~~Hon~~ Pre-Calculus

Test Chapter 4 Name [redacted] 51/56

Circle All Final Answers!! Show All Work For Full Credit!! No Calculators!! Simplify Completely and Leave All Answers in Calculator Ready Form!!

Short Answer

1. Determine two co-terminal angles (one positive and one negative) for the given angle:

$\theta = \frac{5\pi}{12}$ $+\frac{24\pi}{12} = \frac{29\pi}{12}$ $-\frac{24\pi}{12} = -\frac{19\pi}{12}$

a) Positive = $\frac{29\pi}{12}$

b) Negative = $-\frac{19\pi}{12}$

2. The diameter of a DVD is approximately 15 cm. The drive motor of the DVD player is controlled to rotate precisely between 200 and 500 revolutions per minute, depending on what track is being read.

a) Find the slowest possible angular speed.

$r = 7.5$

$= 200 * 2\pi$

$= 400\pi$ rev/min

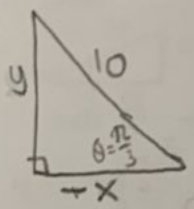
b) Find the fastest possible linear speed for a point 2 cm in from the outermost track of the DVD..

$= \frac{500}{1} * \frac{2\pi}{1} * \frac{5.5}{1}$

$= 1000\pi * 5.5$

$= 5500\pi$ in/min

3. Find the coordinates of the point that corresponds to a $\frac{2\pi}{3}$ radian angle on a circle of radius 10 inches.



$= \sin\frac{\pi}{3} = \frac{y}{10}$ $\cos\frac{\pi}{3} = \frac{x}{10}$

$= \frac{\sqrt{3}}{2} = \frac{y}{10}$ $-\frac{1}{2} = \frac{x}{10}$

$y = \frac{10\sqrt{3}}{2}$ $x = -5$

$y = 5\sqrt{3}$

$(-5, 5\sqrt{3})$

4. Find the arc length in a circle formed by a $\frac{\pi}{6}$ central radian angle and a sector area of $\frac{3}{\pi}$ cm^2.

$s = \theta r$ $\theta = \frac{\pi}{6}$ $\frac{1}{2}r^2\theta = \frac{3}{\pi}$ $r = \frac{6}{\pi}$

$s = \frac{\pi}{6} \cdot \frac{6}{\pi}$ $r^2\frac{\pi}{6} = \frac{6}{\pi}$

$\sqrt{r^2} = \sqrt{\frac{36}{\pi^2}}$

$s = 1$ cm $\frac{1}{2} \cdot \frac{6}{\pi} \cdot \frac{6}{\pi} \cdot \frac{2}{6} = \frac{3}{\pi}$

$\frac{3}{\pi} = \frac{3}{\pi}$ ✓

5. Convert 11.71° to degrees, minutes, seconds.

11.71

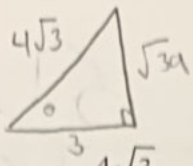
$0.71 * 60$

42.6

$0.6 * 60$

36

$11^\circ \; 42' \; 36''$



$3^2 + b^2 = (4\sqrt{3})^2$

$9 + b^2 = 48$

$\sqrt{b^2} = \sqrt{39}$

$b = \sqrt{39}$

6. Given: $\sec t = \frac{4\sqrt{3}}{3}$, Evaluate: $\cos t = \frac{3}{4\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)$

a) $\cos(t+\pi) = -\cos t$

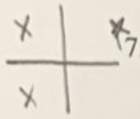
$= \frac{3\sqrt{3}}{12}$

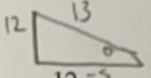
$= \frac{\sqrt{3}}{4}$

$-\frac{\sqrt{3}}{4}$

b) $-\sec(-t)$

$-\left(\frac{4\sqrt{3}}{3}\right) = -\frac{4\sqrt{3}}{3}$





7. Given: $\left(\frac{\pi}{2} < \theta < \frac{3\pi}{2}\right)$ and $\cot\theta = -\frac{12}{5}$ find:

$\tan\theta = -\frac{5}{12}$

a) $\csc\theta$

$\frac{13}{12}$, 2 $\frac{13}{5}$

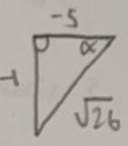
b) $\sec\theta$ ← $\cos = -\frac{5}{13}$

