Hon Pre-Calc

Quiz 12.1 - 12.3 Name [redacted]

$\frac{55}{60}$

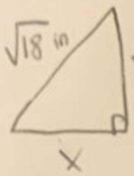
Show All Work!!! Circle All Final Answers!!

Short Answer

a b must be equal to maximize area

1. You are given wire and are asked to form a right triangle with a hypotenuse of $\sqrt{18}$ inches whose area is as large as possible.

   a) Find a function for the area of the triangle in terms of x, where x = the base of the triangle.



$A=\frac{1}{2}x\sqrt{18-x^2}$ $\boxed{A=\frac{1}{2}x^2}$

2

$x^2+x^2=(\sqrt{18})^2$

$2x^2=18$

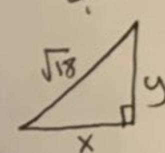
$x^2=9$

$x=3$

   b) Using a graphing calculator determine what should x be for the area of the triangle to be at a maximum?

$\boxed{x=3}$

1. a)



$A=\frac{1}{2}xy$

$x^2+y^2=18$

$y=\sqrt{18-x^2}$

$\boxed{A=\frac{1}{2}x\left(\sqrt{18-x^2}\right)}$

   c) What is the maximum area?

$=\frac{1}{2}\,3\cdot 3$

$=\frac{9}{2}=\boxed{4.5u^2}$

2. Given: $\lim_{x\to -4}\frac{\frac{x}{x+2}-2}{x+4}$

   a) Create a table of values for the function to determine the limit numerically.

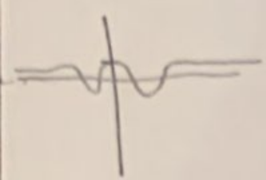
x	-4.1	-4.01	-4	-3.99	-3.9
$f(x)$	0.4762	0.4975	Error	0.5025	0.5263

   b) What is the limit?

$\lim_{x\to -4}\frac{\frac{x}{x+2}-2}{x+4}=\boxed{0.5}$

2

3. Use a graph to determine $\lim_{x\to 0} f(x)$ if $f(x)=\cos\frac{1}{x}$



~~lim cos~~

$\boxed{\text{limit d.n.e}}$

4. Given: $\lim_{x\to c} f(x)=3$ and $\lim_{x\to c} g(x)=6$ Evaluate the following:

a) $\lim_{x\to c}\frac{3\bullet f(x)}{\sqrt{g(x)}}$

$\frac{3\cdot 3}{\sqrt6} = \frac{9}{\sqrt6}\left(\frac{\sqrt6}{\sqrt6}\right)$

$= \frac{9\sqrt6}{6} = \boxed{\frac{3\sqrt6}{2}}$

b) $\lim_{x\to c}\left[-2g(x)\bullet f(x)\right]$

$= [-2\cdot 6\cdot 3]$

$= \boxed{-36}$

5. Find the following limits exactly:

a) $\lim_{x\to 1}\arccos\frac{x}{2}$

$= \cos^{-1}\frac{1}{2}$

$= \boxed{\frac{\pi}{3}}$

b) $\lim_{x\to 5\pi/6}\tan x$

$= \tan\frac{5\pi}{6}$

$= \boxed{-\frac{\sqrt3}{3}}$

$= \frac{\sin}{\cos}$

$= \frac{\frac12}{-\frac{\sqrt3}{2}}$

$= \frac{1}{2}\cdot\frac{2}{-\sqrt3}$

$= \frac{1}{-\sqrt3}\left(\frac{\sqrt3}{\sqrt3}\right)$

c) $\lim_{x\to 0}\frac{\sqrt{7-x}-\sqrt7}{x}\left(\frac{\sqrt{7-x}+\sqrt7}{\sqrt{7-x}+\sqrt7}\right)$

$= \frac{\cancel{7}-\cancel{x}^{1}-\cancel{7}}{\cancel{x}(\sqrt{7-x}+\sqrt7)} = \frac{-1}{\sqrt7+\sqrt7}$

