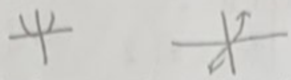
Hon Pre-Calc
Test Chapter 1 Name [redacted]

84/90

Show All Work!!! Circle All Final Answers!! No Calculators!!!

Short Answer

even odd



1. Use the algebraic test to determine which type of symmetry the following equation has and if it is even or odd or neither.:

a) $xy^2 + 8y^4 = 0$ X-axis symmetry

$-xy^2 + 8y^6 = 0$ $x(-y)^2 + 8(-y)^6 = 0$

not y-axis $xy^2 + 8y^6 = 0$

x-axis symmetry

neither

b) $y = \dfrac{x^5 + x^3}{x^2 + 1}$ Origin Symmetry

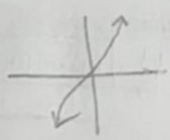
$-y = \dfrac{(-x)^5 + (-x)^3}{(-x)^2 + 1}$

odd

$-y = \dfrac{-x^5 + -x^3}{x^2 + 1}$ $\to y = \dfrac{x^3(\cancel{x^2+1})}{\cancel{x^2+1}}$

$+y = + \dfrac{(x^5 + x^3)}{x^2 + 1}$

Origin symmetry



odd

2. Given: $y = |3x - 7|$

a) Find all x-intercepts

$0 = |3x - 7|$

$\pm 0 = 3x - 7$

$\frac{3x}{3} = \frac{7}{3}$ $x = \frac{7}{3}$

$\boxed{(\frac{7}{3}, 0)}$

b) Find all y-intercepts

$y = |3(0) - 7|$

$y = |-7|$

$y = 7$

$\boxed{(0, 7)}$

$(0, \frac{7}{4})$ $\frac{\frac{7}{4} - \frac{7}{4}}{1 - 0} = \frac{-\frac{3}{4}}{1} = -\frac{3}{4}$

$(1, 1)$

$m = -\frac{3}{4}$

3. Given: $3x + 4y = 7$. Write in **point slope form** the equation of the line passing through $\left(-\frac{2}{3}, \frac{7}{8}\right)$........

$4y = -3x + 7$

$y = -\frac{3}{4}x + \frac{7}{4}$

(1,1)

a) parallel to the given line.

$y - \frac{7}{8} = -\frac{3}{4}(x + \frac{2}{3})$ $y - y_1 = m(x - x_1)$

$\boxed{y - \frac{7}{8} = -\frac{3}{4}(x + \frac{2}{3})}$

b) perpendicular to the given line.

⊥ slope $= \frac{4}{3}$

$\boxed{y - \frac{7}{8} = \frac{4}{3}(x + \frac{2}{3})}$

4. Use interval notation to write the domain of the following:

a) $y = \dfrac{1}{x} - \dfrac{3}{x + 2}$

No 0 ⇒ ARN
No −2 except zero and −2

$(-\infty, 0) \cup (0, \infty)$ $(-\infty, -2) \cup (-2, \infty)$

$\boxed{(-\infty, -2) \cup (-2, 0) \cup (0, \infty)}$

b) $y = \dfrac{\sqrt{x + 6}}{3 + x}$

$x \le -6$ $[-6, \infty)$

$\ne -3$ $(-\infty, -3) \cup (-3, \infty)$

$\boxed{[-6, -3) \cup (-3, \infty)}$

5. Find the average rate of change function using the difference quotient for the following function:

$f(x) = \frac{5}{x^2}$

$$\frac{\frac{5x^2}{x^2(x+h)^2} - \frac{5(x+h)^2}{x^2(x+h)^2}}{h}$$

$$= \frac{5x^2 - 5(x+h)^2}{hx^2(x+h)^2}$$

$$\frac{5x^2 - 5(x^2+2hx+h^2)}{hx^2(x+h)^2}$$

$$= \frac{\cancel{5x^2 - 5x^2} - 10hx - 5h^2}{hx^4 + 2h^2x^3 + h^3x^2}$$

$$\frac{-5h(2x+h)}{hx^2(x+h)^2}$$

$$= \boxed{\frac{-5(2x+h)}{x^2(x+h)^2}}$$

6. Find the average rate of change formula from $x = \frac{\pi}{3}$ to $x = \frac{\pi}{3} + h$ using the difference quotient for the following function:

$1 \quad \frac{\sqrt{3}}{2} \quad 60^\circ \quad \frac{1}{2}$

$f(x) = \cos x \qquad \frac{f(x+h) - f(x)}{h} = \frac{\cos(\frac{\pi}{3}+h) - \cos\frac{\pi}{3}}{h}$

$$= \frac{\cos\frac{\pi}{3}\cosh - \sin\frac{\pi}{3}\sinh - \cos\frac{\pi}{3}}{h}$$

$$= \frac{\frac{1}{2}\cosh - \frac{\sqrt{3}}{2}\sinh - \frac{1}{2}}{h}$$

$$= \frac{\frac{1}{2}(\cosh - \sqrt{3}\sinh - 1)}{h}$$

$$\boxed{\frac{\cosh - \sqrt{3}\sinh - 1}{2h}}$$

7. Find the average rate of change function using the difference quotient for the following function: (Rationalize the numerator)

$f(x) = 2\sqrt{x-4} \qquad \frac{2\sqrt{x+h-4} - 2\sqrt{x-4}}{h}$

$$= \frac{2\sqrt{x+h-4} - 2\sqrt{x-4}}{h}\left(\frac{2\sqrt{x+h-4} + 2\sqrt{x-4}}{2\sqrt{x+h-4} + 2\sqrt{x-4}}\right)$$

$$= \frac{4(x+h-4) - 4(x-4)}{h(2\sqrt{x+h-4} + 2\sqrt{x-4})}$$

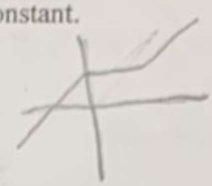
$$= \cancel{4x} + 4h - \cancel{16} - \cancel{4x} + \cancel{16}$$

$$\frac{2\cancel{4}h}{\cancel{h}(\cancel{2}\sqrt{x+h-4} + \cancel{2}\sqrt{x-4})}$$

$$\boxed{\frac{2}{\sqrt{x+h-4} + \sqrt{x-4}}}$$

8. Determine over which intervals the function is increasing, decreasing, or constant.

$$f(x) = \begin{cases} x+3, x \le 0 \\ 3, 0 < x \le 2 \\ 2x+1, x > 2 \end{cases}$$



a) Increasing = $(-\infty, 0]$ $(2, \infty)$ -1

b) Decreasing = None

c) Constant = $(0, 2]$

$\left(\frac{1-\sqrt{53}}{2}, \frac{1+\sqrt{53}}{2}\right)$ $\quad -6x^2+6x+78 \quad -6(x^2-x-13)$

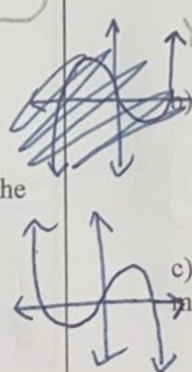
9. Given: $f(x) = -2x^3 + 3x^2 + 78x - 12$

a) Identify the interval(s) where f is increasing.

$x = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(-13)}}{2}$

$x = \frac{1 \pm \sqrt{53}}{2}$

$\boxed{\left(-\infty, \frac{1-\sqrt{53}}{2}\right) \cup \left(\frac{1-\sqrt{53}}{2}, \frac{1+\sqrt{53}}{2}\right)}$ -1



b) Identify the interval(s) where f is decreasing.

