Kinematic equations and projectile motion

Software 2 – Python Labs for Mathematics and Physics

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Contents

Constant velocity and acceleration

Kinematic equation (1D)

$$x(t) = x_0 + v_0 t + \frac{1}{2} a t^2$$

- x_0 = initial position (m)
- v_0 = constant velocity (m/s)
- a = constant acceleration (m/s^2)

Travelled distance

How to calculate, when

(a) velocity is constant?

(b) acceleration is constant?

(c) in general (i.e. $v(t)$ continuous)?

(a) If v constant, then $\boxed{x = v \cdot t}$

(b) If acceleration is constant, then

$$\boxed{v(t) = v_0 + at}$$

$$\boxed{x = v_{ave} \cdot t = v_0 \cdot t + \frac{1}{2} a t^2}$$

Source: Salin, Timo. Physics lecture notes

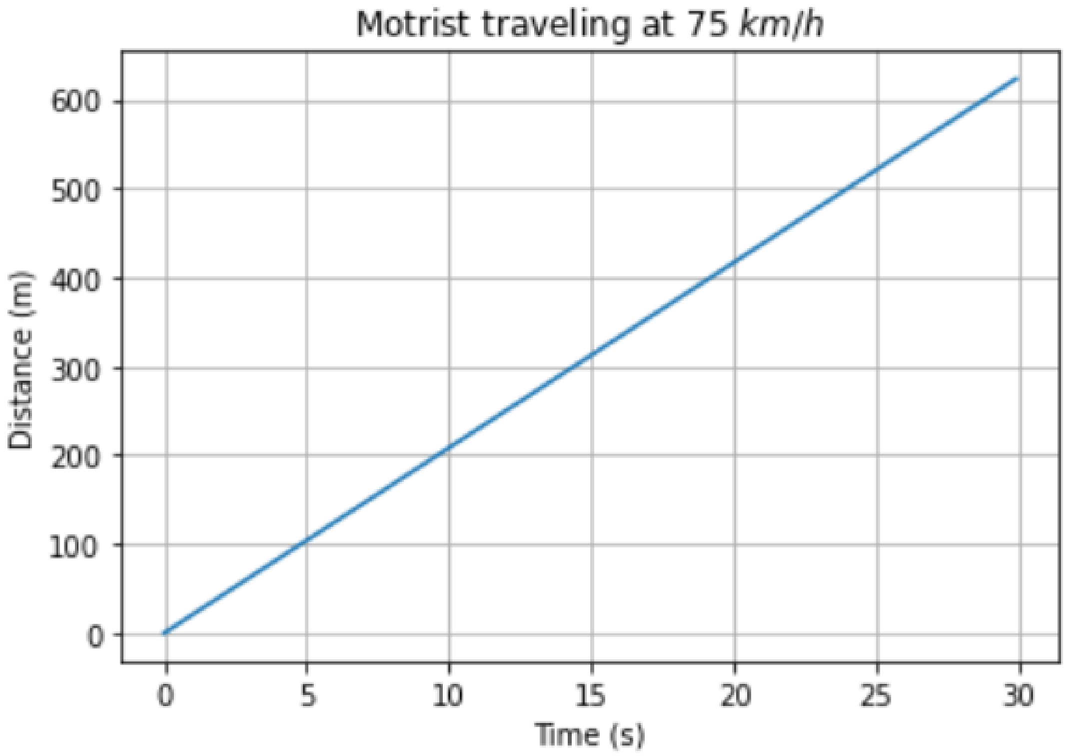
Example – a motorist and a police car

<u>EXAMPLE</u> A speeding motorist zooms through a 50 $\frac{km}{h}$ zone at 75 $\frac{km}{h}$ without noticing a stationary police car. The police officer immediately heads after the speeder at $a = 2{,}5 \frac{m}{s^2}$. When the officer catches up to the speeder, how far down the road are they, and how fast is the police car going?

Salin, T. (2022) . Orientation Physics, Ch 1.