$= \boxed{\frac{-1}{2\sqrt7}}$

d) $\lim_{x\to 0}\frac{\frac{1}{2+x}-\frac{1}{2}}{x}$

$\frac{\cancel{2-2}-\cancel{x}^{1}}{2(2+x)\cancel{x}} = \frac{-1}{2(2+x)}$

$= \frac{-1}{4+2x} = \boxed{\frac{-1}{4}}$

e) $\lim_{x\to 1/2}\frac{2x^5-x^4-16x^3+8x^2-18x+9}{2x-1}$

$x^4 \cdot 0x^3 \cdot -8x^2 \cdot 0x - 9$

$2x-1 \overline{)\,2x^5-x^4-16x^3+8x^2-18x+9}$

$-2x^5+x^4$

$-16x^3+8x^2$

$+16x^3-8x^2$

$-18x+9$

$+18x-9$

0

$\frac{\cancel{2x-1}(x^4-8x^2-9)}{\cancel{2x-1}}$

$= \left(\frac12\right)^4-8\left(\frac12\right)^4-9$

$= \frac{1}{16}-\frac{1}{2}-9$

-9.4375

$= \boxed{\cancel{\frac{-151}{16}}}$

2

f) $\lim_{x\to 0}\frac{\sec x}{\tan x}$

$\frac{\frac{1}{\cos x}}{\frac{\sin x}{\cos x}} = \frac{1}{\cos x}\cdot\frac{\cos x}{\sin x}$

~~$= \frac{1}{\sin x}$~~

~~$\lim_{x\to 0} = \csc x$~~

$\boxed{\text{limit d.n.e}}$

2

6. Evaluate:

a) $\lim_{x \to 16^+} \frac{4-\sqrt{x}}{x-16}$

$\cancel{\frac{4-\sqrt{16}}{16-16} = 0}$

$= \frac{-(\sqrt{x}-4)}{x-16}\left(\frac{\sqrt{x}+4}{\sqrt{x}+4}\right)$

$= -\frac{\cancel{(x-16)}}{\cancel{x-16}\,\sqrt{x}+4} = \frac{-1}{\sqrt{x}+4}$

$\boxed{\frac{-1}{8}}$

b) $\lim_{x \to 1} f(x)$ where $f(x) = \begin{cases} 4-x^2, x \le 1 \\ 3-x, \ x > 1 \end{cases}$ 3 2

$\boxed{\text{limit d.n.e}}$

c) $\lim_{x \to 2^-} \frac{|x-2|}{x-2}$ $\frac{|-2-2|}{-2-2} = \frac{4}{-4}$

limit $= \boxed{-1}$

7. Use the difference quotient to find the slope of the tangent line to the function $h(x) = \sqrt{x+10}$ at the point $(-1,3)$

$\lim_{h \to 0} h'(x) = \frac{\sqrt{x+h+10} - \sqrt{x+10}}{h}\left(\frac{\sqrt{x+h+10}+\sqrt{x+10}}{\sqrt{x+h+10}+\sqrt{x+10}}\right)$

$= \frac{\cancel{x}+\overset{1}{\cancel{h}}+\cancel{10}-\cancel{x}-\cancel{10}}{\sqrt{x+h+10}+\sqrt{x+10}}$

$h'(x) = \frac{1}{2\sqrt{x+10}}$ $(-1,3)$

$\boxed{m = \frac{1}{6}}$

$h'(x) = \frac{1}{2\sqrt{-1+10}} = \frac{1}{2\sqrt{9}}$ 6

8. Find the derivative of the following:

a) $f(x) = -1$

$f'(x) = 0$

b) $f(x) = -5x^1+2$

$f'(x) = -5$

c) $f(x) = \frac{1}{\sqrt{x-9}}$

$f(x) = x-9^{-\frac{1}{2}}$

$f'(x) = -\frac{1}{2}(x-9)^{-\frac{3}{2}}$

$-\frac{1}{2\sqrt[4]{(x-9)^3}}$

$= \boxed{\frac{-1}{2\sqrt{(x-9)^3}}}$

9. Find the equation of the tangent line to the function $f(x) = x^3 - x$ at the point $(2,6)$.

$f'(x) = 3x^2 - 1$

$m = 3(2)^2 - 1$

$m = 11$

$y = 11x + b$

$6 = 11(2) + b$

$6 = 22 + b$

$b = -16$

$\boxed{y = 11x - 16}$

$1_x^3 \quad 3_x^2 h \quad 3xh^2 \quad 1h^3$

10. Given function: $f(x) = -\frac{1}{2}x^3$ and line: $6x + y + 4 = 0$

Find an equation of the line that is tangent to the graph of f and parallel to the given line.

