Hon Pre-Calc Quest 10.6 - 10.9 *2016 - 2017*

Show All Work For FULL Credit!!!!

1. Given: $\begin{cases} x = 2 + 3\cos\theta \\ y = -3 + 4\sin\theta \end{cases}$

   a) Write the equation in the appropriate standard rectangular form by eliminating the parameter.

   b) Name the rectangular equation

2. Given : $\begin{cases} x = t^3 \\ y = 3\ln t \end{cases}$

   a) Write the equation in the appropriate standard rectangular form by eliminating the parameter.

   b) Name the rectangular equation

3. Given: $\begin{cases} x = h + a\csc\theta \\ y = k + b\cot\theta \end{cases}$

   Write the equation in the appropriate standard rectangular form by eliminating the parameter.

4. Find a set of parametric equations for the rectangular equation using $t = \frac{1-x^2}{x^2}$ and $y = 1 - x^2$

5. Convert the polar point to an exact rectangular point or vice versa. (in c and d, $0 \leq \theta < 2\pi$ and $r > 0$)

a) $\left[-2,\ \frac{5\pi}{6}\right]$

b) $\left[-3,\ \frac{7\pi}{6}\right]$

c) $\left(\sqrt{3},\ -\sqrt{3}\right)$

d) $\left(-\sqrt{3},\ 1\right)$

6. Given: $\left[2,\ \frac{-11\pi}{6}\right]$

Find **3** other representations of the polar point when r can be positive or negative and $-2\pi < \theta < 2\pi$

7. Convert the following rectangular equations to polar solved for r.

a) $y = 7x - 3$

b) $x = 10$

c) $y^2 - 8x - 16 = 0$

d) $y^3 = x^2$

e) $\frac{(y-2)^2}{1} - \frac{x^2}{3} = 1$

8. Convert the polar equations to their standard form rectangular and name the equation.

a) $\theta = \frac{5\pi}{3}$

b) $r = 5\,sin\,\theta$

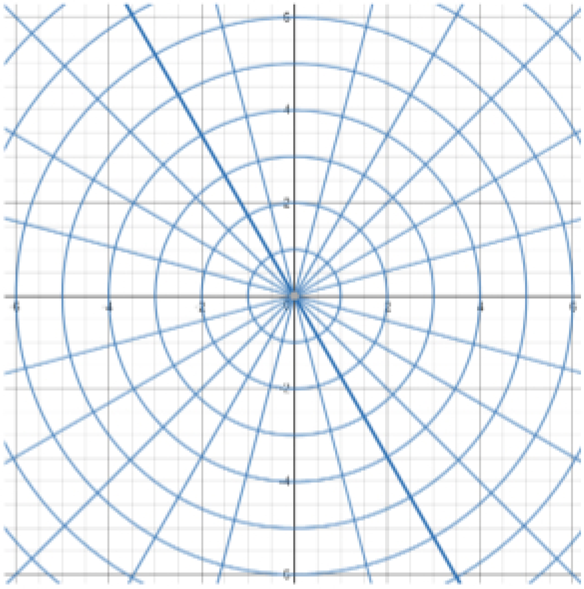
c) $r = \frac{8}{4\cos\theta - 2\sin\theta}$