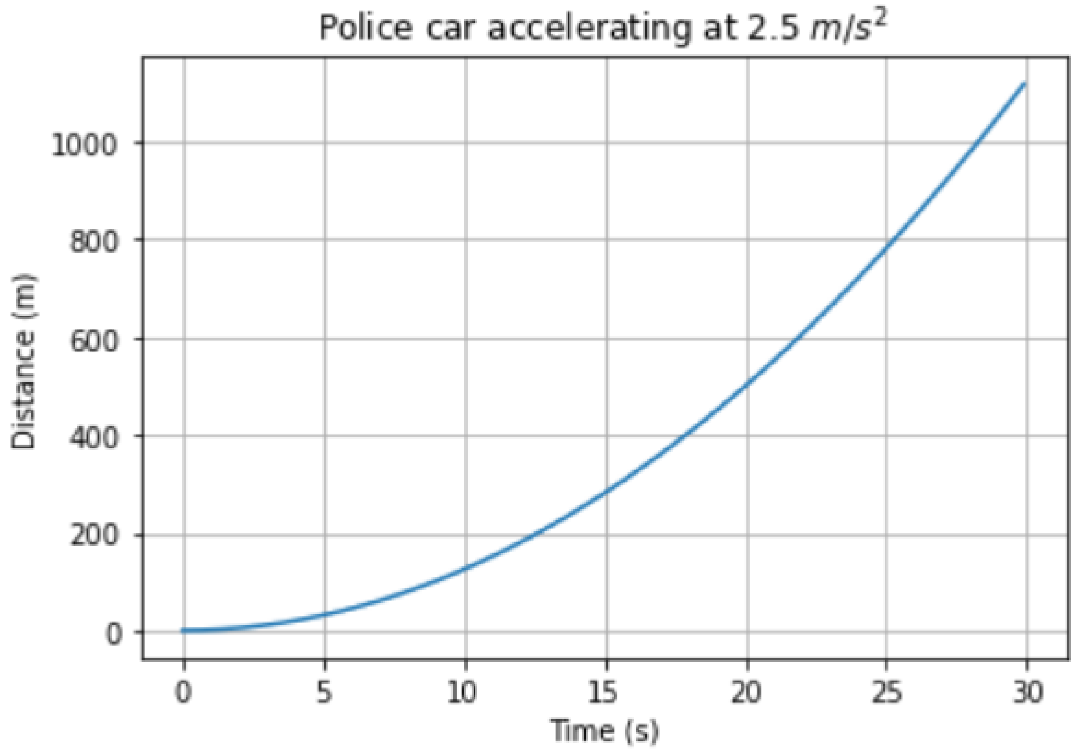
Constant velocity – the motorist

```python
v0 = 75 * 1000/3600 # km/h => m/s
t = np.arange(0, 30, 0.1)
x = v0*t
plt.plot(t, x)
plt.xlabel('Time (s)')
plt.ylabel('Distance (m)')
plt.title('Motrist traveling at 75
plt.grid()
plt.show()
```



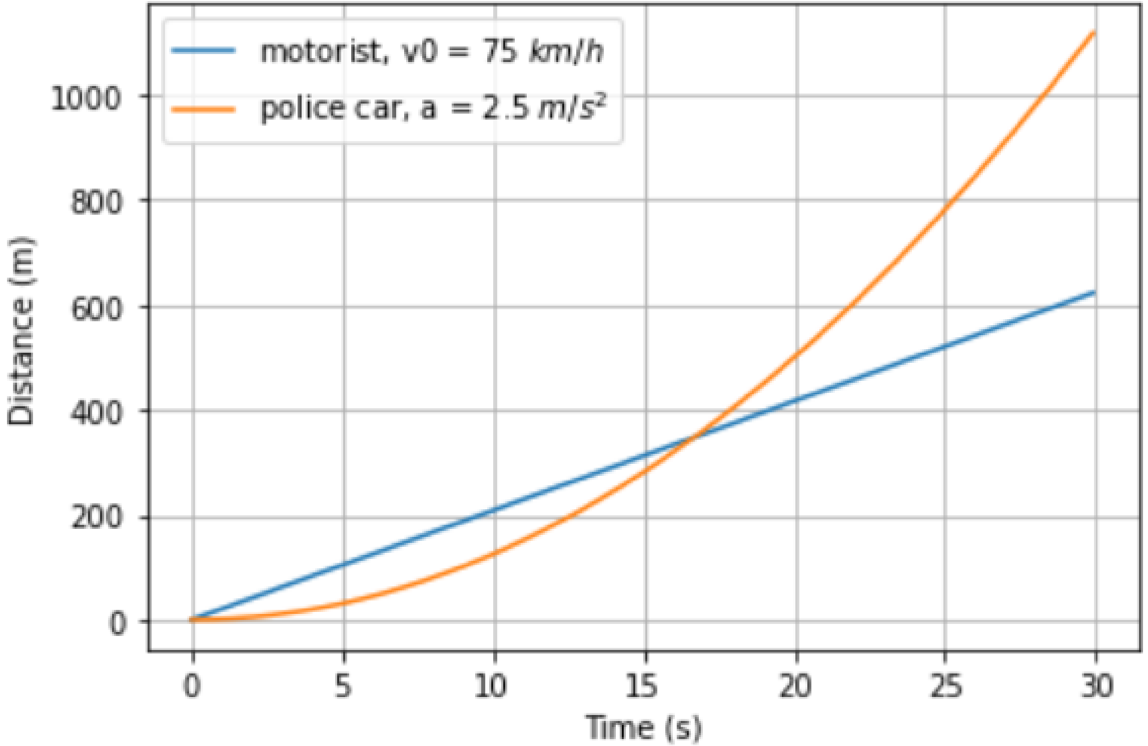
Constant acceleration – the police car

```
a = 2.5 # m/s^2
x2 = 1/2*a*t**2
plt.plot(t, x2)
plt.xlabel('Time (s)')
plt.ylabel('Distance (m)')
plt.title('Police car accelerating
plt.grid()
plt.show()
```



