Hon Pre-Calc

Quest Chapter 10

Special Polar Graphs   Part 1

Name [redacted]

No Graphing Calculators!!! Make sure tables are complete with exact and approximate values!!!

Short Answer

$33/36$

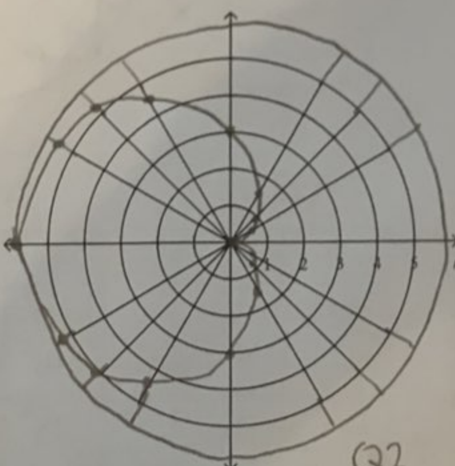
$r = 3 - 3\cos\theta$

1. Given: $r = 3(1 - \cos\theta)$

   a) Specifically name the graph Limaçon Cardioid

   b) Create an appropriate table of values.

   c) Plot the points on the given graph and use symmetry to complete the graph.



Q1

Θ	r
$0°$	0
$30°$	$3-\frac{3\sqrt{3}}{2} \approx 0.40$
$45°$	$3-\frac{3\sqrt{2}}{2} \approx 0.88$
$60°$	1.5
$90°$	3

Q2

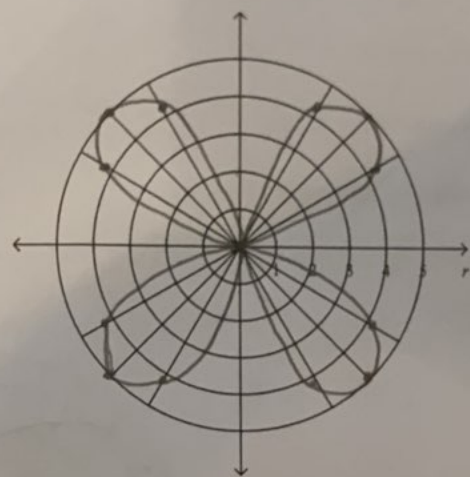
Θ	r
$90°$	3
$120°$	4.5
$135°$	$3+\frac{3\sqrt{2}}{2} \approx 5.12$
$150°$	$3+\frac{3\sqrt{3}}{2} \approx 5.598$
$180°$	6

2. Given: $r = 5\sin 2\theta$

a) Specifically name the graph 4 petal rose

b) Create an appropriate table of values.

c) Plot the points on the given graph and use symmetry to complete the graph.



Q1

θ	r
0°	0
30°	$\frac{5\sqrt{3}}{2} \approx 4.33$
45°	5
60°	$\frac{5\sqrt{3}}{2} \approx 4.33$
90°	0

Q2

θ	r
90°	0
120°	$-\frac{5\sqrt{3}}{2} \approx -4.33$
135°	-5
150°	$-\frac{5\sqrt{3}}{2} \approx -4.33$
180°	0

Q3

θ	r
180°	0
210°	$\frac{5\sqrt{3}}{2} \approx 4.33$
225°	5
240°	$\frac{5\sqrt{3}}{2} \approx 4.33$
270°	0

Q4

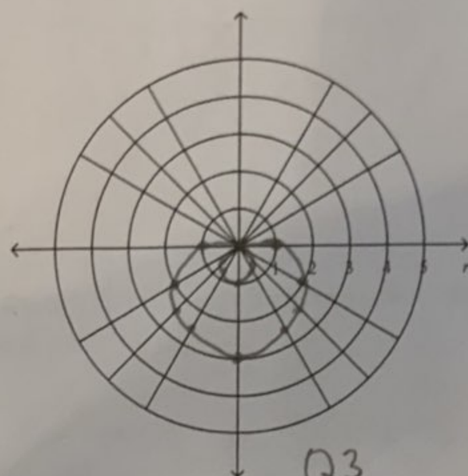
θ	r
270°	0
300°	$-\frac{5\sqrt{3}}{2} \approx -4.33$
315°	-5
330°	$-\frac{5\sqrt{3}}{2} \approx -4.33$
360°	0

3. Given: $r = 1 - 2\sin\theta$

a) Specifically name the graph Limaçon With Inner Loop

b) Create an appropriate table of values.

c) Plot the points on the given graph and use symmetry to complete the graph.



