

## Hon Pre-Calc Test Chapter 5 2017-2018

1. Given:  $\sin(-x) = -\frac{1}{3}$ ,  $\tan x = -\frac{\sqrt{2}}{4}$

a)  $\cos x = ?$

b)  $\csc x = ?$

2. Given:  $\csc \theta = -5$ ,  $\cos \theta < 0$

a)  $\cos \theta = ?$

b)  $\tan \theta = ?$

3. Simplify completely to one single trigonometric function:

$$\frac{1}{1 + \cos x} + \frac{1}{1 - \cos x}$$

4. Simplify completely:

$$\ln|\cos^2 t| + \ln|1 + \tan^2 t|$$

5. Simplify completely to one single trigonometric function:

$$\frac{\cos \theta \cot \theta}{1 - \sin \theta} - 1$$

6. Simplify completely to one single trigonometric function:

$$\csc^4 x - 2 \csc^2 x + 1$$

7. Simplify completely to one single trigonometric function:

$$\frac{\sec \theta - 1}{1 - \cos \theta}$$

8. Solve over the interval  $[0, 2\pi)$  (Answers must be exact)

$$\cos x - \frac{\cos x}{1 - \tan x} = 0$$

9. Solve over the interval  $[0, 2\pi)$  (Answers must be exact)

$$\sin t \csc\left(\frac{\pi}{2} - t\right) - \tan t = 0$$

10. Solve over the interval  $[0, 2\pi)$  (Answers must be exact)

$$\frac{1 + \sin x}{\cos x} + \frac{\cos x}{1 + \sin x} = 4$$

11. Solve over the reals. (Answers must be exact)

$$\sec 4x = 2$$

12. Solve over the interval  $[0, 2\pi)$  (Round to nearest 100th)

$$2 \tan^2 x + 7 \tan x + 2 = 0$$

13. Find the exact value of the expression:

$$\sin \frac{\pi}{12} \cos \frac{\pi}{4} + \cos \frac{\pi}{12} \sin \frac{\pi}{4}$$

14. Find the exact value of the expression:

$$\frac{\tan \frac{5\pi}{36} + \tan \frac{11\pi}{18}}{1 - \tan \frac{5\pi}{36} \tan \frac{11\pi}{18}}$$

15. Given:  $\sin u = -\frac{7}{25}$  and  $\cos v = -\frac{4}{5}$ ,  
(Both  $u$  and  $v$  are in Quadrant III)

Find:  $\csc(u - v)$

16. Write the trigonometric expression as an algebraic expression.

$$\cos(\arccos x - \arctan x)$$

17. Simplify the expression:

$$\tan\left(\frac{3\pi}{2} - x\right)$$

18. Solve over the interval  $[0, 2\pi)$  (Answers must be exact)

$$\tan 2x - \cot x = 0$$

19. Given:  $\sin u = \frac{5}{13}, \frac{\pi}{2} < u < \pi$

a)  $\cos\left(\frac{u}{2}\right) = ?$

b)  $\tan\left(\frac{u}{2}\right) = ?$

20. Write the product as a sum or difference:

$$7 \cos(-5\beta) \sin(3\beta)$$

21. Solve over the interval  $[0, 2\pi)$  (Answers must be exact)

$$\frac{\cos 2x}{\sin 3x - \sin x} - 1 = 0$$

22. Solve over the interval  $[0, 2\pi)$  (Round to nearest 100th)

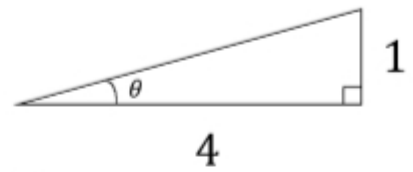
$$2 \tan(3x - 1) = 2$$

23. Solve over the interval  $[0, 2\pi)$  (Round to nearest 100th)

$$-5 \cos\left(3\theta - \frac{\pi}{4}\right) = 2$$

24. Evaluate:  $\csc\left(2 \cot^{-1} \frac{a}{b}\right)$

25. Use the figure to find the exact value of the following:



- a)  $\sec 2\theta$

- b)  $\cot 4\theta$

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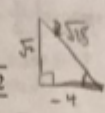
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Show All Work!!! Circle All Final Answers!!! Scientific Calculators Only!!! Happy Holidays!!!

