Hon Pre-Calc Quiz 12.1 – 12.3 *2016 – 2017*

Show All Work For FULL Credit!!! Circle All Final Answers!!!!

1. You create an open box from a square piece of material 12 cm on a side. You cut equal squares from the corners and turn up the sides..

   a) Find a function for the volume of the box in terms of x, where x = length of one side of the corner being cut out.

   b) What should x be in order to get the largest volume?

2. Determine $\lim_{x \to 0} \frac{\sin 4x}{x}$

3. Given: $\lim_{x \to c} f(x) = 6$ and $\lim_{x \to c} g(x) = 25$ Evaluate the following:

   a) $\lim_{x \to c} \frac{3 * f(x)}{\sqrt{g(x)}}$

   b) $\lim_{x \to c} [-2g(x) * f(x)]$

4. Find the following limits exactly:

a) $\lim\limits_{x \to -c} \frac{|x+c|}{x+c}$

b) $\lim\limits_{x \to 0} \frac{e^{3x}-1}{3x}$

c) $\lim\limits_{x \to 1} \cos^{-1} \frac{x}{2}$

d) $\lim\limits_{x \to \frac{5\pi}{6}} tan^2 x$

e) $\lim\limits_{x \to 0} \frac{\sqrt{8-x}-\sqrt{8}}{x}$

f) $\lim\limits_{x \to 2} \frac{\frac{1}{x+1}-\frac{1}{3}}{\frac{2}{x}-1}$

g) $\lim\limits_{x \to \frac{1}{2}} \frac{2x^5-x^4-16x^3+8x^2-18x+9}{2x-1}$

h) $\lim\limits_{x \to 0} \frac{\sin x}{\tan x}$

i) $\lim\limits_{x \to 2} \frac{x^5-32}{x-2}$

j) $\lim\limits_{x \to \frac{\pi}{2}} \frac{1-\sin x}{\cos x}$

5. Evaluate:

a) $\lim_{x \to 9^+} \frac{3-\sqrt{x}}{x-9}$

b) $\lim_{x \to 1} f(x)$ where $f(x) = \begin{cases} 4 - x^2, & x \leq 1 \\ 3 - x, & x > 1 \end{cases}$

c) $\lim_{x \to 2^-} \frac{|x-2|}{x-2}$

6. Find the equation of all the tangent line(s) to the function $f(x) = x^3 - x^2$ that are parallel to the line $x - y = 7$

7. Find the derivative of the following:

a) $f(x) = 5$

b) $f(x) = -7x + 2$

c) $f(x) = \frac{2}{x^2}$

8. Sketch the graph of a function $f(x)$ that satisfy the following conditions: ($f'(x)$ is the derivative of f)

1) $f'(x) < 0$ for $x < -1$
2) $f'(x) < 0$ for $x > 1$
3) $f'(x) > 0$ for $-1 < x < 1$
4) $f'(x) = 0$ for $x = 1$ and $x = -1$

9. Use the function and its derivative to determine any points on the graph of f at which the tangent line is horizontal on the interval $[0, 2\pi)$

$f(x) = 2\cos x + x$
$f'(x) = -2\sin x + 1$

10. Use the difference quotient to find the slope of the tangent line to the function h(x) = $\frac{1}{\sqrt{x+10}}$ at the point (-1, $\frac{1}{3}$)

Hon Pre-Calc

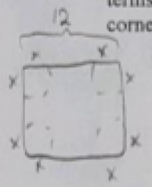
Quiz 12.1 - 12.3 Name 58/65

Show All Work!! Circle All Final Answers!!

Short Answer

1. You create an open box from a square piece of material 12 cm on a side. You cut equal squares from the corners and turn up the sides..

   a) Find a function for the volume of the box in terms of x, where x = the length of one side of the corner being cut out.



$V(x)=x(12-2x)^2$

   b) What should x be in order to get the largest volume?

$V(x)=x(12-2x)^2$

$V(x)=x(144-48x+4x^2)$

$V(x)=4x^3-48x^2+144x$

$V'(x)=12x^2-96x+144$

$(0=12x^2-96x+144)\cdot\frac{1}{12}$

$0=x^2-8x+12$

$0=(x-2)(x-6)$

$x=\cancel{6}$ cm

2

-2

2. Determine $\lim_{x\to 0}\frac{\sin 4x}{x}$ (4)

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(x)$	0.0698	0.0698	0.0698	X	0.0698	0.0698	0.0698

