~~Hon~~ Pre-Calculus
Test Chapter 3

Name [redacted] 93

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Leave Answers In Simplified Calculator Ready Form!!! Circle All Final Answers!!!

Short Answer

1. Solve for x:

$$\frac{4}{3-e^{3x}} = 8$$

$\frac{4}{8} = \frac{8(3-e^{3x})}{8}$

$\frac{1}{2} = 3 - e^{3x}$

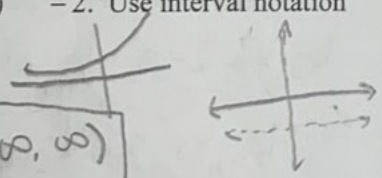
$-\frac{6}{2}$

$\ln e^{3x} = \ln \frac{5}{2}$

$\frac{3x}{3} = \frac{\ln \frac{5}{2}}{3}$

$x = \frac{\ln \frac{5}{2}}{3}$

2. Given: $f(x) = 3(4)^{5-x} - 2$. Use interval notation to write:



a) Domain = $(-\infty, \infty)$

b) Range = $(-2, \infty)$

3. Solve for x: $6e^{2x} + 13e^{x} = 5$

$6e^{2x} + 13e^{x} - 5 = 0$

$6e^{2x} + 15e^{x} - 2e^{x} - 5 = 0$

$3e^{x}(2e^{x} + 5) - 1(2e^{x} + 5) = 0$

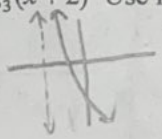
$(3e^{x} - 1) = 0$ $\quad 2e^{x} + 5 = 0$

$\ln e^{x} = \frac{1}{\ln 3}$ $\quad 2e^{x} = -5$

$\ln e^{x} = \frac{-5}{\ln 2}$

$x = \ln \frac{1}{3}$ $\quad x = \frac{-5}{2}$

4. Given: $f(x) = -\log_3(x+2)$ Use interval notation to determine:



a) Domain = $(-2, \infty)$

b) Range = $(-\infty, \infty)$

c) Write the equations of any asymptotes.