$\boxed{\left(\frac{1+\sqrt{53}}{2}, \infty\right)}$ -2 $\quad \left(-\infty, \frac{1-\sqrt{53}}{2}\right)$ $\left(\frac{1+\sqrt{53}}{2}, \infty\right)$

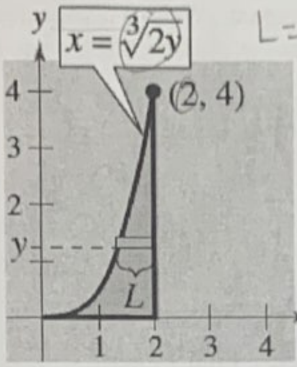
c) Identify the location(s) of any relative maximums or minimums

Minimum: $\left(\frac{1-\sqrt{53}}{2}, 0\right)$ $\quad \frac{1-\sqrt{53}}{2}$

Maximum: $\left(\frac{1+\sqrt{53}}{2}, 0\right)$ $\quad \frac{1+\sqrt{53}}{2}$

.4

10. Write the length L of the rectangle as a function of y.

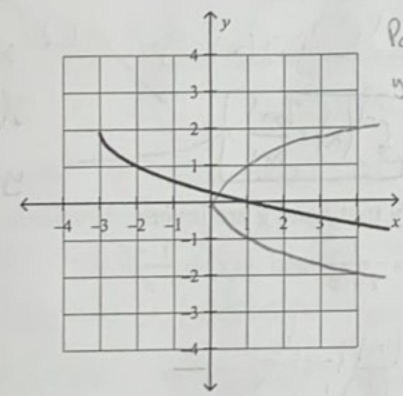


$L = 2 - 3\sqrt{2y}$

$L =$ $2 - 3\sqrt{2y}$

11. Determine the equation for the following graphs:

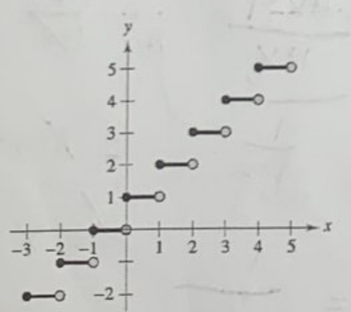
a)



Parent
$y = +\sqrt{x}$

$y = -\sqrt{x+3} + 2$

b)

