Hon Pre-Calculus

Test Chapter 5 Name [redacted]

89/88

Only Scientific Calculators Allowed!!! Show ALL Work!!! Circle ALL final answers!!!

Short Answer

$\sin^2 x + \cos^2 x = 1$
$1 + \cot^2 x = \csc^2 x$
$\tan^2 x + 1 = \sec^2 x$

1. Simplify completely to one single trig function:

$$\frac{\tan x}{1} - \frac{\sec^2 x}{\tan x}$$

$\tan^2 x + 1 = \sec^2 x$
$\tan^2 x - \sec^2 x = -1$

$$\frac{\tan^2 x - \sec^2 x}{\tan x} \rightarrow \frac{-1}{\tan x} = \boxed{-\cot x}$$

2. Simplify completely to one single trigonometric function: $\sec x \csc x - \tan x$

$$\frac{1}{\cos x} \cdot \frac{1}{\sin x} - \tan x$$

$$\frac{1}{\cos x \sin x} + \frac{-\sin x}{\cos x}$$

$$= \frac{\cos x - \cos x \sin^2 x}{\cos x(\cos x \sin x)}$$

$$\frac{\cos x(\cancel{1-\sin^2 x})^{\cos^2 x}}{\cos x(\cos x \sin x)} = \frac{\cos^2 x}{\cos x \sin x} = \boxed{\cot x}$$

3. Use trigonometric substitution to write the algebraic expression as a function of θ, where θ is in the interval $\left(0, \frac{\pi}{2}\right)$.

$\sqrt{4x^2 + 9}$, $x = \frac{3\tan\theta}{2}$

$$\sqrt{\frac{4}{1}\left(\frac{9\tan^2\theta}{4}\right) + 9}$$

$$\sqrt{9(\tan^2\theta + 1)}$$

$$\sqrt{9\sec^2\theta}$$

$\boxed{3\sec\theta}$

$\cos^2 x \cdot \frac{\sin^2 x}{\cos^2 x}$

4. Simplify Completely:

$$\ln\left|\tan^2 x\right| - \ln\left|\frac{1-\sin^2 x}{\csc^2 x - 1}\right| + \ln\left|\frac{1}{\tan^2 x + 1}\right|$$

$\cos^2 x \sin^2 x$, $\cot^2 x$, $\cos^2 x$, $\sec^2 x$

$$\ln\left|\frac{\sin^2 x}{\cos^2 x} \cdot \frac{1}{\sin^2 x}\right| + \ln\left|\cos^2 x\right|$$

$$\ln\left|\frac{1}{\cos^2 x}\right| + \ln\left|\cos^2 x\right|$$

$$\ln|1| = \boxed{0}$$

5. Simplify completely to one single trigonometric function:

$$\frac{\cos\theta\cot\theta}{1-\sin\theta} - \frac{1}{1}$$

$$\frac{\cos\theta\cot\theta - 1 + \sin\theta}{1-\sin\theta}$$

$$= \frac{\frac{\cos^2\theta}{\sin\theta} - 1 + \sin\theta}{1-\sin\theta}\left(\frac{\sin\theta}{\sin\theta}\right) = \frac{\cos^2\theta - \sin\theta + \sin^2\theta}{\sin\theta(1-\sin\theta)}$$

$$\frac{\cos^2\theta + \sin^2\theta - \sin\theta}{\sin\theta(1-\sin\theta)} = \frac{\cancel{1-\sin\theta}}{\sin\theta(\cancel{1-\sin\theta})} = \boxed{\csc\theta}$$