d) $r = \frac{3}{1+2\,sin\,\theta}$

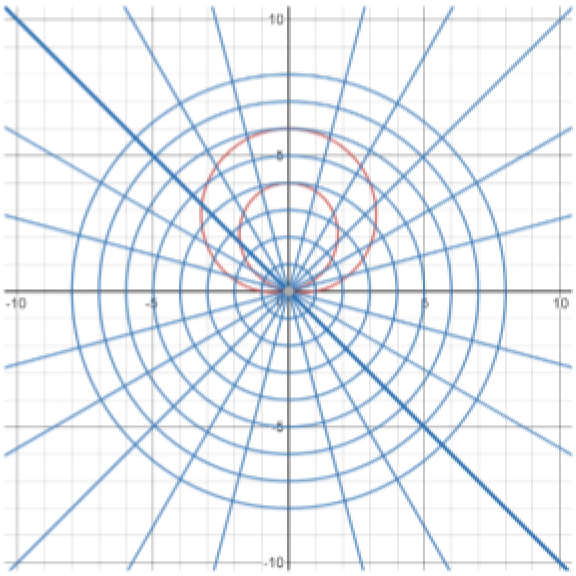
e) $r = \frac{7}{1+sin\,\theta}$

9. Graph and name the following: (Be sure to include a table with exact values):

$r = 3 + 2\cos\theta$



10. State a possible polar equation for the following graph.



11. Graph and name the following: (Be sure to include a table with exact values)

$r = 5\sin 3\theta$

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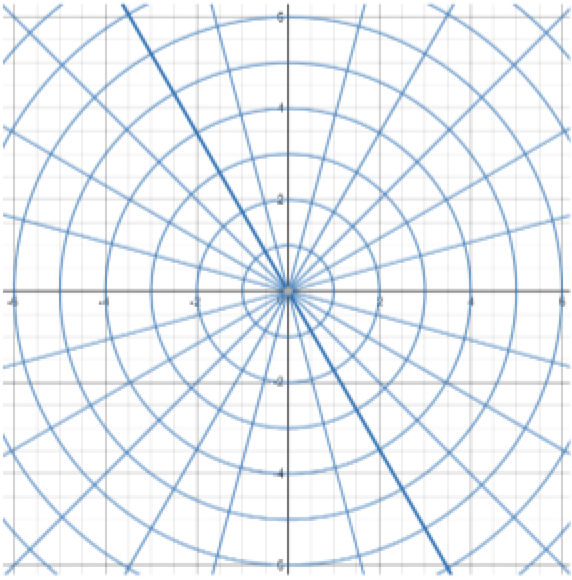

12. Graph and name the following: (Be sure to include a table with exact values)

$r = 2 + 2\sin\theta$

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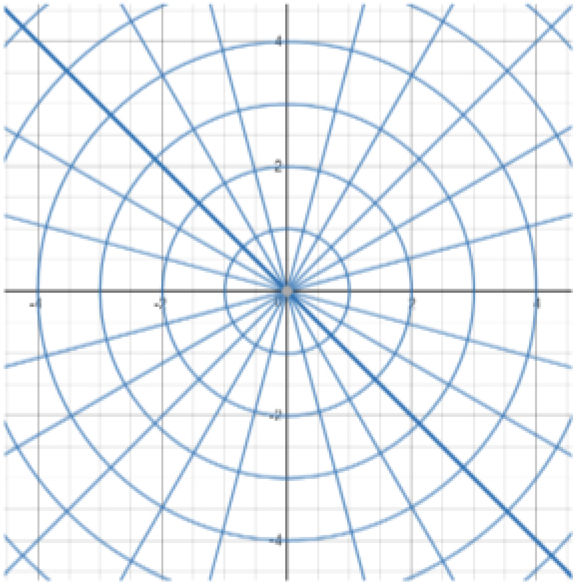


Hon Pre Calculus

Quest 10.6 -10.9 Name 61/70

Only Scientific Calculators Allowed!!!! Show all work!! Circle final answers!!!!

1. Given: $\begin{cases} x = 2 + 3\cos\theta \\ y = -3 + 4\sin\theta \end{cases}$

a) Write the equation in the appropriate standard rectangular form by eliminating the parameter.

$x = 2 + 3\cos\theta$ $\quad y = -3 + 4\sin\theta$

$x - 2 = 3\cos\theta$ $\quad y + 3 = 4\sin\theta$

$\frac{x-2}{3} = \cos\theta$ $\quad \sin\theta = \frac{y+3}{4}$

$\sin^2\theta + \cos^2\theta = 1$ $\quad \frac{(x-2)^2}{9} + \frac{(y+3)^2}{16} = 1$

$\left(\frac{x-2}{3}\right)^2 + \left(\frac{y+3}{4}\right)^2 = 1$

b) Name the rectangular equation

Ellipse

2. Given: $\begin{cases} x = t^3 \\ y = 3\ln t \end{cases}$

a) Write the equation in the appropriate standard rectangular form by eliminating the parameter.

