Hon Pre-Calculus
Test Chapter 1 Name ________

No Calculators!!! Show All Work!!!! Circle All Final Answers!!!!

84/90

Short Answer

y-axis even

1. Use the algebraic test to determine which type of symmetry the following equation has:

a) $xy^2 + 10 = 0$

$x(-y)^2+10=0$

$xy^2+10=0$

$\boxed{\text{x-axis symmetry}}$

b) $y = \dfrac{x}{x^2+1}$

$y = \dfrac{-x}{(-x)^2+1}$

$-y = \dfrac{-x}{x^2+1}$

$\boxed{\text{origin symmetry}}$

2. Find the equation in point-slope form of the perpendicular bisector of the segment with endpoints $(-2,5)$ and $(6,7)$

slope: $\dfrac{7-5}{6+2} = \dfrac{2}{8} = \dfrac{1}{4}$ -4

midpt: $\left(\dfrac{-2+6}{2}, \dfrac{5+7}{2}\right) = (2,6)$

$\boxed{y-6=-4(x-2)}$

3. Find the average rate of change function using the difference quotient for the following function: (Rationalize the numerator)

$f(x) = 3\sqrt{x-1}$

$\dfrac{3\sqrt{x+h-1} - 3\sqrt{x-1}}{h} \cdot \dfrac{\sqrt{x+h-1}+\sqrt{x-1}}{\sqrt{x+h-1}+\sqrt{x-1}}$

$\dfrac{3(\cancel{x}+\cancel{h}\cancel{-1}\cancel{-x}\cancel{+1})}{\cancel{h}(\sqrt{x+h-1}+\sqrt{x-1})}$

$\boxed{\dfrac{3}{\sqrt{x+h-1}+\sqrt{x-1}}}$

4. Find the average rate of change from $x = \dfrac{\pi}{3}$ to $x = \dfrac{\pi}{3} + h$ using the difference quotient for the following function:

$f(x) = \cos x$

$\boxed{\dfrac{\cos h - \sqrt{3}\sin h - 1}{2h}}$

$\dfrac{\cos\left(\frac{\pi}{3}+h\right) - \cos\frac{\pi}{3}}{h}$

$\dfrac{\cos\frac{\pi}{3}\cos h - \sin\frac{\pi}{3}\sin h - \cos\frac{\pi}{3}}{h}$

$\dfrac{\frac{1}{2}\cos h - \frac{\sqrt{3}}{2}\sin h - \frac{1}{2}}{h}$

5. Find the average rate of change function using the difference quotient for the following function:

$f(x) = 3x^3 + 4x^2 - x$

1 2 1
1 3 3 1

$\dfrac{3(x+h)^3 + 4(x+h)^2 - (x+h) - 3x^3 - 4x^2 + x}{h}$

$\dfrac{\cancel{3x^3} + 9x^2h + 9xh^2 + 3h^3 + \cancel{4x^2} + 8xh + 4h^2 \cancel{-x} - h \cancel{-3x^3} \cancel{-4x^2} \cancel{+x}}{h}$

$\dfrac{9x^2h + 9xh^2 + 3h^3 + 8xh + 4h^2 - h}{h}$

$\boxed{9x^2 + 9xh + 3h^2 + 8x + 4h - 1}$

$f(x) = 2x^2 + 5x - 3$

$(2x^2 + 6x)(-x - 1)$

$2x(x+3) - 1(x+3)$

$x = \frac{1}{2}, -3$

6. Use interval notation to determine the domain and range of the given function:

a) $f(x) = -\frac{2}{3}|x-2| + 3$

Domain = $(-\infty, \infty)$

Range = ~~$(-\infty, 3)$~~ $(-\infty, 3]$

b) $f(x) = \sqrt{25 - x^2}$

$x \neq \pm 5$

Domain = ~~$(-\infty, -5) \cup (-5, 5) \cup (5, \infty)$~~ $[-5, 5]$

Range = ~~$(0, \infty)$~~ $[0, 5]$

7. Find the zeros of the function algebraically.

$f(x) = x^3 - 4x^2 - 9x + 36$

$x^2(x-4) - 9(x-4)$

$(x+3)(x-3)(x-4)$

$x = -3, 3, 4$

$\boxed{(-3,0)\ (3,0)\ (4,0)}$

8. Given: $f(x) = \frac{2}{3}x^3 + \frac{5}{2}x^2 - 3x + 4$. Find the following:

a) relative maximum point.

