Exponential and logarithm functions

Software 2 – Python Labs for Mathematics and Physics

Sakari Lukkarinen

Metropolia University of Applied Sciences

Contents

- Power function and floating point number presentation
- Common prefixes and pitfalls with them
- Exponential functions
- Logarithmic functions
- Semilog and loglog scales in the graphs
- Typical engineering applications of log-scales
- Next steps

Power function and floating point numbers

10^2

```
[2]: 10**2
```

```
[2]: 100
```

2.0×10^2

```
[3]: 2.0*10**2
```

```
[3]: 200.0
```

```
[4]: 2e2
```

```
[4]: 200.0
```

10^{-3}

```
[5]: 10**(-3)
```

```
[5]: 0.001
```

3.0×10^{-3}

```
[6]: 3.0*10**(-3)
```

```
[6]: 0.003
```

```
[7]: 3.0e-3
```

```
[7]: 0.003
```

Common prefixes

Common Prefixes used with SI Units			
Prefix	Symbol	Meaning	Order of Magnitude
giga-	G	1 000 000 000	10^{9}
mega-	M	1 000 000	10^{6}
kilo-	k	1 000	10^{3}
hecto-	h	100	10^{2}
deka-	da	10	10^{1}
	base unit	1	10^{0}
deci-	d	0.1	10^{-1}
centi-	c	0.01	10^{-2}
milli-	m	0.001	10^{-3}
micro-	μ	0.000 001	10^{-6}
nano-	n	0.000 000 001	10^{-9}

Common pitfalls with prefixes

One milli-Volt $1.0mV$ vs. one mega-Volt $1.0MV$

[8]:
```
V = [1.0e-3, 1.0e6]
V
```

[8]:
```
[0.001, 1000000.0]
```

Ten micro-Amps 10 uA = $10\mu A$

Note the prefix u = μ

[9]:
```
I = 10e-6
I
```

[9]:
```
1e-05
```

Common pitfalls – square mm

Twenty square centimeters $A = 20mm^2$ in square meters?

[10]:
```
# One square mm in square m
(1e-3)**2
```

[10]:
```
1e-06
```

[11]:
```
# Twenty square mm in square m
20*(1e-3)**2
```

[11]:
```
1.9999999999999998e-05
```

Exponential functions

Power functions

[12]:
```
x = np.arange(8)
y = 2**x
print(y)
```

```
[  1   2   4   8  16  32  64 128]
```

[13]:
```
x = np.arange(5)
y = 10**x
print(y)
```

```
[    1    10   100  1000 10000]
```

What if you use negative indices for 2**x or 10**x?

How can you fix the error?

Exponential function

There is a function for $y = e^x$

[14]:
```
x = np.arange(5)
y = np.exp(x)
print(y)
```

```
[ 1.          2.71828183  7.3890561  20.08553692 54.59815003]
```