6. Find all EXACT solutions to the equation on the interval $[0, 2\pi)$

$6\sin^2 x = 2 - 5\cos x$

$6\sin^2 x + 5\cos x - 2 = 0$

$6(1-\cos^2 x) + 5\cos x - 2 = 0$

$6 - 6\cos^2 x + 5\cos x - 2 = 0$

-24, 3, -8, -5

$6\cos^2 x - 5\cos x - 4 = 0$

$3\cos x - 8\cos x$

$6\cos^2 x - 8\cos x + 3\cos x - 4 = 0$

$3\cos x(2\cos x + 1) - 4(2\cos x + 1) = 0$

$(3\cos x - 4)(2\cos x + 1) = 0$

$\cancel{\cos x = \frac{4}{3}}$ $\quad \cos x = -\frac{1}{2}$

$\cancel{x}$

$$\boxed{x = \frac{2\pi}{3}, \frac{4\pi}{3}}$$

7. Find the EXACT value for the $\sin\frac{17\pi}{12}$

$\frac{17\pi/6}{2}$? $\frac{5\pi}{6}$

$\sin\frac{17\pi}{12} = \left(-\sqrt{\frac{1+\cos\frac{5\pi}{6}}{2}}\right)$

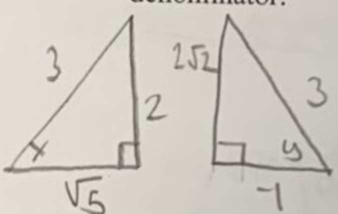
-0.965925826

$= -\sqrt{\frac{2+\sqrt{3}}{4}}$

$\sin\frac{17\pi}{12} = -\frac{\sqrt{2+\sqrt{3}}}{\sqrt{4}}$

$\boxed{\sin\frac{17\pi}{12} = -\frac{\sqrt{2+\sqrt{3}}}{2}}$

8. Find the EXACT value of $\tan(x-y)$ given that $\sin x = \frac{2}{3}$, where $0 < x < \frac{\pi}{2}$ and $\cos y = \frac{-1}{3}$, where $\frac{\pi}{2} < y < \pi$. You do not need to rationalize denominator.



$\tan x = \frac{2}{\sqrt{5}}\left(\frac{\sqrt{5}}{\sqrt{5}}\right) = \frac{2\sqrt{5}}{5}$

$\frac{\tan x - \tan y}{1+\tan x \tan y} = \frac{\frac{2\sqrt{5}}{5} - \left(\frac{10\sqrt{2}}{5}\right)}{1+\left(\frac{2\sqrt{5}}{5}\cdot\frac{-2\sqrt{2}}{1}\right)}$

$\left(\frac{5}{5}\right)\frac{\frac{2\sqrt{5}+10\sqrt{2}}{5}}{1-\frac{4\sqrt{10}}{5}} = \boxed{\frac{2\sqrt{5}+10\sqrt{2}}{5-4\sqrt{10}}}$

9. Find the EXACT solutions over the interval $[0,2\pi)$

$\sin 4x = -2\sin 2x$

$\sin(2x+2x) = -2\sin 2x$

$\cancel{\sin 2x\cos 2x - \cos 2x\sin 2x} = -2\sin 2x$

$\cancel{\sin 2x\cos 2x - \sin 2x\cos 2x}$

$-2\sin 2x = 0$

$\sin 2x = 0$

$2\sin x\cos x = 0$

$\sin x = 0 \qquad \cos x = 0$

$\boxed{x = 0, \pi \quad x = \frac{\pi}{2}, \frac{3\pi}{2}}$

10. Given: $\theta = 22°30'$. Use a half angle formula to find the following: $22.5°$ $\frac{45}{2}$

a) $\sin\theta$

$\sin 22.5° = \sqrt{\frac{1-\cos 45}{2}}$

$= \sqrt{\frac{2-\sqrt{2}}{4}}$

$\boxed{\frac{\sqrt{2-\sqrt{2}}}{2}}$

b) $\tan\theta$

$\tan 22.5° = \frac{1-\cos 45}{\sin 45}$

$= \frac{\frac{2}{2}-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$

$= \frac{2-\sqrt{2}}{\cancel{2}}\cdot\frac{\cancel{2}}{\sqrt{2}} = \frac{2-\sqrt{2}}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$