$y = -6x - 4$

$m = -6$

$-4 = -12 + b$

$b = 8$

$\boxed{y = -6x + 8}$

$f'(x) = -\frac{3}{2}x^2$

$-\frac{2}{3} \cdot -6^2 = -\frac{3}{2}x^2$

$4 = 12 + b$

$b = -8$

$\boxed{y = -6x - 8}$

$\sqrt{x^2} = \sqrt{4}$

$x = \pm 2$

$(2, -4)$

$(-2, 4)$

11. Use the following function and its derivative to determine any points on the graph of f at which the tangent line is horizontal. Points need to be exact.

$f(x) = x^2e^x$ and $f'(x) = x^2e^x + 2xe^x$

$0 = x^2e^x + 2xe^x$

$0 = e^x(x^2 + 2x)$

$0 = e^x \cdot x \cdot (x+2)$

$x = 0$

$\cancel{e^x = 0}$

$x = -2$

$\frac{4}{e^2}$

$\boxed{(0, 0) \quad (-2, \frac{4}{e^2})}$

12. Given: $f(x) = 3x^3 - 9x$

a) Use the difference quotient to find a formula for the average rate of change of f. include h

$\cancel{\frac{3(x+h)^3 - 9(x+h)}{h}}$

$\cancel{= \frac{3(x^3 + 3x^2h + 3xh^2 + h^3) - 9x - 9h}{h}}$

$\cancel{= \frac{3x^3 + 9x^2h + 9xh^2 + 3h^3 - 9x - 9h}{h}}$

Seperate sheet

$\boxed{\frac{9x^2h + 9xh^2 + 3h^3 + 9x}{h}}$

-1

b) Find the derivative function of f.

$\boxed{f'(x) = 9x^2 - 9}$

c) Use the derivative function to determine any points on the graph where the tangent line is horizontal.

$0 = 9x^2 - 9$

$9 = 9x^2$

$x^2 = 1$

$x = \pm 1$

$\boxed{(1, -6) \quad (-1, 6)}$

-1

[redacted]

12.

a) $f(x) = 3x^3 - 9x$

$$\frac{3(x+h)^3 - 9x}{h}$$

$$= \cancel{\frac{3x^3 + 9x^2h + 9xh^2 + 3h^3 - 9x}{h}}$$

$$\frac{3(x^2 + 2hx + h^2)(x+h) - 9x}{h}$$

$$\frac{3(x^3 + 2hx^2 + xh^2 + hx^2 + 2h^2x + h^3) - 9x}{h}$$

$$\frac{3(x^3 + 3hx^2 + 3xh^2 + h^3) - 9x}{h}$$

5.

e)

$$\begin{array}{r|l} & x^4 - 8x^2 - 9 \\ 2x-1 & 2x^5 - x^4 - 16x^3 + 8x^2 - 18x + 9 \\ & -2x^5 + x^4 \\ & 0 \quad -16x^3 + 8x^2 \\ & +16x^3 - 8x^2 \\ & 0 \quad -18x + 9 \\ & +18x - 9 \\ & 0 \end{array}$$

$$\frac{\cancel{2x-1}\,(x^4 - 8x^2 - 9)}{\cancel{2x-1}}$$

$= x^4 - 8x^2 - 9$

$= \left(\frac{1}{2}\right)^2 - 8\left(\frac{1}{2}\right)^2 - 9$

$= \frac{1}{16} - \frac{8}{16} - \frac{144}{16}$

$= \frac{-151}{16}$

12.

a) $= \dfrac{3(x+h)^3 - 9x}{h}$

$= \dfrac{3x^3 + 3hx^2 + 3xh^2 + h^3 - 9x}{h}$

$= 3(x^3 + hx^2 + xh^2 + h^3 - 9x)$

$= \cancel{\dfrac{3h(x^3 + x^2 + xh +}{h}}$

12.

a) $= \dfrac{3(x+h)^3 - 9x}{h}$

1
1 1
1 2 1
1 3 3 1

$= \dfrac{3(x^3 + 3hx^2 + 3xh^2 + h^3) - 3x^3 - 9x}{h}$

$= \cancel{3x^3} + 3hx^2 + 3xh^2 + h^3 \cancel{- 3x^3} - 9x$

$\cancel{3x^3} + 9x^2h + 9xh^2 + 3h^3 \cancel{- 3x^3} + 9x$

h