Honors Pre-Calc
Test Chapter 12: 2016-2017

Short Answer

1. Evaluate the following limits. Give an exact answer when possible.

a) $\lim_{x \to 0} \frac{3x}{\tan 5x}$

x	-0.10	-0.01	0.01	0.10
f(x)	0.549	0.599	0.599	0.549

0.6

b) $\lim_{x \to 1} \frac{\ln(6x-5)}{x-1}$

x	-0.10	-0.01	0.01	0.10
f(x)	9.163	6.188	5.827	4.7

6

c) $\lim_{x \to \sqrt{3}} \text{arccot}(-x)$

$\arctan(-\sqrt{3}/3) = -\pi/6$

$(\frac{1}{2} / \frac{\sqrt{3}}{2})$

d) $\lim_{x \to 2} \frac{x^4-16}{x-2}$

$(x^4-16) = \cancel{(x-2)}(x^3+2x^2+4x+8)$

$8+8+8+8$

32

e) $\lim_{x \to -5} \frac{\sqrt{x+9}-2}{x+5}$

$\frac{x+9-4}{(x+5)(\sqrt{x+9}+2)}$

$\frac{\cancel{x+5}}{\cancel{(x+5)}(\sqrt{x+9}+2)}$

$1/\sqrt{x+9}+2 = 1/4$

f) $\lim_{x \to -4} \frac{x^3+64}{x+4}$

$(x^3+64) = \cancel{(x+4)}(x^2-4x+16)$

$16+16+16$

48

g) $\lim_{x \to 2} \frac{\frac{1}{x+1}-\frac{1}{3}}{\frac{2}{x}-1}$

$\frac{3-x-1}{(\frac{2}{x}-1)(3x+3)}$

$\frac{\cancel{2-x}}{(\cancel{\frac{2-x}{x}})(3x+3)}$

$1/\frac{3x+3}{x} = 2/9$

h) $\lim_{x \to 0} (1+3x)^{2/x}$

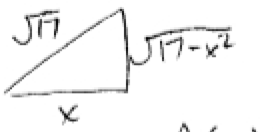
x	-0.01	0.001	-0.0001
f(x)	442.24	407.08	403.79

x	0.0001	0.001	0.01
f(x)	403.07	399.82	369.36

~ 403.43

2. You are given wire and asked to form a right triangle with a hypotenuse of $\sqrt{17}$ inches whose area is as large as possible.

a) Write a function for the area in terms of x, the length of the side of the triangle



$$A(x) = \frac{1}{2}x\sqrt{17-x^2}$$

b) What should x be in order to maximize the area?

$x = \sqrt{17-x^2}$

$x^2 = 17 - x^2$

$x^2 = 17/2$

$x = \sqrt{34}/2$ in

c) What's the maximum area?

$A\left(\frac{\sqrt{34}}{2}\right) = \frac{\sqrt{34}}{4}\sqrt{17-\frac{34}{4}}$

$= \frac{\sqrt{34}}{4} \cdot \frac{\sqrt{34}}{2}$

$= \frac{34}{8}$

$= 17/4 \text{ in}^2$

3. Given: $f(x) = \frac{3}{3-x}$ and $g(x) = \sin\pi x$

a) Find $\lim_{x\to 2}(f(x)g(x))$

$\frac{3\sin\pi x}{3-x}$

$\frac{3(0)}{3-2} = 0$

b) Find $\lim_{x\to 2}(g(x) - f(x))$

$\sin\pi x - 3/3-x$

$0 - 3/1$

-3

4. Given: $f(x) = \frac{1}{x-5}$ find $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$f'(x) = \frac{1/x+h-5 - 1/x-5}{h}$

$= \frac{x-5-x-h+5}{h(x+h-5)(x-5)}$

$= -h/h(x+h-5)(x-5)$

$= -1/(x-5)^2$

5. Find the slope of $h(x) = \sqrt{x+5}$ at (-1, 2)

$h'(x) = \frac{\sqrt{x+h+5} - \sqrt{x+5}}{h}$

$= \frac{x+h+5-x-5}{h(\sqrt{x+h+5} + \sqrt{x+5})}$

$= h/h(\sqrt{x+h+5} + \sqrt{x+5})$

$= 1/2\sqrt{x+5}$

$= 1/2\sqrt{-1+5}$

$= 1/4$

6. Find the derivative of $f(x) = \sqrt{x-4}$

$f'(x) = \frac{\sqrt{x+h-4} - \sqrt{x-4}}{h}$

$= \frac{x+h-4-x+4}{h(\sqrt{x+h-4}+\sqrt{x-4})}$

$= \frac{h}{h}(\sqrt{x+h-4}+\sqrt{x-4})$

$= \frac{1}{2\sqrt{x-4}}$

7. Find the equation of the tangent line that is tangent to $f(x) = x^3 - x$ at the point (2, 6).

$f'(x) = 3x^2 - 1$

$= 3(4) - 1$

$= 11$

$y - 6 = 11(x-2)$

8. Use the derivative of $f(x) = -2x^3 + 24x$ to find any points on the graph where the tangent line is horizontal.

$f'(x) = -6x^2 + 24$

$0 = -6x^2 + 24$

$x^2 = 4$

$x = \pm 2$

$f(2) = -16 + 48$ $\quad$ $f(2) = 32$

$f(-2) = 16 - 48$ $\quad$ $f(-2) = -32$

$(2, 32), (-2, -32)$

9. Use the function and its derivative to determine any points on the graph of f at which the tangent line is horizontal.

