

Comparing general equilibrium modeling in Julia and GAMS: An example using CSAVE

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January 23, 2026

Message from GAMS...

“The reason for GAMS superior performance in [having the shorter model generation time of] this example [IJKLM model] is the use of relational algebra... to process complex queries efficiently.”

— Broihan (2023), GAMS Corporation

Response from Julia...

“... it is not difficult to address it [the bottle neck of Broihan’s benchmark Julia model], given that general-purpose languages like Julia and Python have libraries specialized for this task.”

— M. Lubin, O. Dowson, J. D. Garcia, J. Huchette, B. Legat (2023), JuMP developers

Questions

- What does a CGE written in Julia look like?
- Which language is faster in running CSAVE?
- How coding strategies may affect Julia's solving time?

Scope

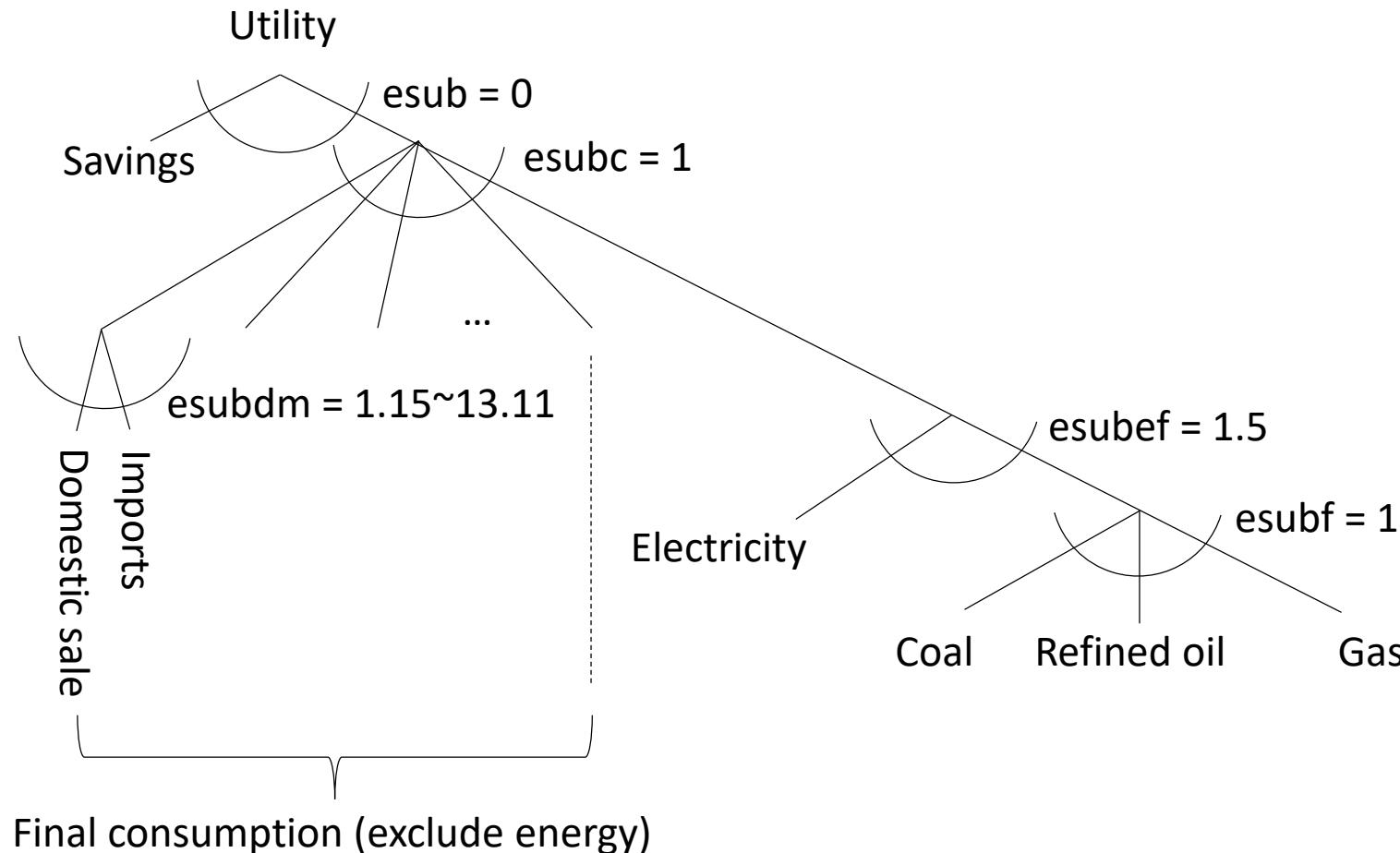
- An exercise based on CSAVE (Chen and Paltsev, 2025): in Julia vs. in GAMS
- Model generation & solving time may vary across models & computers
- Technical details/coding tricks are left for a modeling workshop

Model

- CSAVE: Chen & Paltsev (2025)
 - Motivated by EPPA
 - Derived from GTAPinGAMS
 - In this exercise: multi-sector & multi-region — ↑ resolution for “stress testing”
 - GTAP9 database

