IMPERIAL

3 MINUTE UROP TALKS

1) Chenyang Zhao

Title: Algebraic Geometry and Cryptography

Supervisor name: Dr Soheyla Feyzbakhsh

University: Imperial College London

Abstract:

This research focused on Elliptic Curve Cryptography (ECC) using algebraic geometry, specifically studying elliptic curves over finite fields. The motivation was to understand how these mathematical structures enhance cryptographic security. The research explored properties of finite fields, number theory, and algebraic geometry to evaluate their role in constructing secure trapdoor functions used in ECC. The outcome provided insights into the reliability and efficiency of ECC as a cryptographic system, demonstrating how algebraic structures contribute to robust encryption.

2) Ariff Johan

Title: Computations with Framed Flow Categories

Supervisor name: Prof. Paul Seidel and Kenneth Blakey

University: Imperial College London

Abstract:

The famous Cohen-Jones-Segal construction produces a Floer homotopy type (i.e. suspension spectrum) from a framed flow category. In the setting of Morse theory, this has the familiar interpretation of reconstructing a manifold as a CW complex using critical points (the objects), the moduli spaces of flow lines between those critical points (the morphisms), and instructions on how to concatenate these spaces together. Crucially, we also require a way to stably frame the moduli spaces, as this induces a coherent system of 'Pontryagin-Thom collapses' relating the stable homotopy groups of spheres to framed cobordism groups. In principle, we should thus be able to recover substantial topological information from framed flow category data. My project pays special attention to computing Steenrod squares, as was achieved in the context of Khovanov homotopy by Lobb-Orson-Schuetz. I will mainly discuss the challenges of replicating these results in the setting of Forman's discrete Morse theory, owing largely to the unwieldy abundance of theories that attempt to combinatorialise diffeo-geometric objects.

3) Yunjia Qi

Title: A Study on Diffusion Models and Their Application

Supervisor name: Dr Cong Ling

University: Imperial College London

Abstract:

In recent years, diffusion models have become a powerful framework in generative AI, with their applications rapidly expanding beyond computer vision. My UROP research focuses on Denoising Diffusion Probabilistic Models (DDPMs), which reverse a noisy process to reconstruct original data. This talk will provide an introduction to the math behind these models and highlight their potential uses in fields like biology and physics.

4) Maxim Vassiliev

Title: DualArrays.jl: Extending the notion of Dual numbers to arrays.

Supervisor name: Dr. Sheehan Olver

University: Imperial College London

Abstract:

This is a Julia package efficiently implements arrays of Dual numbers in Julia. e.g. a 'Dual Vector' storing its dual parts as a matrix and thus enabling automatic differentiation when passed through array-valued functions. Because the Dual parts can be stored in sparse array structures (as provided by BandedMatrices.jl, SparseArrays.jl for example), this can be an efficient way to do forward and mixed mode automatic differentiation, with applications in Mathematical Physics and Machine Learning.

5) Xialu Zheng

Title: Advancing Robust Filtering: From Uniform to Symmetric Beta Distributions

Supervisor name: Professor Alessandra Luati

University: Imperial College London

Abstract:

This project extends robust filtering methods by transitioning from uniformly distribution to a symmetric Beta one. Building on the work of Catania, Harvey, and Luati, which developed robust filtering for location parameters, this research investigates the implications of using Beta-distributed random variables. This shift not only enhances the robustness of filtering in time-varying parameter models but also lays the foundation for future generalizations to more complex distributions defined on compact intervals and high-dimensional settings.

6) Sidharth Hariharan

Title: A Formalisation of the Cohn-Elkies Linear Programming Bound in Lean

Supervisor name: University: Dr Maryna Viazovska

University: École Polytechnique Fédérale de Lausanne - EPFL

Abstract:

In 2022, Maryna Viazovska was awarded the Fields Medal for her work on the Sphere Packing Problem in dimensions 8 and 24. The solutions to these problems depend strongly on an intermediate result proved by Henry Cohn and Noam Elkies that gives an upper-bound on the optimal sphere packing density in □ⁿ in terms of a certain specific type of function, if such a function exists. Indeed, the solutions in dimensions 8 and 24 involve constructing "magic functions" that satisfy these conditions and whose Cohn-Elkies bounds are the sphere packing densities of the E□ and Leech lattices respectively. I spent the summer in Lausanne working with Professor Viazovska and other collaborators from across the globe on formalising the Cohn-Elkies bounds in Lean, taking a much-needed first step towards a formalisation of her groundbreaking work. I will divide my talk into two parts: the first will be an overview of the Cohn-Elkies bound and an explanation of some of its dependencies and the second will be an overview of what we have formalised, what choices we made in the process and what remains to be formalised to complete the proof that the E□ Sphere Packing is optimal in □⁸.

