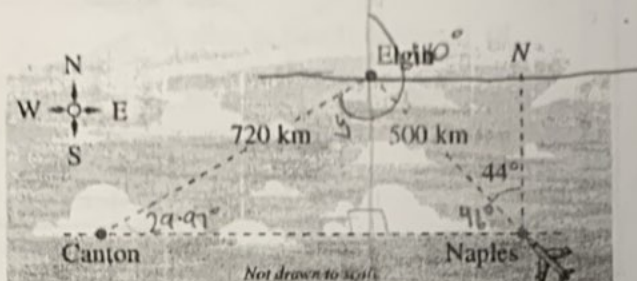


Only Scientific Calculators Allowed!! Show All Work!! Circle All Final Answers!!

80

Short Answer

1. A plane flies 500 kilometers with a bearing of 316° from Naples to Elgin (see figure). The plane then flies 720 kilometers from Elgin to Canton. (Canton is due west of Naples). Find the bearing of the flight from Elgin to Canton.



$$\frac{\sin 46}{720} = \frac{\sin x}{500}$$

$$x = 29.97^\circ$$

$$y = 90 + 29.96 = 119.96$$

$$y = 60.03^\circ$$

$$\text{Bearing} = 60.03^\circ + 180^\circ$$

Bearing = 240.03°

Polar Form

Division $\Rightarrow \frac{[r_1, \theta_1]}{[r_2, \theta_2]} \Rightarrow \left[\frac{r_1}{r_2}, \theta_1 - \theta_2 \right]$

Multiplication $\Rightarrow [r_1, \theta_1][r_2, \theta_2] \Rightarrow [r_1 r_2, \theta_1 + \theta_2]$

Bearing = 240.03°

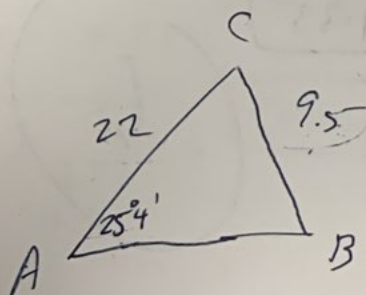
2. Find the area of the triangle having the indicated angle and sides.

$A = 5^\circ 15'$, $b = 4.5$, $c = 22$
 5.25°

$A = \frac{1}{2} bc \sin A$

$A = \frac{1}{2} \times 4.5 \times 22 \times \sin 5.25^\circ$

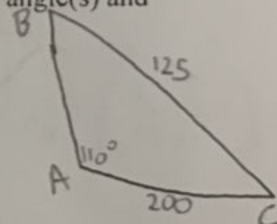
$A = 4.53$



3. Solve the triangles with the given angle(s) and side(s).

a) $A = 110^\circ$, $a = 125$, $b = 200$

Not a Possible Triangle



b) $A = 25^\circ 4'$, $a = 9.5$, $b = 22$

Two Triangles 21.23

$\frac{\sin B}{22} = \frac{\sin 25^\circ 4'}{9.5}$

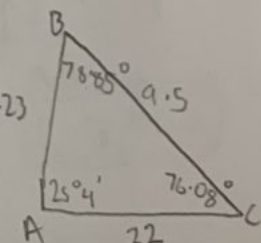
$B = 78.85^\circ$

$C = 76.08^\circ$

or

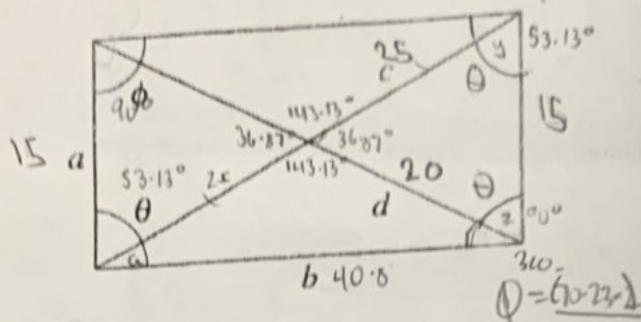
$\frac{C}{\sin 76.08} = \frac{9.5}{\sin 25^\circ 4'}$