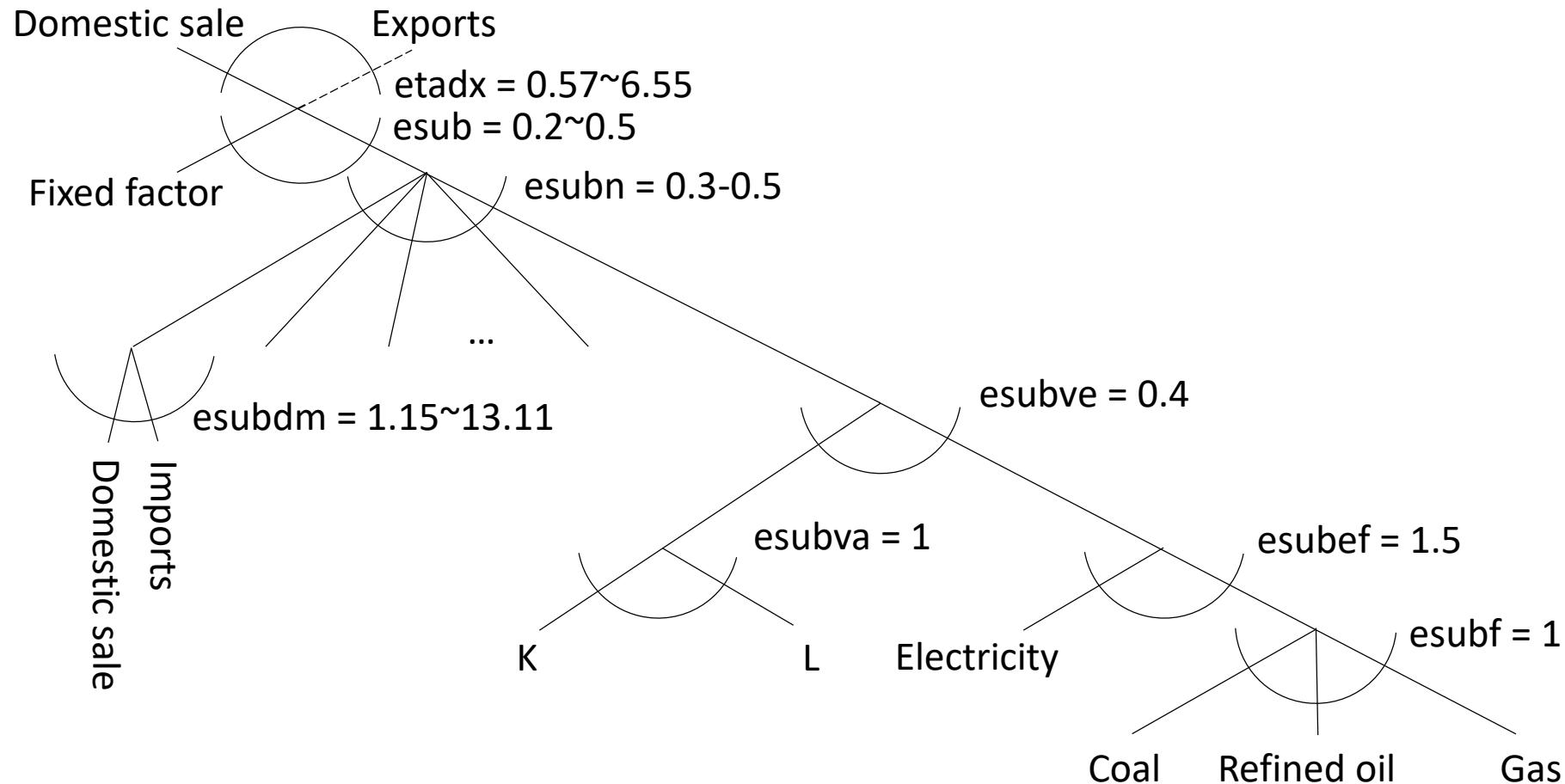
Model

Expenditure function



Model

Cost function for a production sector



Julia vs. GAMS

<ul style="list-style-type: none">• Julia<ul style="list-style-type: none">• Developed by J. Bezanson, S. Karpinski, V. B. Shah, A. Edelman in 2012• Open source/free• General purpose w/ packages for extension• MPSGE.jl is a package of Julia• Free PATH license until 12/31/2026 for now• MPSGE.jl<ul style="list-style-type: none">• D. Anthoff, E. Lazarus, M. Phillipson	<ul style="list-style-type: none">• GAMS<ul style="list-style-type: none">• Developed by Alex Meeraus at the World Bank in the 1970s• Proprietary• DSML for optimization problems• MPSGE is a sub-system of GAMS• Access PATH via GAMS license• MPSGE<ul style="list-style-type: none">• Tom Rutherford
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Coding in Julia

- Packages used directly include

- I/O & data processing

- CSV
 - Dataframes
 - JLD2
 - *CSVtoDIC*
 - *GTAP9data*

- Model construction and solving

- JuMP
 - MPSGE
 - PATHSolver

- Generating figures

- StatsBase
 - StatsPlots
 - Plots
 - PyPlot
 - Measures
 - Distributions

- Time measurement

- BenchmarkTools

Coding in Julia

- CSAVEinJulia is packaged and defines its own environment
 - Project.toml — names & identities of the direct dependencies
 - Manifest.toml — the dependency graph, dependency version, where to load

Project.toml

```
[deps]
CSV = "336ed68f-0bac-5ca0-87d4-7b16caf5d00b"
DataFrames = "a93c6f00-e57d-5684-b7b6-d8193f3e46c0"
GTAPdata = "130c15d4-ad42-4bd4-91d3-6787eb393e58" (highlighted)
Ipopt = "b6b21f68-93f8-5de0-b562-5493be1d77c9"
JLD2 = "033835bb-8acc-5ee8-8aae-3f567f8a3819"
JuMP = "4076af6c-e467-56ae-b986-b466b2749572"
MPSGE = "d5dc2f44-7ae2-49e9-bc77-b47b6bca565d"
```

Manifest.toml

```
[[deps.GTAPdata]]
deps = ["CSV", "CSVtoDIC", "DataFrames", "JLD2"]
git-tree-sha1 = "d5cb8e98efe4a2d2de8cfabcb153f6ba5ac84558" (highlighted)
repo-rev = "master"
repo-url = "https://github.com/chenyhmitedu/GTAPdata"
uuid = "130c15d4-ad42-4bd4-91d3-6787eb393e58" (highlighted)
version = "0.1.0"

[[deps.HashAnnavMannedTrios]]
```

Pin package “GTAPdata” to a specific commit

CSAVEinJulia is available on <https://github.com/chenyhmitedu/>

Coding in Julia

GAMS' set operation is much simpler and cleaner:

```
vtw(j)      = sum(r, vst(j,r));  
  
vafm(i,g,r) = vdfm(i,g,r)*(1+rtfd(i,g,r))+vifm(i,g,r)*(1+rtfi(i,g,r));
```

Julia uses dictionary (key-value mapping) to accomplish the same task:

```
vtw      = Dict(  
| | | | j => sum(vst[(j, r)] for r ∈ set_r)  
| | | | for j ∈ set_i  
| | | )  
  
vafm    = Dict(  
| | | | (i, g, r) => vdfm[(i, g, r)]*(1+rtfd0[(i, g, r)])+vifm[(i, g, r)]*(1+rtfi0[(i, g, r)])  
| | | | for i ∈ set_i, g ∈ set_g, r ∈ set_r  
| | | )
```

Coding in Julia

GAMS/MPSGE

```
$sectors:  
y(g,r)$vom(g,r)  
m(i,r)$vim(i,r)  
yt(j)$vtw(j)  
E(i,s,r)$vxmd(i,s,r)  
A(i,g,r)$vafm(i,g,r)  
  
$commodities:  
p(g,r)$vom(g,r)  
pm(j,r)$vim(j,r)  
pt(j)$vtw(j)  
pf(f,r)$(evom(f,r)$mf(f))  
ps(f,g,r)$(sf(f) and vfm(f,g,r))  
PX(i,s,r)$vxmd(i,s,r)  
PA(i,g,r)$vafm(i,g,r)  
PE(g,r)$vxm(g,r)  
  
$consumers:  
ra(r)
```

```
! Supply  
! Imports  
! Transportation services  
! Exports  
! Armington good i used k  
  
! Domestic output price  
! Import price  
! Transportation services  
! Primary factors rent  
! Sector-specific primary  
! Price index for exports  
! Price for Armington goo  
! Price index for exports  
  
! Representative agent
```

Julia/MPSGE.jl

```
@sectors(MGE, begin  
    Y[set_g, set_r],  
    M[set_i, set_r],  
    YT[set_i],  
    E[set_i, set_r, set_r],  
    A[set_i, set_g, set_r],  
end)  
  
@commodities(MGE, begin  
    P[set_g, set_r],  
    PM[set_i, set_r],  
    PT[set_i],  
    PF[set_mf, set_r],  
    PS[set_sf, set_g, set_r],  
    PX[set_i, set_r, set_r],  
    PA[set_i, set_g, set_r],  
    PE[set_i, set_r],  
end)  
  
@consumers(MGE, begin  
    RA[set_r],  
end)
```

```
(description = "Supply")  
(description = "Imports")  
(description = "Transportation services")  
(description = "Subsidy and transport service")  
(description = "Armington good")  
  
(description = "Domestic output price")  
(description = "Import price")  
(description = "Transportation services")  
(description = "Non-sector-specific primary f")  
(description = "Sector-specific primary facto")  
(description = "Price index for exports (incl")  
(description = "Price index for Armington goo")  
(description = "Price index for exports (excl")  
  
(description = "Representative agent")
```

Coding in Julia

GAMS/MPSGE

```
$prod:y(g,r)$vom(g,r)  t:etadx(g)  s:esub(g)  sn(s):esubn(g)  sve(sn):esubve(g)  sva(sve):esubva(g)  sef(sve):esubef(g)  sf(sf):esubf(g)
  o:P(g,r)          q:(vom(g,r)-vxm(g,r))  a:RA(r)  t:rto(g,r)
  o:PE(g,r)         q:vxm(g,r)            a:RA(r)  t:rto(g,r)
  i:PA(i,g,r)$fe(i) q:vafm(i,g,r)        sf:
  i:PA(i,g,r)$elec(i) q:vafm(i,g,r)        sef:
  i:PA(i,g,r)$ne(i) q:vafm(i,g,r)        sn:
  i:ps(sf,g,r)      q:vfm(sf,g,r)  p:(1+rtf0(sf,g,r))  a:ra(r)  t:rtf(sf,g,r)
  i:pf(mf,r)        q:vfm(mf,g,r)  p:(1+rtf0(mf,g,r))  a:ra(r)  t:rtf(mf,g,r)  sva:
```

Julia/MPSGE.jl

```
for g ∈ set_i, r ∈ set_r
    @production(MGE, Y[g, r], [t = etadx[g], s = esub[g], sn => s = esubn[g], sve => sn = esubve[g], sva => sve = esubva[g], sef => sve = esubef[g], sf =
        @output(P[g, r],           vhm[g, r], t, taxes = [Tax(RA[r], rto[g, r])], reference_price = 1 - rto0[g, r])
        @output(PE[g, r],          vxm[g, r], t, taxes = [Tax(RA[r], rto[g, r])], reference_price = 1 - rto0[g, r])
        [@input(PA[i, g, r],       vafm[i, g, r], sf) for i ∈ set_fe]...
        [@input(PA[i, g, r],       vafm[i, g, r], sef) for i ∈ set_elec]...
        [@input(PA[i, g, r],       vafm[i, g, r], sn) for i ∈ set_ne]...
        [@input(PS[sf, g, r],      vfm[sf, g, r], s, taxes = [Tax(RA[r], rtf[sf, g, r])], reference_price = 1 + rtf0[sf, g, r]) for sf ∈ set_sf]...
        [@input(PF[mf, r],          vfm[mf, g, r], sva, taxes = [Tax(RA[r], rtf[mf, g, r])], reference_price = 1 + rtf0[mf, g, r]) for mf ∈ set_mf]...
    end)
end
```

