Hon Pre-Calc
Test Chapter 1   Name _____

Show All Work!!! Circle All Final Answers!! No Calculators!!!

Short Answer

1. Determine the symmetry of the following:

   a) $y = \dfrac{3x^2}{x^4 - 1}$

   b) $|y| + x^2 = x$

2. Given: $y = 6\sqrt{x} - x\sqrt{x}$

   a) Find all x-intercepts

   b) Find all y-intercepts

3. Given: $3x + 4y = 7$. Write in **point slope form** the equation of the line passing through $\left(-\dfrac{2}{3}, \dfrac{7}{8}\right)$ ....

   a) parallel to the given line.

   b) perpendicular to the given line.

4. Use interval notation to write the domain of the following:

   a) $y = \dfrac{1}{x} - \dfrac{3}{x+2}$

   b) $y = \dfrac{\sqrt{6-x}}{x-6}$

5. Find the average rate of change function using the difference quotient for the following function:

$$f(x) = \frac{5}{x^2}$$

6. Find the average rate of change formula from $x = \frac{\pi}{3}$ to $x = \frac{\pi}{3} + h$ using the difference quotient for the following function:

$$f(x) = \cos x$$

8. Using interval notation, determine over which intervals the function is increasing, decreasing, or constant.

$$f(x) = \begin{cases} 3-x, x \leq 0 \\ 3, 0 < x \leq 2 \\ 2x+1, x > 2 \end{cases}$$

a) Increasing =

b) Decreasing =

c) Constant =

9. Given: $f(x) = 2x^3 - 3x^2 - 36x + 18$

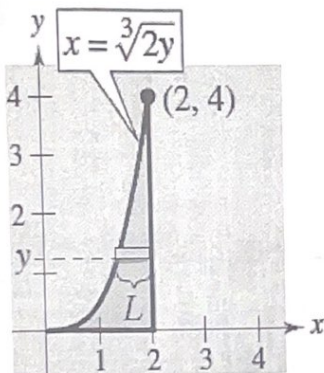a) Use interval notation to state where the function is increasing and decreasing.

Inc =

Dec =

b) Determine any relative minimum(s).

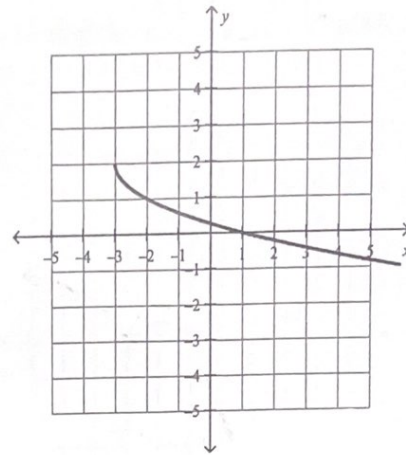10. **Algebraically** determine if the function is even, odd, or neither.

a) $f(x) = -x^2 - 8$

b) $f(x) = -|x - 5|$

11. Write the length L of the rectangle as a function of y.

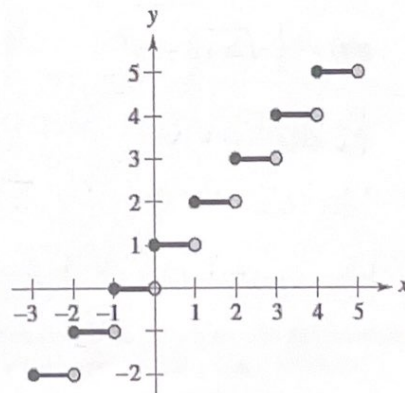12. Determine the equation for the following graphs:

a)

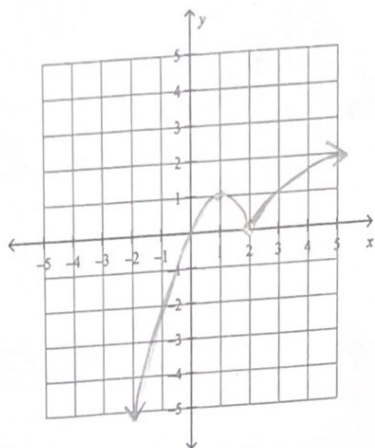b)

3

13. Graph the following:

$$f(x) = \begin{cases} 1-(x-1)^2, & x \le 2 \\ \sqrt{x-2}, & x > 2 \end{cases}$$

14. Given $f(x) = \sqrt{x}$. Describe the sequence of transformations from f to g if

$$g(x) = -\frac{1}{2}\sqrt{2x+3} - 1.$$

15. Let $b(t)$ be the number of births in the United States in year t, and let $d(t)$ represent the number of deaths in the US in year t. If $p(t)$ is the population of the US in year t, find a function the function $c(t)$ that represents the percent change in the population of the US in year t.