$x = t^3$ $\quad y = 3\ln x^{1/3}$

$t = x^{1/3}$

$y = 3 \cdot \frac{1}{3}\ln x$

$y = \ln x$

b) Name the rectangular equation

Logarithm

3. Given: $\begin{cases} x = h + a\csc\theta \\ y = k + b\cot\theta \end{cases}$

Write the equation in the appropriate standard rectangular form by eliminating the parameter.

$x = h + a\csc\theta$ $\quad y = k + b\cot\theta$

$\frac{x-h}{a} = \frac{1}{\sin\theta}$ $\quad \frac{y-k}{b} = \frac{\cos\theta}{\sin\theta}$

$(x-h)\sin\theta = a$ $\quad b\cos\theta = (y-k)\sin\theta$ $\quad \cos^2\theta = 1 - \sin^2\theta$

$\sin\theta = \frac{a}{x-h}$ $\quad b^2\cos^2\theta = (y-k)^2\sin^2\theta$

$b^2\left(1 - \left(\frac{a}{x-h}\right)^2\right) = (y-k)^2\left(\frac{a}{x-h}\right)^2$

$\frac{(x-h)^2}{a^2} \cdot \left(b^2 - \frac{a^2b^2}{(x-h)^2} = (y-k)^2 \cdot \frac{a^2}{(x-h)^2}\right) \cdot \frac{(x-h)^2}{a^2}$

$\frac{b^2(x-h)^2}{a^2} - b^2 = (y-k)^2$

$b^2\left(\frac{(x-h)^2}{a^2} - 1\right) = (y-k)^2$

$\frac{(x-h)^2}{a^2} - 1 = \frac{(y-k)^2}{b^2}$

$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

4. Find a set of parametric equations for the rectangular equation using $t = \frac{1-x^2}{x^2}$, and $y = 1 - x^2$.

$tx^2 = 1 - x^2$ $\quad y = 1 - \frac{1}{t-1}$

$tx^2 - x^2 = 1$ $\quad y = \frac{t-1-1}{t-1}$

$x^2(t-1) = 1$ $\quad y = \frac{t-2}{t-1}$

$x^2 = \frac{1}{t-1}$

$x = \pm\sqrt{\frac{1}{t-1}}$

$y = \frac{t [illegible]}{t-1}$ − 1

−1

5. Convert the polar point to an exact rectangular point or vice versa. (in c and d, $0 \le \theta < 2\pi$ and $r > 0$)

a) $\left[-2, \frac{5\pi}{6}\right]$ $x = r\cos\theta = -2\cos\frac{5\pi}{6} = 2 \cdot \frac{\sqrt{3}}{2} = \sqrt{3}$

$y = r\sin\theta = -2\sin\frac{5\pi}{6} = -2 \cdot \frac{1}{2} = -1$

$(\sqrt{3}, -1)$

b) $\left[-3, \frac{7\pi}{6}\right]$ $x = -3\cos\frac{7\pi}{6} = -3 \cdot -\frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2}$

$y = -3\sin\frac{7\pi}{6} = -3 \cdot -\frac{1}{2} = \frac{3}{2}$

$\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$

c) $(\sqrt{3}, -\sqrt{3})$ $r = \sqrt{x^2 + y^2} = \sqrt{3+3} = \sqrt{6}$

$\tan\theta = \frac{y}{x} = \frac{-\sqrt{3}}{\sqrt{3}} = -1$ $-\frac{\pi}{4} + 2\pi = \frac{7\pi}{4}$

$\left[\sqrt{6}, \frac{7\pi}{4}\right]$

d) $(-\sqrt{3}, 1)$ $r = \sqrt{3+1} = \sqrt{4} = 2$

$\tan = \frac{1}{-\sqrt{3}} = \frac{-\sqrt{3}}{3}$

$\left[2, \frac{5\pi}{6}\right]$

6. Given: $\left[2, \frac{-11\pi}{6}\right]$

Find **3** other representations of the polar point when r can be positive or negative and $-2\pi < \theta < 2\pi$

$\left[2, \frac{\pi}{6}\right]$ $\left[-2, \frac{-5\pi}{6}\right]$ $\left[-2, \frac{7\pi}{6}\right]$