Coding in Julia

GAMS/MPSGE

```
$demand:ra(r)
  d:p("c",r)           q:vom("c",r)
  e:p("c","USA")      q:vb(r)
  e:p("g",r)           q:(-vom("g",r))
  e:p("i",r)           q:(-vom("i",r))
  e:ps(sf,j,r)        q:vfm(sf,j,r)
  e:pf(mf,r)          q:evom(mf,r)
```

Julia/MPSGE.jl

```
for r ∈ set_r
  @demand(MGE, RA[r], begin
    @final_demand(P[:c, r],      vom[:c, r])
    @endowment(P[:c, :USA],       vb[r])
    @endowment(P[:g, r],         -vom[:g, r])
    @endowment(P[:i, r],         -vom[:i, r])
    [@endowment(PF[f, r],        evom[f, r]) for f ∈ set_mf]...
    [@endowment(PS[f, j, r],     vfm[f, j, r]) for f ∈ set_sf, j ∈ set_i]...
  end)
end
```

Setting

- Data resolution

• 56x2	— 2-region	<i>USA, ROW</i>
• 56x4	— 4-region	<i>USA, EUR, CHN, ROW</i>
• 56x8	— 8-region	<i>USA, EUR, ROE, JPN, IND, CHN, ANZ, ROW</i>
• 56x16	— 16-region	<i>USA, ..., ANZ, TWN, CAN, MEX, BRA, RUS, KOR, IDZ, ASI, ROW</i>
• 56x32	— 32-region	<i>USA, [disaggregated EUR: DEU, FRA, GBR, ITA, ESP, ...], ..., ROW</i>
• 56x50	— 50-region	<i>Disaggregate 56x32</i>
• 56x61	— 61-region	<i>Disaggregate 56x50</i>
• 56x74	— 74-region	<i>Disaggregate 56x61</i>

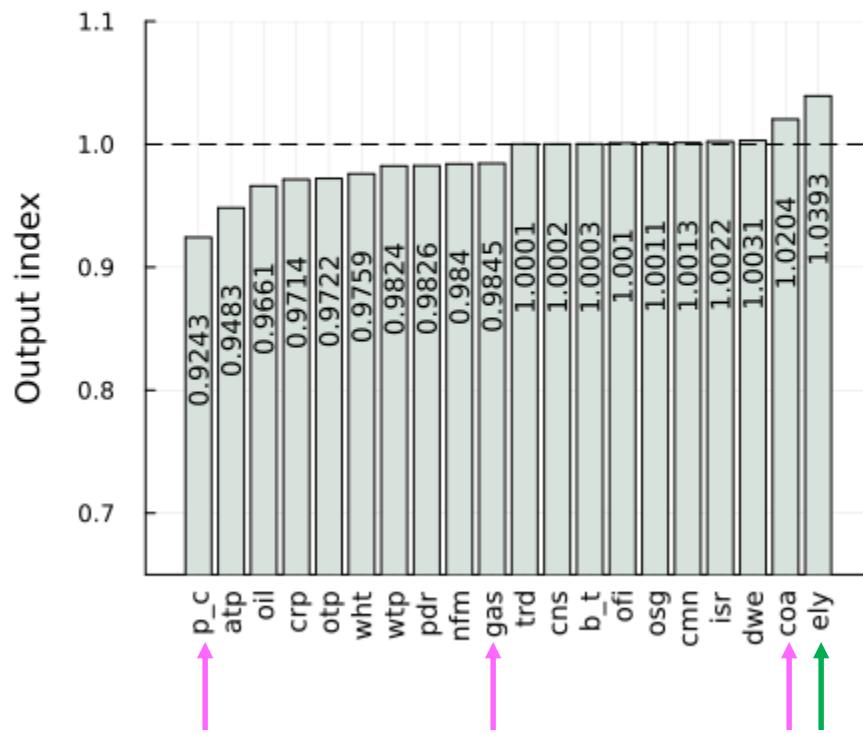
Setting

- Scenario
 - US doubles tax rates on fossil fuels use:
 $\text{rtfd0}[i, g, r] \times 2.0 \text{ & } \text{rtfi0}[i, g, r] \times 2.0 ; r \in \text{USA}, i \in \text{coa, p_c, gas}$