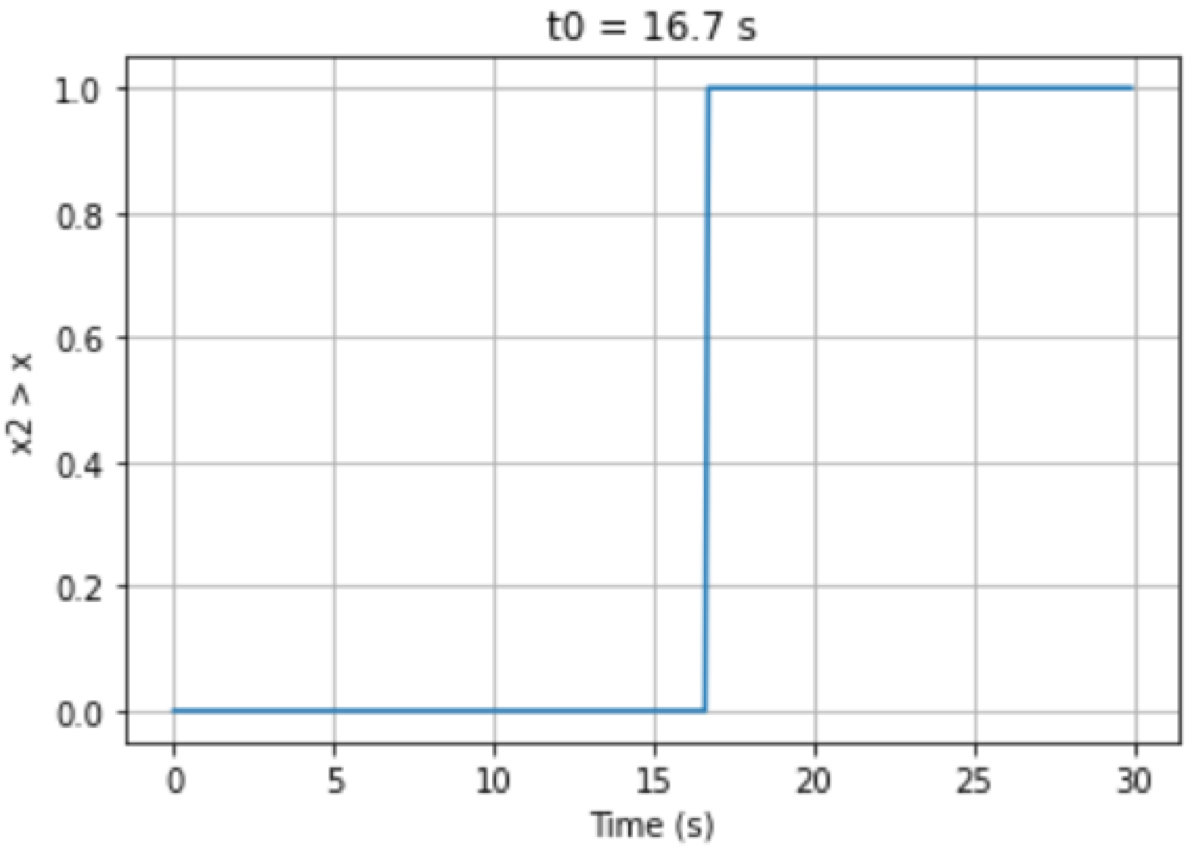
Comparison

```
plt.plot(t, x, label = 'motorist,
plt.plot(t, x2, label = 'police c
plt.xlabel('Time (s)')
plt.ylabel('Distance (m)')
plt.legend()
plt.grid()
plt.show()
```



