Lecture\_2\_Assignment

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# Exponential Population Growth with Constant Immigration

pacman::p\_load(dplyr,tidyr,  
 ggplot2,  
 deSolve)

## Q1

You are a curious student in the Introduction to Theoretical Ecology course. After the class, you decide to do a small experiment on population growth. You set up a “massive” fish tank and introduce flatworm individuals. Also, each day you add I new individuals into the tank, hoping that the population will increase faster. Assuming that the intrinsic rate of increase is r (per day) and there is no factor limiting the growth and reproduction of these flatworms, the population dynamics can be described by the following differential equation:

The analytical solution to this differential equation is:

Please use what you have learned in the lecture to derive the solution for this differential equation step by step. (You can either write down the answer on a paper and embed a picture of it or directly type the equations in Word.)

### Solution

To find , I first rearranged the variables and then integrated both sides of the equation.

For the left hand side, I substituted the term with to get a new differentials ; for the right hand side, the integration is simply all the increments being summed up, thereby ;

After the method of substitution and lifting the natural log, the result is clear

Rearranging the equation, we can get the equation of population size of all time .

Despite the effort for deriving the equation, there is an ambiguous term in the equation which can not help us generalize the result. To resolve the ambiguity of the result, we can plugin for calculating the initial condition of the population.

If is a known constant then the ambiguous term is equal to . After defining all these terms, I can finally close this question by plug in all the terms:

Rearranging the above equation we can get the solution:

This equation tells us that the population size at time is related to the initial condition , the population growth rate , and the constant immigration rate . The behavior of this equation is explored in the subsequent questions.

P.S. Thanks Chueh-Chen for brushing up my Calculus.

## Q2

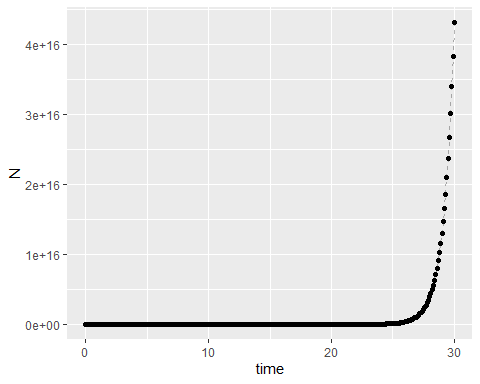
Suppose that = 10,, = 1.2, and = 3. How will the flatworm population change over a week? Solve the differential equation numerically and visualize the population trajectory. Please show the figure along with the R code you used to generate the results.

# set up an equation  
exp\_g <- function(time, y, parms){  
 with(as.list(c(parms, y)), {  
 dN\_dt <- r\*N  
 return(list(c(dN\_dt)))  
 })  
   
 }  
  
time <- seq(0, 30, 0.1)  
y <- c(N = 10)  
parms <- c(r = 1.2,  
 I = 3)  
  
pop\_size <-  
 ode(func = exp\_g,  
 y = y,  
 times = time,  
 parms = parms)  
head(pop\_size)

## time N  
## [1,] 0.0 10.00000  
## [2,] 0.1 11.27497  
## [3,] 0.2 12.71250  
## [4,] 0.3 14.33330  
## [5,] 0.4 16.16075  
## [6,] 0.5 18.22120

### Visualization

ggplot(as.data.frame(pop\_size), aes(x = time, y = N)) +  
 geom\_line(linetype = 2, alpha = 0.3) +  
 geom\_point()



## Q3

Compare the population growth with and without constant immigration and explain the model dynamics in your own words. How does the constant immigration term affect the population dynamics? Do you think your daily addition of new flatworm individuals make a big difference?

