Hon Pre-Calc
Quiz 2.1 - 2.5

Name ______________________

Show All Work!!! Circle All Final Answers!!! NO Calculators!!!

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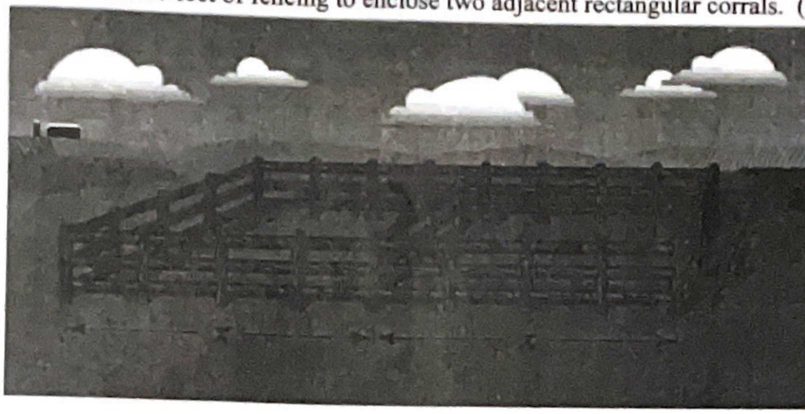
Short Answer

1. A rancher has 200 feet of fencing to enclose two adjacent rectangular corrals. (see figure)



a) Write the area of the corrals as a function of *x*.

$A = -\frac{8}{3}x^2 + \frac{400}{3}$

$A = 2xy$ $P = 200$

$A = 2x\left(\frac{200}{3} - \frac{4}{3}x\right)$

$= \frac{400}{3}x - \frac{8}{3}x^2$

$P = 4x + 3y$

$200 = 4x + 3y$

$\frac{3y = 200 - 4x}{3}$

$y = \frac{200}{3} - \frac{4}{3}x$

b) State the domain of the area function.

$2x > 0$

$x > 0$

$\frac{200}{3} - \frac{4}{3}x > 0$

$-\frac{3}{4} \cdot -\frac{4}{3}x > -\frac{200}{3} \cdot -\frac{3}{4}$

$x < 50$

$(0, 50)$

c) Find the dimensions of the corrals that will produce the maximum area.

$\frac{-400/3}{2\left(-\frac{8}{3}\right)}$ 25

$+\frac{400}{3} \cdot +\frac{3}{16} = 25$

$x = 25$ ft
$y = \frac{100}{3}$ ft

25 × 25 = 625, 625 × 4 = 2500

200 × 25 = 5000, 5000 − 2500 = 2500

$200 = 100 + 3y$

$y = \frac{100}{3}$

$\frac{100}{3} = \frac{200}{3} - \frac{100}{3}$ ✓

2. Find all zeros of the function and write the polynomial as a product of linear factors.

$f(x) = x^4 - 4x^3 + 8x^2 - 16x + 16$

$\pm 4, \pm 2, \pm 8, \pm 16, \pm 1$

$2 \mid 1 \quad -4 \quad 8 \quad -16 \quad 16$
$\downarrow \quad 2 \quad -4 \quad 8 \quad -16$

$(1x^3 - 2x^2)(4x - 8) \quad 0$

$x^4 + 4x^3 + 8x^2 + 16x + 6$ $\quad 0$

$x^2(x-2)4(x-2)$

$\boxed{(x^2+4)(x-2)^2}$

$\boxed{(x+2i)(x-2i)(x-2)^2}$

3. Find two positive real numbers whose product is a maximum if the sum of the first and three times the second is 42.

$P = xy$ $\quad$ $x + 3y = 42$ $\quad$ 21

$x = 42 - 3y$

$P = -3y^2 + 42y$

$\frac{-42}{2(-3)} = \frac{42}{6} = 7$

$\boxed{x = 21 \quad y = 7}$

$21 + 21 = 42$

4. State the end behavior using limit notation of the following functions:

a) $f(x) = 6 - 2x + 4x^2 - 5x^3$

$\lim_{x \to \infty} f(x) = -\infty$

$\lim_{x \to -\infty} f(x) = \infty$

b) $f(x) = \frac{-3x^4 + 2x - 5}{-4}$

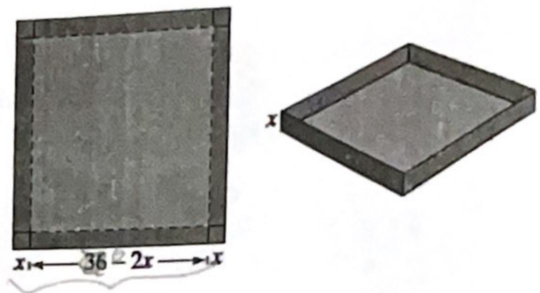
$\lim_{x \to \infty} f(x) = -\infty$

$\lim_{x \to -\infty} f(x) = -\infty$

$\lim_{x \to \infty} f(x) = \infty$

$\lim_{x \to -\infty} f(x) = \infty$

5. An open box is to be made from a square piece of material, 6 inches on a side, by cutting equal squares with sides of length x from the corners and turning up the sides. (see figure)



a) Write a function for the volume in terms of x.

$\boxed{V = (6-2x)(6-2x)(x)}$

b) State the domain of the volume function.

$6 - 2x > 0$ $\quad$ $x > 0$
$-2x > -6$
$x < 3$

$\boxed{(0, 3)}$

c) Find the maximum volume.

