Hon Pre-Calc Quiz Chapter 5.1-5.3 2018 - 2019

1. Simplify the expression: $\csc\alpha\tan\alpha+\sec\alpha$

2. Factor the expression completely: $\csc^3 x-\csc^2 x-\csc x+1$

3. Use trigonometric substitution to write the algebraic expression as a trigonometric function of θ, where θ is in the interval $\left(0,\frac{\pi}{2}\right)$

   $\sqrt{4x^2+9}, 2x=3\tan\theta$

4. Rewrite the following so that it is not in fractional form.

   $$\frac{3}{\sec x-\tan x}$$

5. Simplify completely: $\tan x+\frac{\cos x}{1+\sin x}$

6. Verify the identity:
$$\frac{1+\sin x}{\cos x}+\frac{\cos x}{1+\sin x}=2\sec x$$

7. Verify the identity:
$$\cos x-\frac{\cos x}{1-\tan x}=\frac{\sin x\cos x}{\sin x-\cos x}$$

8. Verify the identity:
$$\tan\left(sin^{-1}\frac{x-1}{4}\right)$$

9. Verify the identity:
$$\cos^3 x\sin^2 x=(\sin^2 x-\sin^4 x)\cos x$$

10. Solve over the reals: $2\sin x + \csc x = 0$

11. Solve over the interval $[0, 2\pi)$: Approximate your answers to three decimal places.
$\tan^2 x + 3\tan x + 1 = 0$

12. Solve over the reals: $\tan^2\left(\frac{\pi x}{6}\right) = 3$

13. Solve over the interval $[0, 2\pi)$: Approximate your answers to three decimal places.
$3\cos(4x - 1) = 2$

14. Solve Over the interval $[0, 2\pi)$: Approximate your answers to three decimal places.

$-5\cos\left(3\theta - \frac{\pi}{4}\right) = 2$

15. Solve over the interval $[0, 2\pi)$: Approximate you answers to three decimal places.

$\cot x - \csc x = 3$

Hon Pre-Calc

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Show all work for credit!!! Calculators allowed!!! Circle all final answers!!!

Short Answer

1. Simplify the expression: $\csc\alpha\tan\alpha+\sec\alpha$

$\frac{\cos\alpha}{\sin\alpha}\cdot\frac{\sin\alpha}{\cos\alpha}=\frac{1}{\cos\alpha}$

$=\frac{2}{\cos\alpha}=\boxed{2\sec\alpha}$

2. Factor the expression completely:
$\csc^3x-\csc^2x-\csc x+1$

$1+\cot^2x=\csc^2x$

$\csc^2x(\csc x-1)-(\csc x-1)$

$=(\csc^2x-1)(\csc x-1)$

$=\boxed{(\csc x+1)(\csc x-1)(\csc x-1)}$

3. Use trigonometric substitution to write the algebraic expression as a trigonometric function of θ, where θ is in the interval $\left(0,\frac{\pi}{2}\right)$.

$\sqrt{4x^2+9},\ (2x=3\tan\theta)^2$

$\sqrt{9\tan^2\theta+9}=3\sqrt{\tan^2\theta+1}$

$=3\sqrt{\csc^2\theta}=\boxed{3\csc\theta}$

4. Rewrite the following so that it is not in fractional form.

$\tan^2x+1=\sec^2x$

$\frac{3}{\sec x-\tan x}\cdot\frac{(\sec x+\tan x)}{(\quad)}$

$=\frac{3(\sec x+\tan x)}{\sec^2x-\tan^2x}=\frac{3\sec x+3\tan x}{1}$

$=\boxed{3\sec x+3\tan x}$

5. Simplify completely: $\tan x+\frac{\cos x}{1+\sin x}$

$\frac{\sin x}{\cos x}+\frac{\cos x}{1+\sin x}=\frac{\sin x+\sin^2x+\cos^2x}{(\cos x)(1+\sin x)}$

$=\frac{1+\sin x}{(\cos x)(1+\sin x)}=\frac{1}{\cos x}=\boxed{\sec x}$

6. Verify the identity:

$$\frac{1+\sin x}{\cos x}+\frac{\cos x}{1+\sin x}=2\sec x$$

$\frac{(1+\sin x)^2+\cos^2 x}{(\cos x)(1+\sin x)} = \frac{1+2\sin x+\sin^2 x+\cos^2 x}{(\cos x)(1+\sin x)}$

$= \frac{1+2\sin x+1}{\cos x(1+\sin x)} = \frac{2(1+\sin x)}{\cos x(1+\sin x)} = \frac{2}{\cos x}$

$= 2\sec x$

7. Verify the identity:

$$\cos x-\frac{\cos x}{1-\tan x}=\frac{\sin x\cos x}{\sin x-\cos x}$$

$\cos x - \frac{\cos x}{1-\frac{\sin x}{\cos x}} = \cos x - \frac{\cos^2 x}{\cos x - \sin x} =$