$C = 21.23$



$B \approx 101.15^\circ$, $C \approx 53.78^\circ$, $c \approx 18.09$

4. Given $a = 15$, $c = 25$, and $d = 20$, Find the values of b , θ , and ϕ . (The figure below is a parallelogram. please show work)



$$20^2 = 25^2 + 15^2 - 2(15)(25)\cos\theta$$

$$\theta = 53.13^\circ$$

$$25^2 = 20^2 + 15^2 - 2(20)(15)\cos\phi$$

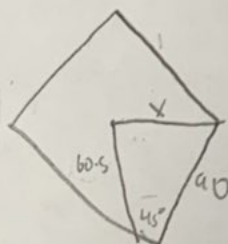
$$\phi = 90^\circ$$

$$b^2 = 25^2 + 20^2 - 2(25)(20)\cos(143.13^\circ)$$

$$b = 40.8$$

$$\phi = 109.77^\circ$$

5. A baseball diamond is a square with 90 foot sides. The pitcher's mound is 60.5 feet from home plate. How far is the pitcher's mound from 1st base?



$$x^2 = 60.5^2 + 90^2 - 2(60.5)(90)\cos 45^\circ$$

$$x = 63.72 \text{ feet}$$

6. Given a vector with initial point (9,3) and terminal point (1,11), find:

$$\text{Vector} = \langle 1-9, 11-3 \rangle$$

$$= \langle -8, 8 \rangle$$

$$\begin{aligned} \text{a) Magnitude (exactly)} &= \sqrt{64+64} \\ &= \sqrt{128} \\ &= \sqrt{64 \cdot 2} \\ &= 8\sqrt{2} \end{aligned}$$

- b) Direction (Exact in radians)

$$\tan^{-1}\left(\frac{8}{-8}\right) = \tan^{-1}(-1)$$

$$= \frac{3\pi}{4}, \frac{5\pi}{4}$$

7. Find the component form of the sum for the following two vectors in polar form.

$$\vec{u} = [50, 30^\circ], \text{ and } \vec{v} = [30, 110^\circ]$$

$$\vec{u} = \langle 50\cos 30^\circ, 50\sin 30^\circ \rangle \quad \vec{v} = \langle 30\cos 110^\circ, 30\sin 110^\circ \rangle$$

$$\vec{u} + \vec{v} = \langle 33.04, 53.19 \rangle$$

8. Three forces with magnitudes of 75 pounds, 100 pounds, and 125 pounds act on an object at angles of 30° , 45° , and 120° , respectively, with the positive x-axis. Find the direction and magnitude of the resultant of these three forces.

- a) Magnitude =

$$\begin{aligned} x &= 75\cos 30^\circ + 100\cos 45^\circ + 125\cos 120^\circ \\ &= 73.1625834 \end{aligned}$$

$$228.49$$

$$\begin{aligned} y &= 75\sin 30^\circ + 100\sin 45^\circ + 125\sin 120^\circ \\ &= 216.4638536 \end{aligned}$$

- b) Direction =

$$71.33^\circ$$

$$\tan^{-1}\left(\frac{y}{x}\right)$$

Use the vectors $\vec{u} = \langle 3, 3 \rangle$, $\vec{v} = \langle -4, 2 \rangle$, and $\vec{w} = \langle 3, -1 \rangle$ to find the indicated quantity.

