Hon Pre-Calc
Test Chapter 12 Name [redacted]

Show All Work!!! Circle All Final Answers!!!

86/85

Short Answer

1. Evaluate the following limit **EXACTLY** (when possible)s:

a) $\lim_{x \to -2} \frac{x-2}{x^2-4}$ $\frac{-4}{0}$

$\boxed{\text{limit dne}}$

b) $\lim_{x \to 0} \frac{\sin 3x}{x}$

$\lim = \boxed{3}$

c) $\lim_{x \to 9} \frac{3-\sqrt{x}}{x-9}$

$-x+9$
$-(x-9)$

$\frac{3-\sqrt{x}}{x-9}\left(\frac{3+\sqrt{x}}{3+\sqrt{x}}\right) = \frac{9-x}{x-9(3+\sqrt{x})} = \frac{-\cancel{(x-9)}}{\cancel{x-9}(3+\sqrt{x})}$

$= \frac{-1}{3+\sqrt{x}} = \boxed{\frac{-1}{6}}$

d) $\lim_{x \to -2} \frac{x^4-16}{x+2}$

$\frac{(x^2+4)\cancel{(x+2)}(x-2)}{\cancel{(x+2)}} = (x^2+4)(x-2)$

$= \boxed{-32}$

e) $\lim_{x \to \pi/2} [(1-\sin x)\sec x]$

$\lim = \boxed{0}$

x	1.4708	1.5708 $\pi/2$	1.6708
y	0.05	Error?	-0.05

f) $\lim_{x \to 0} (1+2x)^{1/x}$

$\lim = \boxed{7.389}$

g) $\lim_{x \to 0} f(x)$ where $f(x) = \begin{cases} 4-x^2, x \le 0 & 4 \\ x+4, \ x>0 & 4 \end{cases}$

$\lim = \boxed{4}$

h) $\lim_{x \to 5^+} \frac{5-x}{25-x^2}$

$\frac{\cancel{5-x}}{(5+x)\cancel{(5-x)}} = \frac{1}{5+x} = \boxed{\frac{1}{10}}$

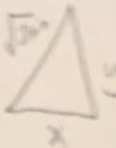
i) $\lim_{x \to 0} \frac{2x}{\tan 4x}$

$\frac{2}{4} = \boxed{\frac{1}{2}}$

j) $\lim_{x \to 0} \frac{\frac{1}{x+4}-\frac{1}{4}}{x}$

$\frac{4-(x+4)}{4(x+4)x} = \frac{\cancel{4}-\overset{1}{\cancel{x}}-\cancel{4}}{4(x+4)\cancel{x}} = \frac{-1}{4(x+4)}$

$= \boxed{\frac{-1}{16}}$



2. You are given wire and are asked to form a right triangle with a hypotenuse of $\sqrt{34}$ inches whose area is as large as possible.

a) Write a function for the area in terms of x, *the length of a side of the triangle.*

$A = \frac{1}{2} x \cdot y$ $\qquad x^2 + y^2 = \sqrt{34}$

$y^2 = 34 - x^2$

$y = \sqrt{34 - x^2}$

$\boxed{A = \frac{1}{2} x\left(\sqrt{34 - x^2}\right)}$

b) What should x be in order to maximize the area?

$x^2 + x^2 = 34$

$2x^2 = 34$

$x^2 = 17$

$\boxed{x = \sqrt{17} \text{ inches}}$

c) What is the maximum area?

$A = \frac{1}{2} \cdot \sqrt{17} \cdot \sqrt{17}$

$A = \boxed{\frac{17}{2} \text{ in}^2}$

3. Given: $f(x) = \frac{3}{3-x}$ and $g(x) = \sin \pi x$ Find....

a) $\lim_{x \to 2} [f(x)g(x)]$

$\lim_{x \to 2} f(x) = 3$

$\lim_{x \to 2} g(x) = 0$

$= 3 \cdot 0$

$= \boxed{0}$

b) $\lim_{x \to 2} [g(x) - f(x)]$

$= 0 - 3$

$= \boxed{-3}$

4. Find the derivative of the function $f(x) = \sqrt{x+8}$

$\lim_{h \to 0} \frac{\sqrt{x+h+8} - \sqrt{x+8}}{h}\left(\frac{\sqrt{x+h+8} + \sqrt{x+8}}{\sqrt{x+h+8} + \sqrt{x+8}}\right)$