Short Answer

1. Given:  $\sin(-x) = -\frac{1}{3}$ ,  $\tan x = -\frac{\sqrt{2}}{4}$

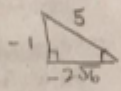


a)  $\cos x = ?$   $\frac{-4}{5} = -\frac{4\sqrt{2}}{16} = -\frac{2\sqrt{2}}{9} = -1$

b)  $\csc x = ?$   $\frac{1}{\sin x} = \frac{\sqrt{2}}{5} = \frac{\sqrt{2}}{2} = \frac{6}{2} = 3$

$\sin \theta = -\frac{1}{5}$

2. Given:  $\csc \theta = -5$ ,  $\cos \theta < 0$



a)  $\cos \theta = ?$   $-\frac{2\sqrt{6}}{5}$

b)  $\tan \theta = ?$   $-\frac{1}{2\sqrt{6}} = \frac{\sqrt{6}}{2\sqrt{6}} = \frac{\sqrt{6}}{12}$

3. Simplify completely to one single trigonometric function:

$$\frac{1}{1+\cos x} + \frac{1}{1-\cos x}$$

$$= \frac{1-\cos x + 1+\cos x}{1-\cos^2 x}$$

$$= \frac{2}{\sin^2 x}$$

$$= 2 \csc^2 x$$

4. Simplify Completely:

$$\ln|\cos^2 t| + \ln|1+\tan^2 t|$$

$$= \ln|\cos^2 t| + \ln|\sec^2 t|$$

$$= \ln|\cos^2 t \cdot \sec^2 t|$$

$$= \ln|1|$$

$$= 0$$

5. Simplify completely to one single trigonometric function:

$$\frac{\cos \theta \cot \theta}{1-\sin \theta} - 1$$

$$= \frac{(1+\sin \theta)(\cos \theta \cot \theta)}{1-\sin^2 \theta} + \frac{-1+\sin^2 \theta}{1-\sin^2 \theta}$$

$$= \frac{\cos \theta \cot \theta + \cos^2 \theta - \cos^2 \theta}{\cos^2 \theta}$$

$$= \frac{\cot \theta}{\cos \theta}$$

$$= \frac{1}{\sin \theta}$$

$$= \csc \theta$$

-1

6. Simplify completely to one single trigonometric function:

$$\begin{aligned} & \csc^4 x - 2\csc^2 x + 1 \\ &= (\csc^2 x - 1)^2 \\ &= (\cot^2 x)^2 \\ &= \cot^4 x \end{aligned}$$

7. Simplify completely to one single trigonometric function:

$$\begin{aligned} & \frac{\sec \theta - 1}{1 - \cos \theta} \\ &= \frac{(\sec \theta - 1)(1 + \cos \theta)}{1 - \cos^2 \theta} \\ &= \frac{\sec \theta - 1 + 1 - \cos \theta}{\sin^2 \theta} \\ &= \frac{\sec \theta - \cos \theta}{\sin^2 \theta} \\ &= \frac{\frac{1}{\cos \theta} - \cos \theta}{\sin^2 \theta} \\ &= \frac{\frac{1 - \cos^2 \theta}{\cos \theta}}{\sin^2 \theta} \\ &= \frac{\sin^2 \theta}{\cos \theta \sin^2 \theta} \\ &= \frac{1}{\cos \theta} = \sec \theta \end{aligned}$$

8. Solve over the interval  $[0, 2\pi)$  (Answers must be exact)

$$\begin{aligned} & \cos x - \frac{\cos x}{1 - \tan x} = 0 \\ & \frac{\cos x - \sin x - \cos x}{1 - \tan x} = 0 \\ & \frac{-\sin x}{1 - \tan x} = 0 \\ & \sin x = 0 \\ & x = 0, \pi \end{aligned}$$

9. Solve over the interval  $[0, 2\pi)$ : (Answers must be exact)

$$\begin{aligned} & \sin t \csc \left( \frac{\pi}{2} - t \right) - \tan t = 0 \\ & \sin t \sec t - \tan t = 0 \\ & \tan t - \tan t = 0 \end{aligned}$$

$$t = [0, 2\pi), t \neq \frac{\pi}{2}, \frac{3\pi}{2}$$

10. Solve over the interval  $[0, 2\pi)$  (Answers must be exact)

$$\begin{aligned} & \frac{1 + \sin x}{\cos x} + \frac{\cos x}{1 + \sin x} = 4 \\ & \frac{1 + 2\sin x + \sin^2 x + \cos^2 x}{\cos x (1 + \sin x)} = 4 \\ & \frac{2(1 + \sin x)}{\cos x (1 + \sin x)} = 4 \\ & 2 \sec x = 4 \\ & \sec x = 2 \end{aligned}$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