16. Given: $f(x) = \dfrac{3}{x^2-1}$ and $g(x) = x+2$

a) Find: $f(g(x))$

b) Write the domain of $f(g(x))$ (Interval notation)

17. Restrict and state the domain keeping the largest possible domain to make the function one to one.

$$f(x) = 2|4x-3| + 7$$

4

18. Given: $f(x) = x^2 - 4$

$g(x) = \dfrac{1}{x}$

Find the domain and range of $g(f(x))$.

Domain = _____

Range = _____

19. Verify that f and g are inverse functions.

$f(x) = \dfrac{1}{1+x}$ and $g(x) = \dfrac{1-x}{x}$

20. Given: $f(x) = \dfrac{x+1}{x-2}$. Find $f^{-1}(x)$

21. Determine if the situation could be represented by a one-to-one function. Explain why or why not.

The depth of the tide d at a beach in terms of the time t over a 24-hour period.

22. Find a mathematical equation for the verbal statement.

The gravitational attraction F between two objects of masses m_1 and m_2 is proportional to the product of the masses and inversely proportional to the square of the distance r between the objects.

23. z varies directly as the square of x and inversely as y. $z = 6$ when $x = 6$ and $y = 4$. Find the constant of variation.

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95/95

Short Answer

1. Determine the symmetry of the following:

   a) $y = \dfrac{3x^2}{x^4 - 1}$

   $y = \dfrac{3(-x)^2}{(-x)^4 - 1} = \dfrac{3x^2}{x^4 - 1}$

   y-axis symmetry

   b) $|y| + x^2 = x$

   $|-y| + (-x)^2 = (-x)$

   $y + x^2 = -x$

   x-axis symmetry

2. Given: $y = 6\sqrt{x} - x\sqrt{x}$

   a) Find all x-intercepts

   $x = 0, \ x = 6$

   $y = \sqrt{x}(6 - x)$

   $\sqrt{x} = 0 \qquad 6 - x = 0$

   $x = 0 \qquad x = 6$

   b) Find all y-intercepts

   $y = 6\sqrt{0} - 0\sqrt{0}$

   $y = 6(0) - 0$

   $y = 0$

   $y = 0$

3. Given: $3x + 4y = 7$. Write in **point slope form** the equation of the line passing through $\left(-\dfrac{2}{3}, \dfrac{7}{8}\right)$ ....

   $4y = 7 - 3x$

   $y = -\dfrac{3}{4}x + \dfrac{7}{4}$

   a) parallel to the given line.

   $y - \dfrac{7}{8} = -\dfrac{3}{4}\left(x + \dfrac{2}{3}\right)$

   b) perpendicular to the given line.

   $y - \dfrac{7}{8} = \dfrac{4}{3}\left(x + \dfrac{2}{3}\right)$

4. Use interval notation to write the domain of the following:

   a) $y = \dfrac{1}{x} - \dfrac{3}{x+2}$

   $x \neq 0, \ x \neq (-2)$

   $(-\infty, -2) \cup (-2, 0) \cup (0, \infty)$

   b) $y = \dfrac{\sqrt{6-x}}{x-6}$ $x \leq 6, \quad x \neq 6 \Rightarrow x < 6$

   $(-\infty, 6)$

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5. Find the average rate of change function using the difference quotient for the following function:

$$f(x) = \frac{5}{x^2}$$

$$\frac{\frac{5}{(x+h)^2} - \frac{5}{x^2}}{h} = \frac{\frac{5x^2 - 5(x+h)^2}{x^2(x-h)^2}}{h}$$

$$\frac{5(x^2 - x^2 - 2xh - h^2)}{hx^2(x-h)^2} = \frac{5(-2xh - h^2)}{hx^2(x+h)^2}$$

$$\frac{-10xh - h^2}{hx^2(x+h)^2} = \frac{h(-10x - h)}{hx^2(x+h)^2} = \boxed{\frac{-10x - h}{x^2(x+h)^2}}$$

6. Find the average rate of change formula from $x = \frac{\pi}{3}$ to $x = \frac{\pi}{3} + h$ using the difference quotient for the following function:

$$f(x) = \cos x$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

$$\cos\frac{\pi}{3} = \frac{1}{2}$$

$$\frac{\cos(\frac{\pi}{3}+h) + \cos\frac{\pi}{3}}{h} = \frac{\cos\frac{\pi}{3}\cos h - \sin\frac{\pi}{3}\sin h + \cos\frac{\pi}{3}}{h}$$

$$\frac{\frac{1}{2}\cos h - \frac{\sqrt{7}}{2}\sin h + \frac{1}{2}}{h} = \boxed{\frac{\cos h - \sqrt{3}\sin h + 1}{2h}}$$