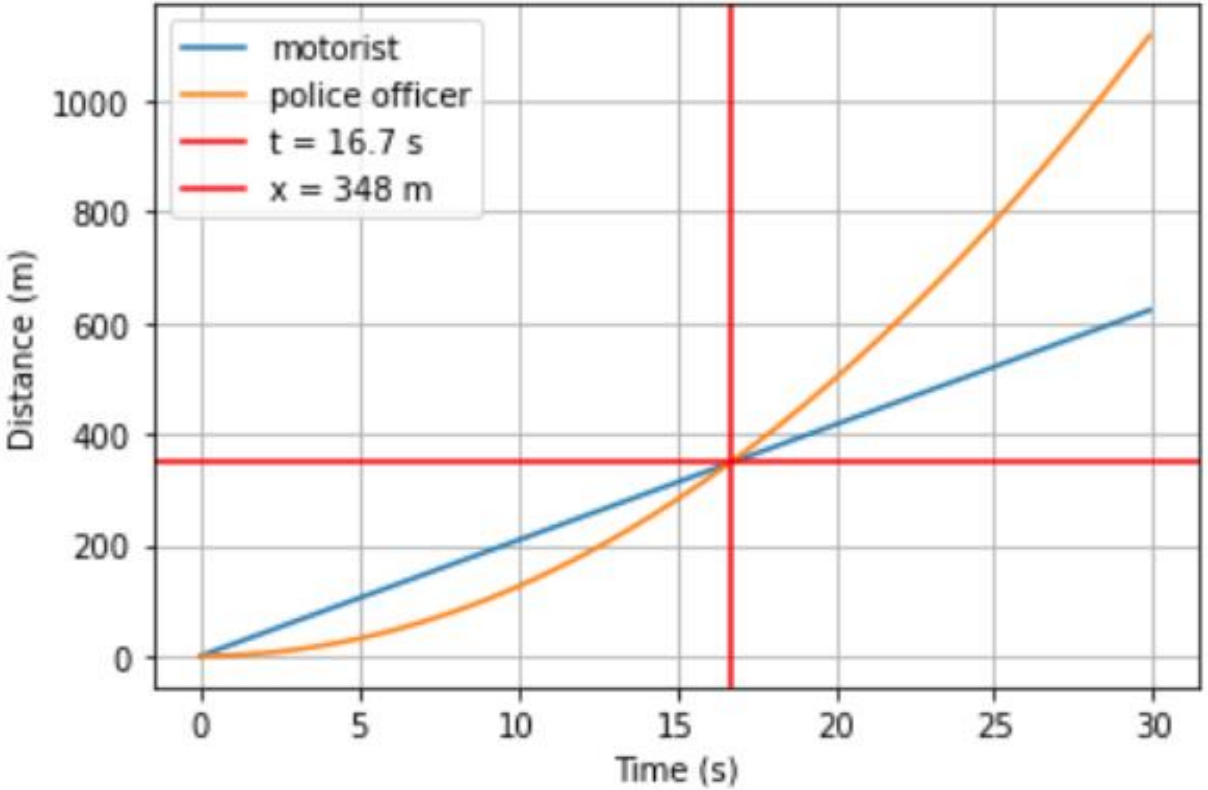
When does the officer catch the speeder?

```
# When is x2 greater than x?
i = (x2 > x)
plt.plot(t, i)
plt.xlabel('Time (s)')
plt.ylabel('x2 > x')
plt.grid()

# When does that happen?
t0 = np.min(t[i])
plt.title(f't0 = {t0} s')
plt.show()
```



