Hon Pre-Calculus
Quiz 4.1 - 4.4 Name [redacted] 50/50

Show All Work For Full Credit!!!! Circle All Final Answers!!! No Calculators!!!

Angular Speed = $\frac{\theta}{t}$

Linear Speed = $\frac{r\theta}{t}$

Short Answer

1. Determine the quadrant the angle lies:

a) $-\frac{29\pi}{4}$ rad

$-\frac{8\pi}{4}$
$+\frac{8\pi}{4}$
$+\frac{8\pi}{4}$ $= -\frac{5\pi}{4}$

II

b) 13 radians

6.28
12.56

I

2. Convert 15.31° to degrees, minutes, and seconds.

15.31
$15^\circ + 0.31 \times 60$
18.6
$18' + 0.6 \times 60''$
$15^\circ + 18' + 36''$

$15^\circ\ 18'\ 36''$

3. A carousel with a 30 foot diameter makes 6 revolutions per minute.

a) Find the angular speed of the carousel in radians per minute.

$\frac{\theta}{t} = \frac{\theta}{1} = \frac{6 \cdot 2\pi}{1} = 12\pi$ /min

b) Find the linear speed (in feet per minute) of the platform rim of the carousel. $r = 15$ ft

$\frac{r\theta}{t} = \frac{15\text{ft}\,(12\pi)}{1\text{ minute}} = 180\pi$ ft/min

$= 180\pi$ ft/min

4. Find the area of a sector defined by a $\frac{5\pi}{3}$ radian central angle that has an arc length of 4 inches.

$S = \theta r$
$A = \frac{1}{2} r^2 \theta$

$4 = \frac{5\pi}{3} r$

$\frac{3}{5\pi} \cdot \frac{4}{1} = r$

$\frac{12}{5\pi} = r$

$\frac{24}{5\pi}$

$A = \frac{1}{2}\left(\frac{12}{5\pi}\right)^2 \left(\frac{5\pi}{3}\right)$

$A = \frac{1}{2} \cdot \frac{144}{25\pi^2} \cdot \frac{5\pi}{3}$

$A = \frac{24}{5\pi}$ in.^{2}

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$A = \frac{24}{5\pi}$ in.^{2}

cos and sec are even

$\sin\left(\frac{\pi}{2}-t\right)=\cos t$ $\sec\left(\frac{\pi}{2}-t\right)=\csc t$ $\sin(\pi-t)$

$\cos\left(\frac{\pi}{2}-t\right)=\sin t$ $\csc\left(\frac{\pi}{2}-t\right)=\sec t$ $\cos(\pi-t)$

$\tan\left(\frac{\pi}{2}-t\right)=\cot t$ $\tan(\pi-t)$

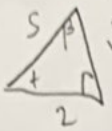
$\sin(\pi+t)=-\sin t$

$\cos(\pi+t)=-\cos t$

$\tan(\pi+t)=\tan t$

Simplify to a si... $(0<\theta<\frac{\pi}{2})$

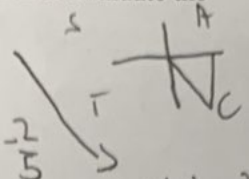
5. Use the value of the function to evaluate the following: $-\cos(-t)=\frac{2}{5}$



$-\cos(+t)=\frac{2}{5}$

a) $\cos(t)=$

$\cos(t)=-\frac{2}{5}$



$\cos(-t)=-\frac{2}{5}$

$\cos(t)=\frac{2}{5}$

$\boxed{-\frac{2}{5}}$

b) $\sin\left(\frac{\pi}{2}-t\right)=\cos(t)$

$\boxed{-\frac{2}{5}}$

test:

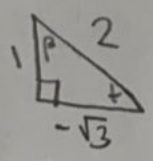
$-\cos\left(-\frac{\pi}{6}\right)=-\frac{\sqrt{3}}{2}$

$\cos\left(\frac{\pi}{6}\right)=\sqrt{3}/2$

$\sin\left(\frac{\pi}{6}\right)=\sqrt{3}/2$

6. Use the value of the function to evaluate the following: $\sin t=\frac{1}{2}$, $\frac{\pi}{2}<t<\pi$

$\csc t=2$



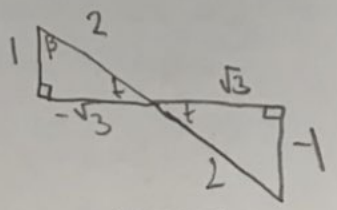
a) $\sec\left(\frac{\pi}{2}-t\right)=\csc t$