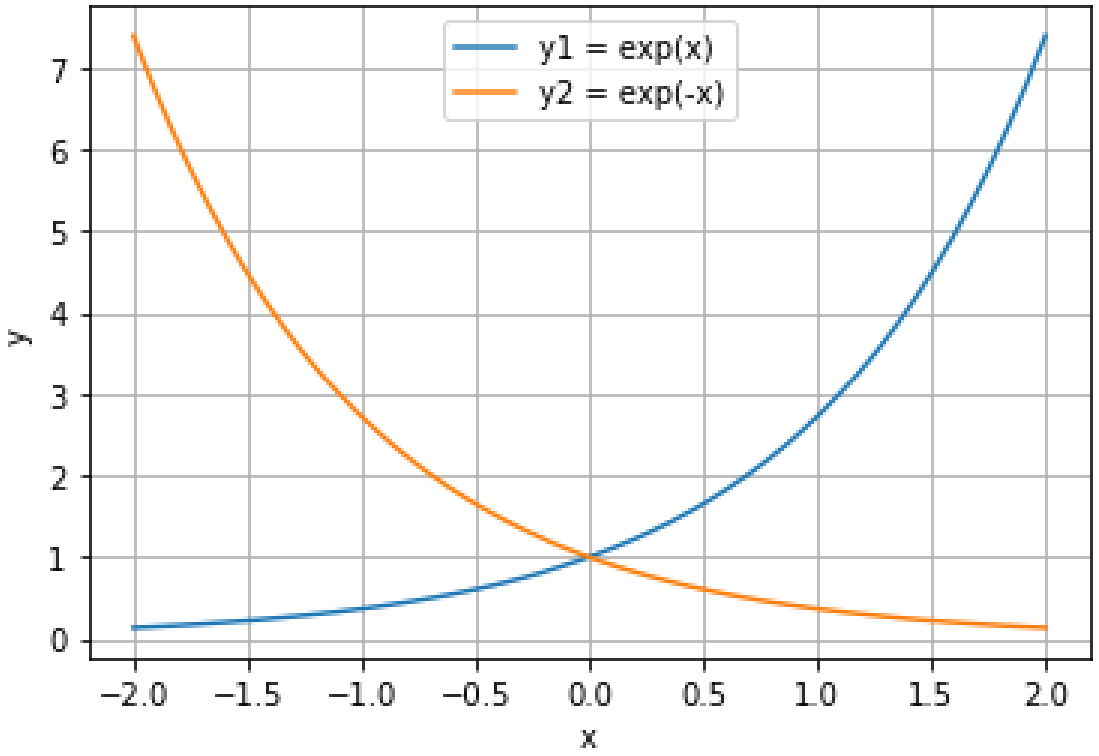
$y = e^{-x}$ can be calculated

[15]:
```
y = np.exp(-x)
print(y)
```

```
[1.         0.36787944 0.13533528 0.04978707 0.01831564]
```

exp(x) vs. exp(-x)

[20]:
```
x = np.linspace(-2, 2)
y1 = np.exp(x)
y2 = np.exp(-x)
plt.plot(x, y1, label = 'y1 = exp(x)')
plt.plot(x, y2, label = 'y2 = exp(-x)')
plt.grid()
plt.xlabel('x')
plt.ylabel('y')
plt.legend()
plt.show()
```



Logarithmic functions

Natural logarithm

$$y = ln(x) \leftrightarrow x = e^y$$

[17]:
```
x = np.array([1, 2, 5, 10, 20, 50])
y = np.log(x)
print(y)

x2 = np.exp(y)
print(x2)
```

```
[0.         0.69314718 1.60943791 2.30258509 2.99573227 3.91202301]
[ 1.  2.  5. 10. 20. 50.]
```

10-base logarithm

$$y = log(x) \leftrightarrow x = 10^y$$

[18]:
```
x = np.array([1, 2, 5, 10, 20, 50])
y = np.log10(x)
print(y)

x2 = 10**y
print(x2)
```

```
[0.       0.30103 0.69897 1.       1.30103 1.69897]
[ 1.  2.  5. 10. 20. 50.]
```

Two-base logarithm

Two-base logarithm $y = log_2(x) \leftrightarrow x = 2^y$

[19]:
```
x = np.array([1, 2, 4, 8, 16])
y = np.log2(x)
print(y)

x2 = 2**y
print(x2)
```