$\lim_{x\to 0}\frac{\sin 4x}{x}=0.0698$

-2

3. Given: $\lim_{x\to c} f(x)=6$ and $\lim_{x\to c} g(x)=25$ Evaluate the following:

   a) $\lim_{x\to c}\frac{3\bullet f(x)}{\sqrt{g(x)}}=\frac{3\cdot 6}{\sqrt{25}}=\frac{18}{5}$

   b) $\lim_{x\to c}[-2g(x)\bullet f(x)]=-2\cdot 25\cdot 6=-300$

-4

4. Find the following limits exactly:

a) $\lim\limits_{x\to -c} \frac{|x+c|}{x+c} = \frac{|-c+c|}{-c+c} = \frac{0}{0}$ ← no limits w/ this type

d.n.e.

b) $\lim\limits_{x\to 0} \frac{e^{3x}-1}{3x} = 1$

x	-0.1	-0.01	-0.001	0	0.001
$f(x)$	0.0086	0.0001	0.0000001	X	0.000001

x	0.01	0.1
$f(x)$	0.0001	0.012

c) $\lim\limits_{x\to 1} \arccos\frac{x}{2} = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

d) $\lim\limits_{x\to 5\pi/6} \tan^2 x = \left(\tan\frac{5\pi}{6}\right)\left(\tan\frac{5\pi}{6}\right) = \frac{\sqrt{3}}{3}\cdot\frac{\sqrt{3}}{3} = \frac{3}{9} = \frac{1}{3}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

e) $\lim\limits_{x\to 0} \frac{\sqrt{8-x}-\sqrt{8}}{x} \cdot \frac{\sqrt{8-x}+\sqrt{8}}{\sqrt{8-x}+\sqrt{8}} = \frac{8-x-8}{x(\sqrt{8-x}+\sqrt{8})} =$

$\frac{-1}{\sqrt{8-x}+\sqrt{8}} = \frac{-1}{\sqrt{8}+\sqrt{8}} = \frac{-1}{2\sqrt{8}}$

f) $\lim\limits_{x\to 2} \frac{\frac{1}{x+1}-\frac{1}{3}}{\frac{2}{x}-1} = \frac{\frac{3-(x+1)}{3(x+1)}}{\frac{2-x}{x}} = \frac{-x+2}{3(x+1)}\cdot\frac{x}{2-x} = \frac{x(2-x)}{3(x+1)(2-x)} =$

$\frac{x}{3(x+1)} = \frac{2}{3(2+1)} = \frac{2}{9}$

g) $\lim\limits_{x\to 1/2} \frac{2x^5-x^4-16x^3+8x^2-18x+9}{2x-1} = \frac{(2x-1)(x^4-8x^2-9)}{2x-1}$

$$
\begin{array}{r}
x^4 - 8x^2 - 9 \\
2x-1\,\overline{)\,2x^5-x^4-16x^3+8x^2-18x+9} \\
-2x^5+x^4 \\
\hline
0 -16x^3+8x^2 \\
+16x^3-8x^2 \\
\hline
0 -18x+9 \\
+18x-9 \\
\hline
0
\end{array}
$$

$\left(\frac{1}{2}\right)^4 - 8\left(\frac{1}{2}\right)^2 - 9 =$

$\frac{1}{16} - 8\cdot\frac{1}{4} - 9 = \frac{1}{16} - 11 =$

$\frac{-176}{16} + \frac{1}{16} = \frac{-175}{16}$

h) $\lim\limits_{x\to 0} \frac{\sin x}{\tan x} = \frac{\sin x}{\frac{\sin x}{\cos x}} = \sin x \cdot \frac{\cos x}{\sin x} = \cos x = \cos 0 = 1$

i) $\lim\limits_{x\to 2} \frac{x^5-32}{x-2} = (2)^4 + 2(2)^3 + 4(2)^2 + 8\cdot 2 + 16 =$

$16+16+16+16+16 = 80$

$$
\begin{array}{r}
x^4 + 2x^3 + 4x^2 + 8x + 16 \\
x-2\,\overline{)\,x^5+0x^4+0x^3+0x^2+0x-32} \\
-x^5+2x^4 \\
\hline
2x^4 \\
-2x^4+4x^3 \\
\hline
4x^3 \\
-4x^3+8x^2 \\
\hline
8x^2 \\
-8x^2+16x \\
\hline
16x-32 \\
-16x+32 \\
\hline
0
\end{array}
$$

j) $\lim\limits_{x\to \pi/2} \frac{1-\sin x}{\cos x} \cdot \frac{1+\sin x}{1+\sin x} = \frac{1-\sin^2 x}{\cos x(1+\sin x)} = \frac{\cos x}{1+\sin x} = \frac{\cos\frac{\pi}{2}}{1+\sin\frac{\pi}{2}} =$