$\cos\left(\frac{\pi}{2}-t\right)=\frac{1}{2}$

$\sec\left(\frac{\pi}{2}-t\right)=\boxed{2}$

b) $\sin(t+\pi)=$

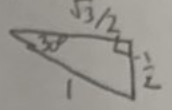
$-\sin t$



$\sin(t+\pi)=\boxed{-\frac{1}{2}}$

7. Evaluate:

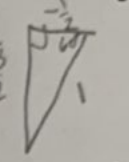
a) $\sin\frac{23\pi}{6}$ $-\frac{12\pi}{6}-\frac{12\pi}{6}$



$\sin-\frac{\pi}{2}$

$\boxed{-\frac{1}{2}}$

b) $\sec\frac{-8\pi}{3}$



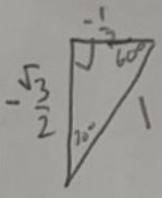
$=\frac{1}{-\frac{1}{2}}$

$=\boxed{-2}$

c) $\csc\frac{16\pi}{3}$ $-\frac{6\pi}{3}-\frac{6\pi}{3}$

$=\frac{4\pi}{3}$

$\csc\frac{4\pi}{3}$



$-\frac{2}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)$

$=\boxed{-2\sqrt{3}/3}$

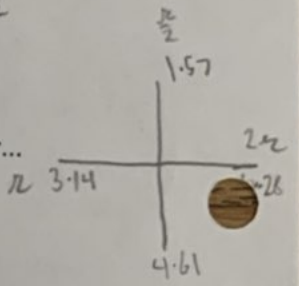
d) $\tan\frac{-10\pi}{3}$ $+\frac{6\pi}{3}+\frac{6\pi}{3}$

$\tan\frac{2\pi}{3}$

$\frac{\sqrt{3}/2}{-1/2}=\boxed{-\sqrt{3}}$

8. Find the exact reference angle for...

$2\pi+\pi=3\pi$

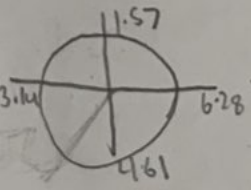


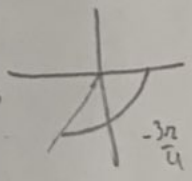
a) 10 radians

$\boxed{10-3\pi}$

b) - 8 radians

$+12.56$





$\boxed{-8+3\pi}$

$\sin^2\theta + \cos^2\theta = 1$ $\quad 1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

9. Simplify to a single trigonometric expression $\left(0 < \theta < \frac{\pi}{2}\right)$

a) $\dfrac{1-\cos^2\theta}{\cos\theta} + \cos\theta$

$\dfrac{\sin^2\theta}{\cos\theta} + \dfrac{\cos\theta}{1}$

$\dfrac{\sin^2\theta + \cos^2\theta}{\cos\theta} = \dfrac{1}{\cos\theta}$

$= \boxed{\sec\theta}$

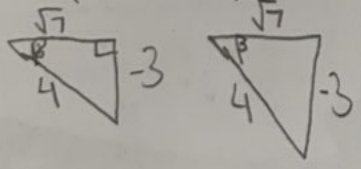
b) $\sin^2\theta\left(1 + \dfrac{1}{\tan^2\theta}\right)$ $\quad 1 + \dfrac{1}{\frac{\sin^2\theta}{\cos^2\theta}}$

$\sin^2\theta + \dfrac{\sin^2\theta}{\tan^2\theta}$ $\quad \dfrac{\cos^2\theta}{\sin^2\theta} + \dfrac{1}{1}$

$= \dfrac{\sin^2\theta}{1}\left(\dfrac{\cos^2\theta + \sin^2\theta}{\sin^2\theta}\right)$

$= \boxed{1}$

10. Given: $\cos\beta = \dfrac{\sqrt{7}}{4}$ and $\left(\dfrac{3\pi}{2} < \beta < 2\pi\right)$ find:



a) $\sin\beta$

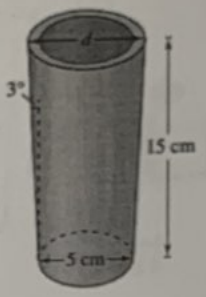
$= \boxed{\dfrac{-3}{4}}$

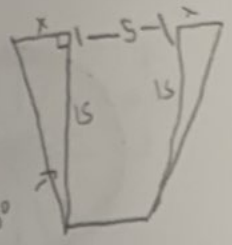
b) $\tan\beta$

$= \dfrac{-3}{\sqrt{7}}\left(\dfrac{\sqrt{7}}{\sqrt{7}}\right)$

$= \boxed{\dfrac{-3\sqrt{7}}{7}}$

11. A tapered shaft has a diameter of 5 centimeters at the small end and is 15 centimeters long (see figure). The taper is 3°. Find the diameter of the large end of the shaft.





