Hon Pre-Calc
Quiz 7.1 - 7.4 Name [redacted] 4/6/50 11

Use Calculator only if the problem can't be solved algebraically. Write all general solutions in terms of Z. Circle all final answers!! No work = No credit!

Short Answer

1. Solve the following system: $\begin{cases} x+y-4=0 \\ x^2+y^2-4x=0 \end{cases}$

$y = 4-x$

$x^2+(4-x)^2-4x=0$

$(4-x)(4-x)$

$x^2+16-4x-4x+x^2-4x=0$

$2x^2-12x+16=0$

$x^2-6x+8=0$

$(x-2)(x-4)=0$

$x=2 \quad x=4$

$y=2$

$y=0$

$(2,2)$

$(4,0)$

2. Solve the following System:

$\begin{cases} 2x+4y+z=1 \\ x-2y-3z=2 \\ x+y-z=-1 \end{cases}$

$2x+4y+z=1$
$+\ x+y-z=-1$
$3x+5y=0$

$6x+12y+3z=3$
$+\ x-2y-3z=2$
$7x+10y=5$

$7x+10y=5$
$-\ 6x+10y=0$
$x=5$

$7(5)+10y=5$
$35+10y=5$
$10y=-30$
$y=-3$

$5-8-z=-1$
$-3-z=2$
$-z=-3$
$z=3$

$(5,-3,3)$

3. Solve the following system:

$\begin{cases} x+2y=8 \\ y=\log_2 x \end{cases}$

$x=8-2y$ ✗

different family

$2^y=x$

graphing calc

$(4,2)$

4. Solve the following system for u and v:

$\begin{cases} v\tan 2x-u\sec 2x=0 \\ u(-2\cot 2x)+v(2\sec 2x)=\tan 2x \end{cases}$

Seperate sheet

$u=\frac{1}{2} \quad v=\frac{1}{2\sin 2x}$

5. Solve the following system: $\begin{cases} x+2y-7z=-4 \\ 2x+y+z=13 \\ 3x+9y-36z=-33 \end{cases}$

$2x+4y-14z=-8$
$2x+y+z=13$
$3y-15z=-21$

$3(5z-7)-15z=-21$

$15z-21-15z=-21$

$0=0$

Infinite general solution

$x+10z=14-7z=-4$

$x=-3z+10$

$6x+3y+3z=39$
$-\ 6x+18y-72z=-66$
$-15y+75z=105$
$\frac{-15y+75z}{15}=\frac{105}{15}$

$-y+5z=7$

$y-5z=-7$

$y=5z-7$

$(-3z+10,\ 5z-7,\ z)$

6. Find the equation of the parabola: $y = ax^2 + bx + c$ that passes through (2,0), (3,-1), and (4,0).

$4a+2b+c=0$ $\quad 5a+b=-1$

$9a+3b+c=-1$ $\quad 7a+b=1$

$16a+4b+c=0$

$b=1-7a$ $\qquad b=1-7(1)$

$b=-6$

$5a+1-7a=-1$

$-2a=-2$

$a=1$

$4(1)+2(-6)+c=0$

$4-12+c=0$

$c=8$

$y=x^2-6x+8$

7. A chemist needs 12 gallons of a 20% acid solution. The solution is to be mixed from three solutions whose concentrations are 10%, 15%, and 25%. How many liters of each solution will be used if the chemist wishes to use the least amount of the 25% solution as possible?

$x=$ # g of 10%

$y=$ # of 15%.

$z=$ # of 25%.

$y=12-z$

$x+y+z=12$

$0.1x+0.15y+0.25z=2.4$

$0.15(12-z)+0.25z=2.4$

$-0.15z+0.25z=0.6$

$0.1z=0.6$

$z=6$

$y=12-6$

$y=6$

0 gallons of 10%.

6 gallons of 15%.

6 gallons of 25%.

8. A mixture of 5 pounds of fertilizer A, 13 pounds of fertilizer B, and 4 pounds of fertilizer C provides the optimal nutrients for a plant. Commercial brand X contains equal parts of fertilizer B and fertilizer C. Commercial brand Y contains one part of fertilizer A and two parts of fertilizer B. Commercial brand Z contains two parts fertilizer A, five parts fertilizer B, and two parts fertilizer C. How much of each fertilizer brand is needed to obtain the desired mixture?

	A	B	C
X		$\frac{1}{2}$	$\frac{1}{2}$
Y	$\frac{1}{3}$	$\frac{2}{3}$	
Z	$\frac{2}{9}$	$\frac{5}{9}$	$\frac{2}{9}$
	5	13	4

$5=\frac{1}{3}y+\frac{2}{9}z$

$13=\frac{1}{2}x+\frac{2}{3}y+\frac{5}{9}z$

$4=\frac{1}{2}x+\frac{2}{9}z$

$45=3y+2z$

$234=9x+17y+10z$

$72=9x+4z$

↓ seperate sheet

4 pounds of X

9 pounds of Y

9 pounds of Z

9. An airplane flying into a headwind travels the 990-mile flying distance between Rochester Hills, Michigan and Atlanta, Georgia in 3 hours and 40 minutes. On the return flight, the distance is traveled in 3 hours. P = plane speed W = wind speed

a) Find the airspeed of the plane.

