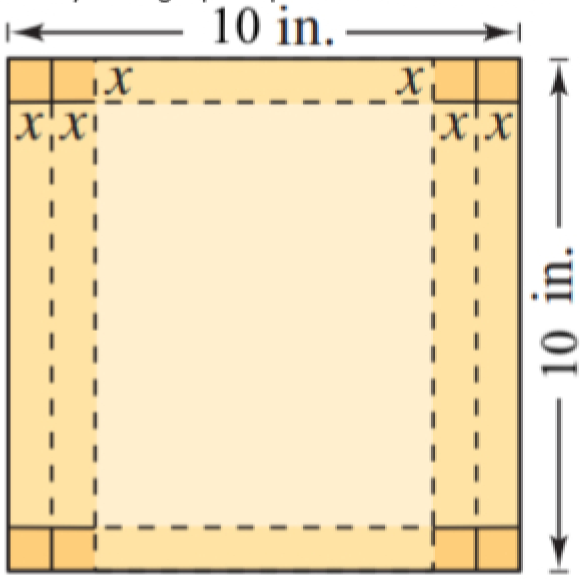
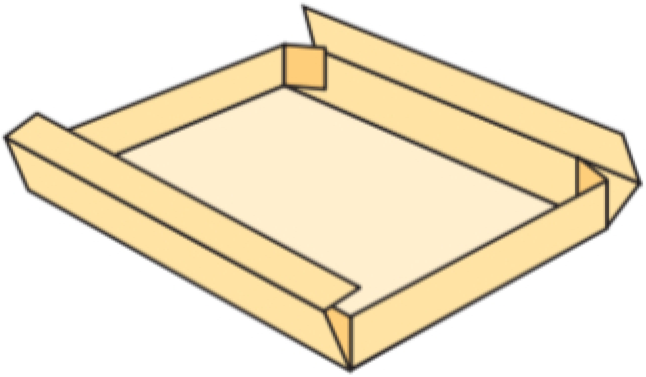
Hon Pre-Calc Quiz 2.1-2.5   2018-2019

Show ALL Work!!! Circle ALL final answers!!! No Calculators!!

1. An open box with locking tabs is to be made from a square piece of material 10 inches on a side. This is to be done by cutting equal squares from the corners and folding along the dashed lines shown in the figure.





a) Write the function $V(x)$ that represents the volume of the box.

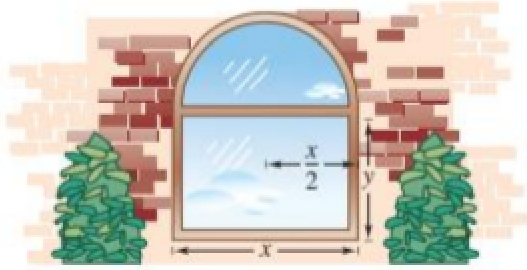
b) Determine the domain of the function.

c) Find the value of x that will maximize the function

2. Find two positive real numbers with a maximum product such that the sum of four times the first and twice the second is 20.

3. Find the remaining factors of $f(x)$ if $f(x) = x^4 - 4x^3 - 15x^2 + 58x - 40$ and $(x - 5)$ and $(x + 4)$ are factors.

4. A Norman window is constructed by adjoining a semicircle to the top of an ordinary rectangular window (see figure). The perimeter of the entire window is 16 feet.



a) Write the area of the entire window as a function of x.

b) What should x be to maximize the area of the window?