$-\frac{13}{5}$, $\sec = -\frac{13}{5}$ $\frac{-13}{12}$

tangent positive quadrant IV

8. Given: $\cot\alpha = 5$ and $\sin\alpha < 0$ find:

$\tan\alpha = \frac{1}{5}$



a) $\sec\alpha$

$-\frac{\sqrt{26}}{5}$

b) $\sin\alpha$

$= -\frac{1}{\sqrt{26}}\left(\frac{\sqrt{26}}{\sqrt{26}}\right)$

$= -\frac{\sqrt{26}}{26}$

9. Simplify to one single trig function. $\left(0 < \beta < \frac{\pi}{2}\right)$

$\frac{\tan\beta + \cot\beta}{\tan\beta}$

$\rightarrow \frac{\frac{\sin}{\cos} + \frac{\cos}{\sin}}{\frac{\sin}{\cos}} = \frac{\frac{\sin^2+\cos^2}{\sin\cos}}{\frac{\sin}{\cos}}$

$= \frac{1}{\sin\cos} \cdot \frac{\cos}{\sin} = \frac{1}{\sin^2} = \csc^2\beta$

10. What are the following exact values:

a) $\cos\frac{17\pi}{3} - \frac{6\pi}{3} - \frac{6\pi}{3}$

$\cos\frac{5\pi}{3}$

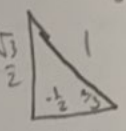
$= \frac{1}{2}$

$\sin\frac{\pi}{3}$

$\frac{\sqrt{3}}{2}$

$\frac{2}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)$ $\csc\frac{\pi}{3}$ $\frac{2\sqrt{3}}{3}$