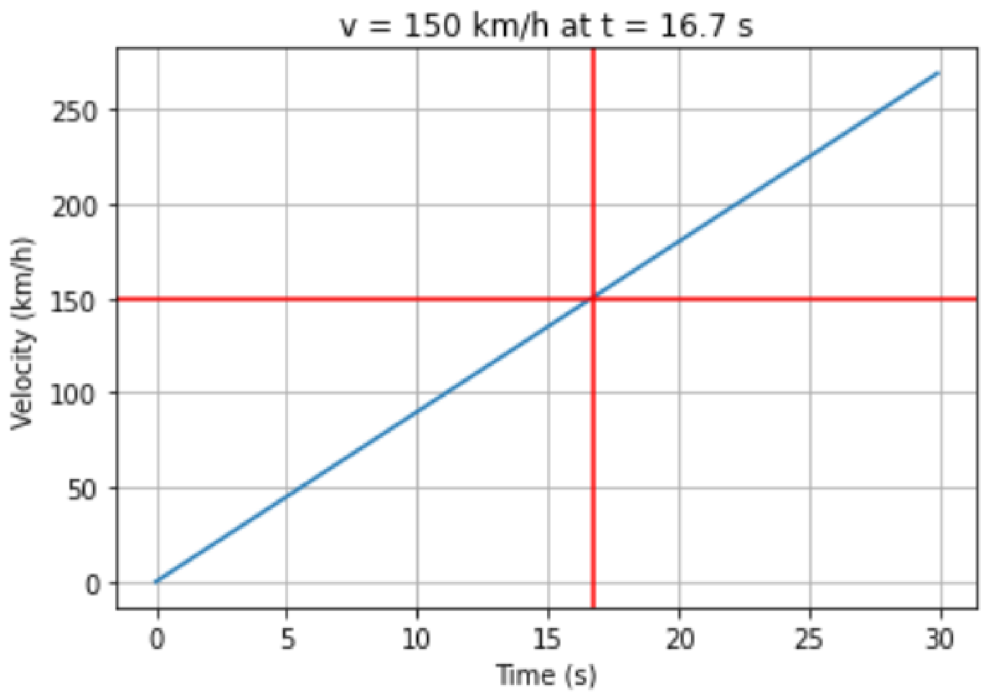
Catching time

```
i2 = (t == t0)
x0 = x[i2]
plt.plot(t, x, label = 'motorist')
plt.plot(t, x2, label = 'police officer'
plt.axvline(t0, color = 'red', label = f
plt.axhline(x0, color = 'red', label = f
plt.xlabel('Time (s)')
plt.ylabel('Distance (m)')
plt.legend()
plt.grid()
plt.show()
```



Police car's velocity at the end

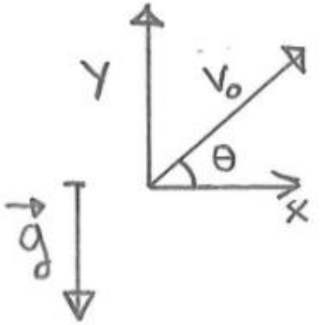
```
v = a*t *3600/1000 # m/s ==> km/h
i2 = (t == t0)
v2 = v[i2]

plt.plot(t, v)
plt.axvline(t0, color = 'red')
plt.axhline(v2, color = 'red')
plt.xlabel('Time (s)')
plt.ylabel('Velocity (km/h)')
plt.title(f'v = {v2[0]:.0f} km/h at t = {t0} s')
plt.grid()
plt.show()
```

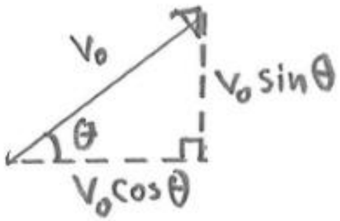


Projectile (2D) motion

Projectile (2D) motion

A projectile is an object that is launched into the air and then moves predominantly under the influence of gravity.





$$
\begin{aligned}
v_x &= v_0 \cos(\theta) \\
v_y &= v_0 \sin(\theta) - gt \\
x &= x_0 + v_0 \cos(\theta) t \\
y &= y_0 + v_0 \sin(\theta) t - \frac{1}{2} g t^2
\end{aligned}
$$

Salin, T. (2022) . Orientation Physics, Ch 1.