11. Solve over the reals. (Answers must be exact)

$$\begin{aligned} & \sec 4x = 2 \quad \theta = \frac{2\pi}{3} = \frac{\pi}{2} \\ & \cos 4x = \frac{1}{2} \\ & 4x = \frac{\pi}{3} \quad 4x = \frac{5\pi}{3} \\ & x = \frac{\pi}{12} \quad x = \frac{5\pi}{12} \\ & x = \frac{\pi}{12} + \frac{\pi}{2}n \quad n \text{ int} \\ & \frac{5\pi}{12} + \frac{\pi}{2}n \quad n \text{ int} \end{aligned}$$

12. Solve over the interval  $[0, 2\pi)$ . (Round to nearest 100th)

$$2\tan^2 x + 7\tan x + 2 = 0$$

$$\tan x = \frac{-7 \pm \sqrt{49 - 16}}{4}$$

$$= \frac{-7 \pm \sqrt{33}}{4}$$

$$\tan x \approx 0.313, -3.186$$

$$x \approx 2.84, 5.98, 1.87, 5.02$$

13. Find the exact value of the expression:

$$\sin \frac{\pi}{12} \cos \frac{\pi}{4} + \cos \frac{\pi}{12} \sin \frac{\pi}{4}$$

$$= \sin \left( \frac{\pi}{12} + \frac{\pi}{4} \right)$$

$$= \sin \frac{\pi}{3}$$

$$= \frac{\sqrt{3}}{2}$$

14. Find the exact value of the expression:

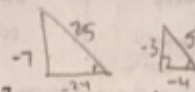
$$\tan \frac{5\pi}{36} + \tan \frac{11\pi}{18}$$

$$1 - \tan \frac{5\pi}{36} \tan \frac{11\pi}{18}$$

$$= \tan \left( \frac{5\pi}{36} + \frac{11\pi}{18} \right)$$

$$= \tan \frac{3\pi}{4}$$

$$= -1$$



15. Given:  $\sin u = -\frac{7}{25}$  and  $\cos v = -\frac{4}{5}$ , (Both  $u$  and  $v$  are in Quadrant III)

Find:  $\csc(u-v)$

$$= \frac{1}{\sin(u-v)}$$

$$= \frac{1}{(-\frac{7}{25})(-\frac{4}{5}) - (-\frac{24}{25})(-\frac{3}{5})}$$

$$= \frac{1}{\frac{28-72}{125}}$$

$$= -\frac{125}{44}$$

16. Write the trigonometric expression as an algebraic expression.

$$\cos(\arccos x - \arctan x)$$

$$= (x) \left( \frac{1}{\sqrt{x^2+1}} \right) + (\sqrt{1-x^2}) \left( \frac{x}{\sqrt{x^2+1}} \right)$$

$$= \frac{x + x\sqrt{1-x^2}}{\sqrt{x^2+1}}$$

$$= \frac{x(1+\sqrt{1-x^2})}{\sqrt{x^2+1}}$$

17. Simplify the expression:

$$\tan \left( \frac{3\pi}{2} - x \right) \rightarrow \frac{\sin \frac{3\pi}{2} \cos x - \cos \frac{3\pi}{2} \sin x}{\cos \frac{3\pi}{2} \cos x + \sin \frac{3\pi}{2} \sin x}$$

$$= \frac{\tan \frac{3\pi}{2} - \tan x}{1 + \tan \frac{3\pi}{2} \tan x}$$

$$= \frac{-\tan x}{1}$$

$$= -\tan x$$

$$= \frac{\sin \frac{3\pi}{2} \cos x}{\sin \frac{3\pi}{2} \sin x}$$

$$= \frac{\cos x}{\sin x}$$

$$= \cot x$$

-4



$$\frac{2 \tan x}{1 - \tan^2 x} = \frac{1}{\tan x} \quad \frac{1 - \tan^2 x}{\tan x} - \frac{2 \tan x}{\tan x} = 0$$

$$2 \tan x = \frac{1 - \tan^2 x}{\tan x} \quad \cot x (1 - 3 \tan^2 x) = 0$$

$$\cot x = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$1 - 3 \tan^2 x = 0$$

$$\tan^2 x = \frac{1}{3}$$

$$\tan x = \pm \frac{1}{\sqrt{3}}$$

$$x = \frac{\pi}{6}, \frac{7\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}$$

18. Solve on the interval  $[0, 2\pi)$  (Answers must be exact)

$$\tan 2x - \cot x = 0$$

$$\frac{2 \tan x}{1 - \tan^2 x} - \frac{1}{\tan x} = 0$$

$$\frac{2 \tan^2 x - 1 + \tan^2 x}{\tan x (1 - \tan^2 x)} = 0$$

$$\frac{3 \tan^2 x - 1}{\tan x (1 - \tan^2 x)} = 0$$

$$\tan^2 x = \frac{1}{3}$$

$$\tan x = \pm \frac{1}{\sqrt{3}}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

