Hon Pre-Calc
Test Chapter 2

95

92

Name [redacted]

No Calculators!!! Circle ALL final answers!!! Good Luck!!

Short Answer

1. Find two positive real numbers whose product is a maximum such that the sum of the twice the first and three times the second is 60..

$A = xy$

$2x + 3y = 60$

$\frac{3y}{3} = \frac{60-2x}{3}$

$y = \frac{60-2(15)}{3} = 10$

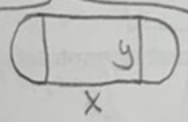
$A = x\left(\frac{60-2x}{3}\right)$

0 30

15

15, 10

2. An indoor physical fitness room consists of a rectangular region with a semicircle on each end. The perimeter of the room is to be a 200 meter single lane running track. Let x represent the length of the rectangular region and y represent the width. What should x and y be to produce a rectangle of maximum area?



$A = xy$

$200 = 2x + \frac{2\pi y}{2}$

$200 = 2x + \pi y$

$\pi y = 200 - 2x$

$y = \frac{200-2x}{\pi}$

$A = x\left(\frac{200-2x}{\pi}\right)$

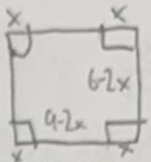
$y = \frac{200-2(50)}{\pi}$

$y = \frac{100}{\pi}$

0 100

50

$x = 50$



3. An open box is to be made from a rectangular piece of material, 9 cm by 6 cm, by cutting equal squares from the corners and turning up the sides.

a) Write a function for the volume of the box in terms of x (the length of one of the sides of the cut out corners.

$V(x) = x(9-2x)(6-2x)$

b) What is the domain of the Volume function?

$9-2x > 0$ $6-2x > 0$

$x > \frac{-9}{-2}$ $x > 3$

D: (0, 3)

c) What should x be to maximize the volume?