\checkmark Dot product

$$= (3\vec{w} \cdot \vec{v}) \vec{u}$$

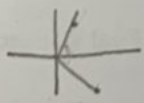
$$= (\langle 9, -3 \rangle \cdot \langle -4, 2 \rangle) \vec{u} \quad \langle -126, -126 \rangle$$

$$= (\langle -36, -6 \rangle) \langle 3, 3 \rangle$$

$$= \boxed{\langle -108, -18 \rangle}$$

10. Find the angle (in degrees) between the vectors.

$$\vec{u} = 3\mathbf{i} - 5\mathbf{j} \quad \langle 3, -5 \rangle$$

$$\vec{v} = \mathbf{i} + 2\mathbf{j} \quad \langle 1, 2 \rangle$$


$$\theta = 59.036^\circ$$

$$\theta = 63.43^\circ = \boxed{122.47^\circ}$$

11. Determine whether \vec{u} and \vec{v} are parallel, orthogonal, or neither.

a) $\vec{u} = \langle -12, 30 \rangle \quad \text{slope } \vec{u} = \frac{30}{-12} = -\frac{5}{2}$

$\vec{v} = \langle \frac{1}{2}, -\frac{5}{4} \rangle \quad \text{slope } \vec{v} = \frac{-5/4}{1/2} = -\frac{5}{2}$

Parallel

b) $\vec{u} = 2\mathbf{i} - 2\mathbf{j}$

$\vec{v} = -\mathbf{i} - \mathbf{j}$

Orthogonal

$$\vec{u} \cdot \vec{v} = -2 + 2 = 0$$

12. Find the exact terminal point of a vector with magnitude of 6, that has the same direction as $\langle -2, 2\sqrt{3} \rangle$, and has an initial point of $(-3, 2)$.

$$\tan^{-1}\left(\frac{2\sqrt{3}}{-2}\right) = 300^\circ$$

$$\langle 6\cos 300, 6\sin 300 \rangle = \langle x - (-3), y - 2 \rangle$$

$$y - 2 = \frac{3\sqrt{3}}{2}$$

$$y = 2 + 3\sqrt{3}$$

$$x + 3 = 3$$

$$x = -6$$

$$\boxed{(-6, 2 + 3\sqrt{3})}$$

13. Find the complex form of the following number:

$$5(\cos 198^\circ 45' + i \sin 198^\circ 45')$$

$$\boxed{-4.74 - 1.59i}$$

14. Perform the indicated operations. Leave answers in polar form.

Division $\frac{r_1}{r_2}, \theta_1 - \theta_2$

a) $\frac{[5, 4.3]}{[4, 2.1]} = [5-4, 4.3-2.1]$

$$= \boxed{[1, 2.2]} = \left[\frac{5}{4}, 2.2\right]$$

b) $\left[\frac{3}{4}, \frac{\pi}{3}\right] \left[4, \frac{3\pi}{4}\right]$

$$\left[3, \frac{13\pi}{12}\right]$$

Multiplication $r_1 \cdot r_2, \theta_1 + \theta_2$

$$\left[\frac{3}{4} + 4, \frac{\pi}{3} + \frac{3\pi}{4}\right]$$

$$\boxed{\left[\frac{19}{4}, \frac{13\pi}{12}\right]}$$

$$\boxed{-4}$$

15. Perform the indicated operation. Leave answer in trig form.

$$\frac{1 + \sqrt{3}i}{6 - 3i} \quad \begin{matrix} \text{mag} = 2 \\ \theta = 60^\circ \end{matrix} \quad \Rightarrow \quad \frac{[2, 60^\circ]}{[3\sqrt{3}, 333.434848^\circ]}$$

$$= [2 \cdot 3\sqrt{3}, 86.565^\circ]$$

$$2 \cdot 3\sqrt{3} (\cos 86.565^\circ + i \sin 86.565^\circ)$$

2
us

16. Write the indicated power in standard complex form.

$$(1 - \sqrt{3}i)^6 \quad \begin{matrix} \text{mag} = 2 \\ \theta = 300^\circ \end{matrix}$$

$$2 [2, 300^\circ]^6$$

$$[64, 1800^\circ]$$

$$64 \cos 1800^\circ + i 64 \sin 1800^\circ$$

$$= 64 + 0i$$

$$= 64 + i$$

$$= \boxed{64}$$

3

17. Find the 3 cube roots of $-8i$. Leave answers in polar form with θ in degrees.

$$-8i = [8, 270^\circ]^{\frac{1}{3}}$$

$$\begin{aligned} &= [2, 90^\circ] \\ &= [2, 210^\circ] \\ &= [2, 330^\circ] \end{aligned}$$