Trigonometric functions and angles

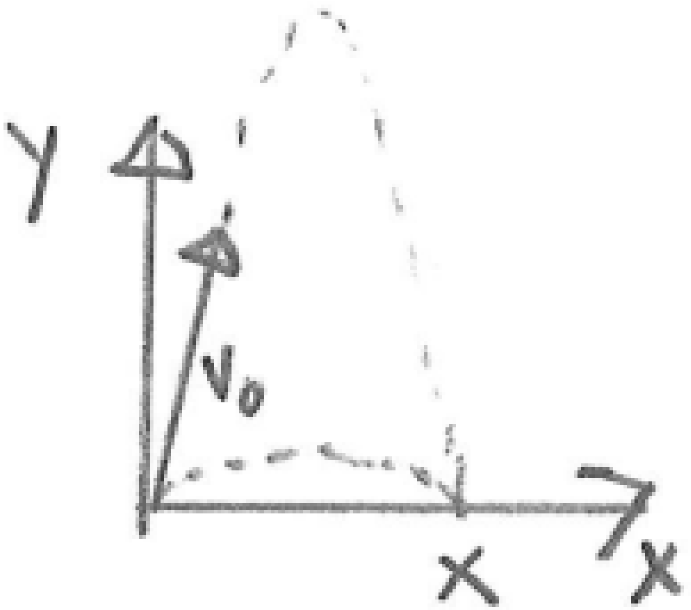
NOTE: The angle for trigonometric functions (sin, cos, tan) should be given in radians.

For that reason we need np.deg2rad() function to convert the degrees to radians. See also: Conversions of angles.

Turns	Radians	Degrees
0 turn	0 rad	0°
1/12 turn	π/6 rad	30°
1/8 turn	π/4 rad	45°
1/6 turn	π/3 rad	60°
1/4 turn	π/2 rad	90°
1/3 turn	2π/3 rad	120°
1/2 turn	π rad	180°
3/4 turn	3π/2 rad	270°
1 turn	2π rad	360°