Q1

θ	r
0°	1
30°	0
45°	$1-\sqrt{2} \approx -0.41$
60°	$1-\sqrt{3} \approx -0.73$
90°	-1

Q3

θ	r
180°	1
210°	2
225°	$1+\sqrt{2} \approx 2.41$
240°	$1+\sqrt{3} \approx 2.73$
270°	3

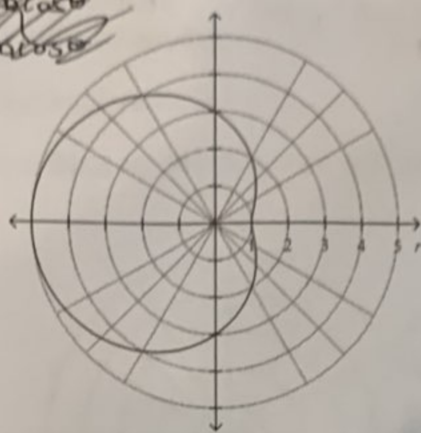
Max = 5 $\quad -(a+b) = -5 \quad a-b=1$

Min = 1 $\quad a+b=5 \quad a=1+b \quad r = a-b\cos\theta$

$1+b+b=5 \quad a=3 \quad r=3-2\cos\theta$ Limaçon

$b=2$ Cardioid Shift Left 1

4. Write a possible polar equation for the following graph.



$r = a-b\cos\theta$ max = 1 min = −6

$-1 = a-b \quad -6 = a+b$

$-1 = -b-b-6 \quad a = -b-6$

$-1 = -2b-6$

$-2b = 5$

$b = -\frac{5}{2}$

$r-1 = \cancel{3-3\cos\theta}$

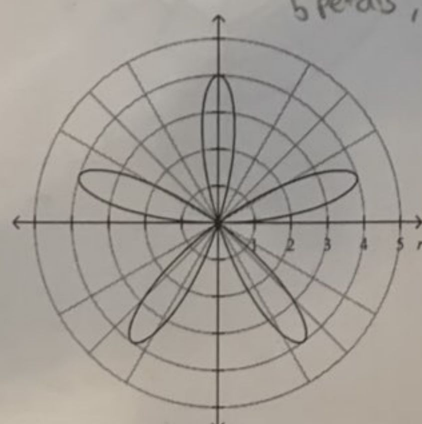
$r = \boxed{3-2\cos\theta}$

, 2

5. Write a possible polar equation for the following graph.

5 petals, length of 4

$r = 4\sin 5\theta$



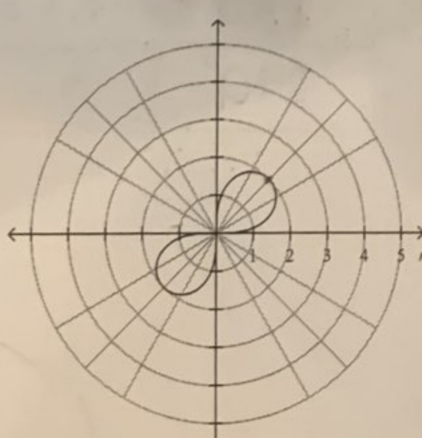
$r = 4\sin 5\theta$

6. Write a possible polar equation for the following graph.

$r^2 = a^2\sin\theta \qquad \sqrt{\frac{4}{\sin 45}} = \sqrt{a^2}\,\frac{\sin(45)}{\sin 45}$

$r^2 = 8\sin\theta$

$a=$



$r^2 = a^2\sin\theta$

−1

$r^2 = 4\sin\theta$

$\boxed{r^2 = 4\sin(2\theta)}$

−3

/

Hon Pre-Calc Part 2
Quest 10.6 - 10.9
& Chapter Review Name [redacted] $\frac{51}{64}$
Graphing Calculators Allowed

Show All Work!!!! All Polar Equations Solved For "r"!!! Circle All Final Answers!!!