18. Find all exact solutions in standard complex form.

$$x^4 - 81i = 0 \quad x^4 = [81, 270^\circ]^{\frac{1}{4}} \quad \frac{360}{4} = 90^\circ$$

$$x^4 = 81i$$

$$x = [3, 67.5^\circ] = \frac{3\sqrt{2+\sqrt{2}}}{2} + i \frac{3\sqrt{2-\sqrt{2}}}{2}$$

$$x = [3, 157.5^\circ] = -\frac{3\sqrt{2+\sqrt{2}}}{2} + i \frac{3\sqrt{2-\sqrt{2}}}{2}$$

$$x = [3, 247.5^\circ] = -\frac{3\sqrt{2+\sqrt{2}}}{2} - i \frac{3\sqrt{2-\sqrt{2}}}{2}$$

$$x = [3, 337.5^\circ] = \frac{3\sqrt{2+\sqrt{2}}}{2} - i \frac{3\sqrt{2-\sqrt{2}}}{2}$$

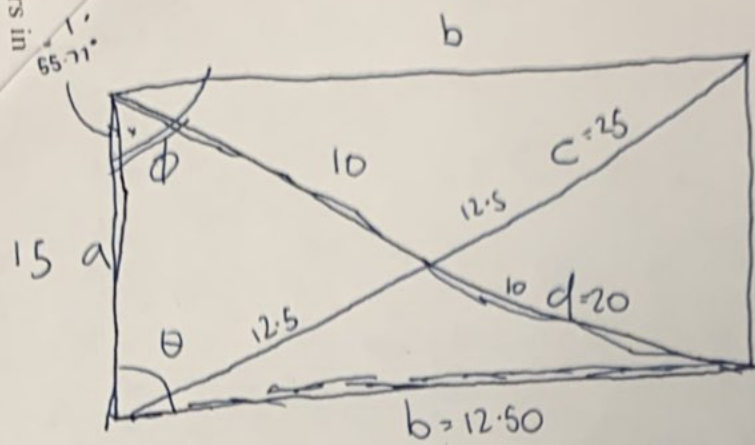
$$\sin 67.5^\circ = \pm \sqrt{\frac{1 - \frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{2 - \sqrt{2}}{4}} = \frac{\sqrt{2 - \sqrt{2}}}{2}$$

$$\cos 67.5^\circ = \pm \sqrt{\frac{1 + \frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{2 + \sqrt{2}}{4}} = \frac{\sqrt{2 + \sqrt{2}}}{2}$$

$$\sin 157.5^\circ = \pm \sqrt{\frac{1 - \frac{\sqrt{2}}{2}}{2}} = \frac{\sqrt{2 - \sqrt{2}}}{2}$$

$$\cos 157.5^\circ = \pm \sqrt{\frac{1 + \frac{\sqrt{2}}{2}}{2}} = \frac{\sqrt{2 + \sqrt{2}}}{2}$$

Answers in
60 = 120
5



c and d are the
entire diagonal

Diagonals bisect each other
in a parallelogram

~~sin 55.77~~

$$b^2 = 15^2 + 10^2 - 2(15)(10)\cos 55.77^\circ$$

$$b = 12.50$$

b =

$$12.5^2 = 15^2 + 10^2 - 2(15)(10)\cos x$$

$$-168.75 = -2(15)(10)\cos x$$

~~12.50~~

$$\cos x = 0.5625$$

$$x = 55.77^\circ$$

18.