$= \frac{\cancel{h}}{\cancel{h}\left(\sqrt{x+h+8} + \sqrt{x+8}\right)} = \boxed{\frac{1}{2\sqrt{x+8}}}$

$f'(x)$

5. Find the equation of the tangent line that is tangent to the graph of $f(x) = x^3 - 2x^2$ at the point (2,0).

$f'(x) = 3x^2 - 4x$

$m = 4$

$y = mx + b$

$y = 4x + b$

$0 = 4(2) + b$

$b = -8$

$\boxed{y = 4x - 8}$

6. Use the function and its derivative to determine any **EXACT** points on the graph of f at which the tangent is horizontal.

$f(x) = x \ln x, \ f'(x) = 1 + \ln x$

$0 = 1 + \ln x$

$f(x) = e^{-1} \ln e^{-1}$ $\quad -1$

$f(x) = e^{-1} \cdot -1 \cdot \cancel{\ln e}$ $\quad -1 = \ln x$

$f(x) = \frac{1}{e} \cdot -1$ $\quad e^{-1} = x$

$f(x) = \frac{-1}{e}$ $\quad x = \frac{1}{e}$

$\boxed{\left(\frac{1}{e}, \frac{-1}{e}\right)}$

7. Use the difference quotient to find the slope of the graph of $f(x) = 2x^2 + 4x - 3$ at the point $(1,3)$.

$$\lim_{h \to 0} \frac{2(x^2 + 2hx + h^2) + 4(x+h) - 3 - (2x^2 + 4x - 3)}{h}$$

$$= \frac{\cancel{2x^2} + 4hx + 2h^2 + \cancel{4x} + 4h \cancel{-3} \cancel{-2x^2} \cancel{-4x} \cancel{+3}}{h}$$

$$= \frac{4hx + 2h^2 + 4h}{h}$$

$$= \frac{\cancel{h}(4x + 2h + 4)}{\cancel{h}}$$

$$\lim_{h \to 0} 4x + 2h + 4$$

$$f'(x) = 4x + 4$$

Slope @ (1,3)

$$= 4(1) + 4$$

$$= \boxed{8}$$

8. Evaluate the following limits at infinity:

a) $\lim_{x\to\infty} \frac{1-2x}{1+3x}$ $\quad \frac{-2x+1}{3x+1}$

$$\lim_{x \to \infty} = \boxed{\frac{-2}{3}}$$

b) $\lim_{x\to\infty} \left(\frac{1}{2}x - \frac{4}{x^2}\right)$ $\quad \frac{x^3}{2x^2} - \frac{8}{2x^2} = \frac{x^3 - 8}{2x^2}$

$$\boxed{\text{d.n.e}}$$

c) $\lim_{x\to\infty} \left[\frac{x}{2x+1} + \frac{3x^2}{(x-3)^2}\right]$ $\quad \frac{3x^2}{x^2 - 6x + 9}$

$$\lim_{x \to \infty} \frac{1}{2} + \cancel{3}\frac{6}{2}$$

$$= \boxed{\frac{7}{2}}$$

$(x^2+x)(x^2+x) = x^4 + 2x^3 + x^2$

d) $\lim_{x\to\infty} \left[\frac{x(x+1)}{x^2} - \frac{1}{x^4}\left(\frac{x(x+1)}{2}\right)^2\right]$ $\quad x^2 + x$ $\quad = \frac{x^4 + 2x^3 + x^2}{2x^4}$

$$= 1 - \frac{1}{4}$$

$$\lim_{x \to \infty} = \boxed{\cancel{2}} \quad \frac{3}{4} \quad 2$$

e) $\lim_{x\to\infty} \left[\frac{(3n-1)!}{(3n)!}\right]$

$$\lim_{x \to \infty} \frac{\cancel{(3n-1)!}}{(3n)\cancel{(3n-1)!}} = \frac{1}{3n}$$

