Hon Pre-Calc

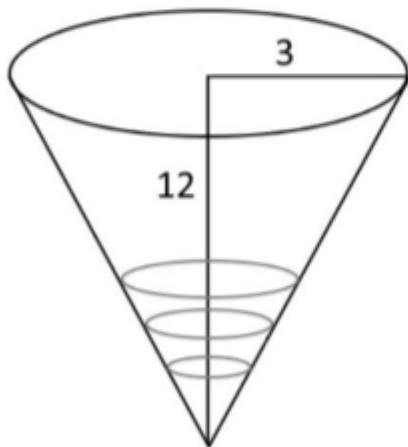Grab Bag – Applications of Differentiation   *2018-2019*

1. At noon, Ship A is 100 km west of ship B. Ship A is sailing south at 35 km/hr and ship B is sailing north at 25 km/hr. How fast is the distance between the ships changing at 4:00 P.M.?

A conical water tower has a height of 12 ft and a radius of 3 ft. Water is pumped into the tank at a rate of 4 ft^3/ min. How fast is the water level rising when the water level is 6 ft?

2. Find the length of the shortest ladder that will reach over a 10 foot high fence to a large wall which is 4 feet behind the fence.

Suppose a large soup company wants to minimize the surface area of a can that is to contain 125 cubic inches. What should the dimensions be?

3. A woman on an island 5 miles from the straight shoreline wishes to reach, as soon as possible, a point 12 miles from the closest point on shore. If she can average 3 miles per hour rowing in a kayak and 6 miles per hour running, at what point on the shore should she land?

A container firm plans to use two different materials to make a rectangular box having a square bottom and a volume of 200 cubic inches. If the cost of the material for the top and the bottom is 8 cents per square inch and the cost of the material for the sides is 4 cents per square inch, find the cost for the most economical dimensions of the box.

$X \ ^{15}/_{15}$

Name _____

Pick ONLY ONE From EACH group!! Show ALL Work!!! Work with ONE Partner!!! Good Luck!!!!

1. Group A (5 points)

   At noon, Ship A is 100 km west of ship B. Ship A is sailing south at 35 km/hr and ship B is sailing north at 25 km/hr. How fast is the distance between the ships changing at 4:00 P.M.?

   A conical water tower has a height of 12 ft and a radius of 3 ft. Water is pumped into the tank at a rate of 4 ft^3 / min. How fast is the water level rising when the water level is 6 ft?

   See the diagram.
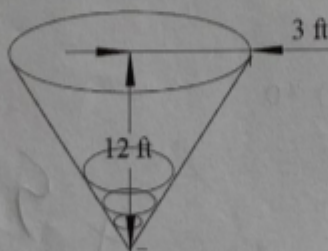
   $h = 12 \quad r = 3$

   $r = \frac{1}{4}h \quad \frac{dr}{dt} = 4$

   $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi (\frac{1}{4}h)^2 h$
   $= \frac{1}{3}\pi (\frac{1}{16}h^2)h$
   $= \frac{1}{48}\pi h^3$

   $\frac{dV}{dt} = \frac{1}{16}\pi h^2 \cdot \frac{dh}{dt}$

   $4 = \frac{1}{16}\pi (6)^2 \cdot \frac{dh}{dt}$

   $X5$

   $\boxed{\frac{dh}{dt} = 0.566 \ ft/min}$

1

2. Group B (5 points)

Find the length of the shortest ladder that will reach over a 10 foot high fence to a large wall which is 4 feet behind the fence.

Suppose a large soup company want to minimize the surface area of a can that is to contain 125 cubic inches. What should the dimensions be?

$V = \pi r^2 h = 125$ $SA = 2\pi r^2 + 2\pi r h$ $SA' = 0$

$h = \dfrac{125}{\pi r^2}$

$SA = 2\pi r^2 + 2\pi r \left(\dfrac{125}{\pi r^2} \right)$

$= 2\pi r^2 + \dfrac{250}{r}$

$SA' = 4\pi r + (-250 r^{-2}) = 0$

$4\pi r + \dfrac{250}{r^2} = 0$

$4\pi r^3 - 250 = 0$

$\pi r^3 = \dfrac{250}{4}$

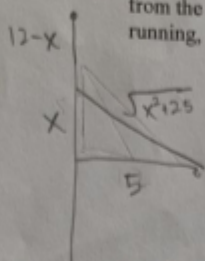$r = \sqrt[3]{\dfrac{250}{4\pi}}$

$h = \dfrac{125}{\pi (2.71)^2} = $

| 2.71 in = r |
| 5.42 in = h |

✗5

3. Group C (5 points)

A woman on an island 5 miles from a straight shoreline wishes to reach, as soon as possible, a point 12 miles from the closest point on shore. If she can average 3 miles per hour rowing in a kayak and 6 miles per hour running, at what point on the shore should she land?

$12-x$

x

$\sqrt{x^2+25}$

5

$t = \frac{d}{r}$

$t = \frac{\sqrt{x^2+25}}{3} + \frac{12-x}{6}$

$t' = \frac{1}{3}(\frac{1}{2})(x^2+25)^{-\frac{1}{2}} \frac{(2x)}{1} + \frac{1}{6}(-1) = 0$

$= \frac{(2x)}{6\sqrt{x^2+25}} - \frac{1}{6} = 0$

$\frac{2x - \sqrt{x^2+25}}{6\sqrt{x^2+25}} = 0$

$2x = \sqrt{x^2+25}$

$4x^2 = x^2 + 25$

$3x^2 = 25$

$x = \frac{5\sqrt{3}}{3}$ mi

$t' = 0$

$\frac{5\sqrt{3}}{3}$ miles from the closest point on the shore

$+ \! \! \int^5$

A container firm plans to use two different materials to make a rectangular box having a square bottom and a volume of 200 cubic inches. If the cost of the material for the top and the bottom is 8 cents per square inch and the cost of the material for the sides is 4 cents per square inch, find the cost for the most economical dimensions of the box.