Hon Pre Calculus
Test Chapter 11 Name [redacted]

**Cross Product:** $\vec{u} \times \vec{v} = (u_2v_3 - u_3v_2, u_3v_1 - u_1v_3, u_1v_2 - u_2v_1)$

95/97

**Show All Work!!! Circle All Final Answers!!**

Seperate Sheet

1. Find the volume of the sphere given by the equation:

$2x^2 + 2y^2 + 2z^2 - 2x - 6y - 4z + 5 = 0$

$r^2 = \frac{7}{4}$ $r = \sqrt{\frac{7}{4}}$ $r = \frac{\sqrt{7}}{2}$

$2x^2 - 2x + 2y^2 - 6y + 2z^2 - 4z = -5$

$2(x^2 - 2x + 1) + 2(y^2 - 3y + \frac{9}{4}) + 2(z^2 - 2z + 1) = -5 + \frac{18}{4} + 2 + 2$

$\frac{2(x-1)^2}{2} + \frac{2(y-\frac{3}{2})^2}{2} + \frac{2(z-1)^2}{2} = \frac{7}{2}$ $V = \frac{4}{3}\pi\left(\sqrt{\frac{7}{4}}\right)^3$

$(x-1)^2 + (y-\frac{3}{2})^2 + (z-1)^2 = \frac{7}{4}$

$\boxed{\frac{4}{3}\pi \text{ units}^3}$

2. Find the second endpoint of a segment given A $(-1,5,-3)$ the first endpoint, and M $(1,6,-2)$ the midpoint of the segment. $B(x,y,z)$

$1 = \frac{x + -1}{2}$ $6 = \frac{y+5}{2}$ $-2 = \frac{z+-3}{2}$

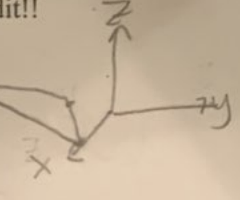
$x = 3$ $y = 7$ $-4 = z - 3$

$-1 = z$

$\boxed{(3, 7, -1)}$

3. Determine if the triangle formed by the following points is right, isosceles, or neither.

A $(1,-2,-1)$, B $(3,0,0)$, C $(3,-6,3)$ ** No work to support your answer = no credit!!



$dAB = \sqrt{4+4+1} = 3$

$dAC = \sqrt{4+16+16} = 6$

$dBC = \sqrt{0+36+9} = \sqrt{45}$

Not Isosceles

$\boxed{\text{Right Triangle}}$

Continued on Seperate Sheet

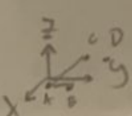
4. Find the EXACT area of a triangle with the given vertices: A $(2,4,0)$, B $(-2,-4,0)$, and C $(0,0,4)$

$\vec{AB} = \langle -4, -8, 0 \rangle$
$\vec{AC} = \langle -2, -4, 4 \rangle$ $\times$ $= \langle -32, 16, 0 \rangle$

$A = \frac{1}{2}\|\vec{AB} \times \vec{AC}\|$ $A = \frac{1}{2}(16\sqrt{5})$

$A = \frac{1}{2}(\sqrt{1280})$ $\boxed{A = 8\sqrt{5} \text{ units}^2}$

5. Find the area of the parallelogram with the given vertices: $A(2,1,1)$, $B(2,3,1)$, $C(-2,4,1)$, and $D(-2,6,1)$



$\vec{AB} = \langle 0, 2, 0 \rangle$
$\vec{AC} = \langle -4, 3, 0 \rangle$

$A = \|\vec{AB} \times \vec{AC}\|$ $A = \sqrt{64}$

$\boxed{A = 8 \text{ units}^2}$

$\vec{AB} \times \vec{AC} = \langle 0, 0, 8 \rangle$

6. Determine the value(s) of $c$ such that $\|c\vec{u}\| = 12$, where $\vec{u} = -2\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$

$\sqrt{144} = \sqrt{(-2c)^2 + (2c)^2 + (-4c)^2}$

$144 = 4c^2 + 4c^2 + 16c^2$

$\frac{144}{24} = \frac{24c^2}{24}$

$\sqrt{c^2} = \sqrt{6}$

$\boxed{c = \pm\sqrt{6}}$

7. Determine whether $\vec{u}$ and $\vec{v}$ are parallel, orthogonal, or neither.

a) $\vec{u} = \langle -1,3,-1 \rangle$ $\quad -2 + -3 + -5 = -10$ Not Orthog

$\vec{v} = \langle 2,-1,5 \rangle$ $\quad \boxed{\text{Neither}}$

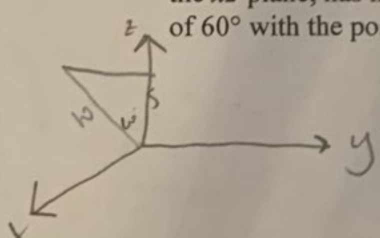
b) $\vec{u} = \langle 2,-3,1 \rangle$ $\quad -2 + 3 + -1 = 0$

