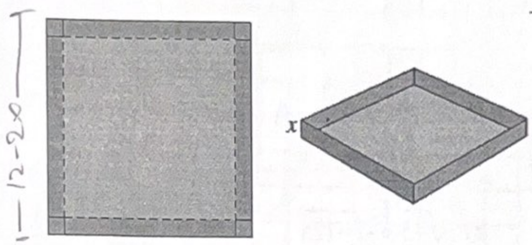
Hon Pre-Calc

Quiz 2.1 - 2.5 Name ______________________

Show All Work!!! Circle All Final Answers!!! NO Calculators!!!

Short Answer

1. An open box is to be made from a square piece of material 12 inches on a side by cutting equal squares with sides of length x from the corners and turning up the sides (see figure)



a) Write a function $V(x)$ that represents the volume of the box.

b) State the domain of the vloume function.

c) Find the value of x that will maximize the volume of the box..

2. Find all complex zeros of the function and write the polynomial as a product of linear factors.

$$f(x) = x^4 + 6x^3 + 10x^2 + 6x + 9$$

3. Find two positive real numbers whose product is a maximum if the sum of the first and three more than twice the second is 27.

4. State the end behavior using limit notation of the following functions:

a) $f(x) = 16 + 32x - 4x^5 - 5x^3$

b) $f(x) = \dfrac{-3x^4 + 2x - 5x^6}{-9}$

5. Use synthetic division to show that x is a solution to the given equation.

$x^3 - x^2 - 13x - 3 = 0,\ x = 2 - \sqrt{5}$

6. Find the real zero of $2x^3 - 11x^2 + \overset{30x - 27}{\cancel{22x} \cancel{-15}}$ if $f\left(2 + i\sqrt{5}\right) = 0$

7. Simplify and write your answer in $a + bi$ form.

a) $\dfrac{1+i}{i} - \dfrac{3}{4-i}$

b) $\sqrt{-5} \bullet \sqrt{-125}$

c) i^{1235}

8. Sketch a possible graph of the following function (Don't worry about y-scale):

$$f(x) = -x^5(x+3)^3(-2x-3)^2(x-5)^5$$

9. Find the values of b such that the function has a minimum value of -50.

$$f(x) = x^2 + bx - 25$$

10. A Norman window is constructed by adjooining a semicircle to the top of an ordinary rectangular window (See figure). The perimeter of the window is 18 feet.

a) Write the area of the entire window as a function of x.

b) What should x be (EXACTLY) that will produce the maximum area?

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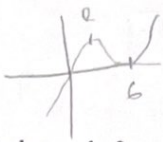
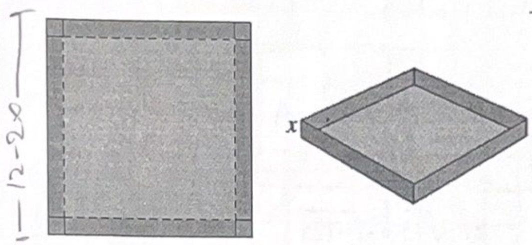
46/50

Short Answer



1. An open box is to be made from a square piece of material 12 inches on a side by cutting equal squares with sides of length x from the corners and turning up the sides (see figure)



a) Write a function $V(x)$ that represents the volume of the box.

$12-2x=0$

$12=2x$

$x=6$

$V(x)=(12-2x)^2x$

$V(x)=(144-48x+4x^2)x$

$\boxed{V(x)=4x^3-48x^2+144x}$

b) State the domain of the vloume function.

$(0,6)$

c) Find the value of x that will maximize the volume of the box..