$x = -2$

5. Find the x and y intercepts of $y = \log_5(x-1)+2$

a) $x =$ $\left(\frac{26}{25}, 0\right)$

$0 = \log_5(x-1)$

$-2 = \log_5(x-1)$

$\frac{1}{25} = x-1$

$x = \frac{26}{25}$

b) $y =$ n/a

$y = \log_5(-1)+2$

6. Evaluate: $\log_8 \sqrt[7]{32}$

$\log_8 \sqrt[7]{2^5}$

$\log_8 2^{5/7}$

$\frac{5}{7}\log_8 2$

$\frac{5}{7}\cdot\frac{1}{3}$ → $\boxed{\frac{5}{21}}$

$8^x = 2$

$(2^{3x}) = 2^1$

$3x = 1$

$x = \frac{1}{3}$

7. Expand: $\ln\left[\frac{\sqrt[3]{x-1}\,(3x-2)^4}{(x+1)\sqrt{x-1}}\right]^2$

$\ln\left[\frac{(x-1)^{2/3}(3x-2)^8}{(x+1)^2 (x-1)^{3/2}}\right]$

$\ln\left[\frac{(x-1)^{-1/3}(3x-2)^8}{(x+1)^2}\right]$

$\ln(x-1)^{-1/3} + \ln(3x-2)^8 - \ln(x+1)^2$

$\boxed{-\frac{1}{3}\ln(x-1) + 8\ln(3x-2) - 2\ln(x+1)}$

8. Condense: $\frac{1}{3}\left[\log_8 y + 2\log_8(y+4)\right] - \log_8(y-$

$\frac{1}{3}\left[\log_8 y(y+4)^2\right]^{1/3} - \log_8(y-1)$

$= \log_8 y^{1/3}(y+4)^{2/3} - \log_8(y-1)$

$= \boxed{\log_8 \frac{y^{1/3}(y+4)^{2/3}}{y-1}}$

9. Given that $\log_b 9 = a$ and $\log_b 2 = c$, find an expression for $\log_b \frac{24}{b^2}$ in terms of a and c.

$\log_b 24 - \log_b b^2$

$2 = \log_b 24$

$b^c = 2$

$\frac{2}{9}b^a = b^c$

$\frac{2}{9}b^a = a\cdot\frac{2}{a}$

$b^a = \frac{9}{2}b^c$

$\frac{2}{9}b^a = 2$

$2 = \log_b 12\cdot 2$

$2 = \log_b 2^2 + \log_b 12$

$2 = c + \log_b a\cdot\frac{2}{a}$

$2 = c + \log_b 9 + \log_b \frac{2}{a}$

10. Solve for x in terms of a: Separate sheet!

$\log_b x = 2 - a + \log_b\left(\frac{a^2 b^a}{b^2}\right)$

$\log_b x = 2 - a + \log_b a^2 + \log_b b^a - \log_b b^2$

$\log_b x = 2 - a + 2\log_b a + a - 2$

$\log_b x = 2\log_b a^2$

$\log_b x = \log_b a^2$

$\boxed{x = a^2}$

-2

Condense: $\dfrac{1+2\log_8 x}{3}$

$\frac{1}{3}(1+2\log_8 x)$

$\frac{1}{3}(1+\log_8 x^2)$

$\frac{1}{3}+\log_8 x^{2/3}$

$\boxed{\frac{1}{3}+\log_8 \sqrt[3]{x^2}}$

12. Solve: $\frac{1}{2}\log_a(x+2)+\frac{1}{2}\log_a(x-2)=\frac{2}{3}\log_a 27$

$\log_a\sqrt{(x+2)(x-2)} = \log_a 9$

$\sqrt{(x+2)(x-2)}^2 = 9^2$

$x^2-4=81$

$\sqrt{x^2}=\sqrt{85}$

$x=\pm\sqrt{85}$

$\boxed{x=\sqrt{85}}$

13. Solve: $3\left(5^{2x+3}\right)=18\left(2^{3x-2}\right)$

$\ln 5^{2x+3} = \ln 6\left(2^{3x-2}\right)$

$(2x+3)\ln 5 = \ln 6 + \ln 2(3x-2)$

$2x\ln 5+3\ln 5 = \ln 6 + 3x\ln 2 - 2\ln 2$

$2x\ln 5 - 3x\ln 2 = \ln 6 - 2\ln 2 - 3\ln 5$

$\dfrac{x(2\ln 5-3\ln 2)}{2\ln 5-3\ln 2} = \dfrac{\ln 6-2\ln 2-3\ln 5}{2\ln 5-3\ln 2}$

$x = \dfrac{\ln 6-\ln 4-\ln 125}{\ln 25-\ln 8} = \dfrac{\ln \frac{6}{500}}{\ln \frac{25}{8}}$