$V = 4x^3 - 24x^2 + 36x$
$= 12x^2 - 48x + 36$ $\quad$ $12(x^2 - 4x + 3)$

$x = \frac{4 \pm \sqrt{16 - 4(3)}}{2}$

$x = \frac{4 \pm 2}{2}$ $\quad$ $x = 3, 1$ $\quad$ $1 - 3$

$(6-2(1))(6-2(1))(1)$

$4(4) = 16$

$\boxed{16 \text{ in}^3}$

6. F[illegible]nd the remaining factors of $f(x)$ if $f(x) = x^4 - 4x^3 - 15x^2 + 58x - 40$ and $(x - 5)$ and $(x + 4)$ are factors.

$$\begin{array}{r|rrrrr} 5 & 1 & -4 & -15 & 58 & -40 \\ & \downarrow & 5 & 5 & -50 & 40 \\ \hline -4 & 1 & 1 & -10 & 8 & 0 \\ & \downarrow & -4 & 12 & -8 & \\ \hline & 1x^2 & -3x & +2 & 0 & \end{array}$$

$\boxed{(x-2),(x-1)}$

$(x-5)(x+4)(x-2)(x-1)$

7. Find the value of k such that $x - 3$ is a factor of $x^3 - kx^2 + 2kx - 12$.

$$\begin{array}{r|rrrr} 3 & 1 & -k & 2k & -12 \\ & \downarrow & 3 & 9-3k & 12 \\ \hline & 1 & 3-k & 9-k & 0 \end{array}$$

$\frac{3(9-k)=12}{3}$

$9-k=4$

$-k=-5$

$k=5$

$$\begin{array}{r|rrrr} 3 & 1 & -5 & 10 & -12 \\ & \downarrow & 3 & -6 & 12 \\ \hline & 1 & -2 & 4 & 0 \end{array} \checkmark$$

8. Simplify and write your answer in $a + bi$ form.

a) $\frac{1+i}{i} - \frac{3}{4-i}$

$\frac{1+i(4-i)}{i(4-i)} - \frac{3i}{i(4-i)}$

$\frac{5+\cancel{3i}-\cancel{3i}}{1+4i}$ $\frac{5}{1+4i} \cdot \frac{(1-4i)}{(1-4i)}$

$\frac{5-20i}{17}$ $\boxed{\frac{5}{17} - \frac{20}{17}i}$

b) $\sqrt{-5} \bullet \sqrt{-10}$

$\sqrt{+5} \cdot \sqrt{-1} \cdot \sqrt{-1} \cdot \sqrt{10}$

$-\sqrt{50}$

$\boxed{-5\sqrt{2}}$

$\sqrt{5}i \cdot \sqrt{10}i$

$\sqrt{50}i$

$5\sqrt{2}i$

c) i^{1233} 308 R1

i^1

$\boxed{=i}$

9. Find a polynomial of least degree in standard form (fully expanded) with integer coefficients that has -5 multiplicity of 2, and $1+i\sqrt{3}$ as zeros.

$(x+5)^2(x-1+i\sqrt{3})(x-1-\sqrt{3}i)$

$(x^2+10x+25)(x^2-x \cancel{-\sqrt{3}i} -x+1 \ldots)$

$(x^2+10x+25)(x^2-2x+4)$

$x^4 \cancel{-2x^3} \cancel{+4x^2} \cancel{+10x^3} \cancel{-20x^2} \cancel{+40x} \cancel{+25x^2} \cancel{-50x} +100$

$\boxed{x^4+8x^3+9x^2-10x+100}$

$-5 \mid 1 \quad 8 \quad 9 \quad -10 \quad 100$
$\quad -5 \quad -15 \quad 30 \quad -100$

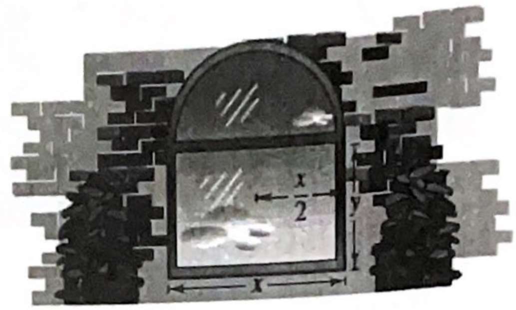
$-5 \mid 1 \quad 3 \quad -6 \quad 20 \mid 0$
$\quad -5 \quad 10 \quad -20$
$\quad 1 \quad -2 \quad 4 \quad 0$

$\frac{2 \pm \sqrt{4-16}}{2}$

$\frac{2 \pm \sqrt{12}}{2}$ $\frac{2 \pm 2\sqrt{3}i}{2}$

$1 \pm \sqrt{3}i$

10. A Norman window is constructed by adjoining a semicircle to the top of an ordinary rectangular window (see - figure). The perimeter of the entire window is 12 feet.



What should x be to maximize area of the window?

$P=12 \quad P=x+2y+\frac{x\pi}{2} \quad A=xy+\frac{x^2}{8}\pi$

$2y=12-x-\frac{x\pi}{2}$

$y=6-\frac{x}{2}-\frac{x\pi}{4}$

$A=x\left(6-\frac{x}{2}-\frac{x\pi}{4}\right)+\frac{x^2}{8}\pi$

$A=6x-\frac{4x^2}{8}-\frac{2x^2\pi}{8}+\frac{x^2\pi}{8}$

$A=6x-\frac{4x^2}{8}-\frac{x^2\pi}{8}$

$A=\frac{-4-\pi}{8}x^2+6x$

$\frac{-6}{2\left(\frac{-4-\pi}{\cancel{8}\,4}\right)} = \boxed{\frac{24}{4+\pi}=x}$