7. Find the zeros of the function:
$$f(x) = 4x^3 - 24x^2 - x + 6$$

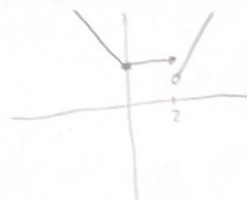$$4x^2(x-6) - 1(x-6)$$
$$(4x^2 - 1)(x-6)$$
$$(2x-1)(2x+1)(x-6)$$

$$2x - 1 = 0$$

$$\boxed{x = \frac{1}{2}, \ x = -\frac{1}{2}, \ x = 6}$$

8. Using interval notation, determine over which intervals the function is increasing, decreasing, or constant.

$$f(x) = \begin{cases} 3 - x, & x \le 0 \\ 3, & 0 < x \le 2 \\ 2x + 1, & x > 2 \end{cases}$$

a) Increasing = $(2, \infty)$

b) Decreasing = $(-\infty, 0]$

c) Constant = $(0, 2]$

9. Given: $f(x) = 2x^3 - 3x^2 - 36x + 18$

a) Use interval notation to state where the function is increasing and decreasing.

Inc = $(-\infty, -2) \cup (3, \infty)$

Dec = $(-2, 3)$

b) Determine any relative minimum(s).

$(3, -63)$

$6x^2 - 6x - 36 = 0$
$x^2 - x - 6 = 0$
$(x-3)(x+2) = 0$
$x = 3, \ x = (-2)$

$2(3)^3 - 3(3)^2 - 36(3) + 18$

$54 - 27 - 108 + 18$

-63

10. **Algebraically** determine if the function is even, odd, or neither.

a) $f(x) = -x^2 - 8$ $\boxed{Even}$
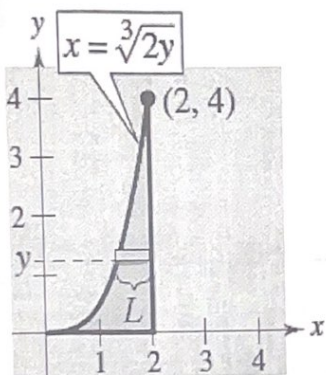
$f(-x) = -(-x)^2 - 8$

$f(-x) = -x^2 - 8$

b) $f(x) = -|x - 5|$ $\boxed{Neither}$

$f(-x) = -|-x - 5|$

11. Write the length L of the rectangle as a function of y.

$x = \sqrt[3]{2y}$

$(2, 4)$

L

$\underline{L = 2 - \sqrt[3]{2y}}$

12. Determine the equation for the following graphs:
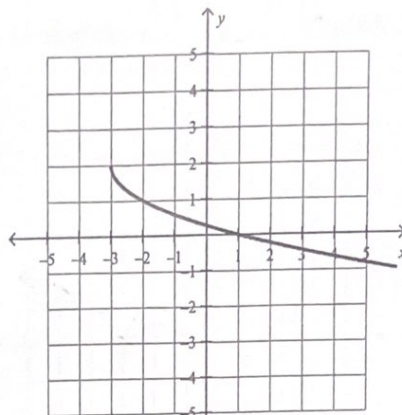
a)

$y = \sqrt{x}$
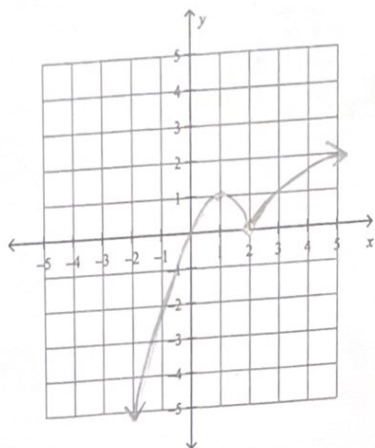
$y = -\sqrt{x+3} + 2$

$\boxed{y = -\sqrt{x+3} + 2}$

b)
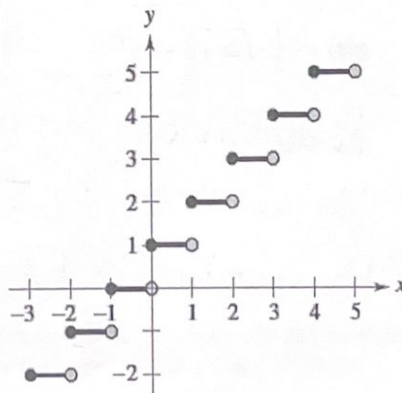
$\boxed{y = \lfloor x \rfloor + 1}$

3

13. Graph the following:

$-(x-1)^2+1$

$$f(x) = \begin{cases} 1-(x-1)^2, & x \le 2 \\ \sqrt{x-2}, & x > 2 \end{cases}$$

14. Given $f(x) = \sqrt{x}$. Describe the sequence of transformations from f to g if

$$g(x) = -\frac{1}{2}\sqrt{2x+3} - 1.$$

- Reflection over x-axis
- Vertical shrink by $\frac{1}{2}$
- Horizontal shrink by $\frac{1}{2}$
- Translation left $\frac{3}{2}$ units
- Translation down 1 unit