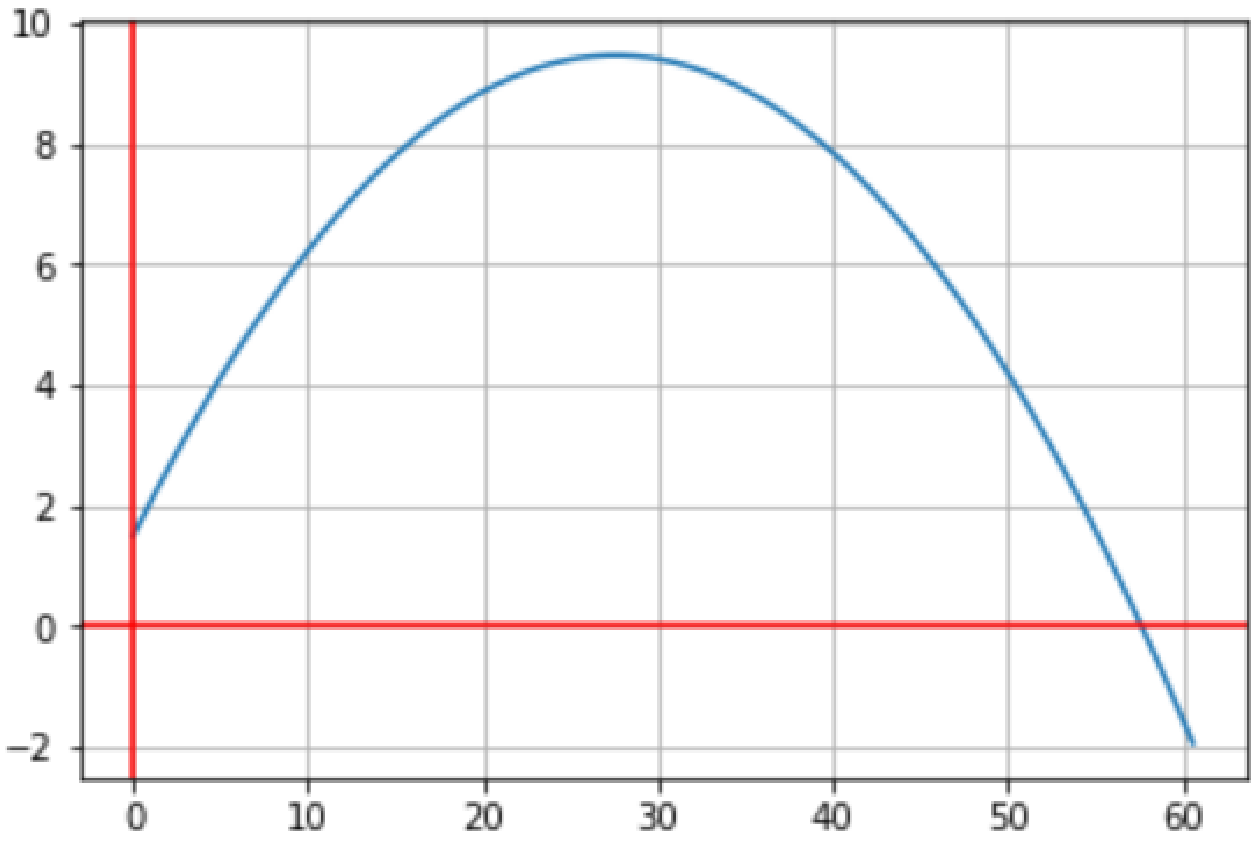
Example

You toss a ball at speed of 25.0 m/s and It leaves your hand at 1.5 m above a floor in angle of 30 degrees. How far does the ball flight? Draw the trajectory of the ball.



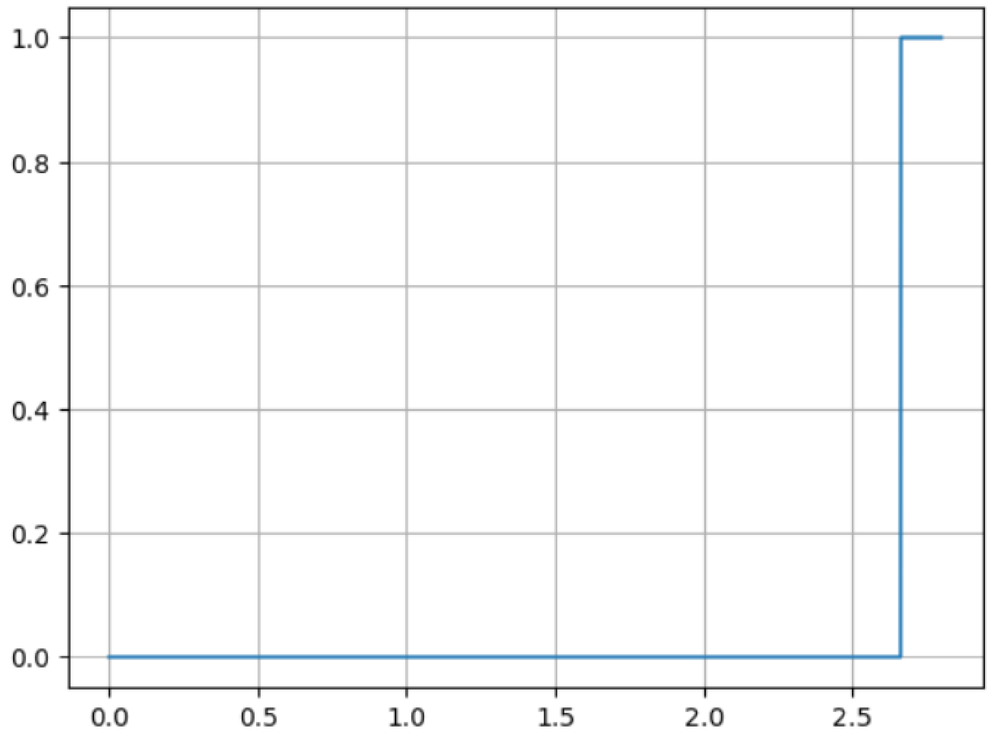
Example – tossing a ball (2D)

```
x0, y0 = 0, 1.5
v0 = 25.0
theta = np.deg2rad(30)
g = 9.81

t = np.arange(0, 2.8, 0.001)

vx = v0*np.cos(theta)
vy = v0*np.sin(theta) - g*t
x = x0 + v0*np.cos(theta)*t
y = y0 + v0*np.sin(theta)*t - 1/2*g*t**2

plt.plot(x, y)
plt.axhline(0, color = 'red')
plt.axvline(0, color = 'red')
plt.grid()
```



When does the ball land to the floor?

```
i = (y <= 0)
plt.plot(t, i)
plt.grid()
```




```
t_end = np.min(t[i])
print(f't_end = {t_end} s')
```

```
t_end = 2.664 s
```

What is the landing location?

```
i_end = (t == t_end)
x_end = x[i_end][0]
print(f'The ball lands at x = {x_end:.1f} m')
```

```
The ball lands at x = 57.7 m
```

Creating animations (BONUS)

FuncAnimation makes an animation by repeatedly calling a given graphics function. The animation is then converted to HTML presentation by using IPython.display module's HTML class.