5. Use limit notation to describe the end behavior of $f(x) = -\left(-\frac{1}{3}x^6 - \frac{1}{2}x^7 - 4x^5\right)$

6. Consider: $f(x) = x^3 + \frac{3}{2}x^2 - 9x - \frac{27}{2}$. Find all zeros.

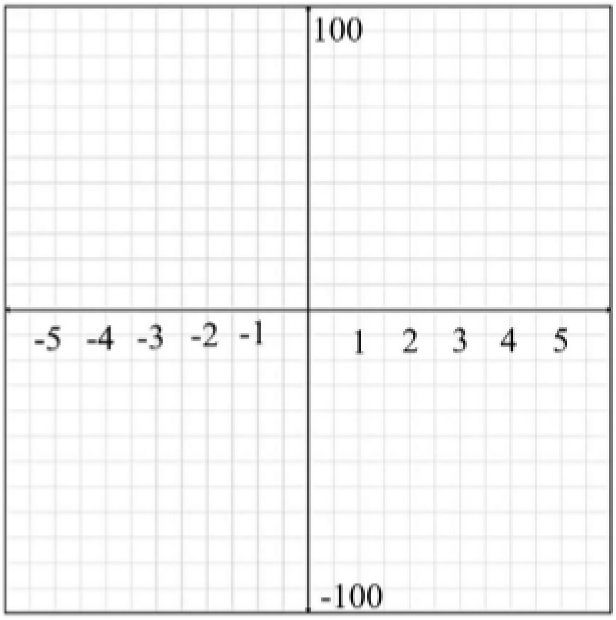
7. Graph the following function using zeros, y-intercept, and relative max and/or mins.
$f(x) = -2x^3 + 3x^2 + 36x - 54$
(**Graph needs to be accurate!)**

Zeros = ________________________

y- int = ________________________

Rel Min = ______________________

Rel Max = ______________________



8. Find a polynomial with real coefficients of least degree with -2 mult 2 and $1 + i\sqrt{3}$ as zeros that also has the following end behavior:
$$\begin{cases} \lim_{x \to \infty} f(x) = -\infty \\ \lim_{x \to -\infty} f(x) = -\infty \end{cases}$$

9. Perform the operation and write your answer in $a + bi$ form.

a) $(3 + \sqrt{-5})(7 - \sqrt{-10})$

b) $\frac{1+i}{i} - \frac{3}{4-i}$

10. Find all zeros of the function and write the polynomial as a product of linear factors.

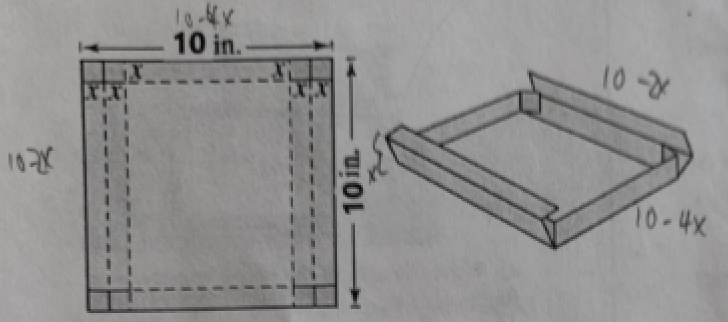
$$f(x) = x^4 - 4x^3 + 8x^2 - 16x + 16$$

Hon Pre Calculus

Quiz 2.1 - 2.5 Name [redacted]

Show All Work!!! Circle All Final Answers!!! NO Calculators!!

1. An open box with locking tabs is to be made from a square piece of material 10 inches on a side. This is to be done by cutting equal squares from the corners and folding along the dashed lines shown in the figure.



$(10-2x)(10-4x)(x)$

43/50

a) Write the function $V(x)$ that represents the volume of the box.

$V(x)=(100-40x-20x+8x^2)\,x$

$=(8x^2-60x+100)x \quad = 8x^3-60x^2+100x$

$\boxed{V(x)=8x^3-60x^2+100x}$

b) Determine the domain of the function.

$8x^3-60x+100x>0$

$x=5,\ \frac{5}{2},\ 0$

$x\neq>5,\ x\neq>\frac{5}{2}$

$D:(0,\frac{5}{3})\cup\cancel{(5,\infty)}$ $\Rightarrow \boxed{D:(0,\frac{5}{2})}$

c) Find the value of x that will maximize the volume.