$\tan 3 = \dfrac{x}{15}$ $\quad \tan 3 = \dfrac{x}{15}$

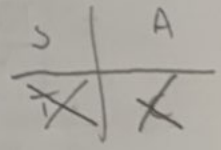
$x = 15\tan 3$

$2x = 30\tan 3$

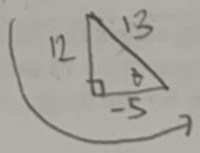
$\boxed{d = 5 + 30\tan 3}$

12. Given: $\cot\theta = -\dfrac{5}{12}$ and $\sin\theta > 0$

Find:

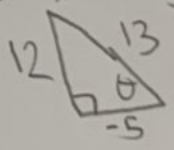


a) $\sec\theta$



$= \boxed{-\dfrac{13}{5}}$

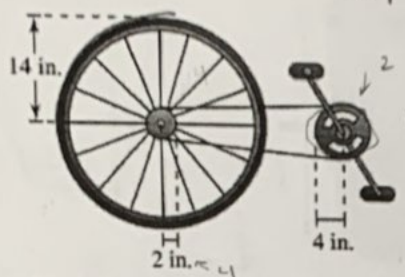
$\tan = -\dfrac{12}{5}$



b) $\sin\theta$

$= \boxed{\dfrac{12}{13}}$

13. The radii of the pedal sprocket, the wheel sprocket, and the wheel of the bicycle in the figure are 4 inches, 2 inches,, and 14 inches respectively. A cyclist is pedaling at a rate of 2 revolutions per second.



a) Find the speed of the bike in feet/second.

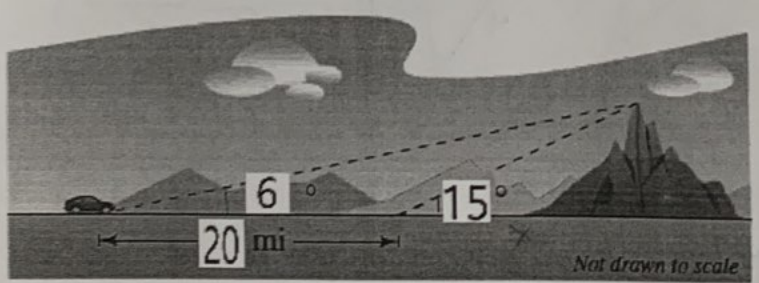
b) Find the speed of the bike in miles per hour.

14. Given: $(0 < \theta \le 2\pi)$ Find all θ that satisfy the following:

a) $\sec\theta = -\sqrt{2}$

b) $3\cot\theta - \sqrt{3} = 0$

15. In traveling across flat land, you notice a mountain directly in front of you. Its angle of elevation (to the peak) is 4°. After you drive 15 miles closer to the mountain, the angle of elevation is 10°. Write a calculator ready expression for the height of the mountain.



15.

$\tan 6 = \frac{h}{20+x}$ $\quad \tan 15 = \frac{h}{x}$

$x = \frac{h}{\tan 15}$

$\tan 6 = \frac{h}{20 + \frac{h}{\tan 15}}$

$\tan 6 * \left(20 + \frac{h}{\tan 15}\right) = h$

$20\tan 6 + \frac{h\tan 6}{\tan 15} = h$

$h - \frac{h\tan 6}{\tan 15} = 20\tan 6$

$h\left(1 - \frac{\tan 6}{\tan 15}\right) = 20\tan 6$

$$h = \frac{20\tan 6}{1 - \frac{\tan 6}{\tan 15}}$$

3. 15 ft = radius

6 rev/min

a) Angular speed = $\frac{\theta}{t}$

$t = 1$ minute

$\theta = 2\pi \cdot 6$ rev/min

$\theta = 12\pi$

$= \frac{12\pi}{1 \text{ min}}$

$= \boxed{12\pi \text{ rad/min}}$

b) linear speed = $\frac{r\theta}{t}$

$r = 15$ ft $\quad t = 1$ minute

$\theta = 12\pi$

$= \frac{15 \text{ ft } (12\pi)}{1 \text{ minute}}$

linear speed = $\boxed{180\pi \text{ ft/min}}$

4. $\frac{5\pi}{3} = \theta \qquad S = 4$ in.

$4 = \frac{5\pi}{3} r$

$\frac{4}{1} \cdot \frac{3}{5\pi} = r$

$\frac{12}{5\pi} = r$

$A = \frac{1}{2} r^2 \theta$

$A = \frac{1}{2} \left(\frac{12}{5\pi}\right)^2 \left(\frac{5\pi}{3}\right)$

$A = \frac{1}{2} \cdot \frac{144}{25\pi^2} \cdot \frac{5\pi}{3}$

$\boxed{A = \frac{24}{5\pi} \text{ in.}^2}$