19. Given:  $\sin u = \frac{5}{13}$ ,  $\frac{\pi}{2} < u < \pi$

a)  $\cos\left(\frac{u}{2}\right) = ?$

$$= \sqrt{\frac{1 + \frac{12}{13}}{2}} = \sqrt{\frac{1 + \frac{12}{13}}{2}}$$

$$= \sqrt{\frac{\frac{13 + 12}{13}}{2}} = \sqrt{\frac{\frac{25}{13}}{2}} = \sqrt{\frac{25}{26}} = \frac{5}{\sqrt{26}}$$

b)  $\tan\left(\frac{u}{2}\right) = ?$

$$\frac{\cos u + 1}{\sin u} = \frac{\frac{12}{13} + 1}{\frac{5}{13}} = \frac{\frac{25}{13}}{\frac{5}{13}} = \frac{25}{5} = 5$$

20. Write the product as a sum or difference:

$$7 \cos(-5\beta) \sin(3\beta)$$

$$= 7 \left[ \frac{1}{2} [\sin(-5\beta + 3\beta) - \sin(-5\beta - 3\beta)] \right]$$

$$= \frac{7}{2} (\sin(-2\beta) - \sin(-8\beta))$$

21. Solve on the interval  $[0, 2\pi)$  (Answers must be exact)

$$\frac{\cos 2x}{\sin 3x - \sin x} - 1 = 0$$

$$\frac{\cos 2x}{2 \cos x \sin x} - 1 = 0$$

$$\frac{1}{2} \cot x = 1$$

$$\cot x = 2$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

22. Solve over the interval  $[0, 2\pi)$  (Round to nearest 100th)

$$2 \tan(3x - 1) = 2$$

$$\tan(3x - 1) = 1$$

$$3x - 1 = \frac{\pi}{4}$$

$$3x = 1 + \frac{\pi}{4}$$

$$x = \frac{1 + \frac{\pi}{4}}{3}$$

$$x \approx 0.60$$

$$x \approx 1.64$$

$$x \approx 2.69$$

$$x \approx 3.74$$

$$x \approx 4.78$$

$$x \approx 5.83$$

23. Solve over the interval  $[0, 2\pi)$  (Round to nearest 100th)

$$-5 \cos\left(3\theta - \frac{\pi}{4}\right) = 2$$

$$\cos\left(3\theta - \frac{\pi}{4}\right) = -\frac{2}{5}$$

$$3\theta - \frac{\pi}{4} \approx 1.98$$

$$\theta \approx 0.92$$

$$3.02$$

$$5.11$$

$$3\theta - \frac{\pi}{4} \approx 4.30$$

$$\theta \approx 1.70$$

$$3.79$$

$$5.88$$

$$\text{per} = \frac{2\pi}{3} \approx 2.094$$

24. Evaluate:  $\csc\left(2\cot^{-1}\frac{a}{b}\right)$

$$= \csc(2\theta)$$

$$= \frac{1}{\sin 2\theta}$$

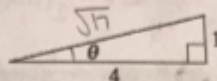
$$= \frac{1}{2 \sin \theta \cos \theta}$$

$$= \frac{1}{2 \left(\frac{b}{\sqrt{a^2+b^2}}\right) \left(\frac{a}{\sqrt{a^2+b^2}}\right)}$$

$$= \frac{a^2+b^2}{2ab}$$



25. Use the figure to find the exact value of the following:



a)  $\sec 2\theta$

$$= \frac{1}{\cos 2\theta}$$

$$= \frac{1}{\cos^2 \theta - \sin^2 \theta}$$

$$= \frac{1}{\left(\frac{4}{\sqrt{5}}\right)^2 - \left(\frac{1}{\sqrt{5}}\right)^2}$$

$$= \frac{1}{\frac{16-1}{5}}$$

$$= \frac{5}{15}$$

$$= \frac{1}{3}$$

b)  $\cot 4\theta = \cot(2 \cdot 2\theta)$

$$= \frac{1}{\frac{2 \tan 2\theta}{1 - \tan^2 2\theta}}$$

$$= \frac{1 - \tan^2 2\theta}{2 \tan 2\theta}$$

$$= \frac{1 - \left(\frac{8}{15}\right)^2}{2 \left(\frac{8}{15}\right)}$$

$$= \frac{1 - \frac{64}{225}}{\frac{16}{15}}$$

$$= \frac{\frac{161}{225}}{\frac{16}{15}}$$

$$= \frac{161}{240}$$

$$= \frac{161}{240}$$

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$$= \frac{161}{240}$$