$V'(x)=24x^2-120x+100=0$

$6x^2-30x+25=0$

$x=\frac{30\pm\sqrt{900-600}}{12}=\frac{30\pm10\sqrt{3}}{12}=\frac{15\pm5\sqrt{3}}{6}$

$\frac{2}{3}$

$\boxed{x=\frac{15-5\sqrt{3}}{6}}$

2. Find two positive real numbers with a maximum product such that sum of four times the first and twice the second is 20.

$4x + 2y = 20 \qquad y = -2x - 10$

$6x = 0$

$x = \frac{10}{3}$

$\left(\frac{10}{3}\right)^2 = \frac{100}{9}$

$4\left(\frac{10}{3}\right) + 2\left(\frac{10}{3}\right) = \frac{60}{3} = 20$

$5, \frac{5}{2}$

-4

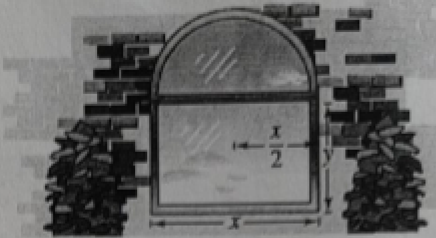
3. Find the remaining factors of $f(x)$ if $f(x) = x^4 - 4x^3 - 15x^2 + 58x - 40$ and $(x - 5)$ and $(x + 4)$ are factors.

5	1	-4	-15	58	-40
		5	5	-50	40
-4	1	1	-10	8	0
		-4	12	-8	
	1	-3	2	0	

$x^2 - 3x + 2 = (x-2)(x-1)$

$(x-2)(x-1)(x+4)(x-5)$

4. A Norman window is constructed by adjoining a semicircle to the top of an ordinary rectangular window (see - figure). The perimeter of the entire window is 16 feet.



a) Write the area of the entire window as a function of x.

$A = x \cdot y + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$

$P = 2y + x + \pi \cdot \frac{x}{2}$

$16 = 2y + x + \frac{x\pi}{2}$

$2y = 16 - x - \frac{x\pi}{2}$

$y = 8 - \frac{2x + x\pi}{4}$

$A(x) = x\left(8 - \frac{2x + x\pi}{4}\right) + \frac{1}{2}\pi \cdot \frac{x^2}{4}$

$= 8x - \frac{2x^2 + \pi x^2}{4} + \frac{\pi x^2}{8}$

$= \frac{64x - 4x^2 - \cancel{2\pi x^2}}{8} \; \pi x^2$

$= 8x - \frac{1}{2}x^2 - \frac{\pi x^2}{8}$

b) What should x be to maximize area of the window?

$A'(x) \Rightarrow -x - \frac{\pi}{4}x + 8 = 0$

$-x\left(1 + \frac{\pi}{4}\right) = -8$

$x = \frac{8}{1 + \frac{\pi}{4}} = \frac{32}{4 + \pi}$

-4

5. Use limit notation to describe the end behavior of $f(x) = -\left(-\frac{1}{3}x^6 - \frac{1}{2}x^7 - 4x^5\right)$

$= \frac{1}{3}x^6 + \frac{1}{2}x^7 + 4x^5$

$$\boxed{\begin{array}{l} \lim_{x \to \infty} f(x) = \infty \\ \lim_{x \to -\infty} f(x) = -\infty \end{array}}$$

6. Consider: $f(x) = x^3 + \frac{3}{2}x^2 - 9x - \frac{27}{2}$. Find all zeros.

P	N	I
1	2	0
1	0	2

$$\begin{array}{c|cccc} -\frac{3}{2} & 1 & \frac{3}{2} & -9 & -\frac{27}{2} \\ & & -\frac{3}{2} & 0 & \frac{27}{2} \\ \hline & 1 & 0 & -9 & 0 \end{array}$$

$(x-\frac{3}{2})(x^2-9) = (x-\frac{3}{2})(x+3)(x-3) = 0$

Zeros: $\frac{3}{2}, \pm 3$

7. Graph he following function using zeros, y intercept, and relative max and/or mins.

$f(x) = -2x^3 + 3x^2 + 36x - 54$ 1, 2, 3, 6, 9, . . .

