Hon Pre-Calc Test Chapter 2 2018-2019

Show ALL Work!!! Circle ALL final answers!!! No Calculators!!

1. Put in $a + bi$ form: $\frac{2}{1+i} - \frac{3}{1-i}$

2. Find the value of k such that $x - 3$ is a factor of $x^3 - kx^2 + 2kx - 12$

3. Find the location of the relative minimum of the function: $f(x) = -2x^3 - 3x^2 + 72x - 7$

4. Find two positive real numbers whose product is a maximum such that the sum of the twice the first and three times the second is 60.

5. Consider: $f(x) = \frac{x^2-3x-4}{2x^2+x-1}$

a) Domain =

Range =

b) Identify any vertical asymptotes

c) Identify any horizontal asymptote

d) Identify any slant asymptotes

e) Identify the location of any holes

6. Given: $f(x) = \frac{2x^2+5x+2}{x^2-4}$

a) Use interval notation to write the domain and range

Domain =

Range =

b) Use limit notation to describe the end behavior.

c) Identify the x and y locations of any holes (write answer as an ordered pair).

d) Use limit notation to describe the behavior around any vertical asymptotes.

7. Write the solution to the inequality using interval notation:

$$\frac{3x}{x-1} \leq \frac{x}{x+4} + 3$$

8. Find the domain of x.

$$\sqrt{\frac{x-1}{2x^2+3x-2}}$$

9. Find a polynomial function of least degree in **STANDARD FORM** with integer coefficients that has $2+3i, 1-\sqrt{2}$, and $\frac{1}{2}$ as zeros and has the following end behavior: $\begin{cases} \lim\limits_{x\to\infty} f(x) = -\infty \\ \lim\limits_{x\to-\infty} f(x) = +\infty \end{cases}$

10. Solve the inequality: $2x^3 + 13x^2 - 8x - 46 \geq 6$

11. A Norman window is constructed by adjoining a semicircle to the top of an ordinary rectangular window (see figure). The perimeter of the entire window is 12 feet.

12. A driver averaged 70 mph on the round trip between Rochester Hills, Michigan, and Grayling, Michigan, 140 miles away. The average speeds for going and returning were x and y miles per hour respectively. Find an equation solved for y in terms of x.

Hon Pre Calculus
Test Chapter 2 Name _____

No Calculators!!! Show ALL Work!!! Circle ALL Final Answers!!!

1. Put in $a+bi$ form: $\frac{2}{1+i}-\frac{3}{1-i}$

$\frac{2(1-i)-3(1+i)}{1-i^2} = \frac{2-2i-3-3i}{1+1}$

$= \frac{-1-5i}{2} = -\frac{1}{2}-\frac{5}{2}i$

2. Find the value of k such that $x-3$ is a factor of $x^3-kx^2+2kx-12$.

	1	$-k$	$2k$	-12
3		3	$9-3k$	$27-3k$
	1	$3-k$	$9-k$	0

$-12+27-3k=0$

$15-3k=0$

$k=5$

	1	-5	10	-12
3		3	-6	12
	1	-2	4	0

3. Find the location of the relative minimum of the function: $f(x)=-2x^3-3x^2+72x-7$

$f'(x)=-6x^2-6x+72=0$

$x^2+6x-12=0$

$x=\frac{-6\pm\sqrt{36+48}}{2}=\frac{-6\pm\sqrt{84}}{2}=\frac{-6\pm2\sqrt{21}}{2}=-3\pm2\sqrt{21}$