$= \frac{2\sqrt{2}-2}{2}$

$= \boxed{\sqrt{2}-1}$

11. Find the exact value of the expression:

$\sin 7° \cos 8° + \sin 8° \cos 7°$

$\sin(7° + 8°)$ $\quad 60-45$

$\sin(15°)$

$\sin(60-45)$

$\sin 60 \cos 45 - \cos 60 \sin 45$

$\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$

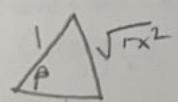
$\frac{\sqrt{6}}{4} + \frac{-\sqrt{2}}{4}$

$\frac{4\sqrt{6} - 4\sqrt{2}}{16} = \frac{4\sqrt{6}}{16} - \frac{4\sqrt{2}}{16} = \boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$

0 ✓

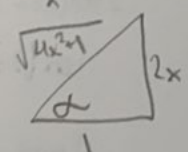
12. Write the trigonometric expression as an algebraic expression. No need to rationalize denominator.

$\sin(\arctan 2x - \arccos x)$



$\sin(\tan^{-1}(2x) - \cos^{-1}(x))$

$\alpha \qquad \beta$



$\sin\alpha \cos\beta - \cos\alpha \sin\beta$

$\frac{2x^2}{\sqrt{4x^2+1}} + \frac{-\sqrt{1-x^2}}{\sqrt{4x^2+1}}$

$\frac{\cancel{2x^2\sqrt{4x^2+1} - (1-x^2)(4x^2+1)}}{\sqrt{4x^2+1}}$

$\boxed{\frac{2x^2 - \sqrt{1-x^2}}{\sqrt{4x^2+1}}}$

13. Solve on the interval $[0, 2\pi)$ (Answers must be exact) $\boxed{0, \frac{\pi}{3}, \frac{5\pi}{3}}$

$\sin\frac{x}{2} + \cos x = \frac{1}{2}$

$\boxed{\cos^{-1}\left(\frac{3-\sqrt{17}}{2}\right)}$

$\left(\sqrt{\frac{1-\cos x}{2}}\right)^2 \quad 2(1-\cos x)$

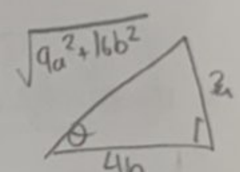
$\frac{1-\cos x}{2} = 1 - 2\cos x \cos^2 x$

$2 - 4\cos x + 2\cos^2 x = \cancel{1 \cos x}$

$2\cos^2 x - 3\cos x - 1 = 0$

$\cos x = \frac{3 \pm \sqrt{9 - 4(2)(-1)}}{2} \rightarrow \frac{3 \pm \sqrt{17}}{2}$

14. Evaluate: $\csc\left(2\tan^{-1}\frac{3a}{4b}\right)$



$\frac{1}{\sin 2\theta}$

$\frac{1}{2\cos\theta \sin\theta}$

$\frac{1}{2 \cdot \left(\frac{4b}{\sqrt{9a^2+16b^2}}\right)\left(\frac{3a}{\sqrt{9a^2+16b^2}}\right)}$

$= \frac{1}{\frac{8b\,3a}{9a^2+16b^2}}$

$= \boxed{\frac{9a^2+16b^2}{24ab}}$

13

$\sin^2 x + \cos^2 x = 1$

15. Simplify to a single trigonometric function.

$$\frac{\cos 4x + \cos 2x}{\sin 4x + \sin 2x}$$

$$\frac{\cancel{2}\cos 3x \cancel{\cos x}}{\cancel{2}\sin 3x \cancel{\cos x}} = \frac{\cos 3x}{\sin 3x}$$

