at which the tangent is horizontal.

$f(x) = x \ln x, \qquad f'(x) = 1 + \ln x$

7. a) Use the definition of a derivative to find the derivative function of $f(x) = 3x^3 - 4x - 1 = 0$

b) Use your derivative to find any relative maximums or minimums.

8. Evaluate the following limits at infinity:

a) $\lim_{x \to \infty} \frac{1-2x}{1+3x^2}$

b) $\lim_{x \to \infty} (\frac{1}{2}x - \frac{4}{x^2})$

c) $\lim_{x \to \infty} (\frac{x}{2x+1} + \frac{3x^2}{(2x-3)^2})$

d) $\lim_{x \to \infty} [\frac{x(x+1)}{x^2} - \frac{1}{(2x)^4}\left(\frac{x(x+1)}{2}\right)^2]$

e) $\lim_{x \to \infty} [\frac{(3n-1)!(4n)}{(3n)!}]$

9. Evaluate the sum: $\sum_{n=1}^{10}(n^3 - 3n^2)$

10. Find the approximate area under the curve $f(x) = 6 + x - x^2$ and above the x-axis from -1 to 2, using 20 rectangles.

11. Find the exact area bounded by the function $f(x) = x^3 - x^2 - x$ and the x-axis on the interval [-1, 3]

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Show All Work!!! Circle All Final Answers!!!

Short Answer

1. Evaluate the following limit **EXACTLY** (when possible)s:

a) $\lim_{x \to 2} \frac{x-2}{x^2-4}$ $= \frac{1}{x+2} = \frac{1}{2+2} = \boxed{\frac{1}{4}}$

b) $\lim_{x \to 0} \frac{\sin \pi x}{x}$ $= \pi$

c) $\lim_{x \to 4} \frac{2-\sqrt{x}}{x-4} \cdot \frac{2+\sqrt{x}}{2+\sqrt{x}}$

$\frac{4-x}{(x-4)(2+\sqrt{x})}$

$= \frac{-1}{2+\sqrt{4}} = \boxed{-\frac{1}{4}}$

d) $\lim_{x \to 2} \frac{x^4-16}{x+2}$

$= \frac{2^4-16}{2+2}$

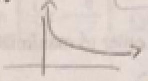
$= \frac{0}{4} = \boxed{0}$

e) $\lim_{x \to \pi/2} \left[(1-\sin x)\sec x\right]$

$= [(1-1)(0)]$

$= \boxed{0}$

f) $\lim_{x \to 0} (1+4x)^{1/x}$ $\boxed{\text{DNE}}$



-2

54.6

g) $\lim_{x \to 1} f(x)$ where $f(x) = \begin{cases} 4-x^2, & x \le 0 \\ x+4, & x > 0 \end{cases}$

$1+4 = \boxed{5}$

h) $\lim_{x \to 5^+} \frac{5-x}{25-x^2}$ $= \frac{5-x}{(5+x)(5-x)} = \frac{1}{5+x} = \boxed{\frac{1}{10}}$

i) $\lim_{x \to 0} \frac{4x}{\sin 4x}$ $= 1$

j) $\lim_{x \to 0} \frac{\frac{1}{x+4} - \frac{1}{4}}{x}$ $= \frac{4-x-4}{4x(x+4)}$

$= \frac{-1}{4(0+4)}$

$= \boxed{\frac{-1}{16}}$

-2

2. You are given wire and are asked to form a right triangle with a hypotenuse of $\sqrt{22}$ inches whose area is as large as possible.

a) Write a function for the area in terms of x, *the length of a side of the triangle.*

$x^2 + y^2 = 22$

$y^2 = 22 - x^2$

$y = \sqrt{22 - x^2}$

$\boxed{A = \frac{1}{2}x\sqrt{22 - x^2}}$

b) What should x be in order to maximize the area?

$x \approx 3.32$ in

c) What is the maximum area?

$A = \frac{1}{2}(3.32)\sqrt{22 - (3.32)^2}$

$= (1.66)(3.31)$

$\approx 5.49\ in^2$

3. Given: $f(x) = \frac{3}{3-x}$ and $g(x) = \sin \pi x$ Find....

$\lim f(x) = 3$ $\lim g(x) = 0$

a) $\lim_{x \to 2} [f(x)g(x)]$

$= (3)(0) = \boxed{0}$

b) $\lim_{x \to 2} [g(x) - f(x)]$

$= 0 - 3 = \boxed{-3}$

4. Find the derivative of the function $f(x) = \sqrt{3x+1}$ using the limit process.

$\frac{f(x+h) - f(x)}{h} = \frac{\sqrt{3(x+h)+1} - \sqrt{3x+1}}{h} \cdot \frac{\sqrt{3(x+h)+1} + \sqrt{3x+1}}{\sqrt{3(x+h)+1} + \sqrt{3x+1}}$

$= \frac{3x + 3h + 1 - 3x - 1}{h(\sqrt{3(x+h)+1} + \sqrt{3x+1})}$

$\lim_{h \to 0} \frac{3}{\sqrt{3(x+h)+1} + \sqrt{3x+1}}$

$= \boxed{\frac{3}{2\sqrt{3x+1}}}$

5. Find the equation of all tangent line(s) that are tangent to the graph of $f(x) = 2x^3 - 6x^2$ perpendicular to the line with the equation $6y - x = 14$. $\frac{1}{6} \to -6$

$f'(x) = 6x^2 - 12x$

$-6 = 6x^2 - 12x$

$x^2 - 2x + 1 = 0$

$(x-1)^2 = 0$

$x = 1$

$f(x) = 2(1)^3 - 6(1)$

$= 2 - 6$

$= -4$

$(1, -4)$

$-4 = -6(1) + b$

$b = 2$

$\boxed{y = -6x + 2}$

6. Use the function and its derivative to determine any **EXACT** points on the graph of f at which the tangent is horizontal.