$990=3\frac{2}{3}(P-W)$ $\quad P-W=270$ $\quad \Rightarrow W=P-270$

$990=3(P+W)$ $\quad P+W=330$ $\quad \Rightarrow P+P-270=330$

$2P=600$

$P=300$ mph

b) Find the wind speed.

$W=300-270$

$W=30$mph

10. Write the partial fraction decomposition of the rational expression:

$$\frac{x+4}{x^3+x^2-4x-4}$$

$x^2(x+1)-4(x+1)$

$(x+2)(x-2)(x+1)$

$$\frac{x+4}{x^3+x^2-4x-4} = \frac{A}{x+2} + \frac{B}{x-2} + \frac{C}{x+1}$$

$A(x-2)(x+1) + B(x+2)(x+1) + C(x+2)(x-2) = x+4$

let $x=2$: $12B = 6$, $B = \frac{1}{2}$

let $x=-1$: $-3C = 3$, $C = -1$

let $x=-2$: $4A = 2$, $A = \frac{1}{2}$

$$\frac{x+4}{x^3+x^2-4x-4} = \frac{1}{2(x+2)} + \frac{1}{2(x-2)} - \frac{1}{x+1}$$

11. Write the partial fraction decomposition of the rational expression.

$$\frac{x}{16x^4-1}$$

$(4x^2)^2 - (1)^2$

$(4x^2+1)(4x^2-1)$

$(4x^2+1)(2x+1)(2x-1)$

$$\frac{x}{16x^4-1} = \frac{Ax+B}{4x^2+1} + \frac{C}{2x+1} + \frac{D}{2x-1}$$

$(Ax+B)(2x+1)(2x-1) + C(2x-1)(4x^2+1) + D(2x+1)(4x^2+1) = x$

let $x=\frac{1}{2}$: $D(2)(2) = \frac{1}{2}$, $4D = \frac{1}{8}$, $D = \frac{1}{8}$

let $x=-\frac{1}{2}$: $C(-2)(2) = -\frac{1}{2}$, $C(-4) = -\frac{1}{2}$, $C = \frac{1}{8}$

Seperate sheet

$\frac{1}{8}\left[\frac{1}{2x+1} + \frac{1}{2x-1} - \frac{4x}{4x^2+1}\right]$

$$\frac{x}{16x^4-1} = \frac{-\frac{1}{2}x}{4x^2+1} + \frac{\frac{1}{8}}{2x+1} + \frac{\frac{1}{8}}{2x-1}$$

3

12. Write the partial fraction decomposition of the improper rational expression:

$$\frac{81x^4}{(3x-1)^3}$$

$(3x-1)(3x-1)(3x-1)$

$(9x^2-6x+1)(3x-1)$

$9x^3-18x^2+3x-9x^2+6x-1$

$9x^3-9x^2-9x-1$

Long division

$$\begin{array}{r} 9x+9 \\ 9x^3-9x^2-9x-1 \overline{)\ 81x^4+0x^3+0x^2+0x+0} \\ -81x^4+81x^3+81x^2+9x \\ \hline 81x^3+81x^2+9x+0 \end{array}$$

$$3x+3+\frac{6}{3x-1}+\frac{4}{(3x-1)^2}+\frac{1}{(3x-1)^3} = \frac{81x^3+81x^2+81x+9}{162x^2+90x+9}$$

$54x^2+30x+3$

$18x^2+10x+1$

$$9x+9+\frac{18x^2+10x+1}{(3x-1)^3}$$

$$\frac{18x^2+10x+1}{(3x-1)^3} = \frac{A}{3x-1} + \frac{Bx+C}{(3x-1)^2}$$

$A(3x-1)^2 + (Bx+C)(3x-1) = 18x^2+10x+1$

$A(9x^2-6x+1) + B3x^2 + C3x - Bx - C = 18x^2+10x+1$

$9x^2 \cdot 6x + A + B3x^2 + C3x - Bx - C = 18x^2+10x+1$

$A - C = 1 \quad 9A + 3B = 18 \quad -A + 3C - B = 10$

seperate sheet

$$9x+9+\frac{-13}{3x-1}+\frac{45x+14}{(3x-1)^2}$$

11.