$(-3, \frac{35}{2})$

$\frac{2}{3}(-27)$

$-18 + \frac{45}{2} + 9 + 4$

$-18 + 17$

$\frac{35}{2}$

b) relative minimum point

$(\frac{1}{2}, \frac{77}{24})$

$\frac{2}{3}(\frac{1}{8}) \quad \frac{1}{12} + \frac{5}{8} - \frac{3}{2} + 4$

c) Where the function is increasing (Interval notation)

$(-\infty, -3) \cup (\frac{1}{2}, \infty)$

d) Where the function is decreasing (Interval notation)

$(-3, \frac{1}{2})$

$\frac{1}{12} + \frac{5}{8} - \frac{3}{2} + \frac{48}{12}$

$\frac{18}{12}$

$\frac{2}{24} + \frac{15}{24} - \frac{36}{24}$

$\frac{60}{24}$

9. Determine which intervals (using interval notation) the function is increasing and/or decreasing:

$$f(x) = \begin{cases} x+3, & x \le 0 \\ 3, & 0 < x \le 2 \\ -2x+1, & x > 2 \end{cases}$$

Increasing = ~~$(-\infty, 0]$~~ $(-\infty, 0)$

Decreasing = $(2, \infty)$

-5

10. Determine if the function is even, odd, or neither. (Show work to support your answer)

$h(x) = x\sqrt{x+5}$

$h(-x) = -x\sqrt{-x+5}$ ×

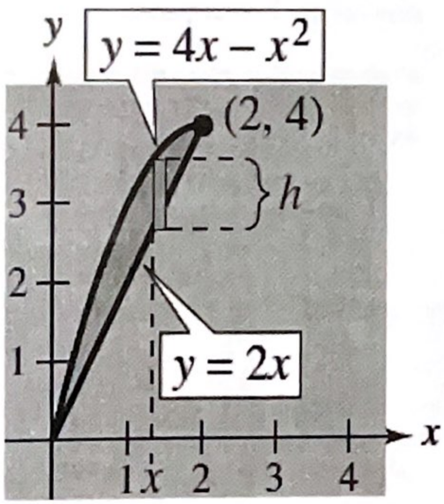
$-h(x) = x\sqrt{x+5}$

no symmetry

↓

$\boxed{\text{neither}}$

11. Write the height h of the rectangle as a function of x.



$h(x) = 4x - x^2 - 2x$

$\boxed{h(x) = 2x - x^2}$

12. Find a linear function that has the indicated function values:

$f\left(\frac{1}{2}\right) = -6$ and $f(4) = -3$

$\frac{-3+6}{4-\frac{1}{2}} = \frac{3}{\frac{7}{2}}$

$\frac{3}{1} \cdot \frac{2}{7} = \frac{6}{7}$

$(\frac{1}{2}, -6)(4, -3)$

$-3 = \frac{6}{7}(4) + b$

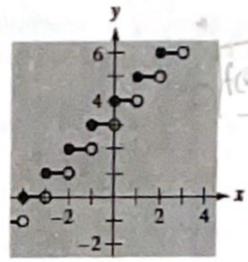
$-\frac{21}{7}$ $-3 = \frac{24}{7} + b$

$b = -\frac{45}{7}$

$\boxed{f(x) = \frac{6}{7}x - \frac{45}{7}}$

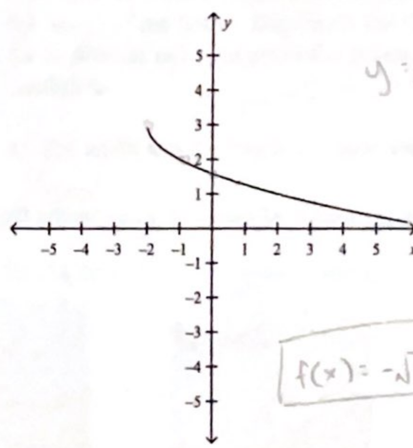
13. Write the function given the graph of the function.

a)



$\boxed{f(x) = [[x+4]]}$

b)

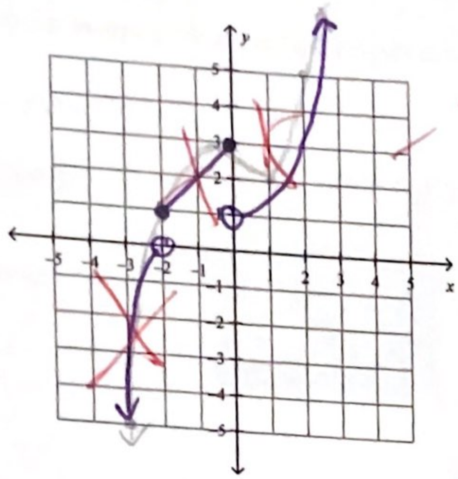


$y = \sqrt{x}$

$\boxed{f(x) = -\sqrt{x+2} + 3}$

14. Sketch the graph of the following:

$$h(x) = \begin{cases} 4 - x^2, & x < -2 \\ 3 + x, & -2 \le x < 0 \\ x^2 + 1, & x > 0 \end{cases}$$