# set up an equation  
exp\_g <- function(time, y, parms){  
 with(as.list(c(parms, y)), {  
 with\_I <- r\*N1 + I  
 no\_I <- r\*N2  
 return(list(c(with\_I, no\_I)))  
 })  
   
 }  
  
time <- seq(0, 100, 0.1)  
y <- c(N1 = 10, N2 = 10)  
parms <- c(r = 1.2,  
 I = 3)  
  
pop\_size <-  
 ode(func = exp\_g,  
 y = y,  
 times = time,  
 parms = parms)  
head(pop\_size)

## time N1 N2  
## [1,] 0.0 10.00000 10.00000  
## [2,] 0.1 11.59371 11.27497  
## [3,] 0.2 13.39062 12.71250  
## [4,] 0.3 15.41662 14.33330  
## [5,] 0.4 17.70094 16.16075  
## [6,] 0.5 20.27649 18.22120

### Visualization

as.data.frame(pop\_size) %>%   
 pivot\_longer(cols = c("N1", "N2"), names\_to = "variables", values\_to = "values") %>%   
 ggplot(aes(x = time, y = values, color = variables)) +  
 geom\_line(linetype = 2, alpha = 0.3) +  
 ylab("Linear scale") +  
 geom\_point() +  
 ggtitle("Whole time frame")

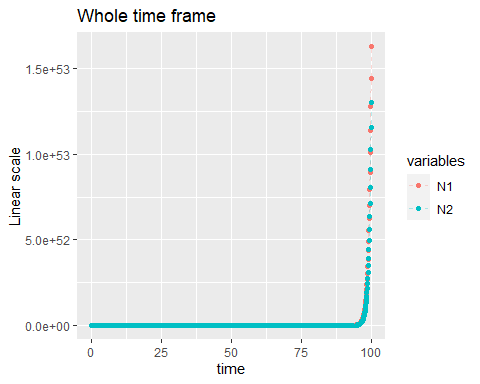
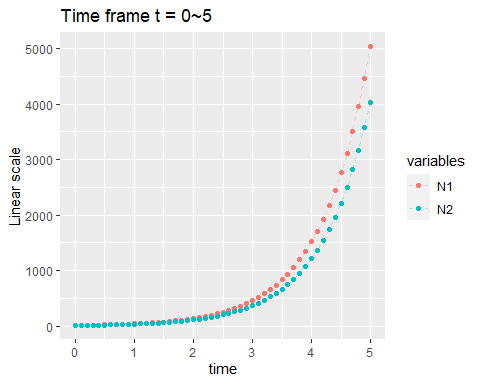


Figure 1.

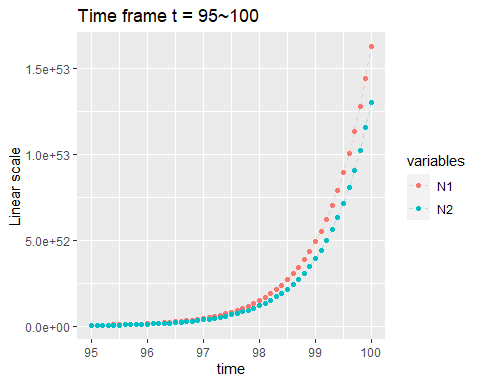
as.data.frame(pop\_size) %>%   
 filter(time <= 5) %>%   
 pivot\_longer(cols = c("N1", "N2"), names\_to = "variables", values\_to = "values") %>%   
 ggplot(aes(x = time, y = values, color = variables)) +  
 geom\_line(linetype = 2, alpha = 0.3) +  
 ylab("Linear scale") +  
 geom\_point() +  
 ggtitle("Time frame t = 0~5")#+



# scale\_y\_log10()

figure

as.data.frame(pop\_size) %>%   
 filter(time >= 95) %>%   
 pivot\_longer(cols = c("N1", "N2"), names\_to = "variables", values\_to = "values") %>%   
 ggplot(aes(x = time, y = values, color = variables)) +  
 geom\_line(linetype = 2, alpha = 0.3) +  
 ylab("Linear scale") +  
 geom\_point() +  
 ggtitle("Time frame t = 95~100")#+



# scale\_y\_log10()

### Personal comments

Let’s review the equations first. The two populations and have the same growth rate and unlimited resources. The first population size receives a constant inflow of immigrants while the second population do not.

The difference of the two equations suggests that will always exceeds individuals whenever . Judging the figures, this observation from the equation is true. The general pattern of the two models both demonstrate a exponential growth pattern (fig. 1), with the exact value of being “slightly larger” than . By judging the visuals of the numerical result, the constant influx contributed by immigration have similar proportions of impacts through different points of time (fig. 2, 3).