$\cos^2 x$

$\frac{0}{2} = 0$

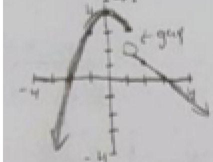
2

5. Evaluate:

a) $\lim_{x \to 9^+} \frac{3-\sqrt{x}}{x-9} \cdot \frac{3+\sqrt{x}}{3+\sqrt{x}} = \frac{9-x}{-1(9-x)(3+\sqrt{x})} = \frac{1}{-(3+\sqrt{x})} =$

$\frac{1}{-(3+\sqrt{9})} = \frac{1}{-(3+3)} = \frac{-1}{6}$

b) $\lim_{x \to 1} f(x)$ where $f(x) = \begin{cases} 4-x^2, & x \le 1 \\ 3-x, & x > 1 \end{cases}$



d.n.e.

c) $\lim_{x \to 2} \frac{|x-2|}{x-2} = \frac{|2-2|}{2-2} = \frac{0}{0}$ ~~d.n.e.~~

-1

2

6. Find the equation of all the tangent line(s) to the function $f(x) = x^3 - x^2$ that are parallel to the line $x - y = 7$.

$f'(x) = 3x^2 - 2x$

$y + 7 = x$

$y = x - 7$

$m = 1$

$1 = 3x^2 - 2x$

$3x^2 - 2x - 1 = 0$

$(3x+1)(x-1) = 0$

$x = \frac{-1}{3}$ or 1

$f(1) = 1^3 - 1^2 = 1 - 1 = 0$

$(1, 0)$

$f(\frac{-1}{3}) = (\frac{-1}{3})^3 - (\frac{-1}{3})^2 =$

$\frac{-1}{27} - \frac{1 \cdot 3}{9 \cdot 3} = \frac{-4}{27}$

$(\frac{-1}{3}, \frac{-4}{27})$

$y - 0 = 1(x-1)$

$y = x - 1$

$y + \frac{4}{27} = 1(x + \frac{1}{3})$

$y = x + \frac{1}{3} - \frac{4}{27}$

$y = x + \frac{5}{27}$

7. Find the derivative of the following:

a) $f(x) = 5$ ← linear is always 0

$f'(x) = 0$

b) $f(x) = -7x + 2 = \frac{f(x+h) - f(x)}{h} = \frac{-7(x+h) + 2 - (-7x+2)}{h} =$

$\frac{-7x - 7h + 2 + 7x - 2}{h} = \frac{-7h}{h} = -7$

c) $f(x) = \frac{2}{x^2} = \frac{f(x+h) - f(x)}{h} = \frac{\frac{2}{(x+h)^2} - \frac{2}{x^2}}{h} = \frac{\frac{2x^2 - 2(x+h)^2}{(x+h)^2 \cdot x^2}}{h} =$

$x^2 + 2xh + h^2$

$\frac{\frac{2x^2 - 2x^2 - 4xh - 2h^2}{(x+h)^2 \cdot x^2}}{h} = \frac{-4xh - 2h^2}{x^2(x^2 + 2xh + h^2)} \cdot \frac{1}{h} = \frac{-2h - 4x}{x^4 + 2x^3h + x^2h^2} =$

let h = 0

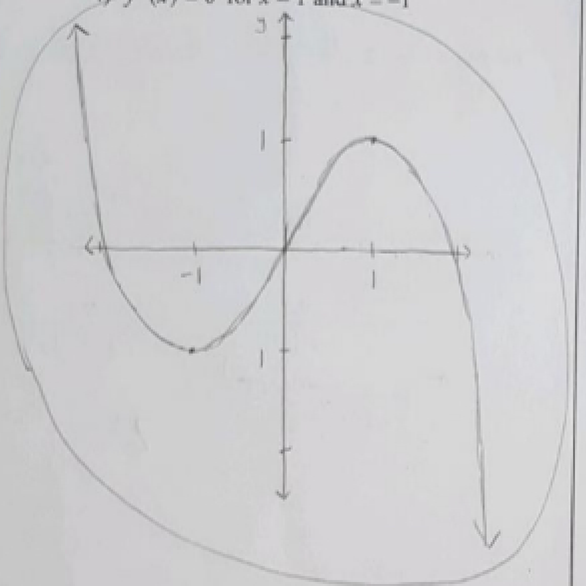
$\frac{-2(0) - 4x}{x^4 + 2x^3(0) + x^2(0)^2} = \frac{-4x}{x^4} = \frac{-4}{x^3}$

2

-4

8. Sketch the graph of a function $f(x)$ that satisfy the following conditions: ($f'(x)$ is the derivative of f)

1) $f'(x) < 0$ for $x < -1$
2) $f'(x) < 0$ for $x > 1$
3) $f'(x) > 0$ for $-1 < x < 1$
4) $f'(x) = 0$ for $x = 1$ and $x = -1$



9. Use the function and its derivative to determine any points on the graph of f at which the tangent line is horizontal on the interval $[0, 2\pi)$.