$y=12x^2-96x+144$

$y=x^2-8x+12$

$y=(x-6)(x-2)$

$x=6,\ x=2$

$\boxed{x=2}$

2. Find all complex zeros of the function and write the polynomial as a product of linear factors.

$+ \quad - \quad + \quad - \quad +$

$f(x)=x^4+6x^3+10x^2+6x+9$

P	N	I
0	4	0
0	2	2
0	0	4

$\frac{1,3,9}{1}=\pm\{1,3,9\}$

$$\begin{array}{r|rrrrr} -3 & 1 & 6 & 10 & 6 & 9 \\ & & -3 & -9 & -3 & -9 \\ \hline & 1 & 3 & 1 & 3 & 0 \end{array}$$

$\boxed{x=-3,\ -1,\ -i\sqrt{3},\ i\sqrt{3}}$

-2

$x^3+3x+x^2+3=0$

$x(x^2+3)+1(x^2+3)=0$

$(x+1)(x^2+3)=0$

$x=(-1),\ x=\pm i\sqrt{3}$

3. Find two positive real numbers whose product is a maximum if the sum of the first and three more than twice the second is 27.

$P=xy$

$27=x+(2y+3)$

$27=x+2y+3$

$x+2y=24$

$x=24-2y$

$P=y(24-2y)$

$P=-2y^2+24y$

-2

$\frac{-24}{2(-2)}=\frac{-24}{-4}=6$

$y=6$

$27=x+2(6)+3$

$27=x+12+3$

$27=x+15$

$x=12$

$\boxed{12,\ 6}$

4. State the end behavior using limit notation of the following functions:

a) $f(x) = 16 + 32x - 4x^5 - 5x^3$

$\lim_{x \to +\infty} f(x) = -\infty$

$\lim_{x \to -\infty} f(x) = +\infty$

$\frac{-3x^4}{-9} = 3x^4$

b) $f(x) = \frac{-3x^4 + 2x - 5x^6}{-9}$

$\lim_{x \to +\infty} f(x) = +\infty$

$\lim_{x \to -\infty} f(x) = +\infty$

5. Use synthetic division to show that x is a solution to the given equation.

$x^3 - x^2 - 13x - 3 = 0,\ x = 2 - \sqrt{5}$

$2-\sqrt{5}$	1	-1	-13	-3
		$2-\sqrt{5}$	$7-3\sqrt{5}$	3
	1	$1-\sqrt{5}$	$-6-3\sqrt{5}$	0

$(2-\sqrt{5})(-6-3\sqrt{5}) = -12 - 6\sqrt{5} + 6\sqrt{5} + 15$
$= 3$

6. Find the real zero of $2x^3 - 11x^2 + \cancel{22x} \cancel{-15}$ $30x - 27$ if $f(2 + i\sqrt{5}) = 0$

$\boxed{x = \frac{3}{2}}$

$(x-(2+i\sqrt{5}))(x-(2-i\sqrt{5}))$

$((x-2)-i\sqrt{5})((x-2)+i\sqrt{5})$

$x^2 - 4x + 4 + 5 = x^2 - 4x + 9$

$$\begin{array}{r}
2x - 3 \\
x^2 - 4x + 9 \overline{)\, 2x^3 - 11x^2 + 30x - 27} \\
-2x^3 + 8x^2 - 18x \\
\hline
-3x^2 + 12x - 27 \\
+3x^2 - 12x + 27 \\
\hline
0
\end{array}$$

$2x - 3 = 0$
$x = \frac{3}{2}$

7. Simplify and write your answer in $a + bi$ form.

a) $\frac{1+i}{i} - \frac{3}{4-i}$

$\frac{(1+i)(4-i) - 3i}{i(4-i)} = \frac{4 - i + 4i + 1}{4i + 1} = \frac{5-3}{1+4i}$

$\frac{5-3i}{1+4i} \times \frac{(1-4i)}{(1-4i)} = \frac{5 - 20i - 3i + 12}{1+16} = \frac{-7-23i}{17}$