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15. g is related to one of the parent functions.
   a) Identify the parent function f, and
   b) Describe the sequence of transformations from f to g.

$$g(x) = 3\sqrt{\frac{1}{2}(x-2)} - 4$$

a. $f(x) = \sqrt{x}$
b. 1. vertical stretch of 3
2. horizontal stretch of 2
3. right 2
4. down 4

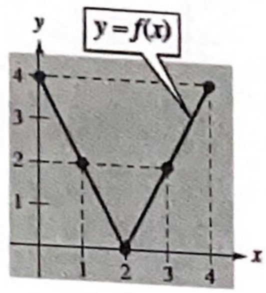
16. Determine if the following situation could be represented by a one-to-one function.

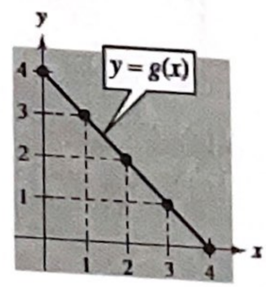
The depth of the tide d at a beach in terms of the time t over a 24 hour period.

If one-to-one then give a statement that describes the inverse function. If not one-to-one explain why not.

it is not one-to-one because the tide goes up & down, meaning there are multiple x values for the same y value so it doesn't pass the horizontal line test.

17. Use the graphs of f and g to evaluate the functions (4 different problems)





a) $(f + g)(3)$
b) $(f/g)(2)$
c) $(f - g)(1)$
d) $(fg)(4)$

a.) $2+1 = 3$
b.) $\frac{0}{2}$ 0
c.) $2-3 = -1$
d.) $4(0) = 0$

-3

18. State the domain and range using interval notation of $g(f(x))$ if:

$f(x) = x^2 - 3x - 4$ and $g(x) = \frac{1}{x}$

Domain = ~~$(-\infty,-1)\cup(-1,4)\cup(4,\infty)$~~ 2

$(-\infty,-1)\cup(-1,4)\cup(4,\infty)$

Range = ~~$(-\infty,-\frac{3}{2})\cup(1,\infty)$~~ 2

$(-\infty,-\frac{4}{25}]\cup(0,\infty)$

On the test correction quiz 2023-2024

$\frac{1}{x^2-3x-4}$

19. Show that f and g are inverses if:

$f(x) = \frac{x-1}{x+5}$ and $g(x) = -\frac{5x+1}{x-1}$

$f(x) = \dfrac{\frac{-5x-1}{x-1} - \frac{x-1}{x-1}}{\frac{-5x-1}{x-1} + \frac{5x-5}{x-1}}$

$f(x) = \dfrac{\frac{-6x+\cancel{0}}{\cancel{x-1}}}{\frac{-6}{\cancel{x-1}}}$

$f(x) = \frac{-6x}{-6}$

$= x$ ✓

4

$g(x) = \dfrac{-\frac{5x+5}{x+5} - \frac{x+5}{x+5}}{\frac{x-1}{x+5} - \frac{x+5}{x+5}}$

$g(x) = \dfrac{\frac{-6x}{\cancel{x+5}}}{\frac{-6}{\cancel{x+5}}}$

$g(x) = \frac{-6x}{-6}$

$= x$ ✓

20. Find the inverse function for $f(x) = \frac{8x-4}{2x+6}$

$x = \frac{8y-4}{2y+6}$

$2xy + 6x = 8y - 4$

$6x + 4 = 8y - 2xy$

$6x + 4 = y(8-2x)$

$\boxed{y = \frac{6x+4}{8-2x}}$

21. Assume y is directly proportional to x, Find a linear model that relates x to y if y = 14 when x = 2.

$y = kx$

$14 = k2$

$k = 7$

$\boxed{y = 7x}$

22. The maximum load that can be safely supported by a horizontal beam varies jointly as the width of the beam and the square of its depth, and inversely as the length of the beam. Determine the changes in the maximum safe load under the following conditions:

$m = \frac{wd^2}{l}$

a) The width and the length are both doubled.

$m = \frac{2wd^2}{2l}$ nothing changes

b) All three dimensions are doubled.

$m = \frac{2w4d^2}{2l}$ quadrupled

c) The depth of the beam is halved.

$m = \frac{w\frac{1}{4}d^2}{l}$ quartered

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