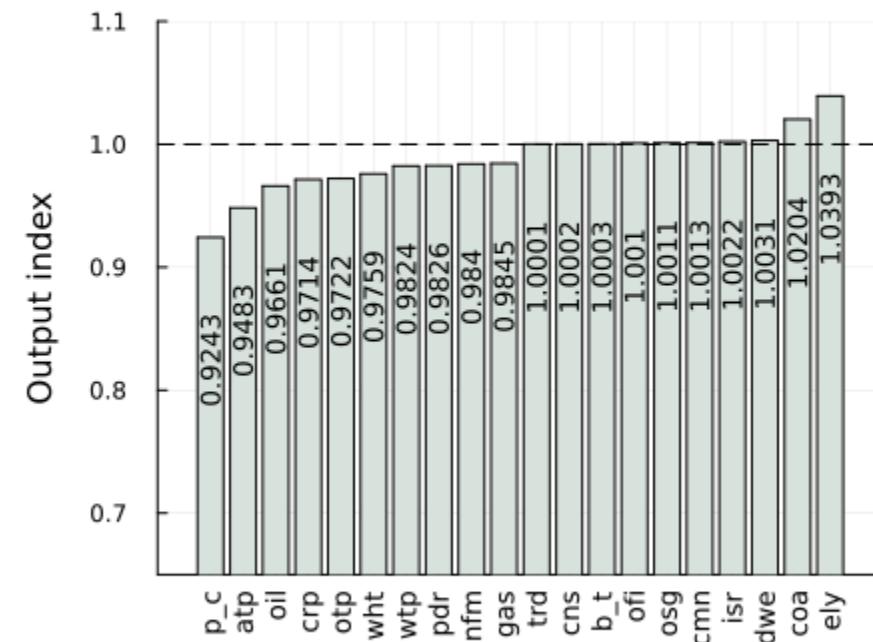
Simulation

Double fossil fuel tax rates

Selected sectoral outputs of USA:
CSAVEinJulia



Selected sectoral outputs of USA:
CSAVEinGAMS



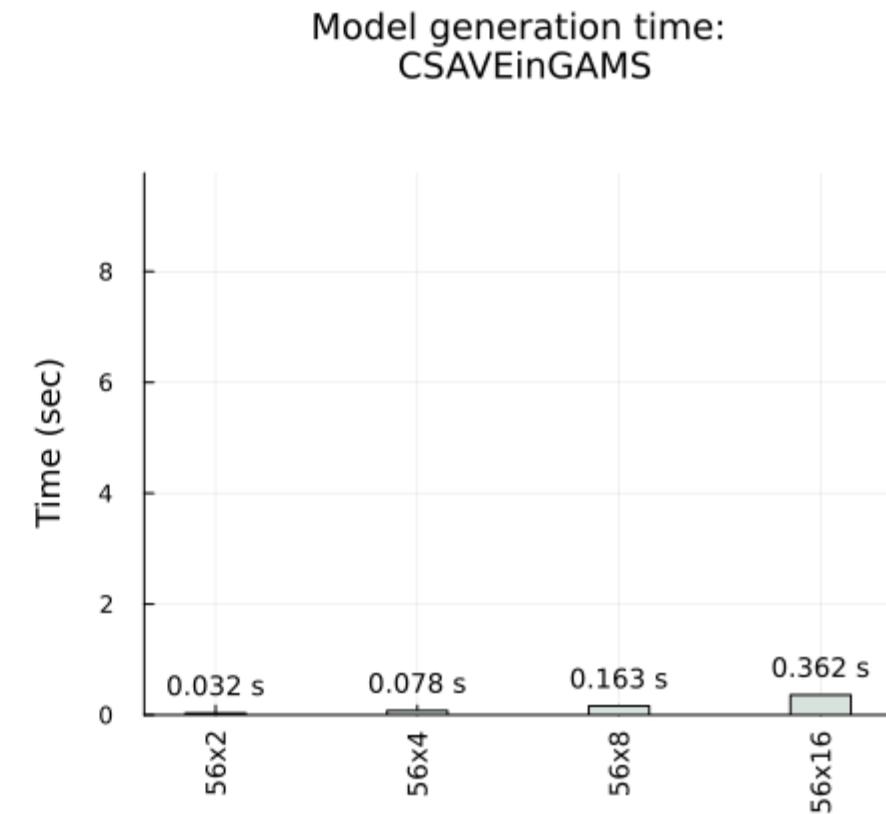
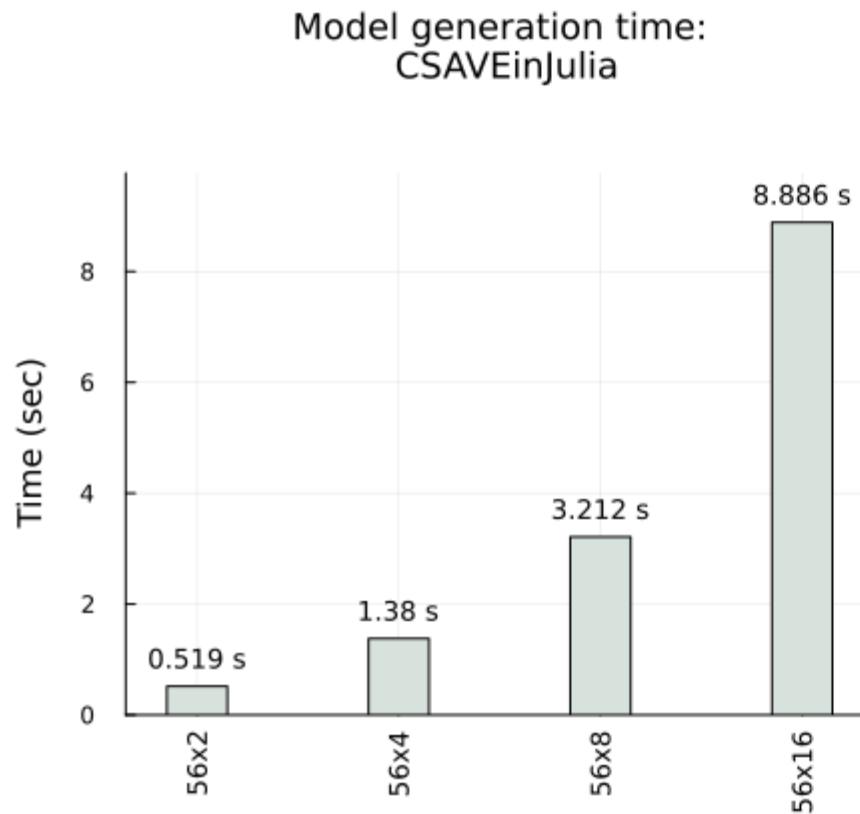
Computer spec

- CPU : AMD Ryzen Threadripper PRO 7955WX 16-Cores 4.5 GHz
- RAM : 128 GB 4800 MT/s

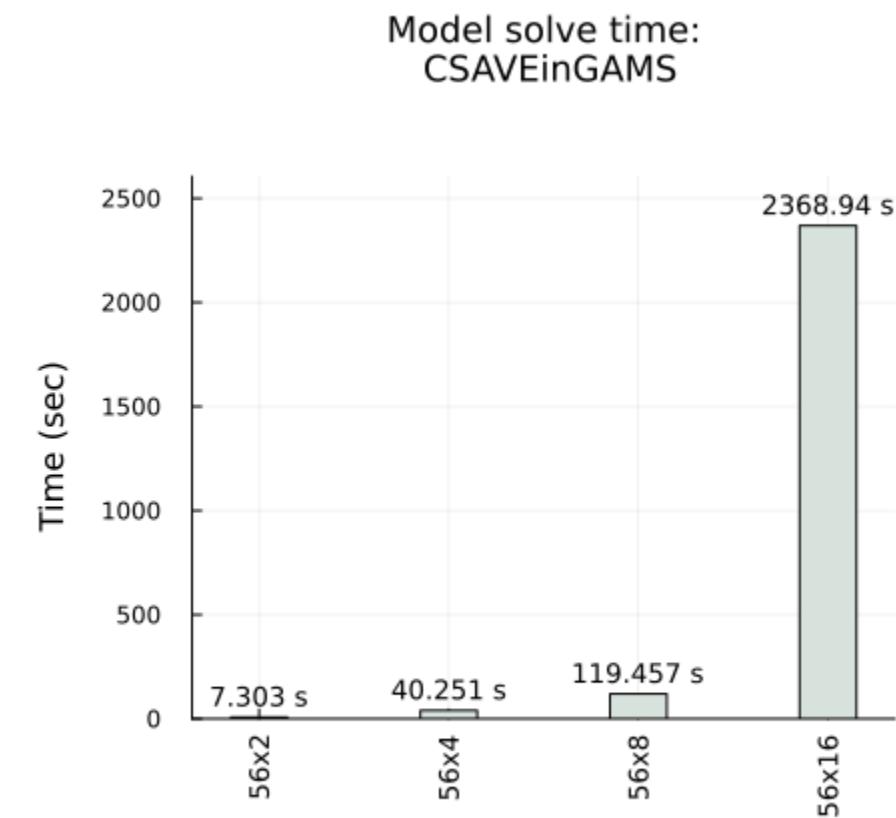
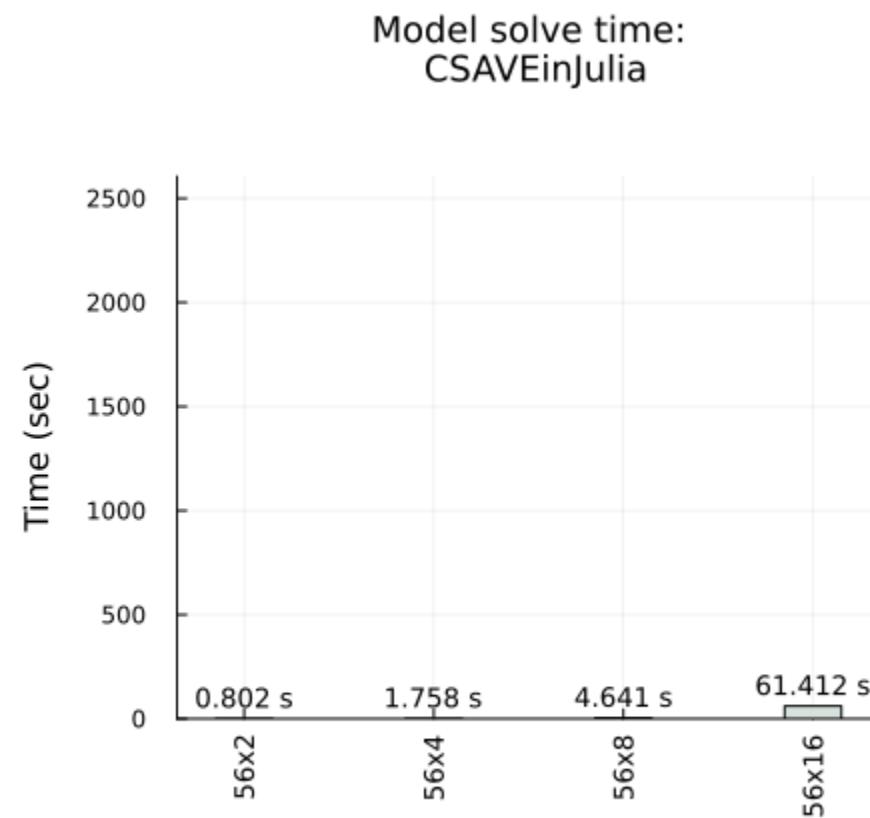