$\boxed{-\frac{7}{17} - \frac{23}{17}i}$ -2

b) $\sqrt{-5} \bullet \sqrt{-125}$

$i\sqrt{5} \times 5i\sqrt{5}$

$5i^2 \times 5$

$-5 \times 5 = -25$

$\boxed{-25}$

c) i^{1235}

$\boxed{= -i}$

$i^1 = i$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

$4 \times 8 = 32$

$\frac{35}{8} = 8\ r\ 3$

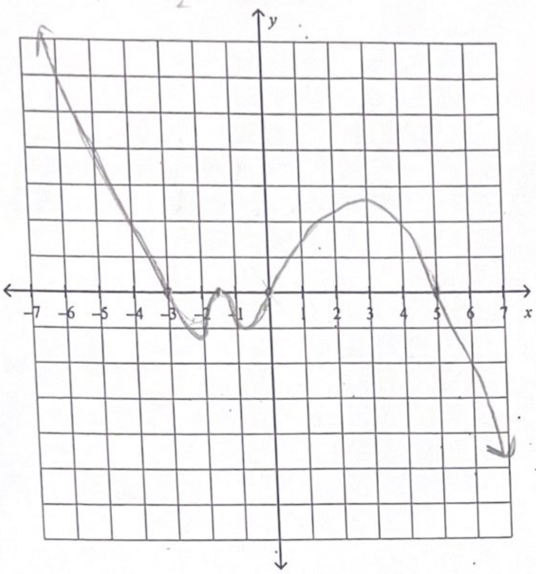
-2

8. Sketch a possible graph of the following function (Don't worry about y-scale):

$-\quad +\quad +\quad +$

$f(x) = -x^5(x+3)^3(-2x-3)^2(x-5)^5$

$-2x-3=0$
$-2x=3$
$x=-\frac{3}{2}$



9. Find the values of b such that the function has a minimum value of -50.

$f(x) = x^2 + bx - 25$

$-50 = x^2 + bx - 25$

$-50 = \left(-\frac{b}{2}\right)^2 + \left(-\frac{b}{2}\right)b - 25$

$-25 = \frac{b^2}{4} - \frac{b^2}{2}$

$-25 = \frac{b^2 - 2b^2}{4}$

$-100 = b^2 - 2b^2$

$-100 = b^2(1-2)$

$-100 = -b^2$

$b^2 = 100$

$\boxed{b = 10, -10}$

$f(-5) = (-5)^2 + (10)(-5) - 25$

$f(-5) = 25 - 50 - 25$

$f(-5) = -50$

$f(5) = 5^2 + (-10)(5) - 25$

$f(5) = 25 - 50 - 25$

$f(5) = 50$

10. A Norman window is constructed by adjooining a semicircle to the top of an ordinary rectangular window (See figure). The perimeter of the window is 18 feet.

a) Write the area of the entire window as a function of x.

$18 = 2y + x + \frac{\pi x}{2}$

$-2y = -18 + x + \frac{\pi x}{2}$

$-y = -9 + \frac{x}{2} + \frac{\pi x}{4}$

$y = 9 - \frac{x}{2} - \frac{\pi x}{4}$

$y = 9 + x\left(-\frac{1}{2} - \frac{\pi}{4}\right)$

$y = 9 + x\left(\frac{-2-\pi}{4}\right)$

$A(x) = xy + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$

$A(x) = x\left(9 + x\left(\frac{-2-\pi}{4}\right)\right) + \frac{1}{2}\pi\left(\frac{x^2}{4}\right)$

$A(x) = 9x + x^2\left(\frac{-2-\pi}{4}\right) + \frac{\pi x^2}{8}$

$A(x) = 9x + \frac{-2x^2 - \pi x^2}{4} + \frac{\pi x^2}{8}$

$A(x) = 9x + \frac{-4x^2 - 2\pi x^2 + \pi x^2}{8}$

$A(x) = 9x + \frac{-4x^2 - \pi x}{8}$

$\boxed{A(x) = x^2\left(\frac{-4-\pi}{8}\right) + 9x}$

b) What should x be (EXACTLY) that will produce the maximum area?

$A(x) = \left(\frac{-4-\pi}{8}\right)x^2 + 9x$

$0 = \left(\frac{-4-\pi}{4}\right)x + 9$

$-9 = \left(\frac{-4-\pi}{4}\right)x$

$\boxed{x = \frac{36}{4+\pi}}$

$-\frac{9}{1} \times \frac{4}{-4-\pi} = \frac{-36}{-4-\pi}$