Hon Pre-Calc Quiz 12.1 – 12.3 *2017 – 2018*

Show All Work For FULL Credit!!! Circle All Final Answers!!!!

1. You create an open box from a square piece of material 24 cm on a side. You cut equal squares from the corners and turn up the sides..

   a) Find a function for the volume of the box in terms of x, where x = length of one side of the corner being cut out.

   b) What should x be in order to get the largest volume?

2. Determine $\lim_{x \to 0} \frac{\sin nx}{x}$

3. Given: $\lim_{x \to c} f(x) = 8$ and $\lim_{x \to c} g(x) = 30$ Evaluate the following:

   a) $\lim_{x \to c} \frac{3 * f(x)}{\sqrt{g(x)}}$

   b) $\lim_{x \to c} [-2g(x) * f(x)]$

4. Find the following limits exactly:

a) $\lim\limits_{x\to -c} \frac{|x+c|}{x+c}$

b) $\lim\limits_{x\to 0} \frac{e^{4x}-1}{4x}$

c) $\lim\limits_{x\to \sqrt{3}} \cos^{-1}\frac{-x}{2}$

d) $\lim\limits_{x\to \frac{5\pi}{6}} sec^2 x$

e) $\lim\limits_{x\to 0} \frac{\sqrt{3-x}-\sqrt{3}}{x}$

f) $\lim\limits_{x\to 2} \frac{\frac{1}{x+1}-\frac{1}{3}}{\frac{2}{x}-1}$

g) $\lim\limits_{x\to -\frac{1}{2}} \frac{2x^5+x^4-6x^3-3x^2-8x-4}{2x+1}$

h) $\lim\limits_{x\to \pi} \frac{\sin x}{\tan x}$

i) $\lim\limits_{x\to 4} \frac{x^3-64}{x-4}$

j) $\lim\limits_{x\to \frac{\pi}{2}} \frac{1-\sin x}{\cos x}$

5. Evaluate:

a) $\lim_{x \to 4^+} \frac{4-\sqrt{x}}{x-4}$

b) $\lim_{x \to 1} f(x)$ where f(x) = $\begin{cases} 4 - x^2, & x \leq 1 \\ 3 - x, & x > 1 \end{cases}$

c) $\lim_{x \to \pi^-} \frac{|x-\pi|}{x-\pi}$

6. Find the equation of all the tangent line(s) to the function $f(x) = 2x^3 - 3x^2$ that are parallel to the line $18x - y = 7$

7. Find the derivative of the following:

a) $f(x) = 0$

b) $f(x) = -5x + 2$

c) $f(x) = \frac{2}{x^3}$

8. Sketch the graph of a function *f(x)* that satisfy the following conditions: (*f'(x)* is the derivative of *f*)

1) $f'(x) < 0$ for $x < -1$
2) $f'(x) < 0$ for $x > 1$
3) $f'(x) > 0$ for $-1 < x < 1$
4) $f'(x) = 0$ for $x = 1$ and $x = -1$

9. Use the function and its derivative to determine any points on the graph of *f* at which the tangent line is horizontal on the interval $[0,2\pi)$

$$f(x) = 2\cos x + x$$
$$f'(x) = -2\sin x + 1$$

10. Use the limit process with the difference quotient to find the slope of the tangent line to the function $h(x) = \frac{1}{\sqrt{x+5}}$ at the point $(-1, \frac{1}{2})$

Hon Pre-Calc

Quiz 12.1 - 12.3 Name ____________ ~~57~~ 56 / 59

Show All Work!! Circle All Final Answers!!

Short Answer

1. You create an open box from a square piece of material 24 cm on a side. You cut equal squares from the corners and turn up the sides..

a) Find a function for the volume of the box in terms of x, where x = the length of one side of the corner being cut out.

$V = (24-2x)(24-2x)x$

$= 4x(12-x)^2$

$= 4x(144-24x+x^2)$

$V = 4x^3 - 96x^2 + 576x$

b) What should x be in order to get the largest volume?

$V' = 12x^2 - 192x + 576$

$0 = x^2 - 16x + 48$

$(x-4)(x-12) = 0$

$x = 4, \cancel{12}$

4cm

2. Determine $\lim\limits_{x \to 0} \frac{\sin(nx)}{x} = \boxed{n}$ $\frac{0}{0}$ *put in radians!

$= \boxed{\sin n}$ -2

Ex: $\frac{\sin(4x)}{x}$

x	[illegible]	[illegible]	0	[illegible]	[illegible]
y	[illegible]	[illegible]	X	[illegible]	[illegible]