$f(x) = 2\cos x + x$

$f'(x) = -2\sin x + 1$

$0 = -2\sin x + 1$

$-1 = -2\sin x$

$\sin x = \frac{1}{2}$

$x = \sin^{-1}\left(\frac{1}{2}\right)$

$x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

$\left(\frac{\pi}{6}, \frac{\pi}{6} + \sqrt{3}\right)$

$\left(\frac{5\pi}{6}, \frac{5\pi}{6} + \sqrt{3}\right)$

2

-2

10. Use the difference quotient to find the slope of the tangent line to the function $h(x) = \dfrac{1}{\sqrt{x+10}}$ at the point $\left(-1, \dfrac{1}{3}\right)$

ok

$$h'(x) = \frac{h(x+h)-h(x)}{h} = \frac{\frac{1}{\sqrt{x+h+10}} - \frac{1}{\sqrt{x+10}}}{h} = \frac{\frac{\sqrt{x+10} - \sqrt{x+h+10}}{(\sqrt{x+10}\cdot\sqrt{x+h+10})}}{h} \cdot \frac{\sqrt{x+10}+\sqrt{x+h+10}}{\sqrt{x+10}+\sqrt{x+h+10}}$$

$$= \frac{\frac{x+h-(x+h+10)}{(\sqrt{x+10}\cdot\sqrt{x+h+10})(\sqrt{x+h+10}+\sqrt{x+10})}}{h} \cdot \frac{\sqrt{x+10}\cdot\sqrt{x+h+10}\,(\sqrt{x+h+10}+\sqrt{x+10})}{\sqrt{x+10}\cdot\sqrt{x+h+10}\,(\sqrt{x+h+10}+\sqrt{x+10})} =$$

$$\frac{-10}{(\sqrt{x+10}\cdot\sqrt{x+h+10})(\sqrt{x+h+10}+\sqrt{x+10})\cdot h} \cdot \frac{(\sqrt{x+10}\cdot\sqrt{x+h+10})^2}{(\sqrt{x+10}\cdot\sqrt{x+h+10})^2} =$$

$$\frac{-10(x+10)(x+h+10)}{(x+10)(x+h+10)\cdot\sqrt{x+10}\cdot\sqrt{x+h+10}}$$

$$\frac{\frac{1}{\sqrt{x+h+10}} - \frac{1}{\sqrt{x+10}}}{h} = \frac{\frac{\sqrt{x+10}-\sqrt{x+h+10}}{\sqrt{x+h+10}\cdot\sqrt{x+10}}}{h} \cdot \frac{\sqrt{x+10}\cdot\sqrt{x+h+10}}{\sqrt{x+10}\cdot\sqrt{x+h+10}} =$$

$$\frac{\frac{(x+10)\sqrt{x+h+10} - (x+h+10)\sqrt{x+10}}{(x+10)(x+h+10)}}{h} \cdot \frac{\overset{(x+10)}{\sqrt{x+h+10}} + \overset{(x+h+10)}{\sqrt{x+10}}}{\underset{(x+10)}{\sqrt{x+h+10}} + \underset{(x+h+10)}{\sqrt{x+10}}}$$

$$\frac{(x+10)^2\cancel{(x+h+10)} - (x+h+10)^2\cancel{(x+10)}}{h\cancel{(x+10)}\cancel{(x+h+10)}((x+10)\sqrt{x+h+10} + (x+h+10)\sqrt{x+10})} =$$

$$\frac{-\cancel{h}}{\cancel{h}((x+10)\sqrt{x+h+10} + (x+h+10)\sqrt{x+10})} =$$

$$\frac{-1}{(x+10)\sqrt{x+10} + (x+10)\sqrt{x+10}} = \frac{-1}{2\sqrt{x+10}\cdot(x+10)}$$

$$h'(-1) = \frac{-1}{2\sqrt{-1+10}\,(-1+10)} = \frac{-1}{2\sqrt{9}\cdot 9} = \frac{-1}{18\cdot 3} = \frac{-1}{54}$$