$\boxed{x=\dfrac{\ln 3/250}{\ln 25/8}}$

14. Solve: $(\ln x)^2 = \ln(x^6)$

$\dfrac{(\ln x)(\ln x)}{\ln x} = \dfrac{6(\ln x)}{\ln x}$

$\ln x = 6$

$\boxed{e^6 = x}$ -2

15. Solve: $e^{-2x}-2xe^{-2x}=0$

$e^{-2x}(1-2x)=0$

$\ln e^{-2x} = \ln 0$

$-2x$

$1-2x=0$

$-2x=-1$

$\boxed{x=\frac{1}{2}}$

$-2x=1$

$\boxed{x=-\frac{1}{2}}$ -1

$\log_x x = \log_{x^3} x^3$

$2\log_x x$

16. Solve: $\log_4 x = \log_{64}\left(-2x^2+x+2\right)$

$-2x^2+x+2=x^3$

$\log_4 x = \log_4 x^1 = \log_{4^3}(x^3)$

$\log_4 x = \log_{64}(-2(1)^2+1+2)$

$\log_4 x = \log_{64}(1)$

$0=0$ ✓

$x^3+2x^2-x-2=0$

$x^2(x+2)-1(x+2)=0$

$(x+1)(x-1)(x+2)=0$

$x=-1 \quad x=1 \quad x=-2$

$\boxed{x=1}$

$\boxed{x=1}$

96 →(5) 48 →(5) 24 →(5) 12

17. A cup of coffee contains approximately 96 mg of caffeine. When you drink the coffee, the caffeine is absorbed into the blood stream and is eventually metabolized by the body. Every 5 hours the amount of caffeine present in the body is reduced by one-half. How many hours does it take for the amount of caffeine to be reduced to 12 mg?

$y = ab^{t/k}$

$\frac{12\text{mg}}{96\text{mg}} = \frac{96\text{mg}}{96\text{mg}}\left(\frac{1}{2}\right)^{t/5}$

$\frac{1}{8} = \left(\frac{1}{2}\right)^{t/5}$

$2^{-3} = 2^{-\left(\frac{t}{5}\right)}$

$-3 = -\left(\frac{t}{5}\right)$

$\frac{t}{5} = 3$

$t = 15$ hours

18. If Brian invested $1000 at 2% compounded continuously, how long would it take before Brian's initial investment (with no withdrawals or deposits) reached $5000?

$A = Pe^{rt}$

$5000 = 1000e^{0.02t}$

$\ln 5 = \ln e^{0.02t}$

$0.02t = \ln 5$

$\frac{1}{50}t = \frac{\ln 5}{1/50}$

$t = 50\ln 5$ years

19. Assume that the number of people infected by a newly discovered virus is growing exponentially. If the number of people infected increases from 200 to 800 in 6 weeks, how much additional time will it take before 12,800 people are infected?

$\frac{800}{200} = 4$

$12{,}800 = 800(4)^{t/6}$

$16 = 4^{t/6}$

$4^2 = 4^{t/6}$

$\frac{t}{6} = 2$

$t = 12$ weeks

20. The populations of two states are growing exponentially. If state A currently has a population of 9 million and state B currently has a population of 11 million, and if the population of the two states increase annually by 3% and 2%, respectively, when will their populations be equal

$9{,}000{,}000(1.03)^t = 11{,}000{,}000(1.02)^t$

$\frac{(1.03)^t}{(1.02)^t} = \frac{11}{9}\frac{(1.02)^t}{(1.02)^t}$

$\ln\left(\frac{1.03}{1.02}\right)^t = \ln\frac{11}{9}$

$t \cdot \ln\left(\frac{103}{102}\right) = \ln 11/9$

$t = \frac{\ln 11/9}{\ln 103/102}$

9. $\log_b 9 = a \quad \log_b 2 = c$

$$\log_b\left(\frac{24}{b^2}\right) = \log_b 8 + \log_b 3 - \log_b b^2$$

$$3\log_b 2 + \frac{1}{2}\log_b 9 - 2$$

$$3c + \frac{1}{2}a - 2$$

11. $$\frac{1 + 2\log_8 x}{3} = \frac{1}{3} + \frac{2}{3}\log_8 x$$

$$\frac{1}{3} + \log_8 x^{2/3}$$

$\log_8 \boxed{2} = \frac{1}{3}$

$$\log_8 2 + \log_8 x^{2/3}$$

$$\boxed{\log_8 2\sqrt[3]{x^2}}$$

14. $$(\ln x)^2 = \ln(x^6)$$

$$(\ln x)^2 = 6\ln x$$

$y^2 = 6y$

$$(\ln x)^2 - 6\ln x = 0$$

$$\ln x(\ln x - 6) = 0$$

$$\ln x = 0 \quad \ln x = 6$$

$$\boxed{x = 1,\ x = e^6}$$