Short Answer

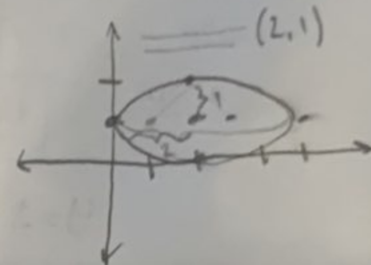
1. Given: An ellipse has vertices at (0,1), (4,1) and the endpoints of its minor axis are (2,0), (2,2).

2a=

a) Find the standard form of the equation.

$$\boxed{\frac{(x-2)^2}{4}+\frac{(y-1)^2}{1}=1}$$

(2,1)



$a=2$
$b=1$

b) Find the foci coordinates.

$\boxed{(2-\sqrt{3},1) \quad (2+\sqrt{3},1)}$

$c^2=a^2-b^2$
$\sqrt{c^2}=\sqrt{3}$
$c=\pm\sqrt{3}$

2. Given: $9x^2-16y^2-18x-32y-151=0$

$9x^2-18x \quad -16y^2-32y = 151+9-16$
$9(x^2-2x+1)-16(y^2-2y+1)=144$
$\frac{(x-1)^2}{16}-\frac{(y-1)^2}{9}=1$

$\boxed{(1,\cancel{1})}$ a) Find the center.

2a) (1,-1)

$\boxed{(1,6) \quad (1,-4)}$ b) Find the foci.

2b) (6,-1), (-4,-1)

$c^2=a^2+b^2$
$c^2=25$
$c=5$

c) Find asymptote equations.

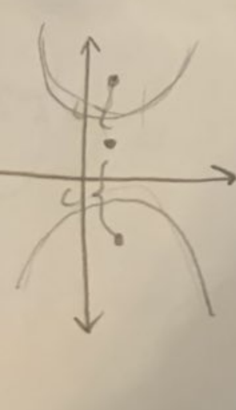
$y=k\pm\frac{A}{2}(x-h)$

$\boxed{y=-2x+3 \quad y=2x-1}$ 2

$y=1\pm2(x-1)$

$y=1+2x-2 \quad y=-2x+3$
$y=2x-1$

2c)
$y+1=\pm\frac{3}{4}(x+1)$



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3. Eliminate the parameter and solve for y.

a) $\sqrt[3]{x} = \sqrt[3]{t^3}$ $t = \sqrt[3]{x}$

$y = 3\ln t$

$y = 3\log_e x^{1/3}$

$\boxed{y = \log_e x}$

b) $x = \cos\theta$

$2\sin\theta\cos\theta$ $y = 2\sin 2\theta$

$y = 2\sin\theta\, x$

$\cancel{x^2 = 1 - \sin^2\theta}$

$x = \cancel{(\sin\theta)}(1-\sin\theta)$

$\boxed{y = 4x^2 - 4x}$

$y = \pm 4x\sqrt{1-x^2}$

$2 \cdot 2\sin\theta\cos\theta$

$\frac{y}{4} = \sin\theta\cos\theta$

$\frac{y}{4} = (1-\cos^2\theta)\cos^2\theta$

$\frac{y}{4} = (1-x)x$

$y = 4(x^2 - x)$

$y = 4x^2 - 4x$

c) Eliminate the parameter "θ" to obtain the standard form of the conic.

$x = h + a\sec\theta$

$y = k + b\tan\theta$

$\frac{(x-h)^2}{a^2} = \sec^2\theta$

$\frac{(y-k)^2}{b^2} = \tan^2\theta$

$$\boxed{\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1}$$

4. Find a set of parametric equations for the rectangular equation using:

$t = 2 - x$ $x = t + 2$

$y = x^2 - 3$

$y = (t+2)^2 - 3$

$$\boxed{\begin{cases} y = t^2 + 4t + 1 \\ x = t + 2 \end{cases}}$$

$$\begin{cases} x = 2 - t \\ y = t^2 - 4t + 1 \end{cases}$$

-4

5. Given: polar point $\left[-2, \frac{7\pi}{6}\right]$

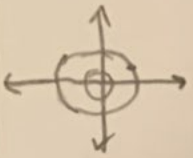
$\cos\frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$

$\sin\frac{7\pi}{6} = -\frac{1}{2}$

a) Convert to an **exact** rectangular point.