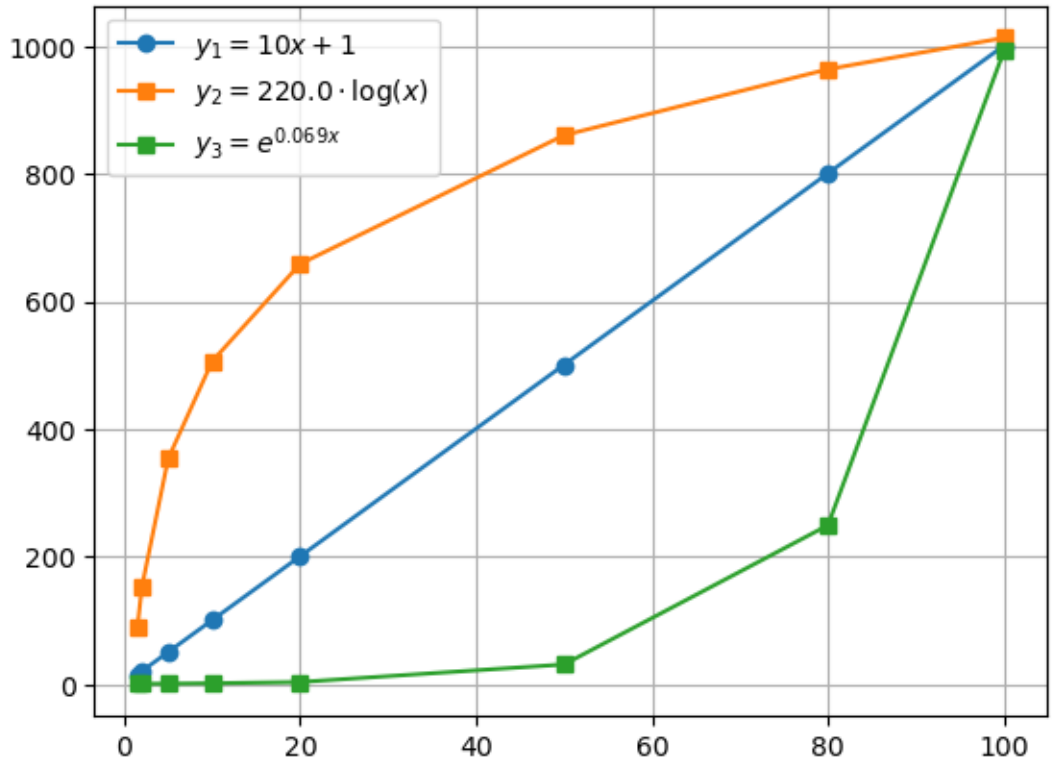
```
[0. 1. 2. 3. 4.]
[ 1.  2.  4.  8. 16.]
```

Semilog and log-log scales

Example functions – linear, logarithmic, and expopential

Three functions in linear scales (plt.plot).

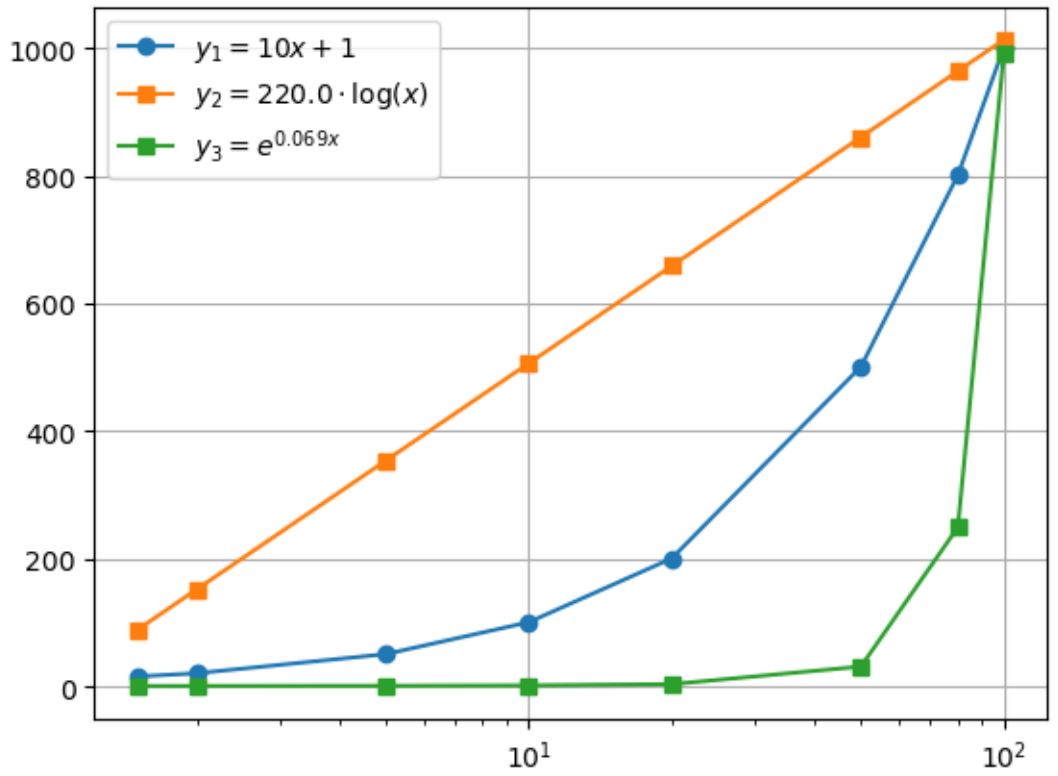
[20]:
```
x = np.array([1.5, 2.0, 5.0, 10.0, 20.0, 50.0, 80.0, 100.0])
y1 = 10*x + 1
y2 = 220.0*np.log(x)
y3 = np.exp(0.069*x)
plt.plot(x, y1, 'o-', label = '$y_1 = 10x + 1$')
plt.plot(x, y2, 's-', label = '$y_2 = 220.0 \cdot \log(x)$')
plt.plot(x, y3, 's-', label = '$y_3 = e^{0.069x}$')
plt.grid()
plt.legend()
plt.show()
```