$$-2 \quad \lim_{x \to \infty} = \boxed{0}$$

9. Evaluate the sum: $\sum_{n=1}^{10}\left(n^3-3n^2\right)$

$\sum_{n=1}^{10} n^3 - 3\sum_{n=1}^{10} n^2$ $n=10$

$= \frac{(n)^2(n+1)^2}{4} - 3\left(\frac{n(n+1)(2n+1)}{6}\right)$

= Plugin 10 for n

$= \boxed{1870}$

10. Find the approximate area under the curve $f(x)=9-x^2$ from 0 to 2, using 20 rectangles.

[0,2]

$W=\frac{2}{n}$ $h=f\left(\frac{2i}{n}\right)$

$\lim_{n\to 20}\sum_{i=1}^{n}\frac{2}{n}\cdot f\left(\frac{2i}{n}\right)$

$=\frac{2}{n}\sum_{i=1}^{n}\left[9-\frac{4i^2}{n^2}\right]$

$=\frac{2}{n}\left[9n-\frac{4(n)(n+1)(2n+1)}{6n^2}\right]$

plug in 20 for n

$= \frac{1513}{100}$ units^2

≈ 15.13 units^2

[-1,2]

11. Given: $f(x)=2+\frac{1}{2}x^3$. Find the completely simplified explicit sum formula of:

$\sum_{i=1}^{n} f\left(-1+\frac{3i}{n}\right)\left(\frac{3}{n}\right)$ $f(x)=2+\frac{1}{2}x^3$

$f\left(-1+\frac{3i}{n}\right)=2+\frac{1}{2}\left(-1+\frac{3i}{n}\right)^3$

$-\frac{1}{2}$ $\frac{3}{2}\cdot\frac{3i}{n}$ $-\frac{3}{2}\cdot\frac{9i^2}{n^2}$ $\frac{1}{2}\cdot\frac{27i^3}{n^3}$

$2-\frac{1}{2}+\frac{9i}{2n}-\frac{27i^2}{2n^2}+\frac{27i^3}{2n^3}$

$\frac{3}{n}\sum_{i=1}^{n}\left(2-\frac{1}{2}+\frac{9i}{2n}-\frac{27i^2}{2n^2}+\frac{27i^3}{2n^3}\right)$

Seperate Sheet

$\frac{351n^4+486n^3+81n^2}{24n^4}$

$\frac{\cancel{n^2}(351n^2+486n+81)}{24n^2}$

$=\frac{(351n^2+486n+81)/3}{24n^2/3}$

$=\boxed{\frac{\cancel{117n^2}+\cancel{162n}+27}{8n^2}}$

$\frac{63n^2+54n+27}{8n^2}$

12. Find the exact area bounded by the function $f(x) = \frac{1}{4}\left(x^2 + 4x\right)$ and the x-axis on the interval $[1,4]$

$\cancel{\int_1^4 4x^2 + 4x \, dx \quad W = \frac{3}{n} \quad h = f\left(1 + \frac{3i}{n}\right)}$

$\cancel{\lim_{n \to \infty} \sum_{i=1}^{n} \frac{3}{n} \cdot f\left(1 + \frac{3i}{n}\right)}$

$\cancel{\frac{3}{n} \sum_{i=1}^{n} \left(4 + \frac{24i}{n} + \frac{36i^2}{n^2} + 4 + \frac{12i}{n}\right)}$

$\cancel{12 + 36 + 36 + 12 + 18}$

$\cancel{= 114}$

$1 \qquad 2 \cdot \frac{3i}{n} \qquad 1 \cdot \frac{9i^2}{n^2}$

$\frac{1}{4} + \frac{6i}{4n} + \frac{9i^2}{4n^2}$

For number 12:

$\cancel{4 \qquad 8 \cdot \frac{3i}{n} \qquad 4 \cdot \frac{9i^2}{n^2}}$

$\cancel{4 + \frac{24i}{n} + \frac{36i^2}{n^2}}$

$\cancel{4x = 4\left(1 + \frac{3i}{n}\right)}$

$\cancel{4x = 4 + \frac{12i}{n}}$

$f(x) = \frac{1}{4}x^2 + x$

$\int_1^4 \frac{1}{4}x^2 + x \, dx \qquad W = \frac{3}{n}$

$h = f\left(1 + \frac{3i}{n}\right)$

$= \frac{3}{n} \sum_{i=1}^{n} \left(\frac{1}{4} + \frac{6i}{4n} + \frac{9i^2}{4n^2} + 1 + \frac{3i}{n}\right)$

$= \frac{3}{4} + \frac{9}{4} + \frac{9}{4} + 3 + \frac{9}{2}$

$= \frac{51}{4} = \boxed{12.75 \text{ units}^2}$

$f(x) = 9-x^2$ $[0,2]$ 20 rectangles

$$\int_0^2 9-x^2 \, dx$$

$w = \frac{2}{n}$ $h = f(0+\frac{2i}{n})$

$h = \frac{2i}{n}$

$$= \frac{2}{n}\sum_{i=1}^{n}\left[9 - \frac{4i^2}{n^2}\right]$$

$$= 18 - \frac{8}{3}$$

$$= \frac{2}{n}\left(9n - \frac{4(n)(n+1)(2n+1)}{6n^2}\right)$$

$$= \frac{2}{20}\left(9(20) - \frac{4(20)(21)(41)}{2400}\right)$$

$$= \frac{1513}{100}$$

$54n^3 + 81n^2 + 27n$ $27n^2(n^2+2n+1)$

$(2n^3 + 3n^2 + n)$ $27n^4 + 54n^3 + 27n^2$

$9n^2+9n$

11.

$$\frac{3}{n}\sum_{i=1}\left(2n - \frac{n}{2} + \frac{9(n)(n+1)}{4n} - \frac{27(n)(n+1)(2n+1)}{12n^2} + \frac{27n^2(n+1)^2}{8n^3}\right)$$

$$= \frac{144n^4}{24n^4} - \frac{36n^4}{24n^4} + \frac{162n^4+162n^2}{24n^4} - \frac{162n^4+243n^3+81n^2}{24n^4} + \frac{243n^4+486n^3+243n^2}{24n^4}$$

$$= \frac{144n^4 - 36n^4 + 162n^4 + 162n^2 - 162n^4 - 243n^3 - 81n^2 + 243n^4 + 486n^3 + 243n^2}{24n^4}$$

$$= \frac{351n^4 + 81n^2 + 486n^3}{24n^4} = \frac{351n^4 + 486n^3 + 81n^2}{24n^4}$$

$$\frac{3}{n}\sum_{i=1}^{n}\left(2\cdot\frac{1}{2}+\frac{9i}{2n}-\frac{27i^2}{2n^2}+\frac{27i^3}{2n^3}\right)$$

$$= 6-\frac{3}{2}+\frac{27}{4}-\frac{27}{2}+\frac{81}{8}$$

$$=$$

$$= \frac{3}{n}\left[2n-\frac{n}{2}+\frac{9(n)(n+1)}{4n}-\frac{27(n)(n+1)(2n+1)}{12n^2}+\frac{27n^2(n+1)^2}{8n^3}\right)$$

$$=$$

11.

$$\frac{3}{n}\sum_{i=1}^{n} f\left(-1+\frac{3i}{n}\right)\cancel{\left(\frac{3}{n}\right)} \qquad f(x)=2+\frac{1}{2}x^3$$

$$\left(-1+\frac{3i}{n}\right)^3$$

$$-1 \qquad 3\left(\frac{3i}{n}\right) \qquad -3\left(\frac{9i^2}{n^2}\right) \qquad 1\left(\frac{27i^3}{n^3}\right)$$

$$-1+\frac{9i}{n}+\frac{-27i^2}{n^2}+\frac{27i^3}{n^3}$$

$$\frac{3}{n}\sum_{i=1}^{n}\left[2+\frac{1}{2}\left(-1+\frac{9i}{n}-\frac{27i^2}{n^2}+\frac{27i^3}{n^3}\right)\right]$$