c) $\csc\frac{25\pi}{3} - \frac{24\pi}{3}$



b) $\sec\frac{31\pi}{4} - \frac{32\pi}{4}$

$\cos -\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ $= -\frac{\pi}{4}$

$\frac{2}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{2\sqrt{2}}{2}$

$\sec = \sqrt{2}$

d) $\cot\frac{-22\pi}{3}$

$-\frac{22\pi}{3} + \frac{18\pi}{3} + \frac{6\pi}{3} = \frac{2\pi}{3}$

$\frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)$

$= -\sqrt{3}/3$

11. What is the **EXACT** reference angle of 15 radians?

6.28 2π

12.56 4π

15.7 5π

$5\pi - 15$

-4

$y = 2\tan\left[\frac{\pi}{3}\left(x - \frac{1}{2}\right)\right] + 3$

12. Given: $(0 < \theta \leq 2\pi)$. Find all theta that satisfy:

a) $\sec\theta = -2$

$\cos\theta = -\frac{1}{2}$

$\frac{2\pi}{3}, \frac{4\pi}{3}$

b) $-\sqrt{3}\cot\theta - 3 = 0$

$-\sqrt{3}\cot = 3$

$\cot = \frac{3}{-\sqrt{3}}$

$\tan = -\frac{\sqrt{3}}{3}$

$\frac{11\pi}{6}$

$\frac{5\pi}{6}, \frac{7\pi}{6}$ ✓

13. Consider: $y = -2\sin(4x + \pi) + 1$. State the following:

Reflection x-axis

$y = -2\sin\left[4\left(x + \frac{\pi}{4}\right)\right] + 1$

a) Amplitude = 2

b) Period = $\frac{\pi}{2}$

$\frac{2\pi}{4} = \frac{1}{2}\pi$

c) Phase Shift = Left $\frac{\pi}{4}$

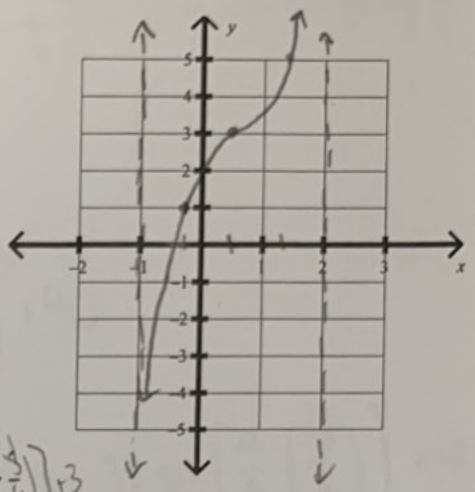
d) Vertical Shift = Up 1

14. Graph the following: $y = 2\tan\left[\frac{\pi}{3}x - \frac{\pi}{6}\right] + 3$

$\frac{\pi}{\pi/3} = \frac{\pi}{1} \cdot \frac{3}{\pi} = 3$

Int width $= \frac{3}{4}$

Asymptote width $= \frac{3}{2}$



$y = 2\tan\left[\frac{\pi}{3}\left(-\frac{3}{4}\right)\right] + 3$

$= 2 \cdot -1 + 3$

$= 1$

15. ***Ferris Wheel***: As you ride the Ferris wheel your distance from the ground varies sinusoidally with time. When the last seat is filled and the Ferris wheel starts, you notice it takes you 5 seconds to get to the top. The platform to load the chairs is 8 feet off the ground and the Ferris wheel towers at a height of 72 feet above the ground. You time the ride and you see that you made 6 revolutions in 1 min and 12 seconds.

1 rev every 12 seconds

Predict your exact height above ground at:

a) 40 seconds

$16\sqrt{3} + 40$ feet

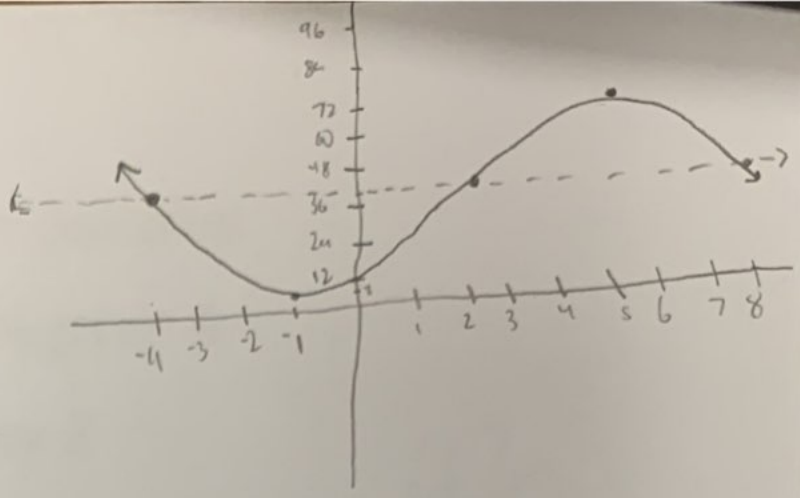
b) 3.5 seconds

$16\sqrt{2} + 40$ feet

1

$k = 8$ $\rangle 64$ $\quad a = 32$

$(5, 72)$ $\quad k = 40$



Per = 12 seconds

$12 = \frac{2\pi}{b}$ = Int width @ 3

$b = \frac{\pi}{6}$

$$y = +32 \sin\left[\frac{\pi}{6}(x-2)\right] + 40$$

a) $y = 32 \sin\left[\frac{\pi}{6}\left(\frac{42}{1}\right)\right] + 40$

$= 32 \sin(7\pi) + 40$

$= 40$ feet

$\frac{14\pi}{3} - \frac{18\pi}{3}$

$= -\frac{\pi}{3}$

$y = 32 \sin\left[\frac{\pi}{6}\left(\frac{38}{1}\right)\right] + 40$

$y = \frac{32}{1}\left(\frac{\sqrt{3}}{2}\right) + 40$

$y = 16\sqrt{3} + 40$ feet

b) $y = +32 \sin\left[\frac{\pi}{6}\left(\frac{11}{2}\right)\right] + 40$

$= +32 \sin\left[\frac{11\pi}{12}\right] + 40$

$=$

$y = 32 \sin\left[\frac{\pi}{6}\left(\frac{8}{2}\right)\right] + 40$

$y = \frac{32}{1}\left(\frac{\sqrt{2}}{2}\right) + 40$

$y = 16\sqrt{2} + 40$ feet

$\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

Hon Pre-Calculus

Test Chapter 4

Day 2

42/44

Name [redacted]

$\frac{2\pi}{3} = \frac{2\pi}{3}$

Int width $\frac{\pi}{6}$ Asm width $\frac{\pi}{3}$ $y = 2\sec\left[3\left(x - \frac{\pi}{12}\right)\right] + 2$

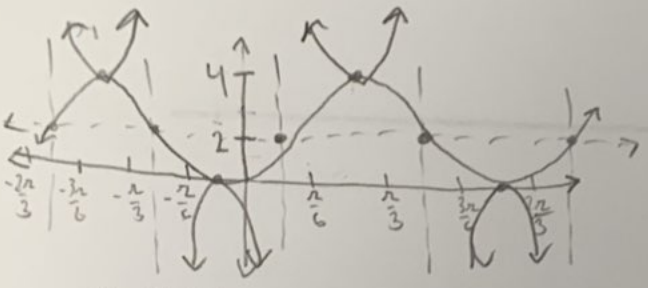
16. Consider: $y = 2\sec\left(3x - \frac{\pi}{4}\right) + 2$

a) State the domain:

$(-\infty, \infty)$, $x \neq \frac{\pi}{12} + \frac{\pi}{3}n$ (mint) $\frac{\pi}{4}$ -1

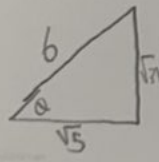
b) State the range:

$(-\infty, 0] \cup [4, \infty)$



17. Find the exact value of the expressions:

a) $\sin\left(\cos^{-1}\left(\frac{\sqrt{5}}{6}\right)\right)$



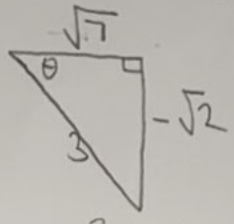
$(\sqrt{5})^2 + b^2 = 36$

$\sqrt{b^2} = \sqrt{31}$

$b = \sqrt{31}$

$\frac{\sqrt{31}}{6}$

b) $\sec\left(\sin^{-1}\left(-\frac{\sqrt{2}}{3}\right)\right)$



$= \frac{3}{\sqrt{7}}\left(\frac{\sqrt{7}}{\sqrt{7}}\right)$

$= \frac{3\sqrt{7}}{7}$

$(\pm\sqrt{2})^2 + b^2 = 9$

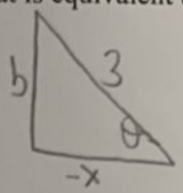
$2 + b^2 = 9$

$\sqrt{b^2} = \sqrt{7}$

$b = \sqrt{7}$

18. Write an algebraic expression that is equivalent to the expression:

$\tan\left(\cos^{-1}\left(\frac{x}{3}\right)\right)$; $x < 0$



$3^2 = (-x)^2 + b^2$

$9 = x^2 + b^2$

$b^2 = 9 - x^2$

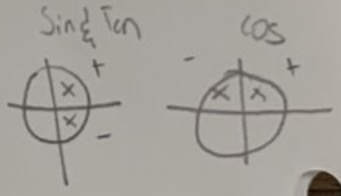
$b = \sqrt{9 - x^2}$

$\tan\theta = \frac{\sqrt{9-x^2}}{-x}$ -1

Correct Answer:

$\frac{\sqrt{9-x^2}}{|x|}$

19. Evaluate the expression:



a) $\sin^{-1}\left(\cos\frac{5\pi}{6}\right)$

$= \arcsin\left(-\frac{\sqrt{3}}{2}\right)$

$= -\frac{\pi}{3}$

b) $\cot^{-1}\left(\tan\frac{-5\pi}{6}\right)$

$\frac{+\frac{1}{2}}{+\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{\sqrt{3}}{3}$

$= \cot^{-1}\left(\frac{\sqrt{3}}{3}\right)$

$= \frac{\pi}{3}$

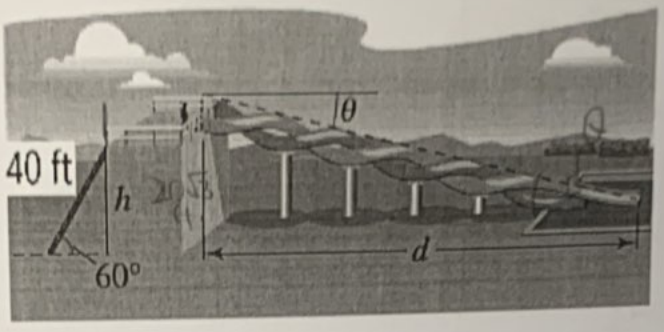
$\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{\sqrt{3}}{3}$

-1

20. The desig
of the ladd

hat is equivalent to

20. The designers of a water park are creating a new slide and have sketched some preliminary drawings. The length of the ladder is 40 feet, and its angle of elevation is 60° (see figure)



a) Find the height of the slide.

$\sin 60 = \frac{h}{40}$

$h = 20\sqrt{3}$ ft.

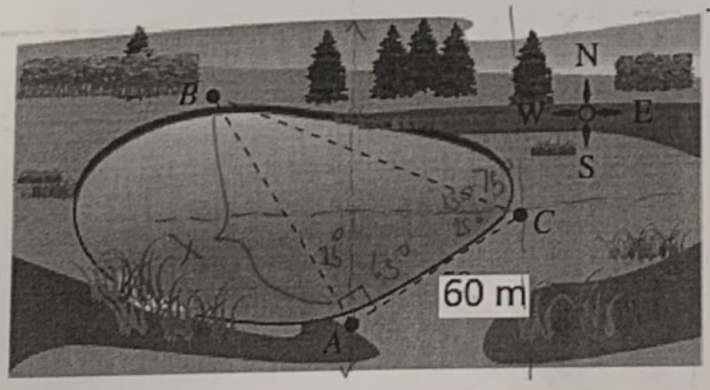
$\frac{40^{20}}{1} \cdot \frac{\sqrt{3}}{2_1} = h$

b) Find the angle of depression θ from the top of the slide to the end of the slide at the ground in terms of the horizontal distance d the rider travels.

$\tan\theta = \frac{20\sqrt{3}}{d}$

$\theta = \tan^{-1}\left(\frac{20\sqrt{3}}{d}\right)$

21. A surveyor wants to find the distance across a swamp (see figure). The bearing from A to B is N 25° W. The surveyor walks 60 meters from A, and at the point C the bearing to B is N 75° W.



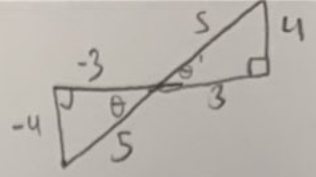
Find:

a) The bearing from A to C.

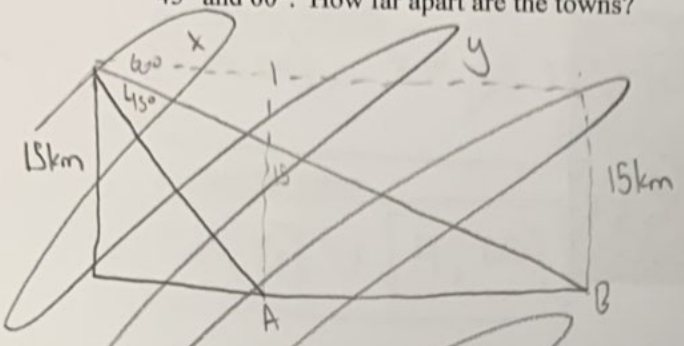
N 65° E

b) The distance from A to B.