Semilogx scale (x in logarithmic, y in linear)

semilogx scale

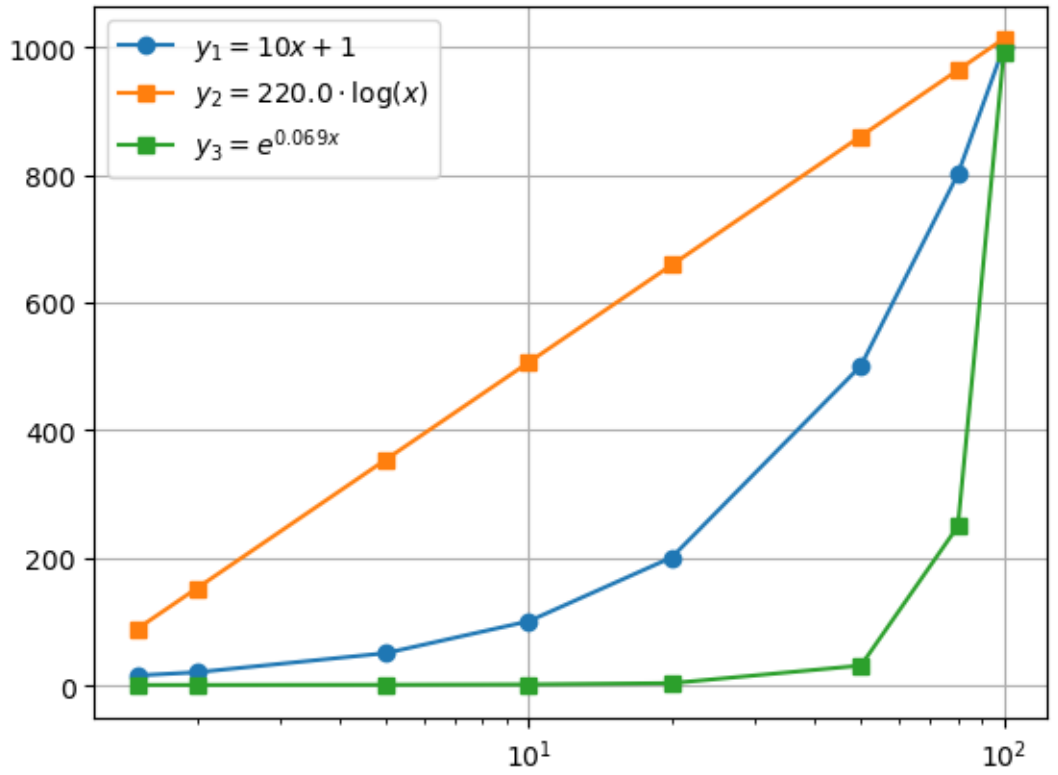
[21]:
```
plt.semilogx(x, y1, 'o-', label = '$y_1 = 10x + 1$')
plt.semilogx(x, y2, 's-', label = '$y_2 = 220.0 \cdot \log(x)$')
plt.semilogx(x, y3, 's-', label = '$y_3 = e^{0.069x}$')
plt.grid()
plt.legend()
plt.show()
```