$$\frac{x}{16x^4-1} = \frac{A}{(2x+1)} + \frac{B}{2x-1} + \frac{Cx+D}{4x^2+1}$$

$(4x^2+1)(4x^2-1)$

$(4x^2+1)(2x+1)(2x-1)$

$$x = A(2x-1)(4x^2+1) + B(2x+1)(4x^2+1) + (Cx+D)(4x^2-1)$$

let $x=\frac{1}{2}$ $B=\frac{1}{8}$

let $x=-\frac{1}{2}$ $A=\frac{1}{8}$

$8Ax^3 + 8Bx^3 + 4Cx^3 = 0x^3$

$1+1+4C=0$

$C=-\frac{1}{2}$

$-A \cancel{-B} + D = 0$

$D=0$

$$= \boxed{\frac{\frac{1}{8}}{(2x+1)} + \frac{\frac{1}{8}}{(2x-1)} + \frac{-\frac{1}{2}x}{(4x^2+1)}}$$

12.

$$\frac{81x^4}{(3x-1)^3}$$

$|(3x)^3(-1)^0 \; 3(3x)^2(-1)^1 \; 3(3x)^1(-1)^2 \; (-1)^3|$

$27x^3 - 27x^2 + 9x - 1$

$$\begin{array}{r} 3x+3 \\ 27x^3-27x^2+9x-1 \;\overline{)}\; 81x^4+0x^3+0x^2+0x+0 \\ -81x^4+81x^3-27x^2+3x \\ \hline 81x^3-27x^2+3x \\ -81x^3+81x^2-27x+3 \\ \hline 54x^2-24x+3 \end{array}$$

$$3x+3+\left[\frac{54x^2-24x+3}{(27x^3-27x^2+9x-1)}\right] = \frac{A}{(3x-1)^3}$$

$$= \frac{A}{(3x-1)} + \frac{B}{(3x-1)^2} + \frac{C}{(3x-1)^3}$$

$45 = 3y + 2z$

$234 = 9x + 12y + 10z$

$72 = 9x + 4z$

$9x = 72 - 4z$

$x = 8 - \frac{4}{9}z$

$x = 8 - \frac{4}{9}\begin{pmatrix}3\\1\end{pmatrix}$

$x = 4$

$3y = 45 - 2z$

$y = 15 - \frac{2}{3}z$

$y = 15 - \frac{2}{3}\begin{pmatrix}3\\1\end{pmatrix}$

$y = 15 - 6$

$y = 9$

$234 = 9\left(8 - \frac{4}{9}z\right) + 12\left(15 - \frac{2}{3}z\right) + 10z$

$234 = 72 - 4z + 180 - 8z + 10z$

$-18 = -4z - 8z + 10z$

$-18 = -2z$

$z = 9$

11.

$(Ax+B)(4x^2-1) + C(8x^3 - 4x^2 + 2x - x) + D(8x^3 + 4x^2 + 2x + 1) = x$

$4Ax^2 + 4Bx^2 - Ax - B + C8x^3 - C4x^2 + C2x - Cx + D8x^3 + D4x^2 + D2x + D = x$

$4Ax^2 + 4Bx^2 - C4x^2 + D4x^2 = 0x^2$

$4A + 4B - \frac{1}{2} + \frac{1}{2} = 0$

$A + B = 0$

$-\frac{1}{2} + B = 0$

$B = \frac{1}{2}$

$-Ax - Cx + 2Dx = x$

$-Ax - \frac{x}{8} + \frac{x}{4} = x$

let $x = 8$

$-8A - 1 + 2 = 8$

$-8A = 7$

$A = -\frac{7}{8}$

12.

$3A + B = 6$

$B = 6 - 3A$

$C = 1 - A$

$-A + 3(1-A) - (6-3A) = 10$

$-A + 3 - 3A - 6 + 3A = 10$

$-A = 13$

$A = -13$

$B = 6 - 3(-13)$

$B = 45$

$C = 1 - (-13)$

$C = 14$

$$\frac{9x+9}{} = \frac{-13}{3x-1} + \frac{45x+14}{(3x-1)^2}$$

4.

$v\tan 2x - u\sec 2x = 0$

$v\tan 2x = u\sec 2x$

$V = \frac{u\sec 2x}{\tan 2x}$

$\sin^2 x + \cos^2 x = 1$

$1 + \cot^2 x = \csc^2 x$

$\tan^2 x + 1 = \sec^2 x$

$\sec^2 x - \tan^2 x = 1$

$\tan^2 x \left(\frac{u(-2\csc 2x)}{1} + \frac{2u\sec^2 2x}{\tan 2x} \right) = (\tan 2x)^{\tan 2x}$

$-2u + 2u\sec^2 2x = \tan^2 2x$

$-2u(1 - \sec^2 2x) = \tan^2 2x$

$\frac{-2u(-\tan^2 2x)}{-\tan^2 2x} = \frac{\tan^2 2x}{-\tan^2 2x}$

$-2u = -1$

$u = \frac{1}{2}$

$V = \frac{\frac{1}{2}\sec 2x}{\tan 2x}$

$V = \frac{1}{2} \cdot \frac{1}{\cos 2x} \cdot \frac{\cos 2x}{\sin 2x}$

$V = \frac{1}{2\sin 2x}$

$u = \frac{1}{2} \quad V = \frac{1}{2\sin 2x}$