CSE 2331 Homework 1
Summer, 2014

1. Give the asymptotic complexity of each of the following functions in simplest terms. Your solution should have the form $\Theta(n^\alpha)$ or $\Theta((\log_\mu(n))^\beta)$ or $\Theta(n^\alpha(\log_\mu(n))^\beta)$ or $\Theta(\gamma^{\delta n})$ or $\Theta(1)$ where $\alpha, \beta, \gamma, \delta, \mu$ are constants. (No need to give any justification or proof.)

   (a) $f_a(n) = \log_3(2n+7) \times \log_4(5n+9) + \log_2(5n^4+3n^2+8)$;
   (b) $f_b(n) = 3\log_4(n^3+7n^2) + 9n^{0.3}$;
   (c) $f_c(n) = 5\log_4(8n+3) + 9\log_6(5n+9)$;
   (d) $f_d(n) = 6\log_{10}(n) + 3n + 2\sqrt{5n}$;
   (e) $f_e(n) = 4n + 5n\log_3(6n^{0.4} + 3n^{0.3})$;
   (f) $f_f(n) = 3\log_3(4n^2+3n+19) + 9\log_7(8n^4+2n)$;
   (g) $f_g(n) = 4^6 + 3^9 \times 2\log_3(5^8)$;
   (h) $f_h(n) = 5(2n+3)\log_3(n^4+16n^2+55) + 4n + 62$;
   (i) $f_i(n) = (6\log_5(n^3+1) + 3\log_3(n+9)) \times (3\sqrt{6n+2} + 2\log_6(2n^2+12))$;
   (j) $f_j(n) = \sqrt{5\log_3(n) + 7n + 26}$;
   (k) $f_k(n) = 2\sqrt{3n^3+5n+20}$;
   (l) $f_l(n) = \sqrt{5(\log_4(n))^3 + 6n^2}$;
   (m) $f_m(n) = 5n^{0.7} + 2n^{0.8}$;
   (n) $f_n(n) = (2n^2+n+1) \times (4n^3+6n^2+n+13) \times (7n+2)$;
   (o) $f_o(n) = 3^n + 6^n + 9^n$;
   (p) $f_p(n) = 9 \times 5^{n+3} + 4 \times 5^{n+7}$;
   (q) $f_q(n) = 5n^3 + 3^{n+5} + 5^{n+3}$;
   (r) $f_r(n) = 3^{5n} + 3 \times 5^n$;
   (s) $f_s(n) = 5 \times 5^{\log_5(3n^2+2n)}$;
   (t) $f_t(n) = 6\log_3(8^n + n^8 + 8)$;

2. Give an example of a function $f(n)$ such that:

   - $f(n) \in O(n^2)$ and $f(n) \in \Omega(n\log_2(n))$ but $f(n) \notin \Theta(n^2)$ and $f(n) \notin \Theta(n\log_2(n))$.

3. Give an example of a function $f(n)$ such that:

   - $f(n) \in O(n)$ and $f(n) \in \Omega(\sqrt{n})$ but $f(n) \notin \Theta(n)$ and $f(n) \notin \Theta(\sqrt{n})$.

4. Prove that $5\sqrt{5n^3+6n^2+7} \in \Theta(n^{1.5})$ using the definition of $\Theta(n^{1.5})$ as functions $f(n)$ such that $c_1 n^{1.5} \leq f(n) \leq c_2 n^{1.5}$ for constants $c_1, c_2 \geq 0$ for all large n.

5. Let $f(n) = 3n(\log_4(7n+5))^3$ and $g(n) = 4n\log_3(n^2+2) \times 5\log_2(n^5+n^4)$. Prove that $f(n) \in \Omega(g(n))$ using $\lim_{n\to\infty} f(n)/g(n)$.