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Hon Pre-Calc

Quiz 5.1 - 5.3 Name [redacted] 49/50

Show All Work!!! Circle All Final Answers!!! Use Exact Values When Possible!!! Round to nearest 100th when not possible!!!!

$\sin^2 x+\cos^2 x = 1$

$1+\cot^2 x = \csc^2 x$

$\tan^2 x + 1 = \sec^2 x$

Short Answer

1. Rewrite the following so that it is not in fractional form.

$$\frac{3}{\sec x-\tan x}\left(\frac{\sec x+\tan x}{\sec x+\tan x}\right)$$

$$\frac{3(\sec x+\tan x)}{\sec^2 x-\tan^2 x} = \frac{3(\sec x+\tan x)}{1}$$

$$= \boxed{3(\sec x+\tan x)}$$

2. Simplify completely to a single trigonometric function:

$\sin\beta\tan\beta+\cos\beta$

$$\frac{\cancel{\sin\beta}}{1}\cdot\frac{\sin^{2}\beta}{\cos\beta}+\frac{\cos\beta}{1}$$

$$= \frac{\sin^2\beta+\cos^2\beta}{\cos\beta} = \frac{1}{\cos\beta} = \boxed{\sec\beta}$$

3. Simplify completely to a single trigonometric function:

$$\tan x+\frac{\cos x}{1+\sin x}$$

$$\frac{\sin x}{\cos x}+\frac{\cos x}{1+\sin x}$$

$$\frac{\sin x+\overbrace{\cancel{\sin^2 x+\cos^2 x}}^{1}}{\cos x(1+\sin x)} = \frac{1+\cancel{\sin x}}{\cos x(1+\cancel{\sin x})}$$

$$= \frac{1}{\cos x} = \boxed{\sec x}$$

4. Simplify completely to a single trigonometric function:

$$\frac{1+\sin x}{\cos x}+\frac{\cos x}{1+\sin x}$$

$$= \frac{1+2\sin x+\overbrace{\cancel{\sin^2 x+\cos^2 x}}^{1}}{\cos x(1+\sin x)} = \frac{2(1+\sin x)}{co\ldots}$$

$$= \frac{2}{\cos x} = \boxed{2\sec x}$$

5. Simplify completely to a single trigonometric function:

$$\frac{\tan x}{1}-\frac{\sec^2 x}{\tan x}$$

$$\frac{\tan^2 x-\sec^2 x}{\tan x} = \frac{-1}{\tan x} = \boxed{-\cot x}$$

6. Factor the expression completely. (leave your answer in terms of just secant.)

$\sec^3 x-\sec^2 x-\sec x+1$

$\sec^2 x(\sec x-1)-1(\sec x-1)$

$(\sec^2 x-1)(\sec x-1)$

$(\sec x+1)(\sec x-1)(\sec x-1)$

$\boxed{(\sec x+1)(\sec x-1)^2}$

7. Use trigonometric substitution to write the algebraic expression as a trigonometric function of θ, where θ is in the interval $\left(0, \frac{\pi}{2}\right)$.

$\sqrt{9x^2+25}$, $(3x)^2 = (5\tan\theta)^2$

$9x^2 = 25\tan^2\theta$

$\sqrt{25\tan^2 x + 25}$

$\sqrt{25(\tan^2 x + 1)}$

$5\sqrt{\sec^2 x}$

$\boxed{5\sec x}$

8. Verify the identity:

$$\frac{\cot\alpha}{\csc\alpha+1} = \frac{\csc\alpha-1}{\cot\alpha}$$

$$\frac{\cot a}{\csc a+1}\left(\frac{\csc a-1}{\csc a-1}\right) = \frac{\csc a-1}{\cot a}$$

$$\frac{\cot a(\csc a-1)}{\csc^2 a-1} = \frac{\csc a-1}{\cot a}$$

$$\frac{\cancel{\cot a}(\csc a-1)}{\cot^2 a} = \frac{\csc a-1}{\cot a}$$

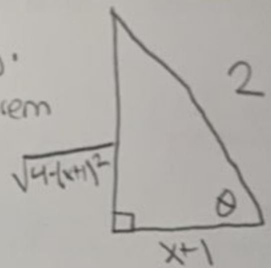
✓ $$\frac{\csc a-1}{\cot a} = \frac{\csc a-1}{\cot a}$$

9. Verify the identity:

$$\tan\left(\cos^{-1}\frac{x+1}{2}\right) = \frac{\sqrt{4-(x+1)^2}}{x+1}$$

Let $\cos^{-1}\frac{x+1}{2} = \theta$

Pythag. Theorem



$\therefore \tan\left(\cos^{-1}\frac{x+1}{2}\right) = \frac{\sqrt{4-(x+1)^2}}{x+1}$