$\tan 40 = \frac{X}{60} \Rightarrow$ $60\tan 40$ m



22. A passenger in an airplane at an altitude of 15 kilometers sees two towns directly to the east of the plane. The angles of depression to the towns are 45° and 60°. How far apart are the towns?



$\tan 45 = \frac{15}{x}$ $\tan 60 = \frac{15}{x+y}$

$x = 15\tan 45°$

$x = 15$ $\tan 60 = \frac{15}{15+y}$

$15\tan 60 + \tan 60\, y = 15$

$\tan 60\,(15 + y) = 15$

$15+y = \frac{15}{\sqrt{3}} \cdot \frac{1}{15}$

$\tan 60 = \sqrt{3}$

$y = \frac{1}{\sqrt{3}}\left(\frac{\sqrt{3}}{3}\right)$

Seperate Sheet -1

$y = 5\sqrt{3} - 15$ km

$15 - 5\sqrt{3}$ km

23. The point (-3,-4) is on the terminal side of an angle θ whose reference angle is θ'. Find:

a) $\tan\theta'$ $\frac{4}{3}$

b) $\sin\theta$ $-\frac{4}{5}$

c) $\sec\theta$

$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{-\frac{3}{5}}$

$= -\frac{5}{3}$

24. Determine if the function is odd, even, or neither.

a) $f(x) = x^3 + x\tan x$

$f(x) = (-x)^3 + -x\tan(-x)$

$= -x^3 + +x(\tan x)$

Neither

b) $f(x) = x^2 + \sec x$

