

Honors Pre-Calc
Test Chapter 12: 2016-2017

Short Answer

1. Evaluate the following limits. Give an exact answer when possible.

a) $\lim_{x \rightarrow 0} \frac{3x}{\tan 5x}$

b) $\lim_{x \rightarrow 1} \frac{\ln(6x-5)}{x-1}$

c) $\lim_{x \rightarrow \sqrt{3}} \operatorname{arccot}(-x)$

d) $\lim_{x \rightarrow 2} \frac{x^4 - 16}{x - 2}$

e) $\lim_{x \rightarrow -5} \frac{\sqrt{x+9} - 2}{x+5}$

f) $\lim_{x \rightarrow -4} \frac{x^3 + 64}{x + 4}$

g) $\lim_{x \rightarrow 2} \frac{\frac{1}{x+1} - \frac{1}{3}}{\frac{x}{2} - 1}$

h) $\lim_{x \rightarrow 0} (1 + 3x)^{2/x}$

2. You are given wire and asked to form a right triangle with a hypotenuse of $\sqrt{17}$ inches whose area is as large as possible.

a) Write a function for the area in terms of x , the length of the side of the triangle

b) What should x be in order to maximize the area?

c) What's the maximum area?

3. Given: $f(x) = \frac{3}{3-x}$ and $g(x) = \sin \pi x$

a) Find $\lim_{x \rightarrow 2} (f(x)g(x))$

b) Find $\lim_{x \rightarrow 2} (g(x) - f(x))$

4. Given: $f(x) = \frac{1}{x-5}$ find $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

5. Find the slope of $h(x) = \sqrt{x+5}$ at $(-1, 2)$

6. Find the derivative of $f(x) = \sqrt{x-4}$

9. Use the function and its derivative to determine any points on the graph of f at which the tangent line is horizontal.

$$f(x) = \frac{\ln x}{x} \qquad f'(x) = \frac{1 - \ln x}{x^2}$$

7. Find the equation of the tangent line that is tangent to $f(x) = x^3 - x$ at the point $(2, 6)$.

8. Use the derivative of $f(x) = -2x^3 + 24x$ to find any points on the graph where the tangent line is horizontal.

10. Find the following limits at infinity:

a) $\lim_{x \rightarrow \infty} \left(\frac{x}{2x+1} + \frac{3x^2}{(2x-3)^2} \right)$

b) $\lim_{x \rightarrow \infty} \left(\frac{x}{2} - \frac{4x}{x^2} \right)$

c) $\lim_{x \rightarrow \infty} \left(\frac{(4n-2)!}{(4n+2)!} \right)$

d) $\lim_{x \rightarrow \infty} \left(\frac{8}{n^5} \left(\frac{n(n+1)(2n+1)(3n+1)(4n+1)}{6} \right) \right)$

11. Given: $\sum_{i=1}^n \left(\frac{4}{n} - \left(\frac{2i}{n} \right)^2 \right) \left(\frac{2i}{n} \right)$

a) Rewrite the sum as a rational function

b) Find the n th partial sum when $n = 100$.

12. Approximate the area of the region bounded by the graph of $f(x) = 9 - x^3$, the x -axis, and the vertical lines $x = 0$ and $x = 2$ using 20 rectangles.

13. Find the exact area of the region between the graph of $f(x) = x^3 - x^2 - x$ and the x -axis over the interval $[2, 5]$.