Model generation time

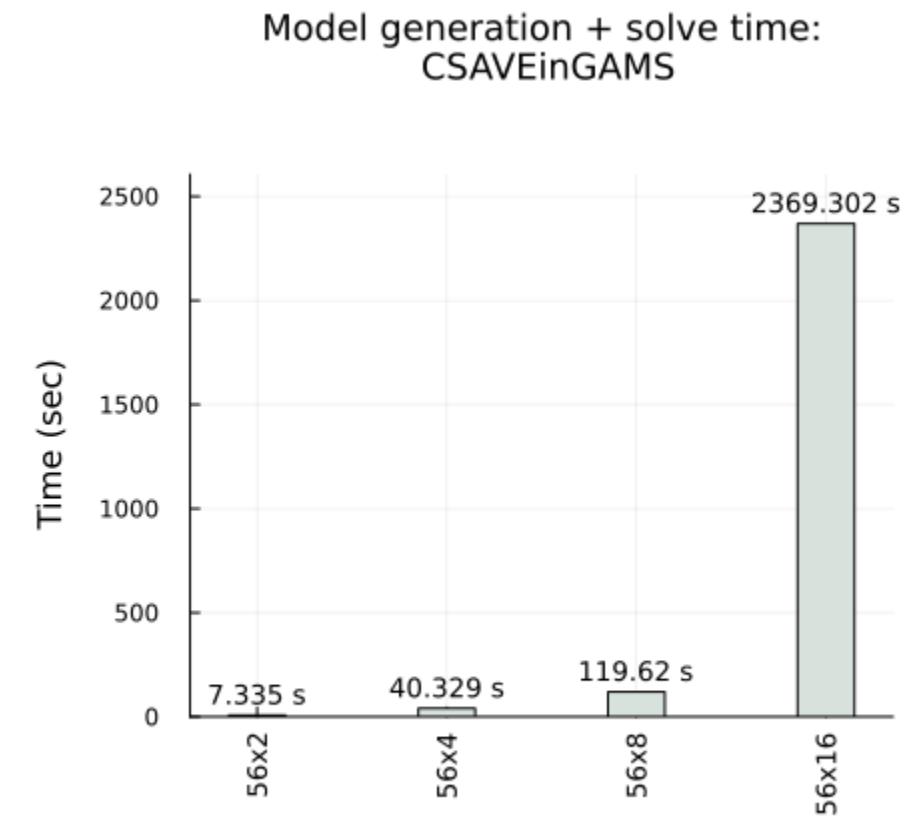
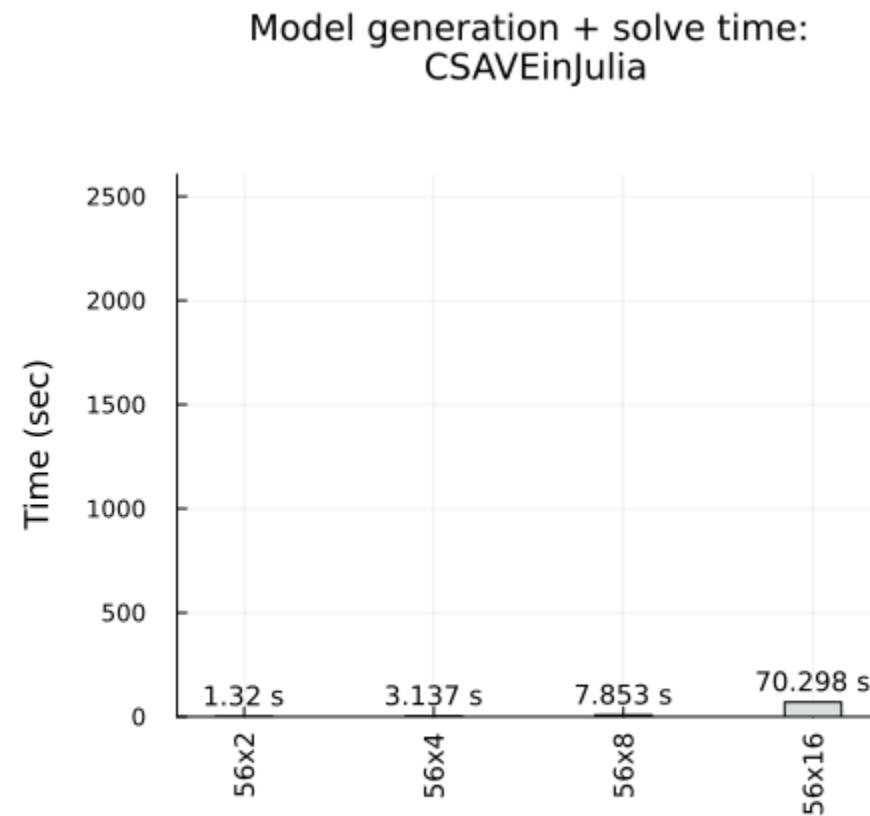
For now, each data resolution setting is run once for demonstration purposes.



Model solving time



Model generation + solving time



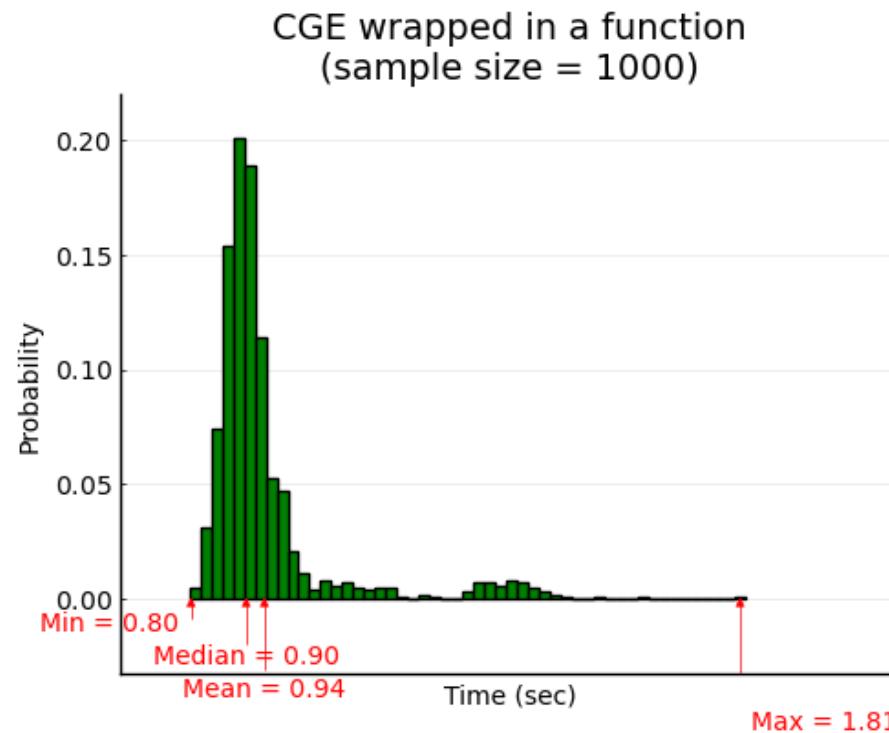
Looking into Julia solving time

“Time distributions are always right-skewed... This phenomena can be justified by considering that the machine noise affecting the benchmarking process is, in some sense, inherently positive...”

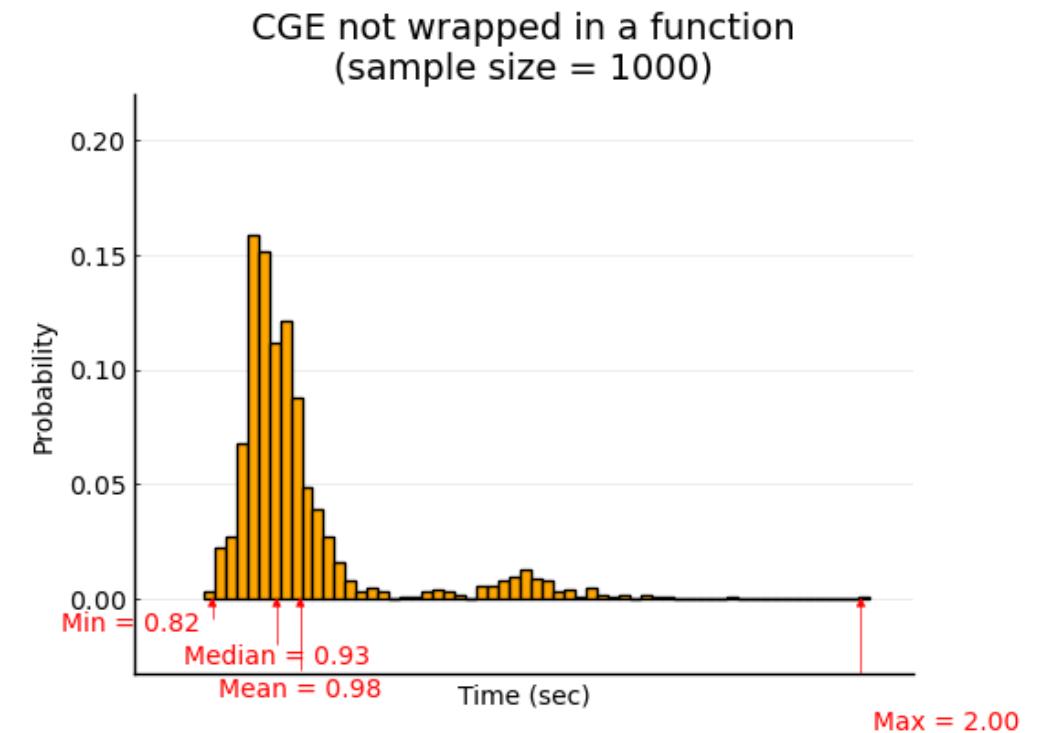
— Chen and Revels (2016)

Looking into Julia solving time

How solving time is affected by coding strategy?



