Hon Pre-Calculus
Test Chapter 1 Name ______

No Calculators!!! Show All Work!!!! Circle All Final Answers!!!!

20/20

Short Answer

1. Find the average rate of change from $x = \frac{\pi}{4}$ to $x = \frac{\pi}{4} + h$ using the difference quotient for the following function:

   $f(x) = \tan x$

   $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

   $\frac{f(x+h) - f(x)}{h}$

   $\frac{\tan(\frac{\pi}{4} + h) - \tan\frac{\pi}{4}}{h}$

   $\frac{\frac{1+\tanh}{1-\tanh} - \frac{1-\tanh}{1-\tanh}}{h}$

   $\frac{\sqrt{2}}{2} / \frac{\sqrt{2}}{2}$ 1

   $\frac{\frac{\tan\frac{\pi}{4} + \tanh}{1 - \tan\frac{\pi}{4}(\tanh)} - \tan\frac{\pi}{4}}{h}$

   $\frac{\frac{2\tanh}{1-\tanh}}{h}$ $\cdot \frac{1}{h}$

   $\boxed{\frac{2\tanh}{h(1-\tanh)}}$

2. State the domain and range using interval notation of $g(f(x))$ if:

   $f(x) = x^2 - 16$ and $g(x) = \frac{1}{x}$

   $(0, -16)$

   Domain = $(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$

   Range = $(-\infty, -\frac{1}{16}] \cup (0, \infty)$

   $x \neq 0$

   $x^2 \neq 16$

   $16 \neq x^2$

   $x \neq 4$

   $f(x): (-\infty, \infty)$

   $f(x): [-16, \infty)$ $[-16, 0) \cup (0, \infty)$

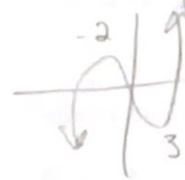
   $g(x): (-\infty, 0) \cup (0, \infty)$

   $g(x): (-\infty, -\frac{1}{16}] \cup (0, \infty)$

   $\frac{1}{x^2 - 16}$ $\frac{1}{-16}$

   $x = 0$

3. Given: $f(x) = 2x^3 - 3x^2 - 36x + 12$. Find the following:

   $f'(x) = 6x^2 - 6x - 36$

   $6(x^2 - x - 6)$

   $(x-3)(x+2)$

   $x = 3, -2$

   a) relative maximum point.

   $f(-2) = 2(-2)^3 - 3(-2)^2 + 36(2) + 12$

   $= -16 - 12 + 72 + 12$

   $\boxed{(-2, 56)}$

   b) relative minimum point

   $f(3) = 2(3)^3 - 3(3)^2 - 36(3) + 12$

   $54 - 27 - 108 + 12$

   $\boxed{(3, -69)}$

   c) Where the function is increasing (Interval notation)

   $\boxed{(-\infty, -2) \cup (3, \infty)}$

   d) Where the function is decreasing (Interval notation)

   $\boxed{(-2, 3)}$

4. Find the inverse function for $f(x) = \frac{3x-2}{4-x}$

$$x = \frac{3y-2}{4-y}$$

$$2+4x-\cancel{xy} = 3y\cancel{-2}+xy$$

$$3y+xy = 4x+2$$

$$y(3+x) = 4x+2$$

$$\boxed{f(x)^{-1} = \frac{4x+2}{3+x}}$$

$$x = \frac{4y+2}{3+y}$$

$$3x+xy = 4y+2$$

$$3x-2 = 4y-xy$$

$$\frac{3x-2}{4-x} = y$$

5. Show that f and g are inverses if:

$f(x) = \frac{x+3}{x-2}$ and $g(x) = \frac{2x+3}{x-1}$

$$f(g(x)) = \frac{\frac{2x+3}{x-1} + \frac{3x-3}{x-1}}{\frac{2x+3}{x-1} + \frac{-2x+2}{x-1}}$$

$$\frac{\frac{\cancel{5}x}{\cancel{x-1}}}{\frac{\cancel{5}}{x-1}} \cdot \frac{\cancel{x-1}}{\cancel{5}} = x$$

$$g(f(x)) = \frac{\frac{2x+6}{x-2} + \frac{3x-6}{x-2}}{\frac{x+3}{x-2} + \frac{-x+2}{x-2}}$$

$$\frac{\frac{\cancel{5}x}{\cancel{x-2}}}{\frac{\cancel{5}}{x-2}} \cdot \frac{\cancel{x-2}}{\cancel{5}} = x$$