$x=-3-2\sqrt{21}$

$x=-4$

4. Find two positive real numbers with a maximum product such that sum of the first and twice the second is 24.

x, y

$x+2y=24$

$x=24-2y$

$y(24-2y)=-2y^2+24y$

max: $\frac{-24}{-4}=6$

$x+12=24$

$x=12$

12 and 6

5. Consider: $f(x)=\dfrac{x^2-3x-4}{2x^2+x-1}$

a) Domain = $(-\infty,-1)\cup(-1,\frac{1}{2})\cup(\frac{1}{2},\infty)$

Range = $(-\infty,\frac{1}{3})\cup(\frac{1}{3},\frac{5}{3})\cup(\frac{5}{3},\infty)$

b) Identify any vertical asymptotes

$2x^2+x-1=0$

$(x+1)(2x-1)=0$

$x=-1,\ \frac{1}{2}$

$x=-\frac{1}{2}$

c) Identify any horizontal asymptotes

$\lim_{x\to\infty} f(x)=\frac{1}{2}$

$y=\frac{1}{2}$

d) Identify any slant asymptotes

None

e) Identify the location of any holes

$\dfrac{(x-4)(x+1)}{(x+1)(2x-1)}=\dfrac{x-4}{2x-1}$

When $x=-1$, there is a hole

$\lim_{x\to-1}\dfrac{x-4}{2x-1}=\dfrac{-5}{-3}=\dfrac{5}{3}$

Removable discontinuity: $(-1,\frac{5}{3})$

6. Consider: $f(x)=\dfrac{2x^2+5x+2}{x^2-4}$

a) Use interval notation to write the domain and range

Domain = $(-\infty,-2)\cup(-2,2)\cup(2,\infty)$

Range = $(-\infty,\frac{3}{4})\cup(\frac{3}{4},2)\cup(2,\infty)$

b) Use limit notation to describe the end behavior.

$\lim_{x\to\infty} f(x)=2$

$\lim_{x\to-\infty} f(x)=2$

c) Identify the x and y location of any holes (write answer as an ordered pair).

$\dfrac{(x+2)(2x+1)}{(x+2)(x-2)}=\dfrac{2x+1}{x-2}$

$\lim_{x\to-2}\dfrac{2x+1}{x-2}=\dfrac{-3}{-4}=\dfrac{3}{4}$

$(-2,\frac{3}{4})$

d) Use limit notation to describe the behavior around any vertical asymptotes.

VA: $x=2$

$\lim_{x\to2^+} f(x)=\infty$

$\lim_{x\to2^-} f(x)=-\infty$

7. Write the solution to the inequality using interval notation:

$$\frac{3x}{x-1} \le \frac{x}{x+4} + 3$$

$x \neq 1$, $x \neq -4$

$$\frac{3x}{x-1} - \frac{x}{x+4} \le 3$$

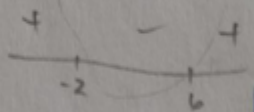
$$3x(x+4) - x(x-1) \le 3(x-1)(x+4)$$

$$3x^2 + 12x - x^2 + x \le 3(x^2 + 3x - 4)$$

$$2x^2 + 13x \le 3x^2 + 9x - 12$$

$$0 \le x^2 - 4x - 12$$

$$(x-6)(x+2) \ge 0$$



$(-\infty, -2] \cup [6, \infty)$

$[-2, 1)$

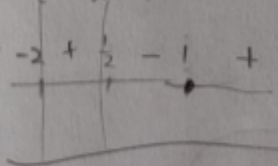
$(-\infty, -4) \cup (-4, -2] \cup [6, \infty)$

8. Find the domain of x.

$$\sqrt{\frac{x-1}{2x^2+3x-2}}$$

$$\frac{x-1}{2x^2+3x-2} \ge 0$$

$$\frac{x-1}{(x+2)(2x-1)} \ge 0$$



$(-2, \frac{1}{2}) \cup [1, \infty)$

9. Find a polynomial function of least degree in **STANDARD FORM** with integer coefficients that has $2+3i$, $1-\sqrt{2}$, and $\frac{1}{2}$ as zeros and has the following end behavior:

$$\begin{cases} \lim_{x \to \infty} f(x) = -\infty \\ \lim_{x \to -\infty} f(x) = +\infty \end{cases}$$