$f(x) = x \ln x,\ f'(x) = 1 + \ln x$

$f'(x) = 1 + \ln x$

$1 + \ln x = 0$

$\ln x = -1$

$x = \frac{1}{e}$

$f(x) = \frac{1}{e}\ln(\frac{1}{e})$

$= \frac{1}{e} \cdot -1$

$= -\frac{1}{e}$

$(\frac{1}{e}, -\frac{1}{e})$

0

7. a) Use the definition of a derivative to find the derivative function of $f(x) = 3x^3 - 4x - 1 = 0$.

$\lim_{h\to0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to0} \frac{3x^3+9x^2h+9xh^2+3h^3-4x-4h-1-3x^3+4x+1}{h}$

$= \lim_{h\to0} \frac{9x^2h+9xh^2+3h^3-4h}{h}$

$= \lim_{h\to0} 9x^2+9xh+h^2-4$

$\boxed{f'(x) = 9x^2-4}$

b) Use your derivative to find any relative maximums or minimums.

$9x^2-4=0$

$(3x+2)(3x-2)=0$

$x = \frac{2}{3}, -\frac{2}{3}$

$f(x)=3(\frac{2}{3})^3-4(\frac{2}{3})-1$ $\quad$ $f(x)=3(-\frac{2}{3})^3-4(-\frac{2}{3})-1$

$=-\frac{25}{9}$ $\quad$ $=\frac{7}{9}$

$(\frac{2}{3}, -\frac{25}{9})$ $\quad$ $(-\frac{2}{3}, \frac{7}{9})$

min $\quad$ max

8. Evaluate the following limits at infinity:

a) $\lim_{x\to\infty} \frac{1-2x}{1+3x^2} = 0$

b) $\lim_{x\to\infty} \left(\frac{1}{2}x - \frac{4}{x^2}\right)$ DNE

c) $\lim_{x\to\infty} \left[\frac{x}{2x+1} + \frac{3x^2}{(2x-3)^2}\right] = \frac{1}{2} + \frac{3}{4} = \boxed{\frac{5}{4}}$

d) $\lim_{x\to\infty} \left[\frac{x(x+1)}{x^2} - \frac{1}{(2x)^4}\left(\frac{x(x+1)}{2}\right)^2\right]$

$= 1 - \frac{1}{2^4 \cdot 2^2}$

$= 1 - \frac{1}{64}$

$= \boxed{\frac{63}{64}}$

e) $\lim_{x\to\infty} \left[\frac{(3n-1)!(4n)}{(3n)!}\right] = \frac{4n}{3n} = \boxed{\frac{4}{3}}$

9. Evaluate the sum: $\sum_{n=1}^{10}\left(n^3-3n^2\right)$

$$=\frac{10^2(10+1)^2}{4}-3\left(\frac{10(10+1)(2\cdot10+1)}{6}\right)$$

$$=3025-1155$$

$$=\boxed{1870}$$

10. Find the approximate area under the curve $f(x)=6+x-x^2$ and above the x-axis from -1 to 2, using 20 rectangles.

$$\sum_{i=1}^{n}\left(\frac{3}{n}\right)f\left(\frac{3i}{n}-1\right)$$

$$=\frac{3}{n}\sum_{i=1}^{n}\left(6+\frac{3i}{n}-1-\left(\frac{9i^2}{n^2}-\frac{6i}{n}+1\right)\right)$$

$$=\frac{3}{n}\sum_{i=1}^{n}\left(-\frac{9i^2}{n^2}+\frac{9i}{n}+4\right)$$

$$=\left(\frac{3}{n}\right)\left(4n+\frac{9n(n+1)}{2n}-\frac{9n(n+1)(2n+1)}{26n^2}\right)$$

$$=\left(12+\frac{(27)(20)(21)}{2(20)^2}-\frac{9(20)(21)(41)}{2(20)^3}\right)$$

$$=12+14.175-9.68625$$

$$=16.48875 \text{ units}^2$$

11. Find the exact area bounded by the function $f(x)=x^3-x^2-x$ and the x-axis on the interval $[-1,3]$

$$\sum_{i=1}^{n}\left(\frac{4}{n}\right)f\left(\frac{4i}{n}-1\right)$$

$$=\frac{4}{n}\sum_{i=1}^{n}\left(\frac{64i^3}{n^3}-\frac{48i^2}{n^2}+\frac{12i}{n}-1-\frac{16i^2}{n^2}+\frac{8i}{n}-1-\frac{4i}{n}+1\right)$$

$$=\frac{4}{n}\sum_{i=1}^{n}\left(\frac{64i^3}{n^3}-\frac{64i^2}{n^2}+\frac{16i}{n}-1\right)$$

$$=\frac{4}{n}\left(\frac{64n^2(n+1)^2}{4n^3}-\frac{64n(n+1)(2n+1)}{6n^2}+\frac{16n(n+1)}{2}-n\right)$$

$$=64-\frac{256}{3}+32-4$$

$$=\frac{20}{3}\text{ units}^2$$