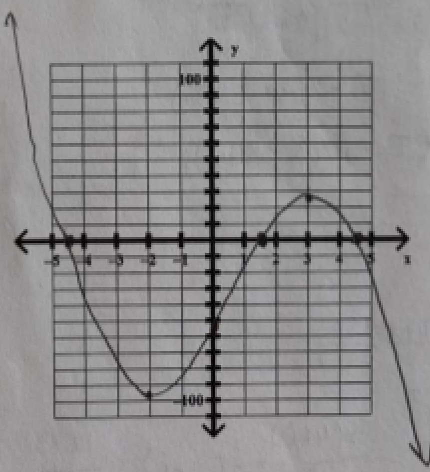
(graph needs to be accurate!!!)

Zeros = $\pm\sqrt{18}, \frac{3}{2}$

y - int = $(0, -54)$

Rel Min = $x = -2, \ y = -98$ $(-2, -98)$

Rel Max = $x = 3, \ y: 27$ $(3, 27)$



7

$f'(x) = -6x^2 + 6x + 36 = 0$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3, -2$

$f(x) = -54, \ x = 0$

$x^2(-2x+3) - 18(-2x+3)$

$(x^2-18)(-2x+3)$

$x = \pm\sqrt{18}, \frac{3}{2}$

$\sqrt{16} < \sqrt{18} < \sqrt{25}$

$4 < \sqrt{18} < 5$

$f(x) = -2(3)^3 + 27 + 108 - 54$

$= -54 - 54 + 27 + 108$

$= -108 + 108 + 27$

$= 27$

$-2(-2)^3 + 3(-2)^2 + 36(-2) - 54$

$= -2(-8) + 3(4) - 72 - 54$

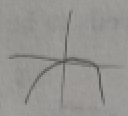
$= 16 + 12 - 72 - 54$

$= 28 - 126$

$= -98$

8. Find a polynomial with real coefficients of least degree with -2 mult 2, and $1+i\sqrt{3}$ as zeros that also has the following end behavior:

$$\begin{cases} \lim_{x\to\infty} f(x) = -\infty \\ \lim_{x\to-\infty} f(x) = -\infty \end{cases}$$



$(x+2)^2(x-(1+i\sqrt{3}))(x-(1-i\sqrt{3}))$

$=(x^2+4x+4)((x-1)^2-(3i)^2)$ $\quad +1+9$

$=(x^2+4x+4)(x^2-2x+10)$

$=x^4-2x^3+10x^2+\cancel{4x^3}-8x^2+40x+4x^2-8x+40$

$=x^4+2x^3+\cancel{6x^2}+\cancel{32x}+40$

$-x^4-2x^3-8x-16$

9. Perform the operation and write your answer in ***a+bi*** form.

a) $\left(3+\sqrt{-5}\right)\left(7-\sqrt{-10}\right)$

$21-3\sqrt{-10}+7\sqrt{-5}-(-\sqrt{50})$

$\boxed{=21+5\sqrt{2}+(7\sqrt{5}-3\sqrt{10})i}$

b) $\frac{1+i}{i}-\frac{3}{4-i}$

$\frac{i+i^2}{i^2}-\frac{3(4+i)}{16-i^2}=-(-1+i)-\frac{12+3i}{17}$

$=1-i-\frac{12}{17}-\frac{3i}{17}$

$=\frac{5}{17}+\left(-\frac{3i}{17}-\frac{17i}{17}\right)$

$=\frac{5}{17}-\frac{20i}{17}$

10. Find all zeros of the function and write the polynomial as a product of linear factors.

$f(x)=x^4-4x^3+8x^2-16x+16$ $\qquad$ 1 2 4 8 16

2	1	-4	8	-16	16
		2	-4	8	-16
2	1	-2	4	-8	0
		2	0	8	
	1	0	4	0	

x^2+4

$=(x-2i)(x+2i)$

$(x-2)(x-2)(x-2i)(x+2i)$

3