$f(x) = (-x)^2 + \sec(-x)$

$= x^2 + \sec x$

even

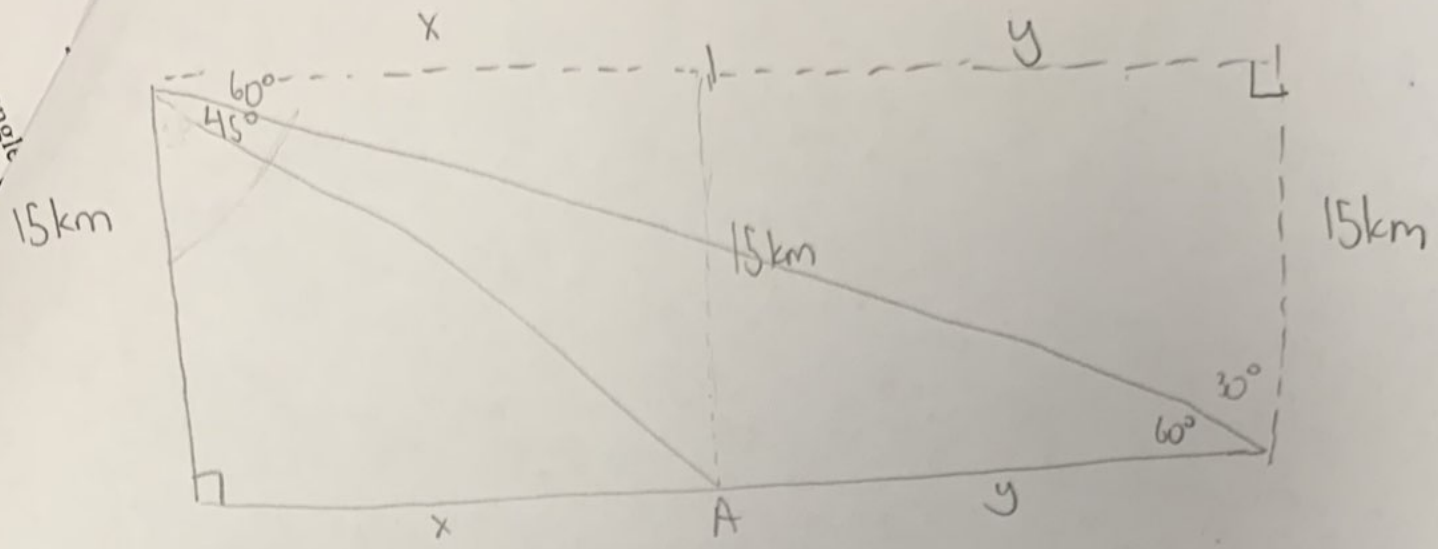
c) $f(x) = x^5\csc(x^4)$

$f(x) = (-x)^5\csc(-x)^4$

$= -(x^5\csc(x^4))$

odd

-1



$\tan 45 = \frac{15\text{km}}{x}$

$x = 15\text{km}$

$\tan 60 = \frac{15}{x+y}$

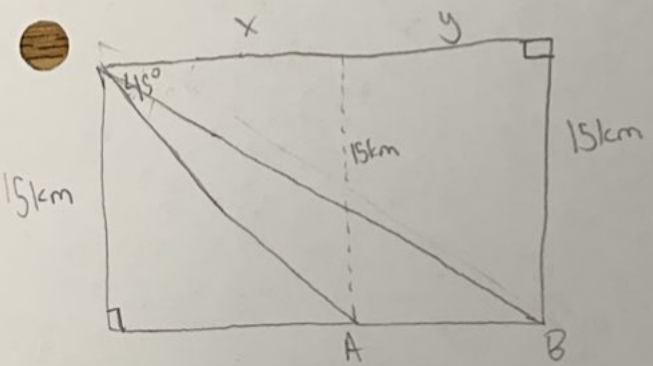
$\tan 60\,x + \tan 60\,y = 15$

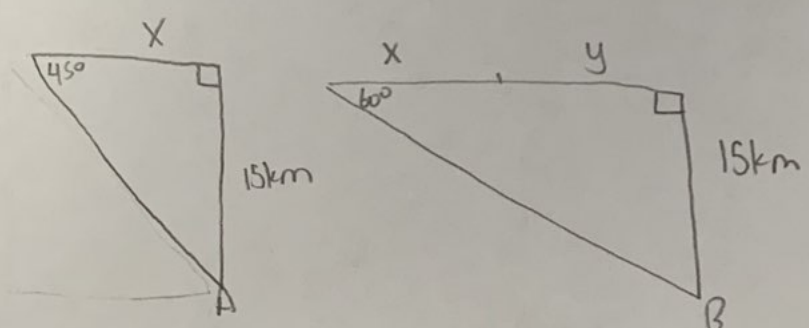
$\frac{15\sqrt{3}}{1} + \tan 60\,y = \left(\frac{15}{1} - \frac{15\sqrt{3}}{1}\right) \cdot \frac{1}{\sqrt{3}}$

$-15\sqrt{3}$

$y = \frac{15}{\sqrt{3}} + \frac{-(15\sqrt{3})}{\sqrt{3}}$

$y = \frac{15}{\sqrt{3}}$





$\tan 45 = \frac{15}{x}$

$x = 15$

$y = 5\sqrt{3} - 15\text{ km}$

$\tan 60 = \frac{15}{x+y}$

$\sqrt{3} = \frac{15}{x+y}$

$\sqrt{3} = \frac{15}{15+y}$

$15\sqrt{3} + \sqrt{3}y = 15$

$\frac{\sqrt{3}(15+y)}{\sqrt{3}} = \frac{15}{\sqrt{3}}$

$15+y = \frac{15\sqrt{3}-45}{3}$

$y = 5\sqrt{3} - 15$

$= \frac{15}{\sqrt{3}} + \frac{(-15)}{1}$

$= \frac{15 - 15\sqrt{3}}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)$

$= \frac{15\sqrt{3} - 45}{3}$

$= 5\sqrt{3} - 15$