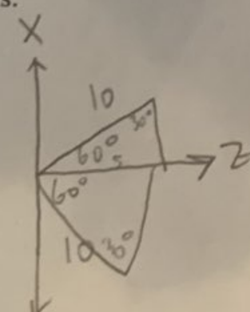
$\vec{v} = \langle -1,-1,-1 \rangle$ $\quad \boxed{\text{Orthogonal}}$

c) $\vec{u} = \langle -12,6,15 \rangle$ $\quad$ Scalars ✓

$\vec{v} = \langle 8,-4,-10 \rangle$ $\quad \boxed{\text{Parallel}}$

8. Find the exact component form of $\vec{v}$. $\vec{v}$ lies in the $xz$ plane, has magnitude 10, and makes an angle of 60° with the positive $z$ axis.




$$\boxed{\langle \pm 5\sqrt{3},\ 0,\ 5 \rangle}$$

$10^{5} \cdot \frac{\sqrt{3}}{2}$

$\pm 5\sqrt{3}$

9. Find a unit vector orthogonal to both $\vec{u}$ and $\vec{v}$ if:

$\vec{u} = \langle 1,-2,2 \rangle$

$\vec{v} = \langle 2,-1,-2 \rangle$

$\vec{u} \times \vec{v} = \langle 6, 6, 3 \rangle$

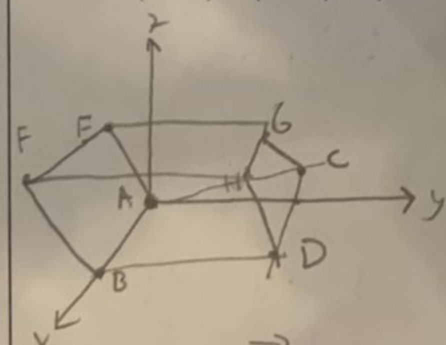
$= \dfrac{\langle 6, 6, 3 \rangle}{\| \vec{u} \times \vec{v} \|}$ $\quad = \sqrt{81} = 9$

$= \boxed{\langle \tfrac{2}{3}, \tfrac{2}{3}, \tfrac{1}{3} \rangle}$

10. Find the volume of the parallelepiped with the given vertices:

$A(0,0,0), B(3,0,0), C(0,5,1), D(3,5,1),$

$E(2,0,5), F(5,0,5), G(2,5,6), H(5,5,6)$




$\vec{AE} = \langle 2, 0, 5 \rangle$

$\vec{AB} = \langle 3, 0, 0 \rangle$

$\vec{AC} = \langle 0, 5, 1 \rangle$

Volume $= \boxed{75 \text{units}^3}$

both $\vec{u}$ and $\vec{v}$ if:

11. Let M be the plane defined by the equation $6x-4y+3z=12$. Find the general equation for the plane N that is parallel to M and passes through $(3,-1,4)$.

$\langle 6 \ \ 3\rangle$

$6x-4y+3z+d=0$

$18+4+12+d=0$

$34+d=0$

$d=-34$

$\boxed{6x-4y+3z-34=0}$

12. Find a set of parametric equations of the line that....

a) passes through (-3, 8, 15) and (1, -2, 16)

A B

$\vec{AB}=\langle 4,-10,1\rangle$

$$\boxed{\begin{aligned} x&=-3+4t\\ y&=8-10t\\ z&=15+t\end{aligned}}$$

b) passes through (2, -3, 5) and is parallel to

$$\begin{cases} x=5+2t\\ y=7-3t\\ z=-7+t\end{cases}$$

$\langle 2,-3,1\rangle$

$$\boxed{\begin{aligned} x&=2+2t\\ y&=-3-3t\\ z&=5+t\end{aligned}}$$

13. Find a set of parametric equations of the line that passes through $(-4,5,2)$ and is perpendicular to $-x+2y+z=5$.

P Q $(0,0,5)$

$\vec{PQ}=\langle 4-5,3\rangle$

$$\boxed{\begin{aligned} x&=-4+4t\\ y&=5-5t\\ z&=2+3t\end{aligned}}$$

$-3$

14. Find the general form of the equation of the plane passing through (5, -1, 4), (1, -1, 2), (2, 1, -3).

A B C

$\vec{AB}=\langle -4,0,-2\rangle$

$\vec{AC}=\langle -3,2,-7\rangle$

$\vec{AB}\times\vec{AC}=\langle 4,-22,-8\rangle$

$4x-22y-8z+d=0$ $\quad 4x-22y-8z-10=0$

$20+22-32+d=0$ $\quad \boxed{2x-11y-4z-5=0}$

$10+d=0$

$d=-10$

15. Find the exact distance between the given point and plane. $\vec{n}=\langle 2,-3,1\rangle$

P $(-1,2,5)$ and $2x-3y+z=6$ $\quad \vec{PQ}=\langle 1,-2,1\rangle$

Q $(0,0,6)$

$D=\dfrac{|\vec{PQ}\cdot n|}{\|n\|}=\dfrac{|2+6+1|}{\sqrt{14}}$

$D=\dfrac{4}{\sqrt{14}}\left(\dfrac{\sqrt{14}}{\sqrt{14}}\right)$ $\quad -1$

1064 $\quad -4$

$D=\dfrac{\cancel{4}^{2}\sqrt{14}}{\cancel{14}\,7}$ $\quad \boxed{D=\dfrac{2\sqrt{14}}{7}}$