$\left(-2\cos\frac{7\pi}{6}, -2\sin\frac{7\pi}{6}\right)$

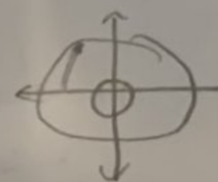
$\boxed{(\sqrt{3}, 1)}$



b) List three other **exact** representations of the point.

$\boxed{\left[2, \frac{\pi}{6}\right] \quad \left[-2, \frac{-5\pi}{6}\right] \quad \left[2, \frac{-11\pi}{6}\right]}$

6. Convert $\left(-1, \sqrt{3}\right)$ to an **exact** polar point with $r \geq 0$ and $-2\pi \leq \theta < 0$



$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{-1}\right)$

$r = \sqrt{1+3}$

$r = -2$

$\left[-2, -\frac{\pi}{3}\right]$

$\theta = \tan^{-1}\left(-\sqrt{3}\right)$

$r = 2$

$\boxed{\left[2, -\frac{4\pi}{3}\right]}$

$\theta = -\frac{\pi}{3}$

7. Convert the following equations from polar to rectangular.

a) $\theta = \dfrac{11\pi}{6}$

$\frac{\sin}{\cos} \cdot \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{\sqrt{3}}{3}$

$\tan^{-1}\left(\frac{y}{x}\right) = \frac{11\pi}{6}$

tan tan

$\frac{y}{x} = -\frac{\sqrt{3}}{3} \cdot x$

$\boxed{y = -\frac{\sqrt{3}}{3}x}$

b) $r = \dfrac{2}{1+\sin\theta}$

$r + r\sin\theta = 2$

$r + y - 2$

$r^2 = (2-y)^2$

$x^2 + \cancel{y^2} = 4 - 4y + \cancel{y^2}$

$x^2 = 4 - 4y$

$\boxed{x^2 = -4(y-1)}$

c) $r = -2\csc\theta$

$r = \frac{-2}{\sin\theta}$

$r\sin\theta = -2$

$\boxed{y = -2}$

d) $r = \dfrac{1}{1+2\cos\theta}$

$1 = r + 2r\cos\theta$

$1 = r + 2x$

$(1-2x)^2 = x^2 + y^2$

$1 - 4x + 4x^2 = x^2 + y^2$

$3x^2 - 4x \quad -y^2 = -1 + \frac{4}{3}$

$3\left(x^2 - \frac{4}{3}x + \frac{4}{9}\right) - y^2 = \frac{1}{3}$

$3\left(x - \frac{2}{3}\right)^2 - y^2 = \frac{1}{3}$

$\frac{\left(x-\frac{2}{3}\right)^2}{\frac{1}{9}} - \frac{y^2}{\frac{1}{3}} =$

$\boxed{\frac{\left(x-\frac{2}{3}\right)^2}{\frac{1}{9}} - \frac{y^2}{\frac{1}{3}} = 1}$

8. Covert the following rectangular equations to polar.

a) $x = 10$

$$\frac{r\cancel{\cos\theta}}{\cancel{\cos\theta}} = \frac{10}{\cos\theta}$$

$$\boxed{r = 10\sec\theta}$$

b) $y^2 - 8x - 16 = 0$

$(r^2\sin^2\theta) - 8r\cos\theta - 16 = 0$

$(\sin^2\theta)\, r^2 + (-8\cos\theta)\, r - 16 = 0$

$$\boxed{r = \frac{4\cos\theta \pm 4}{\sin^2\theta}}$$

$$64(\cos^2\theta + \sin^2\theta)$$

$$r = \frac{8\cos\theta \pm \sqrt{64\cos^2\theta + 64\sin^2\theta}}{2\sin^2\theta}$$

$$r = \frac{4\cos\theta \pm 4}{\sin^2\theta} \qquad r = \frac{4}{1-\cos\theta} \text{ or } r = \frac{-4}{1+\cos\theta}$$

c) $\cancel{xy^2} = \cancel{16}\ 256$

$r^4\sin^2\theta\cos^2\theta = \cancel{16}\ 256$

$$r^4 = \frac{\cancel{16}\ 256}{\sin^2\theta\cos^2\theta}$$

$$r^2 = \frac{16}{\sin\theta\cos\theta}$$

$$\boxed{r = 4 \pm \sqrt{\sec\theta\csc\theta}}$$

$$r^2 = 32\csc(2\theta)$$

d) $\dfrac{x^2}{\frac{1}{3}} + \dfrac{\left(y+\frac{1}{3}\right)^2}{\frac{4}{9}} = 1$

