Hon Pre-Calc Quiz 9.1 – 9.3 *2016 – 2017*

Show All Work!! Circle Final answers!!

1. Write an expression for the apparent n^{th} term of the sequence.

a) 1, 3, $\frac{9}{2}, \frac{27}{6}, \frac{81}{24}, \frac{243}{120}$

b) 1, 7, 1, 7, 1, 7, 1, 7

2. Simplify: $\frac{1000!}{998!6!}$

3. Use sigma notation to write the sum.

$$\frac{-1}{2} + \frac{1}{3} + \frac{3}{4} + \cdots + \frac{27}{16}$$

4. Find the a_{12} term in terms of x and y if a is arithmetic and, $a_6 = x$, and $a_8 = y$

5. Find the sum of the multiples of 5 from -2025 to 5015

6. Evaluate: $\sum_{n=0}^{100}(100-2n)$

7. Given the following recursive formula.

$$\begin{cases} a_1 = 384n \\ a_{k+1} = -\frac{1}{2}a_k \end{cases}$$

Find a_{20} in simplest fraction form.

8. Find all infinite sums for the geometric sequence if $g_4 = 4$ and $g_8 = \frac{1}{4}$

9. Use summation notation to write the sum:

$$2 - \frac{1}{2} + \frac{1}{8} - \cdots + \frac{1}{2048}$$

10. A side of an equilateral triangle is 30 inches long. A second equilateral triangle is inscribed in it by joining the midpoints of the sides of the first triangle. The process is continued. Find the perimeter of the 13th inscribed triangle in reduced fraction form.

11. Filled to capacity, a tank contains 20 gallons of pure antifreeze. Two gallons of liquid is drawn out and the tank is filled with water. If this operation is repeated several times, after how many operations will there be less than 0.5 gallons of pure antifreeze left in the tank?

12. Evaluate: $\sum_{i=0}^{\infty}\left(\frac{1}{10}\right)^{i}$

13. Alex collected 14 aluminum cans the first day, 20 cans the second day, 26 the third day, and so on, until he had collected a total of 3030 cans.

a) How many cans did Alex collect in just his second week of collecting?

b) How many days did he spend total collecting cans?

14. A ball is dropped from a helicopter hovering at 2 miles above sea level. If the ball retains 90% of its previous height after each bounce, find the following:

a) Find the first bounce for which the ball will be under 5 feet.

b) The height of the ball after it has traveled a total vertical distance of 200,000 feet.

Hon Pre-Calc

Quiz 9.1 - 9.3 Name _____

54/52

Show All Work!!! Circle All Final Answers!!!

Short Answer

1. Write an expression for the apparent n^{th} term of the sequence:

a) $1, 3, \frac{9}{2}, \frac{27}{6}, \frac{81}{24}, \frac{243}{120}, \ldots$

$a_n = \frac{3^{n-1}}{(n-1)!}$

b) $1, 7, 1, 7, 1, 7, 1, 7, \ldots$

$3(-1)^n + 4 = a_n$

2. Simplify: $\frac{1000!}{998!6!} = \frac{1000 \cdot 999 \cdot 998!}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 998!} = \frac{2775}{2}$

3. Use sigma notation to write the sum.

$\frac{-1}{2} + \frac{1}{3} + \frac{3}{4} + \ldots + \frac{27}{16}$

$\frac{-1+2(n-1)}{1+n} = \frac{2n-3}{n+1} = \frac{27}{16} \quad n = 15$

$\sum_{i=1}^{15} \frac{2i-3}{i+1}$

4. Find the a_{12} term in terms of x and y if a is arithmetic and, $a_6 = x$, and $a_8 = y$

$a_n = a_1 + d(n-1)$

$x = a_1 + d(6-1)$

$x = 5d + a_1$

$y = 7d + a_1$

$x - y = -2d$

$d = \frac{-x+y}{2}$

$x = 5\left(\frac{-x+y}{2}\right) + a_1$

$a_1 = \frac{2x}{2} + \frac{5x-5y}{2}$

$a_1 = \frac{7x-5y}{2}$

$a_{12} = \frac{7x-5y}{2} + \left(\frac{-x+y}{2}\right)(12-1) = \frac{7x-5y-11x+11y}{2}$

$\frac{-4x+6y}{2} = -2x+3y$

5. Find the sum of the multiples of 5 from -2025 to 5015.

$a_1 = -2025 \quad d = 5 \quad a_n = -2025 + 5(n-1) = 5n - 2030$

$a_n = 5015 \quad 5015 = 5n - 2030 \quad 5n = 7045$

$n = 1409$

$S_n = \frac{n}{2}(a_1 + a_n) = \frac{1409}{2}(-2025 + 5015) = 1409 \cdot \frac{1}{2} \cdot 2990^{1495} = 2106455$

6. Evaluate: $\sum_{n=0}^{100}(100-2n)$

$a_0 = 100-2\cdot0 = 100$ $\quad S_n = \frac{101}{2}(100-100) = \frac{101}{2}(0)$

$a_{100} = 100-2\cdot100 = -100$ $\quad = 0$

7. Given the following recursive formula.

$$\begin{cases} a_1 = 384n \\ a_{k+1} = -\frac{1}{2}a_k \end{cases}$$

Find a_{20} in simplest fraction form.