$$x^4 - 81i = 0$$

$$x^4 = 81i \quad (0, 81)$$

$$\frac{360}{4} = 90^\circ$$

$$\cos 22.5^\circ = \sqrt{\frac{1 + \frac{\sqrt{2}}{2}}{2}} = \frac{\sqrt{2 + \sqrt{2}}}{2}$$

$$\sin 22.5^\circ = \sqrt{\frac{1 - \frac{\sqrt{2}}{2}}{2}} = \frac{\sqrt{2 - \sqrt{2}}}{2}$$

$$[81, 90^\circ] = [3, 22.5^\circ]$$

$$= \frac{3\sqrt{2+\sqrt{2}}}{2} + \frac{3\sqrt{2-\sqrt{2}}}{2}i \quad (-y, x) \leftarrow \text{rotate } 90^\circ$$

$$= -\frac{3\sqrt{2-\sqrt{2}}}{2} + \frac{3\sqrt{2+\sqrt{2}}}{2}i$$

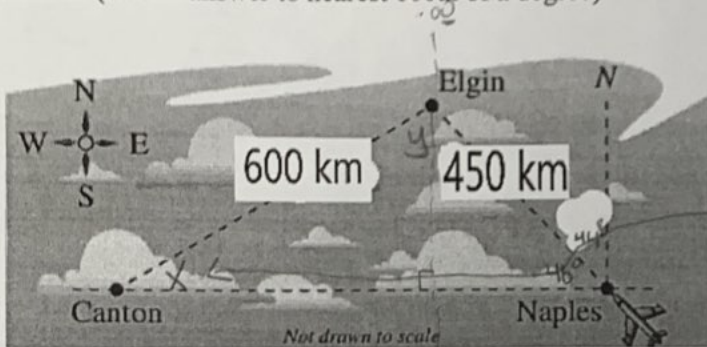
$$= -\frac{3\sqrt{2+\sqrt{2}}}{2} - \frac{3\sqrt{2-\sqrt{2}}}{2}i$$

$$= \frac{3\sqrt{2-\sqrt{2}}}{2} - \frac{3\sqrt{2+\sqrt{2}}}{2}i$$

Only Scientific Calculators Allowed!! Show All Work!! Circle All Final Answers!!

Short Answer

1. A plane flies 450 kilometers with a bearing of 316° from Naples to Elgin (see figure). The plane then flies 600 kilometers from Elgin to Canton (Canton is due west of Naples). Find the bearing of the flight from Elgin to Canton. (Round answer to nearest 100th of a degree)



$$\frac{\sin x}{450} \times \frac{\sin 46}{600}$$

$$\sin x = 0.53950485$$

$$x = 32.64993822^\circ$$

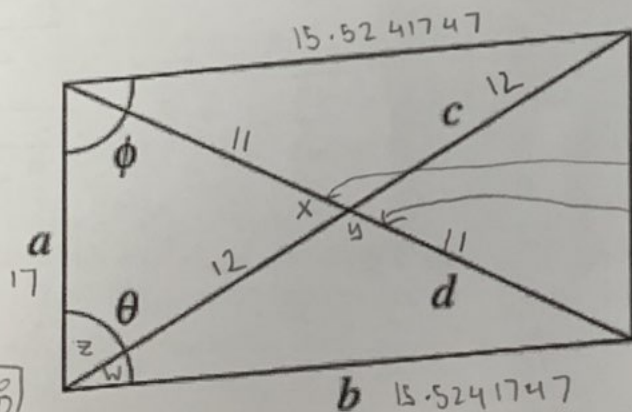
$$y = 180 - (90 + x)$$

$$y = 57.35006178$$

$$\text{Bearing} = 180 + y$$

$$= \boxed{237.35^\circ}$$

2. Given $a = 17$, $c = 24$, and $d = 22$, Find the values of b , θ , and ϕ . (The figure below is a parallelogram). (Round all answers to nearest tenth)