$$= \boxed{\cot 3x}$$

17. Solve over the interval $[0, 2\pi)$

$\cot x - \csc x = 3$

$$\frac{\cos x}{\sin x} + \frac{-1}{\sin x} = 3$$

$$\frac{\cos x \sin x - \sin x}{\sin^2 x} = 3$$

$$\frac{\sin x(\cos x - 1)}{\sin x \cdot \sin x} = 3$$

$$(3\sin x)^2 = (\cos x - 1)^2$$

$$9\sin^2 x = \cos^2 x - 1$$

$$9\sin^2 x = \sin^2 x$$

$$8\sin^2 x = 0$$

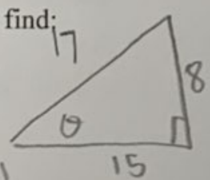
$$\sin^2 x = 0$$

$$\sin x = 0$$

$$\boxed{\cancel{x = 0, \pi}}$$

-3

16. If $\tan\theta = \frac{8}{15}$, and $0° < \theta < 90°$, find:



a) $\cot\frac{\theta}{2}$

$$\frac{1}{\tan\frac{\theta}{2}}$$

$$\tan\frac{\theta}{2} = \frac{1 - \frac{15}{17}}{\frac{8}{17}} = \frac{2}{17} \cdot \frac{17}{8}$$

$$\frac{1}{\tan\frac{\theta}{2}} = \frac{1}{\frac{1}{4}} = \boxed{4}$$

b) $\sec 2\theta$

$$\frac{1}{\cos 2\theta} = \frac{1}{1 - 2\sin^2\theta} = \frac{1}{1 - \frac{2}{1}\left(\frac{8}{17}\right)^2}$$

$$= \frac{1}{1 - \frac{128}{289}}$$

$\frac{161}{289}$

0.55

$$= \frac{289}{161}$$

$$= \boxed{\frac{289}{161}}$$

18. Solve over the interval $[0, \pi)$ (Round to nearest 100th)

let $\theta = 2x - 1$

$2\tan(2x - 1) = 2$

$\tan(2x-1) = 1$

$\tan\theta = 1$

$2x - 1 = \frac{\pi}{4}$ $\qquad$ $2x - 1 = \frac{5\pi}{4}$

$\boxed{x \approx 0.89}$ $\qquad$ $\boxed{x \approx 2.46}$

-3

19. Find all EXACT solutions over the interval $\left[0,2\pi\right)$

$\tan(x+\pi)+2\sin(x+\pi)=0$

$\frac{\sin(x+\pi)}{\cos(x+\pi)}+\frac{2\sin(x+\pi)}{1}=0$

$\frac{+\sin x}{+\cos x}+\frac{-2\sin x}{1}=0$

$\frac{-2\sin^2 x}{\cos x}=0$

$\sin^2 x=0$

$\boxed{x=0,\pi}$

$0,\frac{\pi}{3},\pi,\frac{5\pi}{3}$

-2

20. Solve over the interval $\left[0,2\pi\right)$

$\frac{\cos 2x}{\sin 3x-\sin x}-1=0$

$\frac{\cancel{\cos 2x}\ 1}{2\cancel{\cos 2x}\ \frac{\cancel{\cos x}}{\sin x}}-1=0$

$\frac{1}{2\frac{\cos x}{\sin x}}\ \cancel{+1}=0\ +1$

$\frac{\sin x}{2\cos x}=1$

$\frac{\sin x}{\cos x}=\frac{1}{2}$

$\boxed{x=\frac{\pi}{6},\frac{5\pi}{6}}$

-3

-5

Product-to-Sum Formulas

$$\sin u \sin v = \frac{1}{2}[\cos(u - v) - \cos(u + v)]$$

$$\cos u \cos v = \frac{1}{2}[\cos(u - v) + \cos(u + v)]$$

$$\sin u \cos v = \frac{1}{2}[\sin(u + v) + \sin(u - v)]$$

$$\cos u \sin v = \frac{1}{2}[\sin(u + v) - \sin(u - v)]$$

Sum-to-Product Formulas

$$\sin u + \sin v = 2 \sin\left(\frac{u + v}{2}\right) \cos\left(\frac{u - v}{2}\right)$$

$$\sin u - \sin v = 2 \cos\left(\frac{u + v}{2}\right) \sin\left(\frac{u - v}{2}\right)$$

$$\cos u + \cos v = 2 \cos\left(\frac{u + v}{2}\right) \cos\left(\frac{u - v}{2}\right)$$

$$\cos u - \cos v = -2 \sin\left(\frac{u + v}{2}\right) \sin\left(\frac{u - v}{2}\right)$$