15. Let $b(t)$ be the number of births in the United States in year t, and let $d(t)$ represent the number of deaths in the US in year t. If $p(t)$ is the population of the US in year t, find a function the function $c(t)$ that represents the percent change in the population of the US in year t.

$$c(t) = \frac{b(t) - d(t)}{p(t)} \times 100$$

$1 = x+2$
$-1 = x$
$-1 = x+2$
$x = -3$

16. Given: $f(x) = \dfrac{3}{x^2-1}$ and $g(x) = x+2$

a) Find: $f(g(x))$

$f(g(x)) = \dfrac{3}{(x+2)^2-1}$

$(x+2)^2 - 1 = 0$
$(x+2)^2 = 1$
$x+2 = 1$
$x = (-1)$

b) Write the domain of $f(g(x))$ (Interval notation)

$D\ (-\infty, -3) \cup (-3, -1) \cup (-1, \infty)$
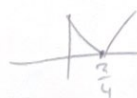$R_g\ (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$
$D_f\ (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

17. Restrict and state the domain keeping the largest possible domain to make the function one to one.

$f(x) = 2|4x-3| + 7$

$4\left(x-\frac{3}{4}\right)$

$\left[\frac{3}{4}, \infty\right)$

$$\frac{1}{x^2}-4 \qquad \left(\frac{1}{x}\right)^2-4$$

$$\frac{1}{2^2}-4=0$$

18. Given: $f(x) = x^2 - 4 = (x-2)(x+2)$

$g(x) = \dfrac{1}{x}$

$\dfrac{1}{x^2-4}$

Find the domain and range of $g(f(x))$.

Domain = $(-\infty,-2)\cup(-2,2)\cup(2,\infty)$

Range = $(-4,\cancel{0})\cup(0,\infty)$ ✓

$Df = (-\infty,\infty) \to (-\infty,-2)\cup(-2,2)\cup(2,\infty)$
$Rf = [0,\infty) \to (0,\infty)$
$Dg = [0,\infty) \to (0,\infty)$
$Rg =$

19. Verify that f and g are inverse functions.

$f(x) = \dfrac{1}{1+x}$ and $g(x) = \dfrac{1-x}{x}$

$g(f(x))$

$$\frac{1}{1+\left(\frac{1-x}{x}\right)} = \frac{1}{\frac{x}{1-x}+\frac{1-x}{x}} = \frac{\frac{1}{x}}{\frac{1}{x}} = x$$

$$g(f(x)) = \frac{1-\left(\frac{1}{1+x}\right)}{1} = \frac{\frac{1+x}{1+x}-\frac{1}{1+x}}{\frac{1}{1+x}} = \frac{x}{\frac{1+x}{1+x}} = x$$

20. Given: $f(x) = \dfrac{x+1}{x-2}$. Find $f^{-1}(x)$

$x = \dfrac{y+1}{y-2}$

$xy - 2x = y+1$

$xy - y = 1 + 2x$

$y(x-1) = 1 + 2x$

$\boxed{y = \dfrac{1+2x}{x-1}}$

21. Determine if the situation could be represented by a one-to-one function. Explain why or why not.

The depth of the tide d at a beach in terms of the time t over a 24-hour period.

No, because the tide comes and goes over 24 hours. The graph would look like waves, so it would not pass the horiz-line test.

22. Find a mathematical equation for the verbal statement.

The gravitational attraction F between two objects of masses m_1 and m_2 is proportional to the product of the masses and inversely proportional to the square of the distance r between the objects.

$$\boxed{F = \frac{k m_1 m_2}{r^2}}$$

23. z varies directly as the square of x and inversely as y. $z = 6$ when $x = 6$ and $y = 4$. Find the constant of variation.

$$z = \frac{kx^2}{y} \qquad 6 = 9k$$
$$k = \frac{2}{3}$$

$$6 = \frac{k6^2}{4}$$

$$\boxed{\dfrac{2}{3}}$$

$$6 = \frac{36k}{4} = 9k$$

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