Hon Pre Calculus

Quiz 9.4 Name [redacted] 30/30

Show All Work For FULL Credit!!!!

1. Evaluate:

$$\sum_{j=1}^{10}\left(2j^3-3j^2+\frac{2}{3}j-\frac{2}{3}\right)$$

$= 2\sum_{j=1}^{10} j^3 - 3\sum_{j=1}^{10} j^2 + \frac{2}{3}\sum_{j=1}^{10} j - \sum_{j=1}^{10} \frac{2}{3}$

$= \frac{2}{1}\left(\frac{100(121)}{4}\right) - \frac{3}{1}\left(\frac{10(11)(21)}{6}\right) + \frac{2}{3}\left(\frac{10(11)}{2}\right) - \frac{10}{1}\left(\frac{2}{3}\right)$

$= \frac{24200}{4} - \frac{6930}{6} + \frac{220}{6} - \frac{20}{3}$

$= \boxed{4925}$

2. Determine an explicit formula to find the nth partial sum.

$1! = 1$

$2! = 2$

$3! = 6$

$$\sum_{x=1}^{n}\left(2x^2+x-1\right)$$

~~$= an^2 + bn + c$~~

-1 2, 9, 20, 35, 54

3 7 11 15 19

4 4 4 4

$2a = 4$

$a = \frac{1}{2}$

$c = -1$

$2\left(\frac{1}{2}n^2 + bn - 1\right) = 0$ 2

$2n^2 +$

$\frac{1}{2}n^2 + bn - 1 =$

Seperate Sheet

2, 11, 31, 66, 120

9 20 35 54

11 15 19

4 4

$\boxed{\frac{2}{3}n^3 + \frac{3}{2}n^2 - \frac{1}{6}n}$

3. Find a formula for the following sequence:

$\{0, 18, 66, 156, 300, 510, 798, 1176, \ldots$

0 18 48 90 144 210 288 378

0 18 30 42 54 66 78 90

18 12 12 12 12 12 12

$d = 0$

$an^3 + bn^2 + cn + d$

$\frac{6a}{6} = \frac{12}{6}$

$a = 2$

$6 + b + c = 0$

$6 + b + c = 0$

×2 $48 + 4b + 2c = 18$

$-12 + 2b + 2c = 0$

$36 + 2b = 18$

-36 -36

$\frac{2b}{2} = \frac{-18}{2}$

$b = -9$

$6 + b + c = 0$

$6 + -9 + c = 0$

$c = 3$

0

Seperate Sheet

$\boxed{n\left(2n^2 - 3n - 5\right)}$

4. **Prove using mathematical induction:**

$$\sum_{i=1}^{n}\left(i \bullet 2^{i-1}\right) = 1 + (n-1) \bullet 2^{n}$$

$S_1 = 1 + (1-1)\cdot 2^1 = 1\cdot 2^{1-1} = 1$ ✓ ✓

$S_k = 1 + 4 + 12 + 32 + \dots + k\cdot 2^{k-1} = 1 + (k-1)\cdot 2^k$

$S_{k+1} = S_k + a_{k+1}$ ✓

$S_{k+1} = 1 + (k-1)2^k + (k+1)2^k$

$S_{k+1} = 1 + 2^k(k-1+k+1) = 1 + 2^k(2k)$

$= 1 + 2^{k+1}(k)$

$S_{k+1} = 1 + ((k+1)-1)\cdot 2^{k+1}$

$= 1 + (k)\cdot 2^{k+1}$ Same ✓

✓

5. **Given:**

$$\frac{1}{(1)(3)} + \frac{1}{(3)(5)} + \frac{1}{(5)(7)} + \dots + \frac{1}{(2n-1)(2n+1)}$$

a) **Determine a formula for the nth partial sum.**

$S_1 = \frac{1}{3}$

$S_2 = \frac{2}{5}$

$S_3 = \frac{3}{7}$

$\boxed{\dfrac{n}{2n+1}}$

$3 + (n-1)2$

$= 3 + 2n - 2$

$= 2n+1$

b) **Prove using mathematical induction:**

$S_1 = \frac{1}{(1)(3)} = \frac{1}{2(1)+1} = \frac{1}{3}$ ✓

$S_k = \frac{1}{(1)(3)} + \frac{1}{(3)(5)} + \frac{1}{(5)(7)} + \dots + \frac{1}{(2k-1)(2k+1)} = \frac{k}{2k+1}$

$S_{k+1} = S_k + a_{k+1}$ ✓

$S_{k+1} = \frac{k}{2k+1} + \frac{1}{(2k+1)(2k+3)}$

$S_{k+1} = \frac{k(2k+3)+1}{(2k+1)(2k+3)} = \frac{2k^2+3k+1}{(2k+1)(2k+3)}$

$S_{k+1} = \frac{\cancel{(2k+1)}(k+1)}{\cancel{(2k+1)}(2k+3)} = \frac{k+1}{2k+3}$

$S_{k+1} = \frac{k+1}{2(k+1)+1} = \frac{k+1}{2k+2+1} = \frac{k+1}{2k+3}$ ✓

2.

3rd degree

$1! = 1$

$2! = 2$

$3! = 6$

$an^3 + bn^2 + cn + \cancel{d}$

0	2	11	31	66	120	
	2	9	20	35	54	
		7	11	15	19	
			4	4	4	

$6a = 4$

$a = \frac{2}{3}$

$= \frac{2}{3}n^3 + bn^2 + cn$

$2 = \frac{2}{3} + b + c$

$11 = \frac{2}{3}\left(\frac{8}{1}\right) + 4b + 2c$

$\Rightarrow \quad 4 = \frac{4}{3} + 2b + \cancel{2c}$

$- \quad 11 = \frac{16}{3} + 4b + \cancel{2c}$

$7 = 4 + 2b$

$\quad -4 \quad -4$

$\frac{2b}{2} = \frac{3}{2}$

$b = \frac{3}{2}$

$2 = \frac{2}{3} + \frac{3}{2} + c$

$c = -\frac{1}{6}$

$= \frac{2}{3}n^3 + \frac{3}{2}n^2 - \frac{1}{6}$

$d=0$

3.

$\frac{6a}{6}=\frac{12}{6}$

$a=2$

$an^3+bn^2+cn+\cancel{d}$

$2n^3+bn^2+cn$

$2+b+c=0$

$\cdot 2\ (16+4b+2c=18$

$4+2b+2c=0$

$12+2b=18$

$-12 \qquad -12$

$\frac{2b}{2}=\frac{6}{2}$

$b=3$

$2+3+c=0$

$c+5=0$

$c=-5$

$2n^3+3n^2-5n$

$n(2n^2+3n-5)$

$2k^2+2k+k+1$

$2k(k+1)+1(k+1)$

$(2k+1)(k+1)$