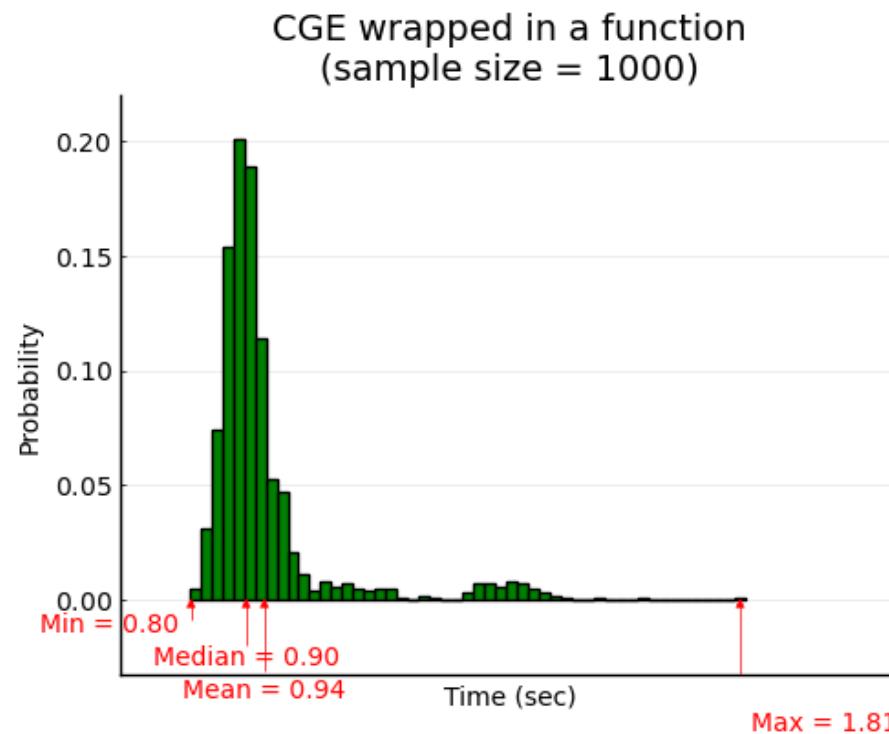
Code lives within the function



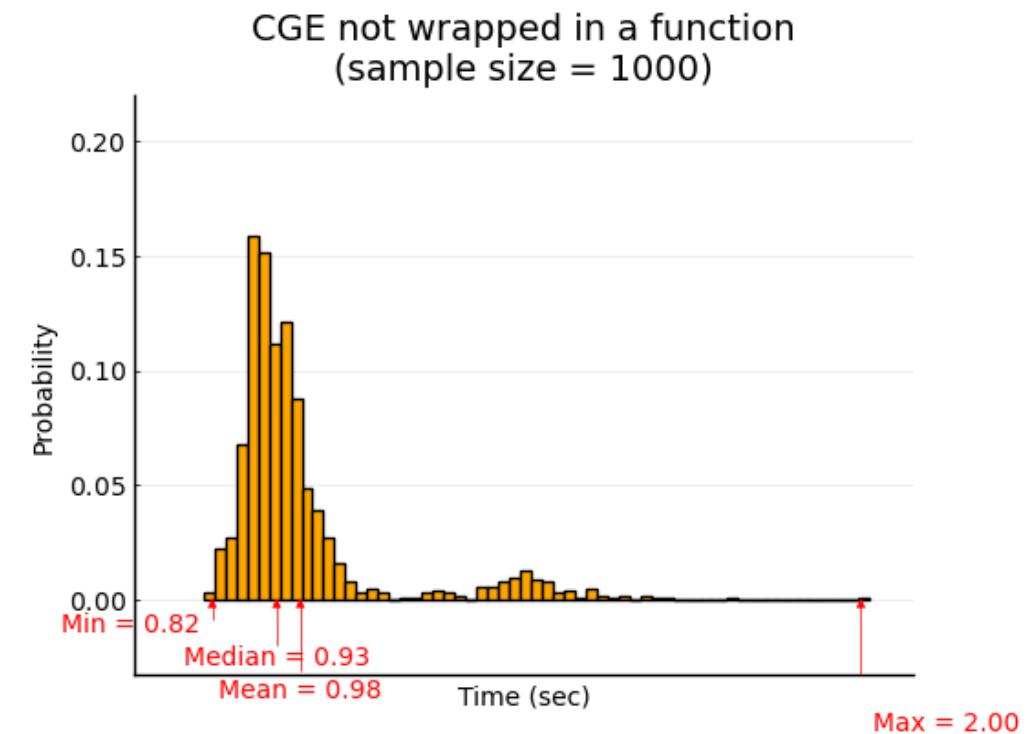
Code lives in global scope

Looking into Julia solve time

Using median to measure central tendency. Question: Is the difference in medians statistically significant?



Code lives within the function



Code lives in global scope

Looking into Julia solve time

Bootstrap resampling with sample size B = 10000 (Efron and Tibshirani, 1994)

```
# 3. Bootstrap Loop
for i in 1:B
    # Resample with replacement
    X1_star = sample(X1, n1, replace=true) → 1000-element Vector{Float64}
    X2_star = sample(X2, n2, replace=true)

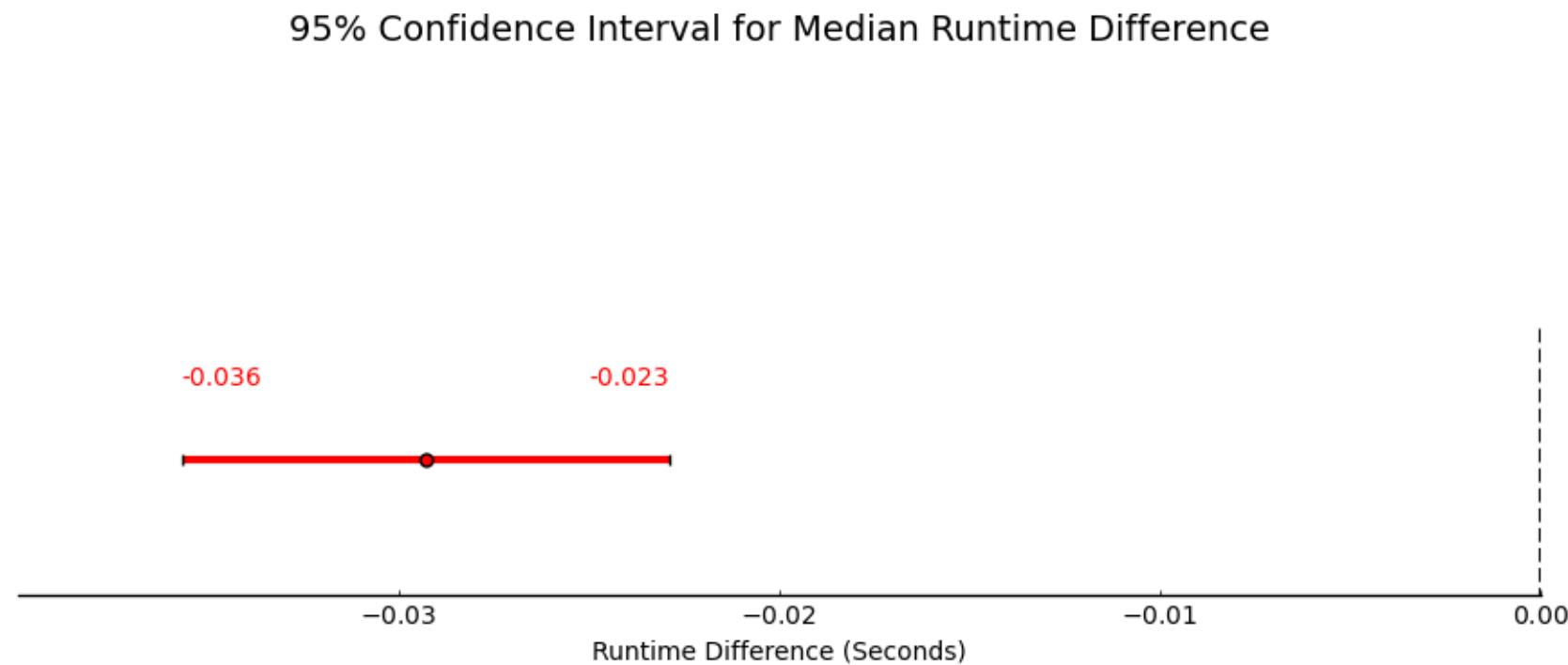
    # Calculate the statistic (difference in medians)
    delta_star = median(X1_star) - median(X2_star) → Float64

    # Append "delta_star" to the end of an array ("bootstrap_differences") one element at a time.
    push!(bootstrap_differences, delta_star)
end      After the for-loop is done, bootstrap_differences is a 10000-element Vector{Float64}

# 4. Construct the Confidence Interval (e.g., 95% CI)
CI_lower = quantile(bootstrap_differences, 0.025)
CI_upper = quantile(bootstrap_differences, 0.975)
```