Semilogy scale (x in linear, y in logarithmic)

semilogy scale

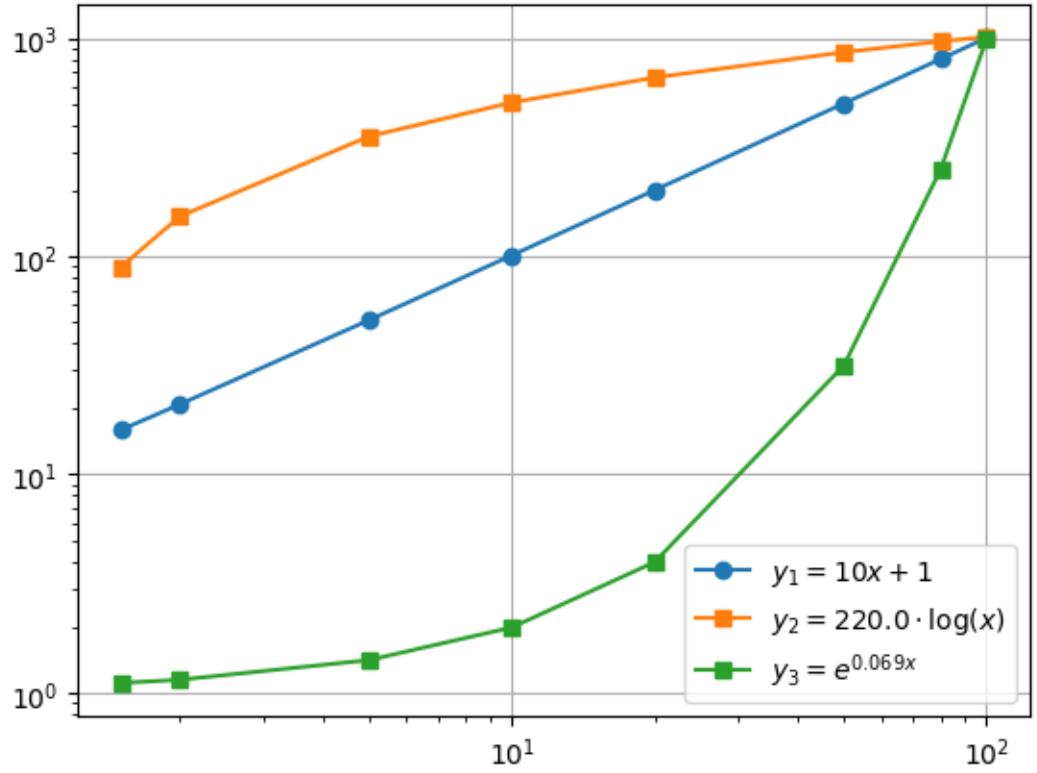
[22]:
```
plt.semilogy(x, y1, 'o-', label = '$y_1 = 10x + 1$')
plt.semilogy(x, y2, 's-', label = '$y_2 = 220.0 \cdot \log(x)$')
plt.semilogy(x, y3, 's-', label = '$y_3 = e^{0.069x}$')
plt.grid()
plt.legend()
plt.show()
```



loglog scale (both x and y in logarithmic)