$$f(x) = (x-2-3i)(x-2+3i)(x-1+\sqrt{2})(x-1-\sqrt{2})(2x-1)$$

$$= -((x-2)^2 - 9i^2)((x-1)^2 - 2)(2x-1)$$

$$= -(x^2 - 4x + 4 + 9)(x^2 - 2x + 1 - 2)(2x-1)$$

$$= -(x^2 - 4x + 13)(x^2 - 2x - 1)(2x-1)$$

$$= -(x^4 - 2x^3 - x^2 - 4x^3 + 8x^2 + 4x + 13x^2 - 26x + 13)(2x-1)$$

$$= -(x^4 - 6x^3 + 20x^2 - 22x + 13)(2x-1)$$

$$= -(2x^5 - x^4 - 12x^4 + 6x^3 + 40x^3 - 20x^2 - 44x^2 + 22x + 26x - 13)$$

$$= -(2x^5 - 13x^4 + 46x^3 - 64x^2 + 48x - 13)$$

$$f(x) = -2x^5 + 13x^4 - 46x^3 + 64x^2 - 48x + 13$$

10. Solve the inequality: $2x^3 + 13x^2 - 8x - 46 \ge 6$

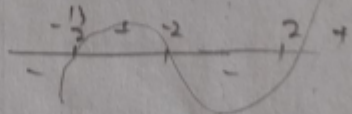
$$2x^3 + 13x^2 - 8x - 52 \ge 0$$

-2	2	13	-8	-52
		-4	-18	52
	2	9	-26	0

$$2x^2 + 9x - 26 = 0$$

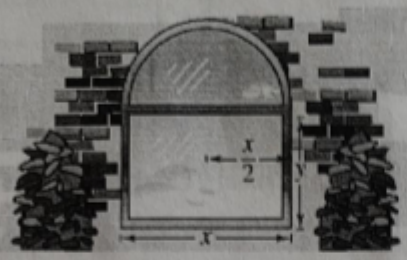
$$(x-2)(2x+13) = 0$$

$$x = 2, -\frac{13}{2}, -2$$



$[-\frac{13}{2}, -2] \cup [2, \infty)$

11. A Norman window is constructed by adjoining a semicircle to the top of an ordinary rectangular window (see - figure). The perimeter of the entire window is 12 feet.



$12 = x + 2y + \pi \cdot \frac{x}{2}$

$12 - x - \frac{\pi x}{2} = 2y$

$y = 6 - \frac{x}{2} - \frac{\pi x}{4}$

$A(x) = x \cdot y + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$

$= x\left(6 - \frac{x}{2} - \frac{\pi x}{4}\right) + \frac{1}{2}\pi\frac{x^2}{4}$

$= 6x - \frac{x^2}{2} - \frac{2\pi x^2}{8} + \frac{\pi x^2}{8}$

$A(x) = 6x - \frac{x^2}{2} - \frac{\pi x^2}{8}$

$A'(x) = 6 - x - \frac{\pi x}{4} = 0$

$x\left(-1 - \frac{\pi}{4}\right) = -6$

$x = \frac{+6}{1 + \frac{\pi}{4}} = \boxed{\frac{24}{4+\pi}}$

12. A driver averaged 70 mph on the round trip between Rochester Hills, Michigan, and Grayling, Michigan, 140 miles away. The average speeds for going and returning were x and y miles per hour respectively. Find an equation solved for y in terms of x.

$70 = \frac{280}{\text{total } t}$

$t = \frac{d}{r}$

$x = \frac{140}{t} \quad t = \frac{140}{x}$

$y = \frac{140}{t} \quad t = \frac{140}{y}$

$70 = \frac{280}{\frac{140}{x} + \frac{140}{y}}$

$\frac{140}{x} + \frac{140}{y} = 4$

$140y + 140x = 4xy$

$35y + 35x = xy$

$35y - xy = -35x$

$y = \frac{-35x}{35 - x} = \frac{35x}{x - 35}$

$\boxed{y = \frac{35x}{x-35}}$