$\sin 4 = \boxed{0.068}$

3. Given: $\lim\limits_{x \to c} f(x) = 8$ and $\lim\limits_{x \to c} g(x) = 30$ Evaluate the following:

a) $\lim\limits_{x \to c} \frac{3 \bullet f(x)}{\sqrt{g(x)}} = \frac{3(8)}{\sqrt{30}} = \frac{24\sqrt{30}}{30} = \frac{4\sqrt{30}}{5}$

b) $\lim\limits_{x \to c} [-2g(x) \bullet f(x)]$

$= -2(30)(8) = -480$

-2

4. Find the following limits exactly:

a) $\lim\limits_{x \to -c} \dfrac{|x+c|}{x+c}$ d.n.e.

b) $\lim\limits_{x \to 0} \dfrac{e^{4x}-1}{4x} = 1$

x	$f(x)$
-0.01	0.961
-0.01	0.9805
0	x
0.01	1.0203
0.02	1.044

c) $\lim\limits_{x \to \sqrt{3}} \arccos \dfrac{-x}{2}$

$\cos^{-1} \frac{-\sqrt{3}}{2}$

$= 150^\circ$

d) $\lim\limits_{x \to 5\pi/6} \sec^2 x$

$= \sec^2\left(\frac{5\pi}{6}\right)$

$= \left(-\frac{2}{\sqrt{3}}\right)^2$

$= \frac{4}{3}$

e) $\lim\limits_{x \to 0} \dfrac{\sqrt{3-x}-\sqrt{3}}{x} \cdot \dfrac{\sqrt{3-x}+\sqrt{3}}{\sqrt{3-x}+\sqrt{3}}$

$= \frac{3-x-3}{x(\sqrt{3-x}+\sqrt{3})}$

$= \frac{-1}{\sqrt{3-x}+\sqrt{3}}$

$= \frac{-1}{2\sqrt{3}}$

$= -\frac{\sqrt{3}}{6}$

f) $\lim\limits_{x \to 2} \dfrac{\frac{1}{x+1}-\frac{1}{3}}{\frac{2}{x}-1} = \dfrac{\frac{3-x-1}{3(x+1)}}{\frac{2-x}{x}} = \frac{2-x}{3(x+1)} \cdot \frac{x}{2-x} = \frac{x}{3x+3}$

$\frac{2}{6+3} = \boxed{\frac{2}{9}}$

g) $\lim\limits_{x \to -1/2} \dfrac{2x^5+x^4-6x^3-3x^2-8x-4}{2x+1}$

$\lim\limits_{x \to -1/2} x^4-3x^2-4$

$= \frac{1}{16}-\frac{3}{4}-4$

$= -\frac{75}{16} = -4.6875$

h) $\lim\limits_{x \to \pi} \dfrac{\sin x}{\tan x} = \dfrac{\sin x}{\frac{\sin x}{\cos x}} = \sin x \cdot \frac{\cos x}{\sin x} = \cos x$

$\cos \pi = \boxed{-1}$

i) $\lim\limits_{x \to 4} \dfrac{x^3-64}{x-4} = \dfrac{(x-4)(x^2+4x+16)}{x-4}$

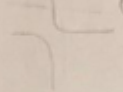
$4^2+4(4)+16 = \boxed{48}$

j) $\lim\limits_{x \to \pi/2} \dfrac{1-\sin x}{\cos x} \cdot \frac{1+\sin x}{1+\sin x} = \frac{1-\sin^2 x}{\cos x(1+\sin x)} = \frac{\cos x}{1+\sin x}$

$\frac{\cos \frac{\pi}{2}}{1+\sin \frac{\pi}{2}} = \boxed{0}$

5. Evaluate:

a) $\lim_{x \to 4^+} \frac{4-\sqrt{x}}{x-4} \cdot \frac{4+\sqrt{x}}{4+\sqrt{x}} = \frac{16-x}{(x-4)(4+\sqrt{x})}$



d.n.e.

x	y
3.99	-1440.3
3.98	-1440.3
4	X
4.01	1449.75
4.02	99.75

b) $\lim_{x \to 1} f(x)$ where $f(x) = \begin{cases} 4-x^2, & x \le 1 \\ 3-x, & x > 1 \end{cases}$

$4-1^2=3$

$3-1=2$

d.n.e.

c) $\lim_{x \to \pi^-} \frac{|x-\pi|}{x-\pi} = -1$

6. Find the equation of all the tangent line(s) to the function $f(x) = 2x^3 - 3x^2$ that are parallel to the line $\boxed{18}x - y = 7$.

