Hon Pre-Calc Quiz 9.4 *2016 – 2017*

Show All Work For FULL Credit!!!!

1. Evaluate: (Answer must be a simple fraction in lowest terms)

$$\sum_{j=1}^{10}\left(6-\frac{1}{3}j+\frac{5}{4}j^2\right)$$

2. Determine an explicit formula to find the nth partial sum.

$$\sum_{x=1}^{n}\frac{1}{x(x+1)}$$

3. Find a formula for the following sequence:

{-4, -5, 4, 35, 100, 211, 380, 619, 940, …}

4. Prove using mathematical induction:

$$\sum_{i=1}^{n} \left(i * 2^{i-1}\right) = 1 + (n-1) * 2^{n}$$

5. Given: $1^3 + 2^3 + 3^3 + 4^3 + \cdots + n^3$

a) Determine a formula for the nth partial sum.

b) Prove using mathematical induction:

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Show All Work For FULL Credit!!!!

1. Evaluate: (Answer must be a simple fraction in lowest terms)

$$\sum_{j=1}^{10}\left(6-\frac{1}{3}j+\frac{5}{4}j^2\right)$$

$\sum_{j=1}^{10} 6 - \frac{1}{3}\sum_{j=1}^{10} j + \frac{5}{4}\sum_{j=1}^{10} j^2$

$6 \cdot 10 - \frac{1}{3}\left(\frac{10(10+1)}{2}\right) + \frac{5}{4}\left(\frac{10(10+1)(2\cdot 10+1)}{6}\right)$

$60 - \frac{1}{3}(55) + \frac{5}{4}(385)$

$60 - \frac{55}{3} + \frac{1925}{4}$

$\frac{720}{12} - \frac{220}{12} + \frac{5775}{12}$

$\frac{6275}{12}$

2. Determine an explicit formula to find the nth partial sum.

$$\sum_{x=1}^{n}\frac{1}{x(x+1)} = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{42} + \dots + \frac{1}{n(n+1)}$$

n	1	2	3	4	5	...
S_n	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{6}$	...

$S_n = \frac{n}{n+1}$

3. Find a formula for the following sequence:

$\{-4, -5, 4, 35, 100, 211, 380, 619, 940, \dots\}$

-1 9 31 65 111 169 239 321

10 22 34 46 58 70 82

12 12 12 12 12 12

3rd degree polynomial

$a_n = an^3 + bn^2 + cn + d$

$-4 = a + b + c + d$

$-5 = 8a + 4b + 2c + d$

$4 = 27a + 9b + 3c + d$

$35 = 64a + 16b + 4c + d$

$35 = 64a + 16b + 4c + d$
$-\ 4 = 27a + 9b + 3c + d$

$31 = 37a + 7b + c$

$-5 = 8a + 4b + 2c + d$
$+4 = a + b + c + d$

$-1 = 7a + 3b + c$

$c = -1 - 7a - 3b$

$211 = 216a + 36b + 6c + d$
$100 = 125a + 25b + 5c + d$

$111 = 91a + 11b + c$

$111 = 91a + 11b + (-1 - 7a - 3b)$

$112 = 84a + 8b$

$28 = 21a + 2b$

$31 = 37a + 7b - 1 - 7a - 3b$

$32 = 30a + 4b$

$16 = 15a + 2b$

$2b = 16 - 15a$

$28 = 21a + 16 - 15a$

$12 = 6a$ $a = 2$

$16 = 15(2) + 2b$

$2b = -14$

$b = -7$

$-1 = 7(2) + 3(-7) + c$

$-1 = 14 - 21 + c$

$-1 = -7 + c$

$c = 6$

$a + b + c + d = -4$

$2 - 7 + 6 + d = -4$

$2 - 1 + d = -4$

$d + 1 = -4$

$d = -5$

$a_n = 2n^3 - 7n^2 + 6n - 5$

4. Prove using mathematical induction:

$1 \cdot 2^{1-1} = 1(1) = 1$

$$\sum_{i=1}^{n}\left(i \bullet 2^{i-1}\right) = 1 + (n-1) \bullet 2^{n} = 1 + 4 + 12 + \ldots + n \cdot 2^{n-1}$$

Let $n = 1$

$S_n = 1 + (n-1) \cdot 2^n$

$S_1 = 1 + (1-1) \cdot 2^1$

$= 1 + 0 \cdot 2$

$= 1$ ✓

Let $n = K$

$S_{K+1} = S_K + a_{K+1}$

$= 1 + (K-1) \cdot 2^K + (K+1) \cdot 2^{K+1-1}$

$= 1 + K \cdot 2^K - 2^K + (K+1)2^K$

$= 1 + K \cdot 2^K - 2^K + K \cdot 2^K + 2^K$

$= 1 + 2K \cdot 2^K$ ✓

Let $n = K+1$

$S_{K+1} = 1 + (K+1-1)2^{K+1}$

$= 1 + K \cdot (2^K)(2^1)$

$= 1 + 2K \cdot 2^K$ ✓

5. Given: $1^3 + 2^3 + 3^3 + 4^3 + \ldots + n^3 = 1 + 8 + 27 + 64 + \ldots + n^3$

a) Determine a formula for the nth partial sum.

$S_n = \frac{n^2(n+1)^2}{4}$ $\quad \sum_{i=1} i^3$ on memorized list

b) Prove using mathematical induction:

Let $n = 1$

$S_n = \frac{n^2(n+1)^2}{4}$

$S_1 = \frac{1^2(1+1)^2}{4}$

$= \frac{1 \cdot 2^2}{4}$

$= \frac{4}{4}$

$= 1$ ✓

Let $n = K$

$S_{K+1} = S_K + a_{K+1}$

$= \frac{K^2(K+1)^2}{4} + (K+1)^3$

$= \frac{(K+1)^2}{4} \cdot (K^2 + 4(K+1))$

$= \frac{(K+1)^2}{4} \cdot (K^2 + 4K + 4)$

$= \frac{(K+1)^2}{4} \cdot (K+2)^2$

$= \frac{(K+1)^2(K+2)^2}{4}$ ✓

Let $n = K+1$

$S_{K+1} = \frac{(K+1)^2(K+1+1)^2}{4}$

$= \frac{(K+1)^2(K+2)^2}{4}$ ✓