$V(x) = x(54 - 12x - 18x + 4x^2)$

$= x(54 - 30x + 4x^2)$

$= 4x^3 - 30x^2 + 54x$

$V'(x) = 12x^2 - 60x + 54$

$= 6(2x^2 - 10x + 9)$

$x = \frac{10 \pm \sqrt{(-10)^2 - 4(2)(9)}}{4} = \frac{10 \pm \sqrt{28}}{4}$

$= \frac{10}{4} \pm \frac{4\sqrt{7}}{4}$

$= \frac{5}{2} \pm \sqrt{7}$

$= \frac{5}{2} - \sqrt{7}$ cm

$\frac{5}{2} - \frac{\sqrt{7}}{2}$ cm

$10 - 4\sqrt{7}$ / 4

4. Find the value of k such that $x - 4$ is a factor of $x^3 - kx^2 + 2kx - 8$

$= (4)^3 - k(4)^2 + 2k(4) - 8$

$= 64 - k16 + 8k - 8$

$= -8k + 56$

$-8k = -56$

$k = 7$

	1	-7	14	-8
4		↓	-12	8
	1	-3	2	0

P N I
4 0 0
2 0 2
0 0 4

5. Perform the operation and write the result in standard form.

$= \frac{2i}{2+i} + \frac{5}{2-i}$

$= \frac{2i(2-i) + 5(2+i)}{4-i^2}$

$= \frac{4i - 2i^2 + 10 + 5i}{5}$

$= \frac{12+9i}{5}$

$= \boxed{\frac{12+9i}{5}}$

6. Write the polynomial as a product of linear factors.

$g(x) = x^4 - 4x^3 + 8x^2 - 16x + 16$

P N I
4 0 0
2 0 2
0 0 4

1, 2, 4, 8, 16

$\pm\{1, 2, 4, 8, 16\}$

2	1	-4	8	-16	16
	↓	2	-4	8	-16
	1	-2	4	-8	0

2	1	-2	4	-8
	↓	2	0	8
	1	0	4	0

$x^2 + 4$

$x^3 - 2x^2 + 4x - 8$

$x^2(x-2) + 4(x-2)$

$= (x^2+4)(x-2)(x-2)$

$x^2 + 4 = 0$

$x^2 = -4$

$x = \sqrt{-4}$

$x = 2i$

$\boxed{(x-2i)(x+2i)(x-2)(x-2)}$

7. Find all complex zeros given $f(4+3i) = 0$:

$f(x) = x^4 - 8x^3 + 29x^2 - 32x + 100$

$(x-(4+3i))$

$(x-(4+3i))$

$(x-4-3i)$

$((x-4)-3i)$

$(x-4)^2 - 9i^2$

$x^2 - 8x + 16 + 9$

$x^2 - 8x + 25$

$x^2 + 4$

$x^2 - 8x + 25 \;\overline{)\; x^4 - 8x^3 + 29x^2 - 32x + 100}$

$-x^4 + 8x^3 - 25x^2$

$4x^2 - 32x + 100$

$-4x^2 + 32x - 100$

0 0 0

$x^2 + 4 = 0$

$x^2 = \sqrt{-4}$

$x = 2i$

$\boxed{x = 4+3i, 4-3i, 2i, -2i}$

8. Given: $f(x) = \frac{x^2 - x - 6}{x^3 + x^2 - 4x - 4}$

$x^2(x+1) - 4(x+1)$

$(x+2)(x-3)$

$(x+2)(x-2)(x+1)$

hole @ $x = -2$

a) Identify all vertical asymptote(s)

$\boxed{x = 2, x = -1}$

b) Identify any horizontal asymptote(s)

$\boxed{y = 0}$

c) Identify any holes

$\frac{(-2-3)}{(-2-2)(-2+1)} = \frac{-5}{4}$

$\boxed{\left(-2, -\frac{5}{4}\right)}$

d) State the domain

$\boxed{(-\infty, -2) \cup (-2, -1) \cup (-1, 2) \cup (2, \infty)}$

e) State the range

$\boxed{(-\infty, 0) \cup (\frac{50}{49}, \infty)}$

f) Use limit notation to describe the behavior around any removable discontinuities.

$\lim_{x \to -2^-} f(x) = -\frac{5}{4}$

$\lim_{x \to -2^+} f(x) = -\frac{5}{4}$

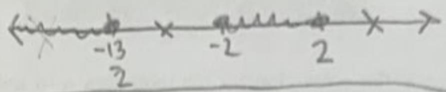
9. Solve the following inequality:
$2x^3 + 13x^2 - 8x - 46 \leq 6$

$2x^3 + 13x^2 - 8x - 52 \leq 0$

$x^2(2x+13) - 4(2x+13) \leq 0$

$(x+2)(x-2)(2x+13) \leq 0$

$-2, 2, \frac{-13}{2}$



$\boxed{(-\infty, -\frac{13}{2}] \cup [-2, 2]}$

10. Solve the following inequality: $\frac{3x}{x-1} \leq \frac{x}{x+4} + 3$

$\frac{3x}{x-1} \leq \frac{4x+12}{x+4}$

$\frac{3x}{x-1}\frac{(x+4)}{(x+4)} - \frac{4x+12(x-1)}{x+4(x-1)} \leq 0$

$\frac{3x^2+12x - (4x^2+8x-12)}{x+4(x-1)}$

$\frac{-(x+2)(x-6)}{(x+4)(x-1)} \leq 0$

$-2, 6, -4, 1$

$\frac{3x^2+12x-4x^2-8x+12}{(x+4)(x-1)} \leq 0$

$\frac{-x^2+4x+12}{(x+4)(x-1)} \leq 0$

$\frac{-(x^2-4x-12)}{(x+4)(x-1)} \leq 0$

$(-\infty, -4) \cup (-4, -2] \cup [-2, 1) \cup (1, 6] \cup [6, \infty)$

$\boxed{(-\infty, -4) \cup [-2, 1) \cup [6, \infty)}$

11. State the domain: $f(x) = \sqrt{\frac{x}{x^2 - 2x - 35}}$

$\frac{x}{x^2-2x-35} \geq 0$

$\frac{x}{(x-7)(x+5)} \geq 0$

$0, 7, -5$

$(-\infty, -5) \cup [-5, 0] \cup (0, 7) \cup (7, \infty)$

$\boxed{[-5, 0] \cup (7, \infty)}$

1,2   1,2,3,4,6,12

12. Given: $f(x) = 2x^5 - 2x^4 + 7x^3 - 11x^2 + 22x - 12$

a) Write a P, N, I chart

P	N	I
5	0	0
3	0	2
1	0	4

b) List all possible rational zeros

$\boxed{\pm \{1, 2, 3, 4, 6, 12, \frac{1}{2}, \frac{3}{2},}$

-1

13. Given: $h(x) = \frac{x^2}{x-1}$

a) State the domain (interval notation)