Symbol for "therefore"

$$\frac{\sqrt{4-(x+1)^2}}{x+1} = \frac{\sqrt{4-(x+1)^2}}{x+1}$$

10. Find all solutions on the interval $[0,2\pi)$ (Answers must be exact)

$2\sec^2 x+\tan^2 x-3=0$

$2\sec^2 x + (\sec^2 x - 1) - 3 = 0$

$3\sec^2 x - 4 = 0$

$\sec^2 x = \frac{4}{3}$

$\sqrt{\cos^2 x} = \pm\sqrt{\frac{3}{4}}$

$\cos x = \pm\frac{\sqrt{3}}{\sqrt{4}}\left(\frac{\sqrt{4}}{\sqrt{4}}\right)$

$\cos x = \pm\frac{\cancel{2}^{1}\sqrt{3}}{\cancel{4}2} = \pm\frac{\sqrt{3}}{2}$

$$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}}$$

11. Find all solutions on the interval $[0,2\pi)$ (Answers must be exact)

$2\sin x+\csc x=0$

$\frac{2\sin x}{1}+\frac{1}{\sin x}=0$

$\frac{\cancel{\sin x}}{1}\cdot\frac{2\sin^2x+1}{\cancel{\sin x}}=0\cdot\sin x$

$2\sin^2x+1=0$
$\quad\;\;-1\;\;-1$

$\frac{2\sin^2x}{2}=\frac{-1}{2}$

$\sin^2x=-\frac{1}{2}$

$\boxed{\text{No Solution!}}$

13. Find all solutions on the interval $[0,2\pi)$ (Answers must be exact)

$2\sin^2x=2+\cos x$

$2(1-\cos^2x)=2+\cos x$

$\cancel{2}-2\cos^2x=\cancel{2}+\cos x$

$2\cos^2x+\cos x=0$

$\cos x(2\cos x+1)=0$

$\cos x=0 \qquad \cos x=-\frac{1}{2}$

$\boxed{x=\frac{\pi}{2},\frac{3\pi}{2} \qquad x=\frac{2\pi}{3},\frac{4\pi}{3}}$

12. Find all solutions on the interval $[0,2\pi)$ (Answers must be exact)

$\sin 2x+\sqrt{2}\sin x=0$

$2\sin x\cos x+\sqrt{2}\sin x=0$

$\sin x(2\cos x+\sqrt{2})=0$

$\sin x=0 \qquad \cos x=-\frac{\sqrt{2}}{2}$

$\boxed{x=0,\pi \qquad x=\frac{3\pi}{4},\frac{5\pi}{4}}$

14. Find all solutions over the reals (answers must be exact):

$\tan^2(3x)=3$

Let $3x=\theta$

$\tan^2\theta=3$

$\tan\theta=\pm\sqrt{3}$

$\theta=\frac{\pi}{3} \quad \theta=\frac{2\pi}{3} \quad \theta=\frac{4\pi}{3} \quad \theta=\frac{5\pi}{3}$

$3x=\frac{\pi}{3} \quad 3x=\frac{2\pi}{3} \quad 3x=\frac{4\pi}{3} \quad 3x=\frac{5\pi}{3}$

$x=\frac{\pi}{9} \quad x=\frac{2\pi}{9} \quad x=\frac{4\pi}{9} \quad x=\frac{5\pi}{9}$

$\boxed{\begin{array}{l} x=\frac{\pi}{9}+\frac{3\pi}{9}n \quad (n\text{-int}) \\ x=\frac{2\pi}{9}+\frac{3\pi}{9}n \quad (n\text{-int}) \end{array}}$

15. Find all solutions on the interval $[0, 2\pi)$ (Round Answers to nearest 100th)

$3\cos(4x-1)=2$

$Per = \frac{2\pi}{b} = \frac{2\pi}{4} = \frac{\pi}{2}$

Let $4x-1=\theta$

$Per = \frac{\pi}{2}$

$3\cos\theta = 2$

$2\pi - 0.84$

$\cos\theta = \frac{2}{3}$

$\theta = 0.84$ $\theta = 5.44$

Subtract Period!

$4x-1 = 0.84$ $4x-1 = 5.442$

$$\boxed{\begin{array}{ll} x = 0.460 & x = 1.611 \\ = 2.03 & = 3.18 \\ = 3.60 & = 4.75 \\ = 5.173 & = 0.04 \end{array}}$$

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