$f(x) = \frac{\ln x}{x}$ $\qquad$ $f'(x) = \frac{1-\ln x}{x^2}$

$0 = \frac{1-\ln x}{x^2}$

$\frac{\ln x}{x^2} = \frac{1}{x^2}$

$\ln x = 1$

$x = e$

$f(e) = \frac{\ln e}{e}$

$= \frac{1}{e}$

$(e, \frac{1}{e})$

10. Find the following limits at infinity:

a) $\lim_{x\to\infty}\left(\frac{x}{2x+1}+\frac{3x^2}{(2x-3)^2}\right)$

$\frac{1}{2}+\frac{3}{4}=\frac{5}{4}$

b) $\lim_{x\to\infty}\left(\frac{x}{2}-\frac{4x}{x^2}\right)$

No limit

c) $\lim_{x\to\infty}\left(\frac{(4n-2)!}{(4n+2)!}\right)$

$\frac{1}{(4n-1)(4n-0)(4n+1)(4n+2)}$

0

d) $\lim_{x\to\infty}\left(\frac{8}{n^5}\left(\frac{n(n+1)(2n+1)(3n+1)(4n+1)}{6}\right)\right)$

$\frac{8n(n)(2n)(3n)(4n)}{\cancel{6}n^5}$

$\frac{32n^5}{n^5}$

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11. Given: $\sum_{i=1}^{n}\left(\frac{4}{n}-\left(\frac{2i}{n}\right)^2\right)\left(\frac{2i}{n}\right)$

a) Rewrite the sum as a rational function

$\sum_{i=1}^{n}\left(\frac{8i}{n^2}-\frac{8i^3}{n^3}\right)$

$\frac{8}{n^2}\left(\sum_{i=1}^{n} i-\sum_{i=1}^{n} i^3/n\right)$

$\frac{8}{n^2}\left(\frac{n(n+1)}{2}-\frac{n^2(n+1)^2}{4n}\right)$

$\frac{8}{n^2}\left(\frac{2n(n+1)}{4}-\frac{n(n+1)^2}{4}\right)$

$\frac{8}{n^2}\left(\frac{\cancel{2n^2}+2n-n^3-\cancel{2n^2}-n}{4}\right)$

$\frac{8}{n^2}\left(\frac{-n^3+n}{4}\right)$

$\frac{-2n^2+2}{n}$

b) Find the nth partial sum when $n = 100$.

$\frac{-20000+2}{100}$

$\frac{-19998}{100}$

$\frac{-9999}{50}$

12. Approximate the area of the region bounded by the graph of $f(x) = 9 - x^3$, the x-axis, and the vertical lines $x = 0$ and $x = 2$ using 20 rectangles.

width: $2/20 = 1/10$

height: $f(i/10)$

$$\sum_{i=1}^{20} f(i/10)(1/10)$$

$$\frac{1}{10}\sum_{i=1}^{20} 9 - (i/10)^3$$

$$\frac{1}{10}\left(180 - \frac{20^2(21)^2}{4000}\right)$$

$$\frac{1}{10}\left(\frac{1359}{10}\right)$$

$$\frac{1359}{10} \text{ units}^2$$

13. Find the exact area of the region between the graph of $f(x) = x^3 - x^2 - x$ and the x-axis over the interval [2, 5].

width: $3/n$

height: $f(2 + 3i/n)$

$$\sum_{i=1}^{n} f\left(2+\frac{3i}{n}\right)\frac{3}{n}$$

$$\frac{3}{n}\left(\sum_{i=1}^{n}\left(8 + \frac{36i}{n} + \frac{54i^2}{n^2} + \frac{27i^3}{n^3} - 4 - \frac{12i}{n} - \frac{9i^2}{n^2} - 2 - \frac{3i}{n}\right)\right)$$

$$\frac{3}{n}\left(\sum_{i=1}^{n}(8-4-2) + (36-12-3)\frac{i}{n} + (54-9)\frac{i^2}{n^2} + \frac{27i^3}{n^3}\right)$$

$$\frac{3}{n}\left(\sum_{i=1}^{n}\left(2 + 21\frac{i}{n} + 45 i^2/n^2 + 27 i^3/n^3\right)\right)$$

$$\frac{3}{n}\left(2n + \frac{21n(n+1)}{2n} + \frac{45n(n+1)(2n+1)}{6n^2} + \frac{27n^2(n+1)^2}{4n^3}\right)$$

$$\frac{3}{n}\left(2n + \frac{21(n+1)}{2} + \frac{15(n+1)(2n+1)}{2n} + \frac{27(n+1)^2}{4n}\right)$$

$$6 + \frac{63(n+1)}{2n} + \frac{45(n+1)(2n+1)}{2n^2} + \frac{81(n+1)^2}{4n^2}$$

$$\lim_{n\to\infty}\left(6 + \frac{63n+63}{2n} + \frac{90n^2+135n+45}{2n^2} + \frac{81n^2+162n+81}{4n^2}\right)$$

$$6 + \frac{63}{2} + 45 + \frac{81}{4}$$

$$\frac{24+126+180+81}{4}$$

$$411/4 \text{ units}^2$$