7. Convert the following rectangular equations to polar solved for r.

a) $y = 7x - 3$

$r\sin\theta = 7r\cos\theta - 3$

$r\sin\theta - 7r\cos\theta = -3$

$r(\sin\theta - 7\cos\theta) = -3$

$r = \frac{-3}{\sin\theta - 7\cos\theta}$

b) $x = 10$

$r\cos\theta = 10$

$r = \frac{10}{\cos\theta}$

c) $y^2 - 8x - 16 = 0$

$r^2\sin^2\theta - 8r\cos\theta - 16 = 0$

$r = \frac{8\cos\theta \pm \sqrt{64\cos^2\theta - 4\sin^2\theta(-16)}}{2\sin^2\theta} = \frac{8\cos\theta \pm \sqrt{64(\cos^2\theta + \sin^2\theta)}}{2\sin^2\theta} = \frac{8\cos\theta \pm 8}{2\sin^2\theta}$

$r = \frac{8(\cos\theta \pm 1)}{2\sin^2\theta}$ $r = \frac{4(\cos\theta \pm 1)}{\sin^2\theta}$ -1

$r = \frac{4\cos\theta \pm 4}{\sin^2\theta}$

d) $y^3 = x^2$

$r^3\sin^3\theta = r^2\cos^2\theta$

$r\sin^3\theta = \cos^2\theta$

$r = \frac{\cos^2\theta}{\sin^3\theta}$

$r = \cot^2\theta \csc\theta$

e) $\frac{(y-2)^2}{1} - \frac{x^2}{3} = 1$

$y^2 - 4y + 4 - \frac{x^2}{3} = 1$

$3(r^2\sin^2\theta - 4r\sin\theta + 4) - r^2\cos^2\theta = 3$

$3r^2\sin^2\theta - 12r\sin\theta + 9 + (-r^2 + r^2\sin^2\theta) = 0$

$4r^2\sin^2\theta - r^2 - 12r\sin\theta + 9 = 0$

$r^2(4\sin^2\theta - 1) - 12r\sin\theta + 9 = 0$

$r = \frac{12\sin\theta \pm \sqrt{144\sin^2\theta - 4(4\sin^2\theta - 1)9}}{2(4\sin^2\theta - 1)} = \frac{12\sin\theta \pm \sqrt{144\sin^2\theta - 144\sin^2\theta + 36}}{2(4\sin^2\theta - 1)}$

$r = \frac{12\sin\theta \pm 6}{2(4\sin^2\theta - 1)} = \frac{6(2\sin\theta \pm 1)}{2(2\sin\theta + 1)(2\sin\theta - 1)}$

$r = \frac{3}{2\sin\theta - 1}$ or $r = \frac{3}{2\sin\theta + 1}$

$r = \frac{3(2\sin\theta + 1)}{(2\sin\theta + 1)(2\sin\theta - 1)}$ or $\frac{3(2\sin\theta - 1)}{(2\sin\theta + 1)(2\sin\theta - 1)}$

-1

−5

8. Convert the polar equations to their **standard form** rectangular and name the equation.

a) $\theta = \frac{5\pi}{3}$ $m = \tan\frac{5\pi}{3} = -\sqrt{3}$ $\frac{-\sqrt{3}/2}{1/2} = -\sqrt{3}$