$a_k = 384n\left(-\frac{1}{2}\right)^{k-1}$

$a_{20} = 384n\left(-\frac{1}{2}\right)^{20-1} = 384n\left(-\frac{1}{2}\right)^{19} = \frac{-3n}{4096}$

8. Find all infinite sums for the geometric sequence if $g_4 = 4$ and $g_8 = \frac{1}{4}$.

$r = \pm\frac{1}{2}$ $\quad 4, 2, 1, \frac{1}{2}$

$a_1 = \pm 32$ $\quad S_\infty = \frac{a_1}{1-r}$

$S_\infty^{+r} = \frac{32}{1-\frac{1}{2}} = \frac{32}{\frac{1}{2}} = 64$

$S_\infty^{-r} = \frac{-32}{1+\frac{1}{2}} = \frac{-32}{\frac{3}{2}} = -32\cdot\frac{2}{3} = \frac{-64}{3}$

64 or $\frac{-64}{3}$

9. Use summation notation to write the sum:

$$2-\frac{1}{2}+\frac{1}{8}-\ldots+\frac{1}{2048}$$

$a_n = 2\left(-\frac{1}{4}\right)^{n-1}$ $\quad S_n = a_1\left(\frac{1-r^n}{1-r}\right)$

$\frac{1}{2048} = 2\left(-\frac{1}{4}\right)^{n-1}$ $\quad = 2\left(\frac{1-\left(-\frac{1}{4}\right)^5}{1-\left(-\frac{1}{4}\right)}\right)$

$\left(-\frac{1}{4}\right)^{n-1} = \frac{1}{4096}$ $\quad = 2\cdot\frac{262145}{262144}\cdot\frac{4}{5}$

$\left(-\frac{1}{4}\right)^{n-1} = \left(-\frac{1}{4}\right)^6$ $\quad = \frac{261245}{98304}$ or 2.667

$n-1 = 6$

$n = 7$

$\sum_{n=1}^{7} 2\left(-\frac{1}{4}\right)^{n-1}$

10. A side of an equilateral triangle is 30 inches long. A second equilateral triangle is inscribed in it by joining the midpoints of the sides of the first triangle. The process is continued. Find the perimeter of the 13th inscribed triangle in reduced fraction form.

$a_n = 30\left(\frac{1}{2}\right)^n$

$a_{13} = 30\left(\frac{1}{2}\right)^{13} = 30\cdot\frac{1}{8192} = \frac{15}{4096}\cdot 3 = \frac{45}{4096}$ in

11. Filled to capacity, a tank contains 20 gallons of pure antifreeze. Two gallons of liquid is drawn out and the tank is filled with water. If this operation is repeated several times, after how many operations will there be less than 0.5 gallons of pure antifreeze left in the tank?

$\frac{18}{20} = 0.9$ $\quad a_n = 18(0.9)^{n-1}$

$0.5 = 18(0.9)^{n-1}$

$\frac{1}{36} = 0.9^{n-1}$

$(n-1)\log 0.9 = \log\frac{1}{36}$

$n-1 = \frac{\log\frac{1}{36}}{\log 0.9}$

$n-1 \approx 34.012$

$n \approx 35.012$

36 operations

2. Evaluate: $\sum_{i=0}^{\infty}\left(\frac{1}{10}\right)^{n}$

$S_\infty = \frac{a_1}{1-r} = \frac{1}{1-\frac{1}{10}} = \frac{1}{\frac{9}{10}} = 1\cdot\frac{10}{9} = \frac{10}{9}$

13. Alex collected 14 aluminum cans the first day, 20 cans the second day, 26 the third day, and so on, until he had collected a total of 3030 cans.

+6

$n=8 \rightarrow n=14$

a) How many cans did Alex collect in just his second week of collecting?

$a_8 = 6\cdot8+8 = 56$

$a_{14} = 6\cdot14+8 = 92$

$S_7 = \frac{7}{2}(56+92) = \frac{7}{2}(148) = 518$ cans

b) How many days did he spend total collecting cans?

$a_1 = 14 \quad d = 6$

$a_n = 14+6(n-1) = 6n+8$

$S_n = \frac{n}{2}(a_1+a_n)$

$3030 = \frac{n}{2}(14+6n+8)$

$3030 = \frac{n}{2}(6n+22)$

$3n^2+11n-3030 = 0$

$3 \cdot -30 = -90 \quad 1 \cdot -30 \quad -90+101 = 11$

$(n-30)(3n+101) = 0$

$n = 30$ days

14. A ball is dropped from a helicopter hovering at 2 miles above sea level. If the ball retains 90% of its previous height after each bounce, find the following:

$5280\cdot2 = 10560$

a) Find the first bounce for which the ball will be under 5 feet.

$10560\cdot0.9 = 9504(0.9)^{n-1}$

$9504(0.9)^{n-1} = 5$

$(0.9)^{n-1} = \frac{5}{9504}$

$(n-1)\log 0.9 = \log\frac{5}{9504}$

$n-1 = 71.659$

$n = 72.659$

73rd Bounce

b) The height of the ball after it has traveled a total vertical distance of 200,000 feet.

$\frac{10560(1-0.9^n)}{0.1} + \frac{9504(1-0.9^n)}{0.1} = 200{,}000$

$20064(1-0.9^n) = 20{,}000$

$20{,}064 - 20{,}064(0.9)^n = 20{,}000$

$64 = 20{,}064(0.9)^n$

$0.9^n = 0.03$

$n = \frac{\log 0.03}{\log 0.9}$ ^54th up

$n = 54.554$

$\frac{10560(1-0.9^{54})}{0.1} + \frac{9504(1-0.9^{54})}{0.1} = 199961.556$

$200{,}000 - 199{,}961.556 = 38.443$ ft

$9504(0.9)^{54-1} = 35.707$ ft

2.735 ft