$$17^2 = 11^2 + 12^2 - 2(11)(12)\cos x$$

$$\cos x = -0.090909091$$

$$x = 95.21540857^\circ$$

$$y = 84.78409143$$

$$b^2 = (12)^2 + (11)^2 - 2(12)(11)\cos 84.78409143$$

$$b = \sqrt{5.5241747}$$

$$\theta = z + w$$

$$\theta = 85.0029563^\circ$$

$$\phi = 180 - 85.0029563^\circ$$

$$\phi = 94.9970437^\circ$$

$$\phi = 95^\circ$$

$$\frac{\sin 95.21540857}{17} = \frac{\sin z}{11}$$

$$z = 40.1191669^\circ$$

$$\frac{\sin w}{11} = \frac{\sin 84.78409143}{15.5241747}$$

$$w = 44.88112873^\circ$$

3. Three forces with magnitudes of 80 pounds, 120 pounds, and 166 pounds act on an object at angles of 60° , 45° , and 120° , respectively, with the positive x-axis. Find the direction and magnitude of the resultant of these three forces. (Round all answers to nearest tenth)

$$\text{Mag} = \sqrt{x^2 + y^2}$$

a) Magnitude =

$$x = 80 \cos 60^\circ + 120 \cos 45^\circ + 166 \cos 120^\circ$$

$$\approx 41.85281374$$

$$= 300.8$$

$$y = 80 \sin 60^\circ + 120 \sin 45^\circ + 166 \sin 120^\circ$$

$$\approx 297.8950631$$

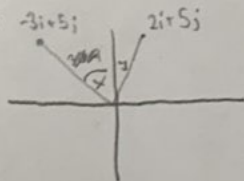
b) Direction =

$$\tan^{-1} \left(\frac{y}{x} \right) = 82.0^\circ$$

4. Find the angle (in degrees rounded to nearest 100th) between the vectors.

$$\vec{u} = -3\mathbf{i} + 5\mathbf{j}$$

$$\vec{v} = 2\mathbf{i} + 5\mathbf{j}$$



$$\cos \theta = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \cdot \|\vec{v}\|}$$

$$x = 30.96375693$$

$$y = 21.80140949$$

$$\theta = 52.77^\circ$$

$$\cos \theta = \frac{19}{\sqrt{34} \cdot \sqrt{29}}$$

$$\theta = 52.77^\circ$$

5. Find all exact solutions in standard complex form.

$$x^4 - 625i = 0$$

$$\tan^{-1} \left(\frac{-1}{5} \right) = 270^\circ$$

$$\left[5, 90^\circ \right]^{\frac{1}{4}}$$

$$\frac{360^\circ}{4} = 90^\circ$$

$$\begin{aligned} [5, 22.5^\circ] &= \frac{5\sqrt{2+\sqrt{2}}}{2} + i \frac{5\sqrt{2-\sqrt{2}}}{2} \\ [5, 112.5^\circ] &= -\frac{5\sqrt{2-\sqrt{2}}}{2} + i \frac{5\sqrt{2+\sqrt{2}}}{2} \\ [5, 202.5^\circ] &= -\frac{5\sqrt{2+\sqrt{2}}}{2} - i \frac{5\sqrt{2-\sqrt{2}}}{2} \\ [5, 292.5^\circ] &= \frac{5\sqrt{2-\sqrt{2}}}{2} - i \frac{5\sqrt{2+\sqrt{2}}}{2} \end{aligned}$$

$$\cos 22.5^\circ = \sqrt{\frac{1 + (\frac{\sqrt{2}}{2})}{2}} = \sqrt{\frac{2+\sqrt{2}}{4}} = \frac{\sqrt{2+\sqrt{2}}}{2}$$

$$\sin 22.5^\circ = \sqrt{\frac{1 - (\frac{\sqrt{2}}{2})}{2}} = \sqrt{\frac{2-\sqrt{2}}{4}} = \frac{\sqrt{2-\sqrt{2}}}{2}$$