$9 - 9\cos^2\theta$

$$\left(\frac{4}{9}x^2 + \frac{1}{3}\left(y+\frac{1}{3}\right)^2\right) = \left(\frac{4}{27}\right)^{27}$$

$12x^2 + 9\left(y+\frac{1}{3}\right)^2 = 4$

$12r^2\cos^2\theta + 9r^2\sin^2\theta + 6r\sin\theta - 3 = 0$

$r^2(9 + 3\cos^2\theta) + (6\sin\theta)r - 3 = 0$

$$\boxed{r = \frac{3}{9+3\cos^2\theta} \qquad r = \frac{-9}{9+3\cos^2\theta}}$$

$9\left(y^2 + \frac{2}{3}y + \frac{1}{9}\right)$

$9y^2 + 6y + 1$

$$r = \frac{1}{2+\sin\theta} \text{ or } r = \frac{-1}{2-\sin\theta}$$

$$r = \frac{-6\sin\theta \pm \sqrt{36\sin^2\theta + 12(9+3\cos^2\theta)}}{2(9+3\cos^2\theta)}$$

$$r = -6\sin\theta \pm 6\sqrt{\sin^2\theta + 3 + \cos^2\theta}$$

$$r = \frac{-6\sin\theta \pm 12}{2(9+3\cos^2\theta)}$$

-4

$$r = \frac{-3 \pm 6}{9+3\cos^2\theta}$$

Correct Work for Missed ~~the~~ Questions:

2.

a) $9x^2-16y^2-18x-32y-151=0$

$9(x^2-2x)-16(y^2+2y)=151$

$9(x^2-2x+1)-16(y^2+2y+1)=151-16+9$

$\frac{9(x-1)^2}{144}-\frac{16(y+1)^2}{144}=\frac{144}{\cancel{144}}\ 1$

$\frac{(x-1)^2}{16}-\frac{(y+1)^2}{9}=1$

$\boxed{\text{Center: } (1,-1)}$

b) Center = $(1,-1)$

Horizontal Hyperbola

$a=4$

$b=3$

$c^2=a^2+b^2$

$c^2=16+9$

$c=5$

Foci points

$(1\pm5,-1)$

$\boxed{(6,-1) \text{ and } (-4,-1)}$

c) $y-k=\pm\frac{b}{a}(x-h)$

$y-(-1)=\pm\frac{3}{4}(x-1)$

$\boxed{y+1=\pm\frac{3}{4}(x-1)}$

3.

b) $x=\cos\theta$

$y=2\sin2\theta$

$y=2(2\sin\theta\cos\theta)$

$y=4\sin\theta\cos\theta$

$\sin^2\theta+\cos^2\theta=1$

$\sin\theta=\pm\sqrt{1-\cos^2\theta}$

$y=4\cos\theta\pm\sqrt{1-\cos^2\theta}$

$\boxed{y=\pm4x\sqrt{1-x^2}}$

4.

$t=2-x$

$y=x^2-3$

$x=-t+2$

$y=(-t+2)^2-3$

$y=t^2-4t+4-3$

$\boxed{\begin{cases}x=-t+2\\y=t^2-4t+1\end{cases}}$

Continued Chapter 10 Correct Work for Misseed Questiocs

8.

b) $y^2 - 8x - 16 = 0$

$x = r\cos\theta$

$y = r\sin\theta$

$(r\sin\theta)^2 - 8(r\cos\theta) - 16 = 0$

$(\sin^2\theta) r^2 + (-8\cos\theta) r - 16 = 0$

$r = \dfrac{8\cos\theta \pm \sqrt{64\cos^2\theta + 64\sin^2\theta}}{2\sin^2\theta}$