Looking into Julia solve time

Bootstrap resampling with sample size $B = 10000$ (Efron and Tibshirani, 1994)



Summary

- Julia
 - CSAVEinJulia: slower in model generation
 - CSAVEinJulia: faster in solving
 - More involved in coding
- GAMS
 - CSAVEinGAMS: faster in model generation
 - CSAVEinGAMS: slower in solving
 - Easier to implement

Beyond the conclusion

“A better brush doesn't make a better painter, but a better painter can do more with a better brush.”

— *Unknown*

Acknowledgement

I am thankful for Mitch Phillipson for sharing his knowledge on CGE modeling in Julia, and for Angelo Gurgel who provides me Julia course materials. All errors are my own.

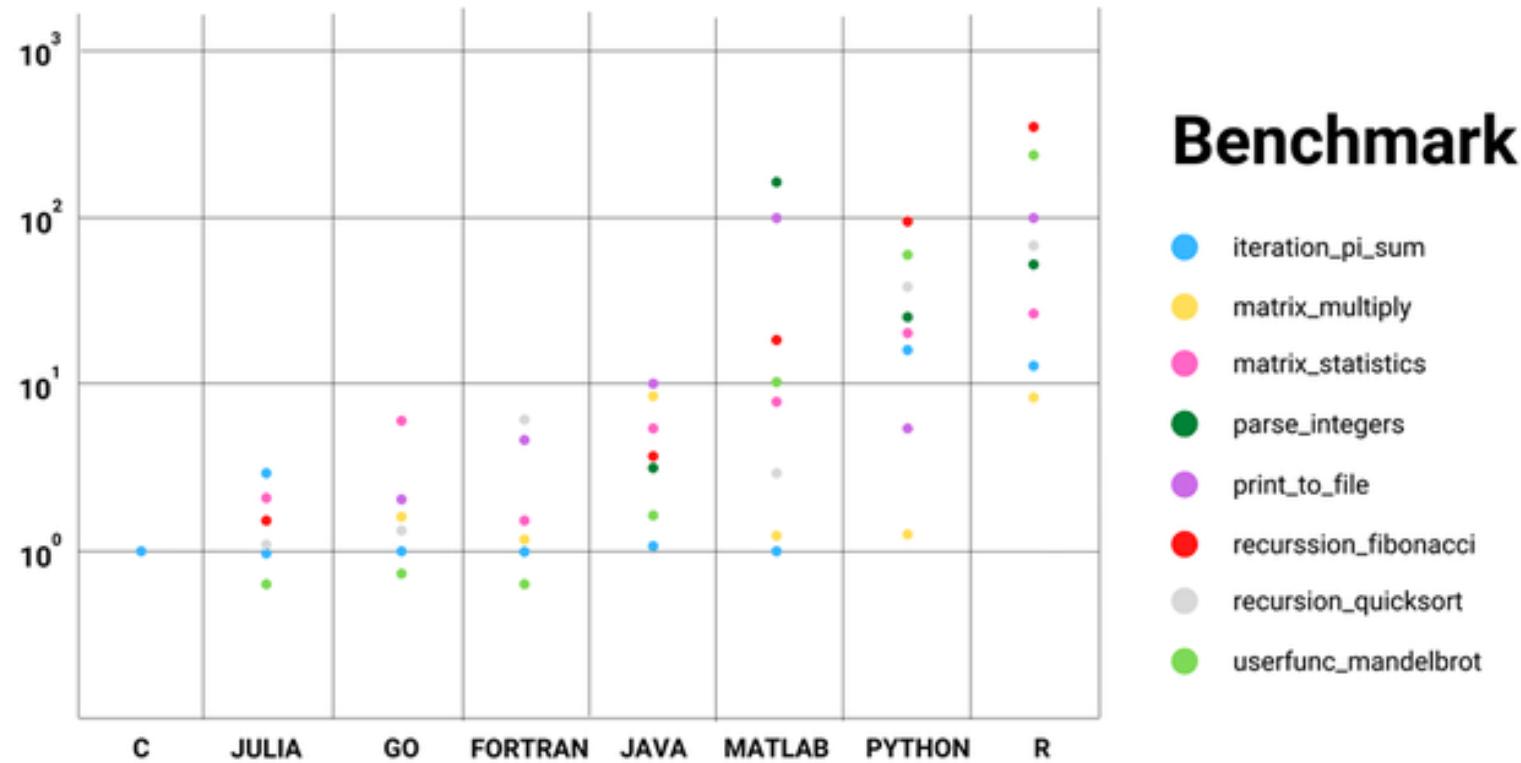
Thank you!

Questions?

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Appendix: A comparison

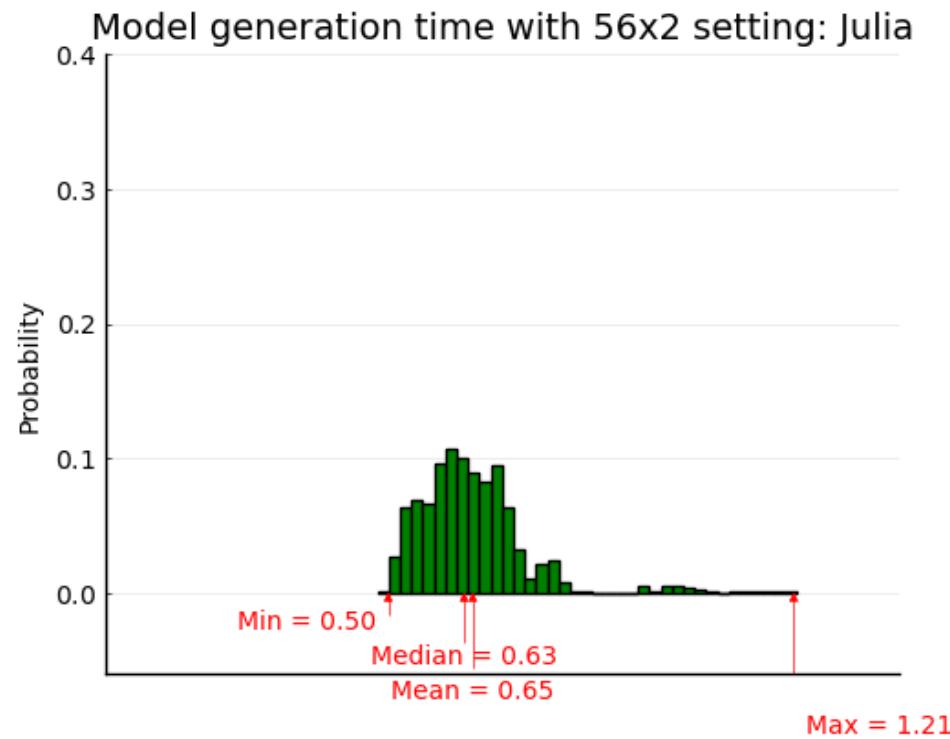
Comparison of Computational Resource for different operations in programming languages