$\boxed{D: (-\infty, 1) \cup (1, \infty)}$

b) Write and LABEL any and all asymptotes.

Vert Asymptote ⇒ $x = 1$

$x - 1 \overline{)\, x^2}$ quotient x; $-x^2 + x$; x

Horizontal Asymptote ⇒ $y = x$

-4

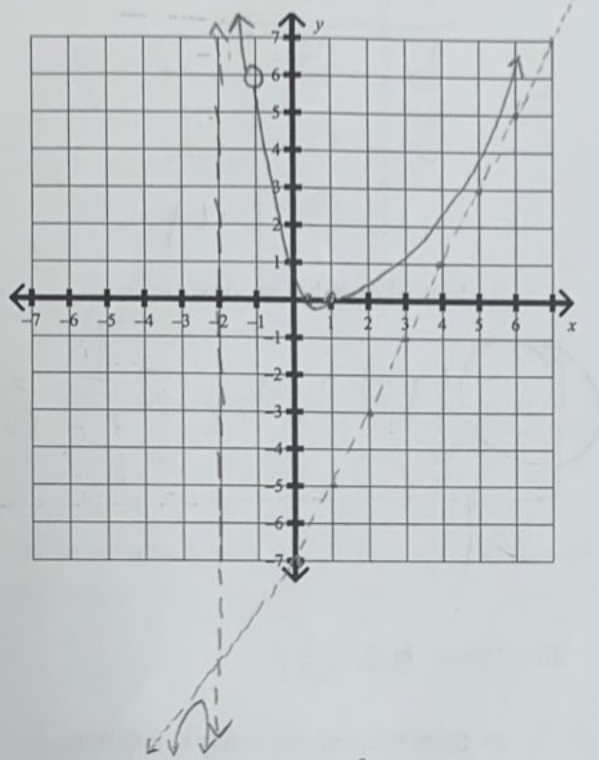
$\frac{(x+1)(x-1)(2x-1)}{(x+1)(x+2)}$ $\frac{x^2(2x-1)-1(2x-1)}{(x+1)(x+2)}$

14. Sketch a graph: $f(x) = \dfrac{2x^3 - x^2 - 2x + 1}{x^2 + 3x + 2}$

Hole @ (-1, 6)

Vert Asymptote $x = -2$

Zeros: $1, \frac{1}{2}$



$$x^2+3x+2 \overline{)\,2x^3 - x^2 - 2x + 1}$$

$2x - 7$

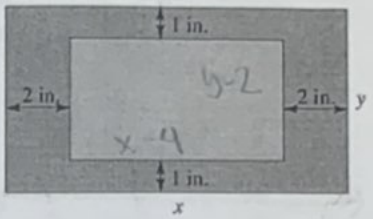
$-2x^3 + 6x^2 + 4x$ ↓

$-7x^2 - 6x + 1$

$-7x^2 + 21x + 14$

$-15x - 15$

15. **PAGE DESIGN** A page that is x inches wide and y inches high contains 30 square inches of print. The top and bottom margins are 1 inch deep, and the margins on each side are 2 inches wide (see figure).



(a) Write a function for the total area A of the page in terms of x.

(b) Determine the domain of the function based on the physical constraints of the problem.

$A = xy$

$30 = (x-4)(y-2)$

$y = \frac{30}{x-4} + 2$

$A = x\left(\frac{30}{x-4} + \frac{2}{1}\right)$

$A = x\left(\frac{2x+22}{x-4}\right)$

$A = \frac{2x^2 + 22x}{x-4}$

$x \neq 4$

$x < 4$ $x > 4$

D: $(4, \infty)$