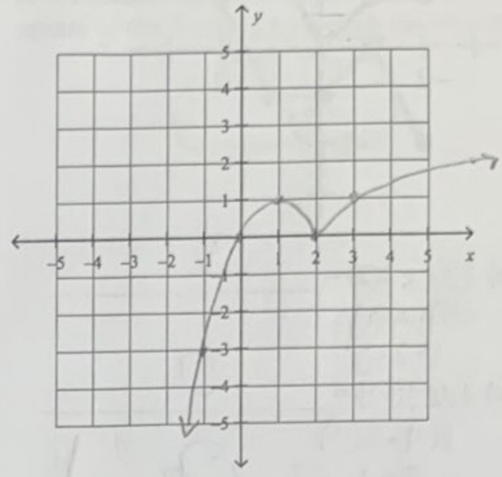


$y = [\![x+1]\!]$

12. Graph the following:

$1 - (x^2 - 2x + 1)$
$1 - x^2 + 2x - 1$

$$f(x) = \begin{cases} 1-(x-1)^2, & x \le 2 \\ \sqrt{x-2}, & x > 2 \end{cases}$$



13. Given $f(x) = \sqrt{x}$. Describe the sequence of transformations from f to g if

$g(x) = -\sqrt{2x+3} - 1.$
$-\sqrt{2(x+\frac{3}{2})} - 1$

Reflection in the x-axis

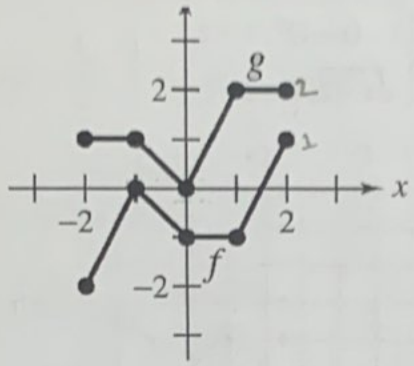
Horizontal shrink by $\frac{1}{2}$

Horizontal shift left $\frac{3}{2}$

Vertical shift down 1

$O = 2x+12$ $M = \frac{1}{2}x+6$ $Y = x$

14. Use the graphs of f and g to find:



a) $(f+g)(2) =$ 3

$f(2)+g(2)$

$1+2=3$

b) $(fg)(-1) =$ 0

$f(-1) \cdot g(-1)$

$0 \times 1 = 0$

c) $(f-g)(-2) =$ ~~3~~ 3, 1

$f(-2) - g(-2)$

$1 - (-2) = 3$

15. Given: $f(x) = \dfrac{3}{x^2-1}$ and $g(x) = x+1$

a) Find: $f(g(x))$

$= \dfrac{3}{(x+1)^2-1}$

$= \boxed{\dfrac{3}{x^2+2x}}$

$x^2+2x \neq 0$

$x \neq -2$

$x \neq 0$

b) Write the domain of $f(g(x))$ $\boxed{(-\infty,-2)\cup(-2,0)\cup(0,\infty)}$

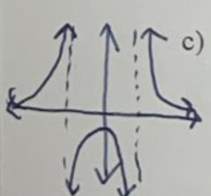
dom g $(-\infty,\infty)$ dom f $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$ 2

$x+1 \neq -1$ $x \neq -2$

$x+1 \neq 1$ $x \neq 0$

$(-\infty,-1)\cup(-1,1)\cup(1,\infty)$

c) Write the range of $f(g(x))$ $\boxed{(-\infty,-3]\cup(0,\infty)}$



? ? 2

range

dom $g(x)$ → $f(g(x))$

~~$(-\infty,\infty)$~~ ~~$(-\infty,\infty)$~~ $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$

$(-\infty,-2)\cup(-2,0)\cup(0,\infty)$

$(-\infty,-1)\cup(-1,1)\cup(1,\infty)$

↑

dom $f(g(x))$

16. Three siblings are of three different ages. The oldest is twice the age of the middle sibling, and the middle sibling is six years older than one half the age of the youngest. Write a composite function that gives the oldest sibling's age in terms of the youngest.