7) Xinyan Wang

Title: correlation analysis between mitochondrial protein-coding DNA'

Supervisor name: Professor Nick Jones

University: Imperial College London

Abstract:

I conducted a correlation analysis of mitochondrial protein-coding DNA. I started by studying the background of single-cell sequencing to understand the dataset, then proceeded with data normalization. Then, I used the Metropolis algorithm to simulate the normalized data by assuming they follow a negative binomial distribution and obtained the necessary parameters. After having the simulated model, I applied a discrete Gaussian copula and got the set of correlations between data for different gene types.

8) Ross Ah-Weng

Title: Bilingual Spiking Neural Networks

Supervisor name: Dr Hardik Rajpal

University: Imperial College London

Abstract:

Mounting experimental results support the Critical Brain Hypothesis, namely that the brain acts as a self-tuning dynamical system near phase change. Existing SNN models all share the dichotomous assumption that neurons are excitatory or inhibitory, also known as Dale's principle - however this is not the case in reality. The speaker will explore the functional implications of "bilingual" neurons violating this principle, and a minimal model resulting in rich dynamics transitioning between asynchronous, avalanche and oscillatory regimes.

9) Jai Joshi

Title: Dynamics of the Unit Circle: Rate-Induced Tipping

Supervisor name: Dr Iacopo Longo

University: Imperial College London

Abstract:

In the past few decades, the phenomenon of rate-induced tipping has become a point interest within the field of non-autonomous dynamics. In an ordinary differential equation, a change in parameters can often lead to dramatic changes in the dynamics of a system, but of late researchers have been investigating how the rate at which these parameters shift over time can be of consequence to the dynamics of the system. Results have become well-established when working within the setting of the reals □, however such behavior has not been investigated on other state spaces such as □ 1 . In this thesis, the dynamics of a family of ODEs defined on □ 1 have been classified, which sets the stage for rate-induced tipping to occur on this novel state space. This research hopes to provide a basis for further investigation into the non -autonomous dynamics of the unit circle, with major applications to the Kuramoto model of synchronization.

10) Qiyu Xie

Title: Bifurcations in set-valued and nonautonomous dynamical systems

Supervisor name: Professor Martin Rasmussen

University: Imperial College London

Abstract:

Set-valued dynamical systems model noise using topological rather than probabilistic methods. Bifurcations in these systems can only occur in specific ways when the system is autonomous. My project generalises this to the nonautonomous case under some extra conditions. Specifically, I identified suitable structures in nonautonomous systems and used them to prove analogous versions of the bifurcation result for both the past behaviour of set-valued processes and set-valued skew product flows when the driving system has periodic limits.

11) Yudai Yamazaki

Title: Formalising Relationships between Group Extensions and Cohomology

Supervisor name: Professor Kevin Buzzard

University: Imperial College London

Abstract:

Given two groups, determining group extensions with those groups as the kernel and the quotient is a non-trivial task. In this UROP project, along with several other theorems, I formalised a theorem stating that when the kernel is an abelian group, there is a bijection between the equivalence classes of extensions and the second cohomology group.

12) Rohan Shenoy

Title: The Variance Gamma Process for Option Pricing

Supervisor name: Dr Peter Kempthorne

University: Imperial College London

Abstract:

The variance gamma process is explored as model for the dynamics of log stock prices. The process is formulated as a Brownian motion evaluated at a random, gamma-distributed, time change. The implementation of a gamma process as stochastic index for market progress is considered as an alternative index to calendar time. Closed forms are obtained for the return density and the prices of European derivatives under the model via an Esscher transform method. The statistical and risk neutral densities are estimated for data on the S&P500 Index and the prices of derivatives on this Index. The additional parameters correct for pricing biases of the Black Scholes model that is a parametric special case of the option pricing model developed