16. Given: $3x - 4y + 5z = 6$ A $\langle 3, -4, 5 \rangle$

$x + y - z = 2$ B $\langle 1, 1, -1 \rangle$

a) Find the angle between them.

$$\cos\theta = \frac{|A \cdot B|}{\|A\| \, \|B\|} = \frac{|-6|}{\sqrt{50}\,\sqrt{3}}$$

$\cos^{-1}$ $\cos^{-1}$

$$\boxed{\theta \approx 60.7^\circ}$$

b) Find parametric equations of their line of intersection. (When solving the system, solve in terms of "z" for your general solution (* NO FRACTIONS *))

$3x - 4y + 5z = 6$ $\qquad$ $x + y - z = 2$

$x = z + 2 - y$

$3z - 3y + \cancel{6} - 4y + 5z = \cancel{6}$

$-7y + 8z = 0$ $\qquad$ $x = z + 2 - \frac{8}{7}z$

$-8z$

$\frac{-\cancel{7}y}{\cancel{-7}} = \frac{-8z}{-7}$

$y = \frac{8z}{7}$

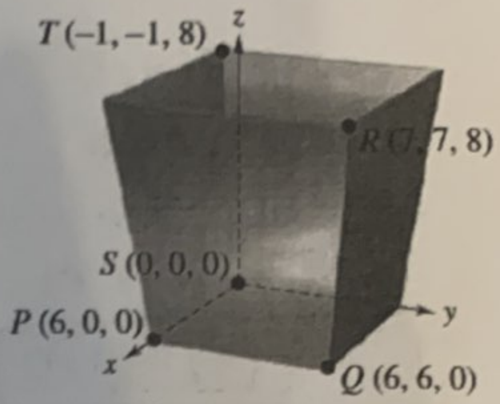
$$\begin{cases} x = -\frac{1}{7}z + 2 \\ y = \frac{8}{7}z \\ z = t \end{cases}$$

$\times 7$

-1

$$\boxed{\begin{aligned} x &= -t + \cancel{14} \\ y &= 8t \\ z &= 7t \end{aligned}}$$

17. A chute at the top of a grain elevator of a combine funnels the grain into a bin, as shown in the figure. Find the angle between two adjacent sides.




$\vec{ST} = \langle -1, -1, 8 \rangle$

$\vec{SP} = \langle 6, 0, 0 \rangle$

A

$\vec{ST} \times \vec{SP} = \langle 0, 48, 6 \rangle$

B

$\vec{QR} = \langle 1, 1, 8 \rangle$ $\qquad$ $\vec{QR} \times \vec{QP} = \langle 48, 0, -6 \rangle$

$\vec{QP} = \langle 0, -6, 0 \rangle$

$\angle$ between planes

$$\cos\theta = \frac{|-36|}{\sqrt{2340}\,\sqrt{2340}}$$

$\cos^{-1}$ $\cos^{-1}$

$$\boxed{\theta \approx 89.11^\circ}$$

-1

$$2x^2 - 2x + 2y^2 - 6y + 2z^2 - 4z = -5$$

$$2\left(x^2 - x + \tfrac{1}{4}\right) + 2\left(y^2 - 3y + \tfrac{9}{4}\right) + 2\left(z^2 - 2z + 1\right) = \cancel{-5} + 2 + \cancel{\tfrac{18}{4} + \tfrac{2}{4}}$$

$$2\left(x - \tfrac{1}{2}\right)^2 + 2\left(y - \tfrac{3}{2}\right)^2 + 2(z-1)^2 = 2$$

$$\left(x - \tfrac{1}{2}\right)^2 + \left(y - \tfrac{3}{2}\right)^2 + (z-1)^2 = 1 \qquad r = 1$$

$$= \tfrac{4}{3}\pi 1^3$$

$$= \boxed{\tfrac{4}{3}\pi \text{ units}^3}$$

3.

$\vec{AB} = \langle 2, 2, 1 \rangle$

$\vec{AC} = \langle 2, -4, 4 \rangle$

$= 4 + -8 + 4$

$= \boxed{0}$

Orthogonal

$\boxed{\text{Right Triangle}}$

6.

$\|\vec{u}\| = \sqrt{4 + 4 + 16}$

$= \sqrt{24}$

9.

$\vec{u} = \langle 1, -2, 2 \rangle$

$\vec{v} = \langle 2, -1, -2 \rangle$

$\vec{u} \times \vec{v} = \langle 6, 6, 3 \rangle$