$\boxed{2x+12}$ 2

x = youngest siblings age

17. Given: $f(x) = \dfrac{x+1}{x-2}$. Find $f^{-1}(x)$

$x = \dfrac{y+1}{y-2}$

$y-2(x) = y+1$

$y-2(x)-y-1 = 0$

?? 2

$\boxed{f^{-1}(x) = \dfrac{2x+1}{x-1}}$

$x = \dfrac{y+1}{y-2}$

$xy-2x = y+1$

$-y \quad -y$

$xy-y = 2x+1$

$y(x-1) = 2x+1$

$y = \dfrac{2x+1}{x-1}$

18. Verify that f and g are inverse functions.

$f(x) = \dfrac{1}{1+x}$ and $g(x) = \dfrac{1-x}{x}$

$g \circ f$ $\dfrac{\frac{(1+x)}{(1+x)}1 - \frac{1}{1+x}}{\frac{1}{1+x}}$

$= \dfrac{\frac{1+x-1}{1+x}}{\frac{1}{1+x}}$

$\dfrac{1+x-1}{\cancel{1+x}} \cdot \dfrac{\cancel{1+x}}{1}$

$= \cancel{1}+x-\cancel{1}$

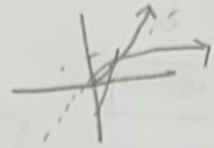
$\checkmark = x$

$f \circ g = \dfrac{1}{\cancel{\frac{x}{x}}+\frac{1-x}{x}}$

$= \dfrac{1}{\frac{1}{x}} = x \checkmark$

-9

19. Given: $f(x) = x^2 - 4, x \le 0$



a) Find $f^{-1}(x)$

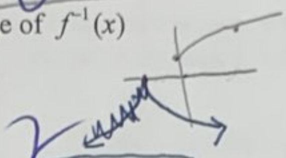
$x = y^2 - 4$

$\sqrt{x+4} = \sqrt{y^2}$

$\boxed{y = \pm\sqrt{x+4}}$

-1

b) Determine the range of $f^{-1}(x)$

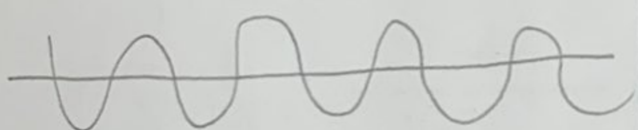


$\boxed{\text{Range } [2, \infty)}$

$\boxed{(-\infty, 0]}$

20. Determine if the situation could be represented by a one-to-one function. Explain why or why not.

The depth of the tide d at a beach in terms of the time t over a 24-hour period.



No, not a one-to-one function because the tide will reach a certain height multiple times per day

21. Find a mathematical model for the verbal statement.

The gravitational attraction F between two objects of masses m_1 and m_2 is proportional to the product of the masses and inversely proportional to the square of the distance r between the objects.

$$\boxed{F = \frac{k\, m_1 m_2}{r^2}}$$

22. z varies directly as the square of x and inversely as y. $z = 6$ when $x = 6$ and $y = 4$.
Find the constant of variation.

$$z = \frac{kx^2}{y}$$

$$6 = \frac{k(6)^2}{4}$$

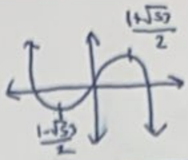
$$\frac{4}{36} \cdot 6 = \frac{36k}{4}$$

$$\frac{24}{36} = \frac{36k}{36}$$

$$\boxed{k = \frac{2}{3}}$$

-3

$\boxed{9.}$ $f(x) = -2x^3 + 3x^2 + 78x - 12$



a) Increasing?

$$f'(x) = -6x^2 + 6x + 78$$
$$= -6(x^2 - x - 13)$$
$$x = \frac{1 \pm \sqrt{53}}{2}$$

$$x = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(-13)}}{2}$$
$$x = \frac{+1 \pm \sqrt{53}}{2}$$

$$\boxed{\left(\frac{1-\sqrt{53}}{2}, \frac{1+\sqrt{53}}{2}\right)}$$

b) Decreasing?

$$\boxed{\left(-\infty, \frac{1-\sqrt{53}}{2}\right) \cup \left(\frac{1+\sqrt{53}}{2}, \infty\right)}$$

c)

$$\boxed{\begin{array}{l} \text{Minimum} \Rightarrow \frac{1-\sqrt{53}}{2} \\ \text{Maximum} \Rightarrow \frac{1+\sqrt{53}}{2} \end{array}}$$

$\boxed{15.}$ $f(x) = \frac{3}{x^2-1}$ $g(x) = x+1$

a) find $f \circ g(x)$

$$\frac{3}{(x+1)^2 - 1} = \boxed{\frac{3}{x^2+2x}}$$

b) domain $f \circ g(x)$ $\rightarrow$ $\boxed{(-\infty, -2) \cup (-2, 0) \cup (0, \infty)}$

dom g range g

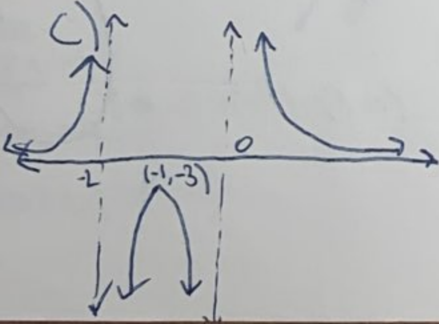
$\cancel{(-\infty, \infty)}$ $\cancel{(-\infty, \infty)}$

dom f

$(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

$-1 \neq x+1$
$x \neq -2$
$1 \neq x+1$
$x \neq 0$

c)



$$= \frac{3}{(-1)^2 + 2(-1)}$$
$$= \frac{3}{1-2} = \frac{3}{-1} = -3$$

$$\boxed{(-\infty, -3) \cup (0, \infty)}$$