Module 08: Searching and Sorting Algorithms

Topics:

- Searching algorithms
- Sorting algorithms

Application: Searching a list

Suppose you have a list **`L`**. How could you determine if a particular value is in that list, if **`L`** is in no particular order?

Algorithm (called Linear Search)

- Check the first element in **`L`**: is it the one?
  - If Yes, return True
  - Else, check the next value
- The value is not in the list if you don't find it (return False)

Implementing Linear Search

```
def linear_search (L, target):
    '''returns True if target is in L,
          False otherwise
       linear_search:(listof X) X -> Bool
       Note: equivalent to: target in L
    '''
    for val in L:
        if val == target:
            return True
    return False
```

Running Time of `linear_search`

- Let `n = len(L)`
- Best Case:
  - If target is in first position, we find it right away → `O(1)`
- Worst Case:
  - It target is not in `L`, we have to check all `n` elements → `O(n)`
  - What is the other worst case?

Alternatives to Linear Search

- If **`L`** is unsorted, we can't do any better than Linear Search.
- How could we improve Linear Search if **`L`** was sorted into increasing order?
  - Are there situations in which we could stop earlier?
  - Is this any faster in the worst case?

A better approach: Binary Search

- Suppose **`L`** is a listing of the taxpayers in Canada, sorted into increasing order by Social Insurance numbers.
- Approximately 22,000,000 entries
- Look at **`L[11000000]`**
- Is it the **`target`** taxpayer?
  - If yes, stop.
  - If not, is **`target`** < **`L[11000000]`** ?
    - If yes, then **`target`** is in the first half of **`L`**
    - If not, then **`target`** is in the second half of **`L`**
  - Repeat this process for the half containing **`target`**

Developing **binary_search**

- We need to determine how to keep track of the section of the list still being searched
  - →Variables **beginning**, **end**
  - →Determine their initial values
- Determine the **middle** position
- If **L[middle]** is target, return **stop**
- Otherwise, update **beginning** and **end**
- Determine when we to continue (or stop) searching

Starting the implementation

```
def binary_search(L, target):
    beginning = …
    end = …
    while …:
        middle = …
        if L[middle] == target:
            return True
        elif L[middle] > target :
            …
        else:
            …
    return False
```

`binary_search` tests should include

- empty list
- list of length 1: target in list and not in list
- small list, both even and odd lengths
- larger list
  - **`target`** “outside” list, i.e. **`target < L[0]`** or **`target > L[len(L)-1]`**
  - **`target`** in the list, various positions (first, last, middle)
  - **`target`** not in the list, value between two list consecutive values

Worst Case running time of `binary_search`

- What is the runtime of each iteration?
- How many iterations are required at most? Suppose $n = 2^k$:
  - First comparison reduces search region size to 2^{k-1}
  - Second comparison reduces region size to 2^{k-2}
  - Third comparison reduces region size to 2^{k-3}
  - …
  - m^{th} comparison reduces region size to 2^{k-m}
  - When is the region reduced to size 1?

Comments on running time for `binary_search`

- Worst case running time is `O(log n)`
  - For n ~ 1000, will consider at most 11 elements (2^{10} = 1024)
  - For n ~100,000, will consider at most 18 elements (2^{17} = 131072)
  - For n ~ 22,000,000, will consider at most 26 elements (2^{25}=33,554,432)
  - Doubling the size of list requires 1 more comparison worst-case!!!!

Comments and Questions on running time for `binary_search`

- Could be modified to return something other than a Boolean:
  - *What would be a good value?*
- Could be written recursively instead in Python and still have worst case run-time of `O(log n)`
  - *Would the worst case for a recursive implementation in Racket still be* `O(log n)`*?*

Application: Sorting a list

```
def sort_list(L):
  '''sorts L into increasing order
     Effects: L is mutated
     sort_list: (listof Int) -> None
     requires: No duplicate values in L

     Example: Suppose lst = [1,4,3,2],
       calling sort_list(lst) => None,
       but reorders lst as [1,2,3,4]
  '''
```

Sorting Algorithms

There are many different approaches to sorting. We will study the following algorithms and their runtimes:

- Selection sort
- Insertion sort
- Mergesort

Selection sort: basic idea

- Place the smallest entry into L[0]
- Place the second smallest entry into L[1]
- Place the third smallest entry into L[2]
- …
- After step n-1, the list is sorted.