$\sqrt{64(\cos^2\theta + \sin^2\theta)}$

$\sqrt{64(1)}$

$\sqrt{64}$

$= 8$

$r = \dfrac{8\cos\theta \pm 8}{2\sin^2\theta}$

$\sin^2\theta = 1 - \cos^2\theta$

$(1+\cos\theta)(1-\cos\theta)$

$r = \dfrac{4\cos\theta \pm 4}{\sin^2\theta}$

$r = \dfrac{4\cos\theta + 4}{(1-\cos^2\theta)}$

$r = \dfrac{4\cancel{(\cos\theta + 1)}}{\cancel{(1+\cos\theta)}(1-\cos\theta)}$

$\boxed{r = \dfrac{4}{1-\cos\theta}}$

$r = \dfrac{4\cos\theta - 4}{(1-\cos^2\theta)}$

$r = \dfrac{4(\cos\theta - 1)}{(1-\cos\theta)(1+\cos\theta)} * \left(\dfrac{-1}{-1}\right)$

$r = \dfrac{-4\cancel{(\cos\theta - 1)}}{\cancel{(-1+\cos\theta)}(1+\cos\theta)}$

$\boxed{r = \dfrac{-4}{1+\cos\theta}}$

Correct Work Chapter 10 Test

8.

c) $xy=16$

$x = r\cos\theta$

$y = r\sin\theta$

$(xy)^2 = 256$

$x^2y^2 = 256$

$(r^2\cos^2\theta)(r^2\sin^2\theta) = 256$

$r^4 \sin^2\theta \cos^2\theta = 256$

$\sqrt{r^4} = \sqrt{\dfrac{256}{\sin^2\theta\cos^2\theta}}$

$\csc(2\theta) = \dfrac{1}{2\cos\theta\sin\theta}$

$r^2 = \dfrac{16}{\sin\theta\cos\theta}$

$r^2 = \dfrac{16}{\sin\theta\cos\theta}\left(\dfrac{2}{2}\right)$

$r^2 = \dfrac{32}{2\sin\theta\cos\theta)} = \dfrac{32}{1} \cdot \dfrac{1}{2\cos\theta\sin\theta)}$

$\boxed{r^2 = 32\csc(2\theta)}$

Correct Work Chapter 10 Test

8.

a) $\frac{x^2}{\frac{1}{3}} + \frac{\left(y+\frac{1}{3}\right)^2}{\frac{4}{9}} = 1$

$x = r\cos\theta$
$y = r\sin\theta$

$4\left(3x^2 + \frac{9\left(y+\frac{1}{3}\right)^2}{4}\right) = 1 \times 4$

$12x^2 + 9\left(y^2 + \frac{2}{3}y + \frac{1}{9}\right) = 4$

$12x^2 + 9y^2 + 6y + 1 = 4$

$\downarrow$

$12(r^2\cos^2\theta) + 9(r^2\sin^2\theta) + 6(r\sin\theta) + 1 = 4$

$(12\cos^2\theta + 9\sin^2\theta)r^2 + (6\sin\theta)r - 3 = 0$

$r = \frac{-6\sin\theta \pm \sqrt{36\sin^2\theta - 4(12\cos^2\theta + 9\sin^2\theta)(-3)}}{24\cos^2\theta + 18\sin^2\theta}$

$r = \frac{-6\sin\theta \pm \sqrt{144(\sin^2\theta + \cos^2\theta)}}{24\cos^2\theta + 18\sin^2\theta}$

$r = \frac{-6\sin\theta \pm 12}{24(1-\sin^2\theta) + 18\sin^2\theta}$

$r = \frac{-6\sin\theta \pm 12}{24 - 6\sin^2\theta}$

$r = \frac{-\sin\theta \pm 2}{4 - \sin^2\theta}$

$r = \frac{-\sin\theta + 2}{4 - \sin^2\theta}$

$r = \frac{-\cancel{\sin\theta + 2}}{\cancel{(2-\sin\theta)}(2+\sin\theta)}$

$\boxed{r = \frac{1}{2+\sin\theta}}$

$r = \frac{-\sin\theta - 2}{4 - \sin^2\theta}$

$r = \frac{-1\cancel{(\sin\theta + 2)}}{\cancel{(2+\sin\theta)}(2-\sin\theta)}$

$\boxed{r = \frac{-1}{2-\sin\theta}}$