$f'(x) = 6x^2 - 6x$

$6x^2 - 6x = 18$

$x^2 - x = 3$

$x(x-1) = 3$

$x = 3, 4$

$f(x) = 2(3)^3 - 3(3)^2$

$= 54 - 27$

$= 27$

$(3, 27)$

$f(x) = 2(4)^3 - 3(4)^2$

$= 80$

$(4, 80)$

$18(x-3) - (y-27) = 7$

$18x - y = 7 - 27 + 54$

$\boxed{18x - y = 34}$

$18(x-4) - (y-80) = 7$

$18x - y = 7 - 80 + 72$

$\boxed{18x - y = -1}$

-2

Solution on back

7. Find the derivative of the following:

a) $f(x) = 0$

$f'(x) = 0$

b) $f(x) = -5x + 2$

$f'(x) = -5$

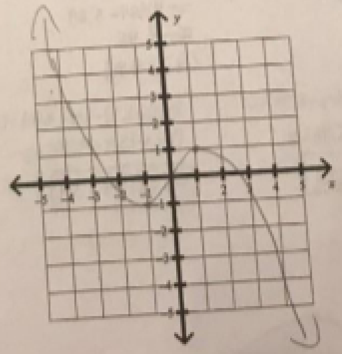
c) $f(x) = \frac{2}{x^3} = 2x^{-3}$

$f'(x) = -6x^{-4}$

$= -\frac{6}{x^4}$

8. Sketch the graph of a function $f(x)$ that satisfy the following conditions: ($f'(x)$ is the derivative of f)

1) $f'(x) < 0$ for $x < -1$
2) $f'(x) < 0$ for $x > 1$
3) $f'(x) > 0$ for $-1 < x < 1$
4) $f'(x) = 0$ for $x = 1$ and $x = -1$



-2

9. Use the function and its derivative to determine any points on the graph of f at which the tangent line is horizontal on the interval $[0, 2\pi)$.

$f(x) = 2\cos x + x$

$f'(x) = -2\sin x + 1$

$-2\sin x + 1 = 0$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

$f(x) = 2\cos\frac{\pi}{6} + \frac{\pi}{6} = 2\left(\frac{\sqrt{3}}{2}\right) + \frac{\pi}{6} = \sqrt{3} + \frac{\pi}{6}$

$\left(\frac{\pi}{6}, \sqrt{3} + \frac{\pi}{6}\right)$

$f(x) = 2\cos\frac{5\pi}{6} + \frac{5\pi}{6} = 2\left(-\frac{\sqrt{3}}{2}\right) + \frac{5\pi}{6} = -\sqrt{3} + \frac{5\pi}{6}$

$\left(\frac{5\pi}{6}, -\sqrt{3} + \frac{5\pi}{6}\right)$

10. Use the limit process with the difference quotient to find the slope of the tangent line to the function

$h(x) = \frac{1}{\sqrt{x+5}}$ at the point $\left(-1, \frac{1}{2}\right)$

$\lim_{h \to 0} h'(x) = \frac{\frac{1}{\sqrt{x+h+5}} - \frac{1}{\sqrt{x+5}}}{h}$

$= \frac{\sqrt{x+5} - \sqrt{x+h+5}}{h(\sqrt{x+h+5})(\sqrt{x+5})} \cdot \frac{\sqrt{x+5} + \sqrt{x+h+5}}{\sqrt{x+5} + \sqrt{x+h+5}}$

$= \frac{x+5-x-h-5}{h(\sqrt{x+h+5})(\sqrt{x+5})(\sqrt{x+5} + \sqrt{x+h+5})}$

$= \frac{-1}{(\sqrt{x+h+5})(\sqrt{x+5})(\sqrt{x+5} + \sqrt{x+h+5})}$

$= \frac{-1}{2(x+5)(\sqrt{x+5})}$

$h'(x) = \frac{-1}{2(-1+5)(\sqrt{-1+5})}$

$= \frac{-1}{2(4)(2)}$

$= \boxed{-\frac{1}{16}}$

6. $f'(x) = 6x^2 - 6x$

$18 = 6x^2 - 6x$

$x^2 - x - 3 = 0$

$x = \frac{1 \pm \sqrt{1+4(3)}}{2}$

$= \frac{1 \pm \sqrt{13}}{2}$

$\approx 2.3, -1.3$

$f(x) = 2(2.3)^3 - 3(2.3)^2$

$= 24.334 - 15.87$

≈ 8.46

$(2.3, 8.46)$

$f(x) = 2(-1.3)^3 - 3(-1.3)^2$

$= -4.394 - 5.07$

≈ -9.46

$(-1.3, -9.46)$

$18(x-2.3) - (y-8.46) = 0$

$18x - 41.4 + 8.46 = y$

$\boxed{y = 18x - 32.9}$

$18(x+1.3) - (y+9.46) = 0$

$18x + 23.4 - 9.46 = y$

$\boxed{y = 18x + 13.9}$