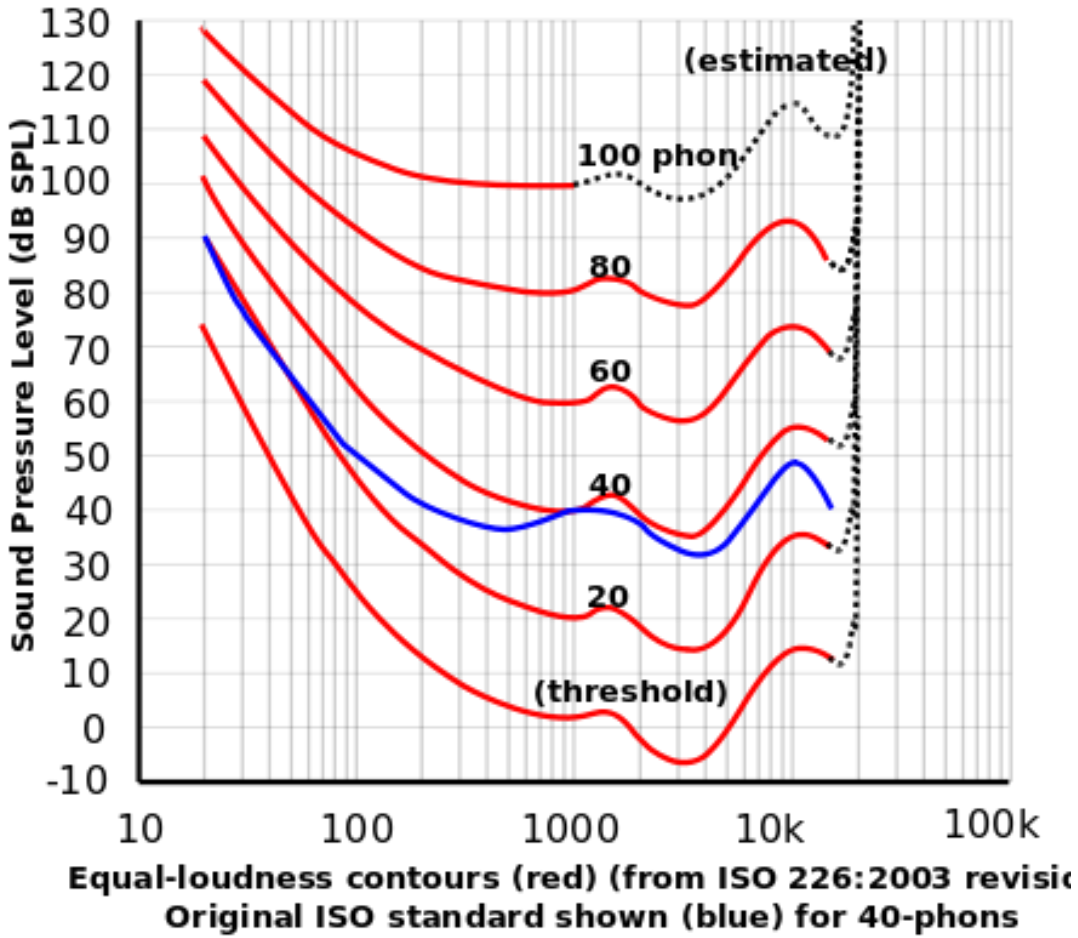
loglog scale

[23]:
```
plt.loglog(x1, y1, 'o-', label = 'Series 1')
plt.loglog(x2, y2, 's-', label = 'Series 2')
plt.grid()
plt.legend()
plt.xlabel('x (log scale)')
plt.ylabel('y (log scale)')
plt.show()
```



Typical engineering applications of log-scales

- Sound pressure level measurements
- Spectral density calculations
- Decibel scale
- Signal-to-noise ratio
- Dynamic range



Next steps

- Practice – Lab 4
  - Notebook can be found from OMA assignments
  - Moodle has code check and verification
- Read more
  - Exponents and Logs with Python - Python for Undergraduate Engineers
    - Note: the same naming convention is used both in basic Python math-package and in numpy functions!
    - If you need numerical arrays, remember use the numpy package (np.exp())
  - numpy.exp — NumPy v1.23 Manual
  - numpy.log — NumPy v1.23 Manual