$y + x\sqrt{3} = 0$ Line

b) $r = 5\sin\theta$

$x^2 + y^2 = 5y$

$x^2 + y^2 - 5y + \frac{25}{4} = 0 + \frac{25}{4}$

$x^2 + (y - \frac{5}{2})^2 = \frac{25}{4}$

Circle

c) $r = \frac{8}{4\cos\theta - 2\sin\theta}$

$4r\cos\theta - 2r\sin\theta = 8$

$4x - 2y = 8$

$2y = 4x - 8$

$y = 2x - 4$

Line

d) $r = \frac{3}{1 + 2\sin\theta}$

$r + 2r\sin\theta = 3$

$\sqrt{x^2+y^2} = 3 - 2y$

$x^2 + y^2 = 4y^2 - 12y + 9$

$x^2 - 3y^2 + 12y = 9$

$x^2 - 3(y^2 - 4y + 4) = 9 - 12$

$x^2 - 3(y-2)^2 = -3$

$\frac{(y-2)^2}{1} - \frac{x^2}{3} = 1$

Hyperbola

e) $r = \frac{7}{1 + \sin\theta}$

$r + r\sin\theta = 7$

$\sqrt{x^2+y^2} = 7 - y$

$x^2 + y^2 = 49 - 14y + y^2$

$x^2 - 49 = -14y$

$y = -x^2 + \frac{7}{2}$ −1

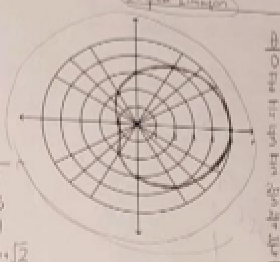
~~$x^2 + 14 =$~~ Parabola

$x^2 = -14(y - \frac{7}{2})$

9. Graph and name the following: (Be sure to include a table with exact values):

$r = 3 + 2\cos\theta$

Dimpled Limaçon



θ	r
0	5
$\frac{\pi}{6}$	$3+\sqrt{3}$
$\frac{\pi}{4}$	$3+\sqrt{2}$
$\frac{\pi}{3}$	4
$\frac{\pi}{2}$	3
$\frac{2\pi}{3}$	2
$\frac{3\pi}{4}$	$3-\sqrt{2}$
$\frac{5\pi}{6}$	$3-\sqrt{3}$
π	1
$\frac{7\pi}{6}$	$3-\sqrt{3}$
$\frac{5\pi}{4}$	$3-\sqrt{2}$

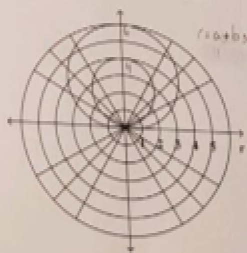
θ	r
$\frac{4\pi}{3}$	2
$\frac{3\pi}{2}$	3
$\frac{5\pi}{3}$	4
$\frac{7\pi}{4}$	$3+\sqrt{2}$
$\frac{11\pi}{6}$	$3+\sqrt{3}$

−1

10. State a possible polar equation for the following graph.

$r = 6 + 4\sin\theta$ $1 + 5\sin\theta = r$

$r = a + b\sin\theta$



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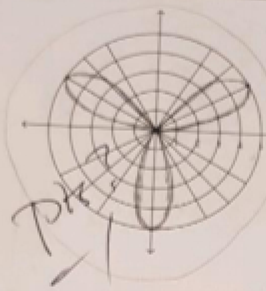
11. Graph and name the following: (Be sure to include a table with exact values):

$r = 5\sin 3\theta$

3 petal rose curve



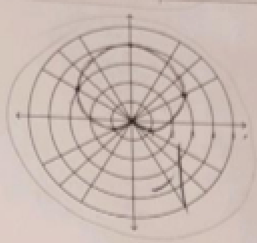
pts
-1

θ	r
0	0
[illegible]	5
[illegible]	$\frac{5\sqrt{2}}{2}$
[illegible]	0
[illegible]	-5
[illegible]	0
[illegible]	$\frac{5\sqrt{2}}{2}$
[illegible]	5
π	0
[illegible]	-5
[illegible]	$-\frac{5\sqrt{2}}{2}$
[illegible]	0
[illegible]	-5
[illegible]	0
[illegible]	$-\frac{5\sqrt{2}}{2}$
[illegible]	-5

12. Graph and name the following (Be sure to include a table with exact values):

$r = 2 + 2\sin\theta$

Cardioid/Heart-Shaped



-1

θ	r
0	0
$\frac{\pi}{6}$	3
$\frac{\pi}{4}$	$2+\sqrt{2}$
$\frac{\pi}{3}$	$2+\sqrt{3}$
$\frac{\pi}{2}$	4
$\frac{2\pi}{3}$	$2+\sqrt{3}$
$\frac{3\pi}{4}$	$2+\sqrt{2}$
$\frac{5\pi}{6}$	3
π	0
$\frac{7\pi}{6}$	1
$\frac{5\pi}{4}$	$2-\sqrt{2}$
$\frac{4\pi}{3}$	$2-\sqrt{3}$
$\frac{3\pi}{2}$	0
$\frac{5\pi}{3}$	$2-\sqrt{3}$
$\frac{7\pi}{4}$	$2-\sqrt{2}$
$\frac{11\pi}{4}$	1

-1

-3