Selection sort implementation

```
def selection_sort(L):
    n = len(L)
    positions = list(range(n-1))
    for i in positions:
        min_pos = i
        for j in range(i,n):
            if L[j] < L[min_pos]:
                min_pos = j
        temp = L[i]
        L[i] = L[min_pos]
        L[min_pos] = temp
```

Selection sort: Runtime

- Before the loop: $O(n)$
- Inner loop: $O(n)$ each iteration
- Outer loop: $O(n)$ iterations

$\rightarrow O(n) + O(n) * O(n)$

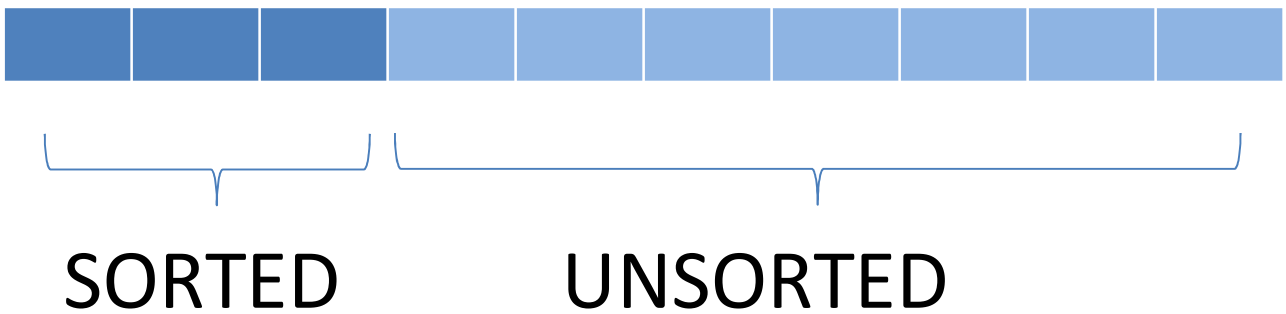
$\rightarrow O(n^2)$

Selection sort is, perhaps, the easiest sorting algorithm, but there are faster algorithms.

Next up: Insertion sort

Insertion Sort: an introduction

- Idea: consider the list to be in two pieces



- Inserting the first item in "Unsorted" into its proper place in "Sorted", shrinks "Unsorted" and enlarges "Sorted"
- Repeat this process until "Unsorted" is empty

Insertion sort: an example Sorting L=[5,8,2,4,3,1,9,6]

- [5] is sorted, insert 8
- [5,8] is sorted, insert 2
- [2,5,8] is sorted, insert 4
- [2,4,5,8] is sorted, insert 3
- [2,3,4,5,8] is sorted, insert 1
- [1,2,3,4,5,8] is sorted, insert 9
- [1,2,3,4,5,8,9] is sorted, insert 6
- [1,2,3,4,5,6,8,9] is sorted. No more to insert.

Insertion Sort

```
def insert(L, pos):
    '''sorts L[0:pos]
       insert: (listof Int) Nat -> None
       requires: L[0:pos-1] is sorted.'''
  while pos > 0 and L[pos] < L[pos-1]:
        temp = L[pos]
        L[pos] = L[pos-1]
        L[pos-1] = temp
        pos = pos-1

def insert_sort(L):
    for i in range(1,len(L)):
        insert(L,i)
```

Running time of **`insert_sort`**

`insert_sort` requires $O(n)$ calls to **`insert`**

`insert` requires at most $O(n)$ while loop iterations

Each while loop body requires $O(1)$ steps

→$O(n) * O(n) * O(1)$

→$O(n^2)$

What is the best-case?

