Chapter I: Fields

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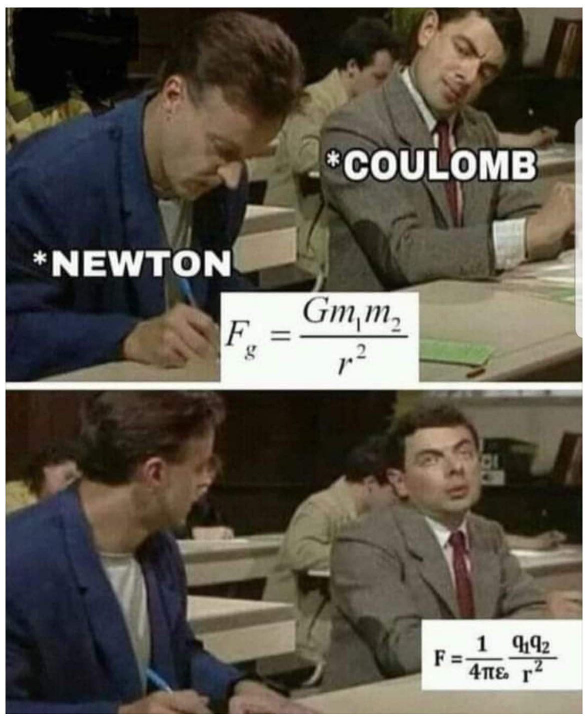

1 Gravitational Field.

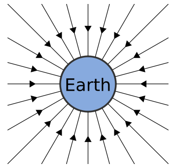

Figure 1: Earth's gravitational field lines.

Gravitational force is defined as:

$$\boxed{\vec{F_g} \equiv -G\frac{Mm}{r^2}\,\hat{r}} \tag{1}$$

from which we can easily derive the gravitational field by setting m to be negligible:

$$\boxed{\vec{g} \equiv \lim_{m\to 0}\frac{\vec{F_g}}{m} = -G\frac{M}{r^2}\,\hat{r}} \tag{2}$$

Note: Near earth's surface, gravitational field $\vec{g}$ is approximately constant.

$$\vec{g} \equiv -g\,\hat{r} \qquad \text{where} \qquad g \equiv G\frac{M}{R_E^2} \approx 9.8\,m/s^2$$

A mass in constant gravitational field (e.g. near Earth's surface) doesn't necessarily move in the direction of the field unless initial velocity moves in the same direction. If initial velocity is perpendicular to the direction of the field, the trajectory will be parabolic.

2 Electric Field.

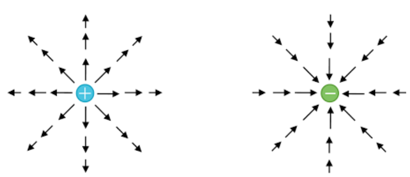

Figure 2: Electric field lines of positive and negatively charged particles.

Force due to an electric field is called **Coulomb's force**, which is defined here as:

$$\boxed{\vec{F_e} \equiv k_e\frac{Qq}{r^2}\,\hat{r}} \tag{3}$$

and we now observe lots of similarity between the forms of the two different fields. Taking the limit as before and we observe:

$$\boxed{\vec{E} \equiv \lim_{q \to 0} \frac{\vec{F_e}}{q} = k_e \frac{Q}{r^2}\,\hat{r}} \tag{4}$$

Now we get a grasp of the link between gravity and electricity. You now know that electric force arises when two or more charged particles interact with each other, and a single charged particle produces its own electric field (recall: you can think of the gravitational field as being produced by a single mass, which bends space and time). The main difference here (other than the constants in the equations) is that gravitational force is **ALWAYS** attractive, while electric force can be **EITHER** attractive or repulsive.

3 Magnetic Field.

Magnetic field is a very special member of this family. It is a little more complex than the other two, and I will not go into the equations just yet. However, I would like to illustrate why it is considered "special".

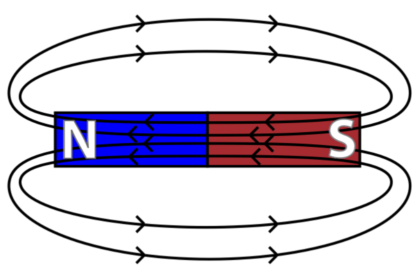

Figure 3: Magnetic field lines.

Simply based on the picture above, we can see a magnet which produces magnetic field going from the North to the South pole. Interesting observation: in the past, we have been saying that we need at least two masses or charged particles to produce a field. Here, however, we only need one magnet to produce a field. There are no such thing as a **monopole** in magnetism, but only **dipole**, and you will learn more about this special identity later on in this course.