Source: Thakur and Satish (2022)

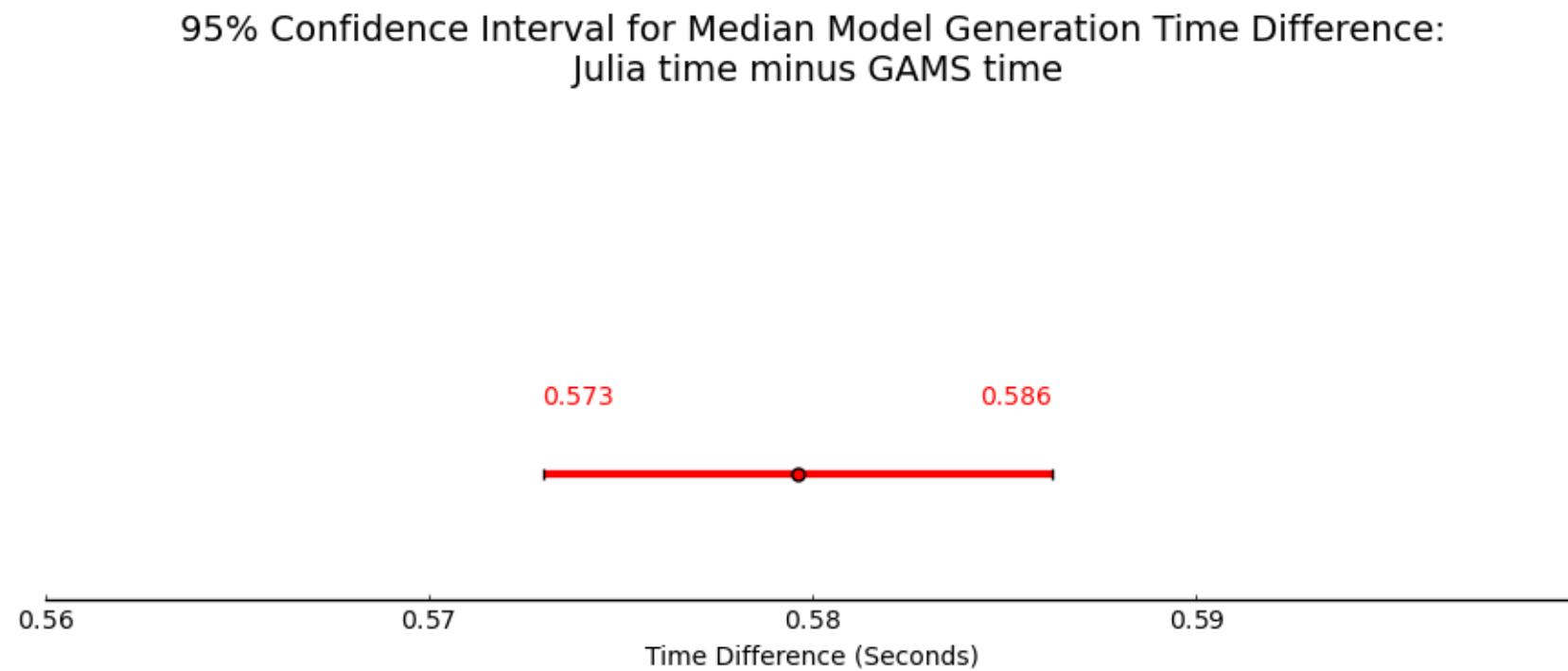
Appendix: Model generation time difference

GAMS is faster!



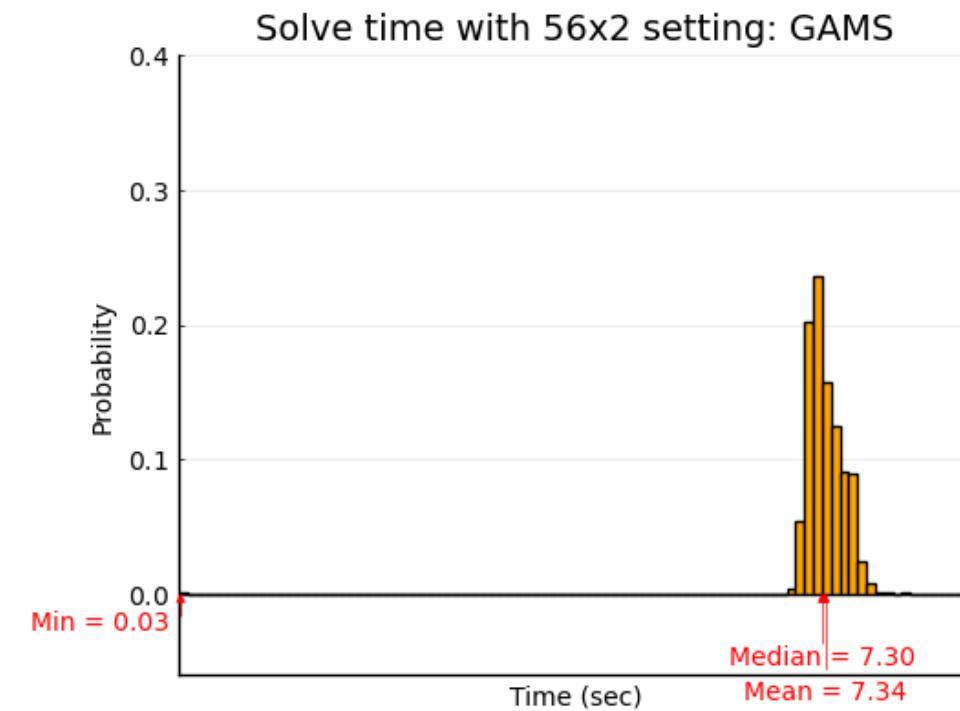
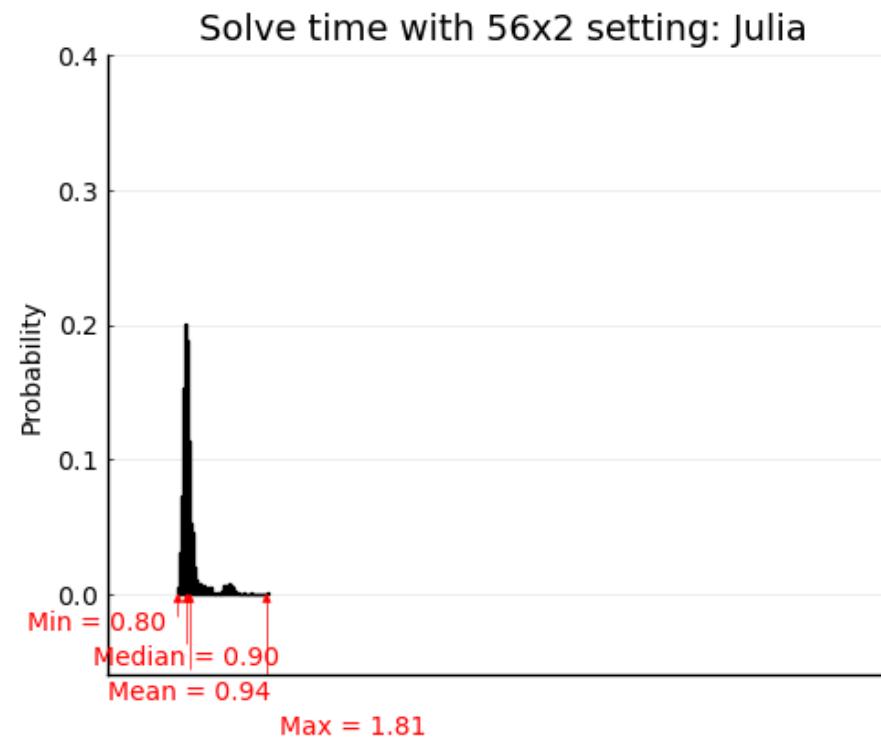
Appendix: Model generation time difference

GAMS is faster!



Appendix: Solve time difference

Julia is faster!



M

Appendix: Solve time difference

Julia is faster!

95% Confidence Interval for Median Solve Time Difference:
Julia time minus GAMS time