Mergesort– another sorting algorithm

Consider the following approach

- Divide the list into two halves
- Sort the first half
- Sort the second half
- Combine the sorted lists together

$\Rightarrow$Done!

Mergesort is a "Divide and Conquer" algorithm.

Mergesort questions

- How to split the list?
  - Find the middle – before and after
- How to sort smaller lists?
  - Use same idea again (mergesort recursively)
- When to stop recursion?
  - When the list is empty
- How to combine the parts?
  - merge

Merge helper function

Suppose **`L1`**, **`L2`** are in increasing order. To merge:

- If **`L1`** is empty, the merged list is **`L2`**
- If **`L2`** is empty, the merged list is **`L1`**
- If **`L1[0] < L2[0]`** , then the merged list is:

  `[L1[0]] + merge(L1[1:], L2)`
- Otherwise, the merged list is:

  `[L2[0]] + merge(L1, L2[1:])`

Note: we will use a modified version of this algorithm to get a better run-time

```python
def merge(L1,L2,L):
    pos1,pos2,posL = 0,0,0
    while (pos1 < len(L1)) and (pos2 < len(L2)):
        if L1[pos1] < L2[pos2]:
            L[posL] = L1[pos1]
            pos1 += 1
        else:
            L[posL] = L2[pos2]
            pos2 += 1
        posL += 1
    while (pos1 < len(L1)):
        L[posL] = L1[pos1]
        pos1, posL = pos1+1, posL+1
    while (pos2 < len(L2)):
        L[posL] = L2[pos2]
        pos2, posL = pos2+1, posL+1
```

Note: `L1` and `L2` must be sorted before `merge` is called, and `L` is combined length of `L1` and `L2`

`pos1`, `pos2`, `posL` are list positions

Running Time of **`merge`**

- Suppose **`len(L1)`** = m and **`len(L2)`** = p

- Maximum number of while loop iterations is $O(m + p)$
- Each loop is $O(1)$
- Total → $O(m + p)$

- Note: if $m = n/2$ and $p = n/2$, then $O(n)$

```
def mergesort(L):
    if len(L) < 2: return
    mid = len(L)//2
    L1 = L[:mid]
    L2 = L[mid:]
    mergesort(L1)
    mergesort(L2)
    merge(L1,L2,L)
```

Split **L** into two pieces

Mutate each list into sorted order

Merge the two parts together, and put back in to **L**

Running time:

$$T(n) = O(n) + 2T\left(\frac{n}{2}\right) \rightarrow O(n \log n)$$

Sorting Run-time Summary

Algorithm	Best Case	Worst Case
Selection Sort	$O(n^2)$	$O(n^2)$
Linear Insertion Sort	$O(n)$	$O(n^2)$
Mergesort	$O(n \log n)$	$O(n \log n)$

Built-in **`sorted`** and **`sort`**

- Python: **`sorted`**
  - Built-in function
  - Consumes a list and returns a sorted copy
- Python: **`sort`**
  - A list method
  - Consumes a list and *modifies* into sorted order
- Additional arguments can be provided to change the sort (e.g. into decreasing order)
- *O(n log n)* runtime for Python's sorting functions

Examples of **`sort`** and **`sorted`**

```
L = [[3,4],[5,1],[0,0],[2,8], [5,0]]
N1 = sorted(L, key= lambda x: x[0]+x[1])
N2 = sorted(L, key= lambda x: x[1],
            reverse=True)

L = ["cs", "116", "1s", "cool"]
L.sort(key = lambda s: len(s))
L.sort(key = lambda s: s.count('1'),
reverse=True)
```

Goals of Module 08

- Understand how linear and binary search work
- Be able to compare running times of searching algorithms
- Understand how insertion sort, selection sort and mergesort work
- Be able to compare running times of sorting algorithms