$\frac{\cos x(\cos x-\sin x)}{\cos x-\sin x} - \frac{\cos^2 x}{\cos x-\sin x}$

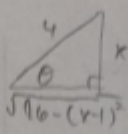
$= \frac{\cos^2 x-\cos x\sin x-\cos^2 x}{\cos x-\sin x} = \frac{-\cos x\sin x}{\cos x-\sin x}$ (-1) (-1)

$= \frac{\sin x\cos x}{\sin x-\cos x}$

8. Verify the identity:

$$\tan\left(\sin^{-1}\left(\frac{x-1}{4}\right)\right)=\frac{x-1}{\sqrt{16-(x-1)^2}}$$

- Let $\sin^{-1}\left(\frac{x-1}{4}\right)=\theta$



-1

$\therefore \tan\theta = \frac{x-1}{\sqrt{16-(x-1)^2}}$

- Substitute θ

$\tan\left(\sin^{-1}\left(\frac{x-1}{4}\right)\right) = \frac{x-1}{\sqrt{16-(x-1)^2}}$

9. Verify the identity:

$$\cos^3 x\sin^2 x=\left(\sin^2 x-\sin^4 x\right)\cos x$$

$\cos x\cdot\cos^2 x\cdot\sin^2 x = \cos x(1-\sin^2 x)\cdot\sin^2 x$

$= \cos x\left(\sin^2 x-\sin^4 x\right)$

-1

10. Solve over the reals: $2\sin x + \csc x = 0$

$2\sin x + \frac{1}{\sin x} = 0$, $2\sin^2 x + 1 = 0$

$\sin^2 x = -\frac{1}{2}$

$\sin x = \pm\sqrt{-\frac{1}{2}}$

No solution

11. Solve over the interval $[0, 2\pi)$: Approximate your answers to three decimal places.

$\tan^2 x + 3\tan x + 1 = 0$ per: π

$\tan x = \frac{-3 \pm \sqrt{9-4}}{2} = \frac{-3 \pm \sqrt{5}}{2}$

$x =$	-0.765	-1.206
$+\pi$	2.777	1.936
$+\pi$	5.918	5.077

$x = 2.777, 1.936, 5.918, 5.077$

12. Solve over the reals: $\tan^2\left(\frac{\pi x}{6}\right) = 3$

$\frac{\pi}{6} \cdot 6 = \frac{7}{6} \cdot 6$

$\tan \frac{\pi}{6} x = \pm\sqrt{3}$

$\frac{\pi}{6} x = \frac{\pi}{3} + n\pi$, $\frac{2\pi}{3} + n\pi$

$x = 2 + 6n$, $\frac{2\pi}{3} \cdot \frac{6}{\pi} = 4 + 6n$

$x = 2 + 6n$, $4 + 6n$, where n is an integer

13. Solve over the interval $[0, 2\pi)$: Approximate your answers to three decimal places.

$3\cos(4x - 1) = 2$ $\frac{2\pi}{4} = \frac{\pi}{2} = 1.57$ per.

$\cos(4x - 1) = \frac{2}{3}$

$4x - 1 = 0.841$, 5.442

$4x = 1.841$, 6.442

$x =$	0.460,	1.611
$+\frac{\pi}{2}$	2.031	3.181
	3.602	4.752
	5.173	6.323
	7.124	

$-\frac{\pi}{2} = 0.040$

$4 \times 2 = 8$

$x = 0.460, 2.031, 3.602, 5.173, 1.611, 3.181, 4.752$

+1

14. Solve over the interval $[0,2\pi)$: Approximate your answers to three decimal places.

$$-5\cos\left(3\theta-\frac{\pi}{4}\right)=2$$

$\frac{2\pi}{3}$ = per.

2.094

$\cos\left(3\theta-\frac{\pi}{4}\right)=-\frac{2}{5}$

$3\theta-\frac{\pi}{4}=1.982$, $3\theta-\frac{\pi}{4}=4.301$

$\theta=0.923$, $\theta=1.695$

3.017, 3.790

5.111, 5.884

7.206, 7.979

$\theta=0.923, 3.017, 5.111, 1.695, 3.790, 5.884$

15. Solve over the interval $[0,2\pi)$: Approximate your answers to three decimal places.

$1+\cot^2 x=\csc^2 x$

$\cot x-\csc x=3+\csc x$

$\cot^2 x=9+6\csc x+\csc^2 x$

$\csc^2 x-1=9+6\csc x+\csc^2 x$

$-10=6\csc x$

$\csc x=-\frac{5}{3}$

$\frac{\cos x}{\sin x}=\frac{1}{\sin x}\cdot 3$

$\sin x=-\frac{3}{5}$

$x=3.785, 5.640$

$\cos x=\frac{4}{5}$

$x=3.785$