We use the same data as in previous example, but now we reduce the time step in order to make the simulation run smoother.

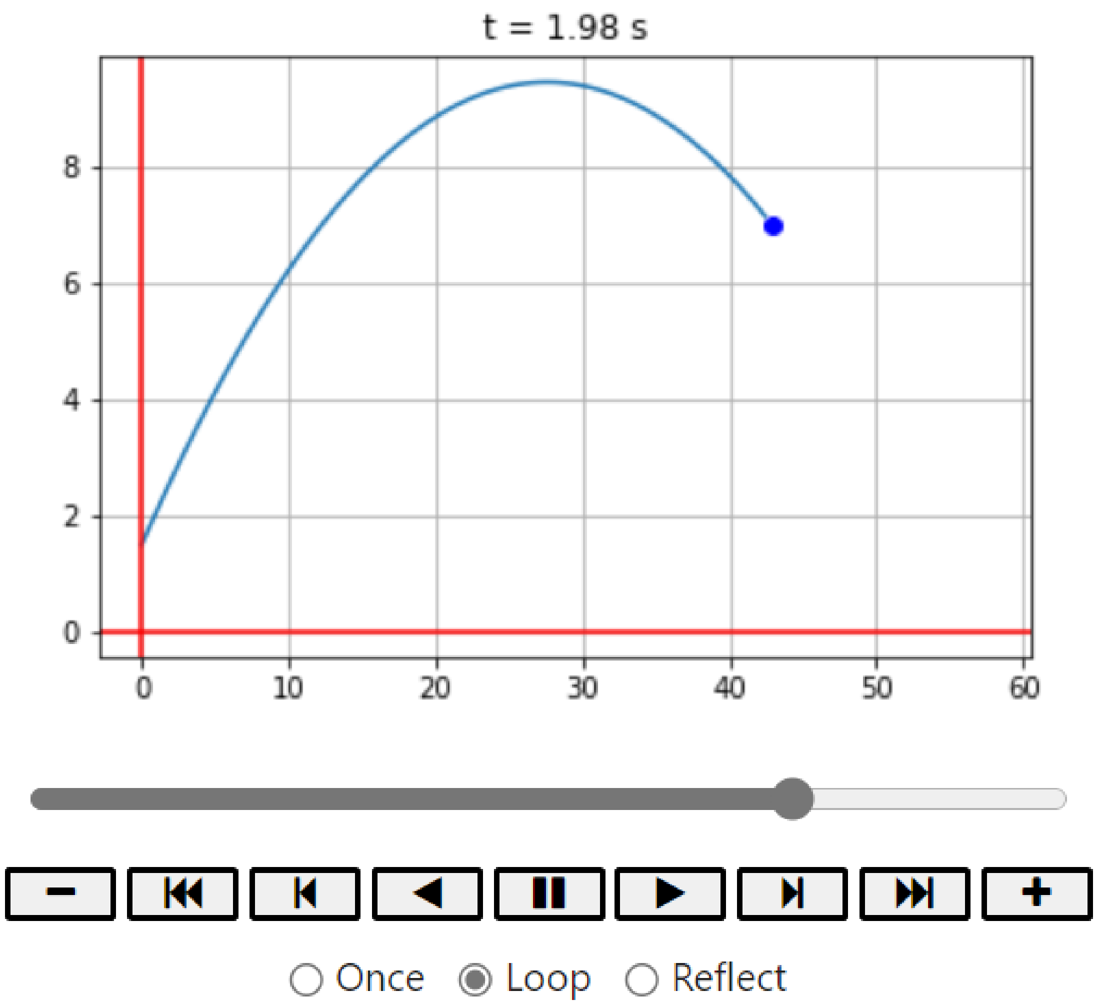
```
from IPython.display import HTML
from matplotlib.animation import FuncAnimation
```

Animation example

```python
t = np.arange(0, 2.67, 0.02)

vx = v0*np.cos(theta)
vy = v0*np.sin(theta) - g*t
x = x0 + v0*np.cos(theta)*t
y = y0 + v0*np.sin(theta)*t - 1/2*g*t**2

# Initialize the graph
fig, ax = plt.subplots()
l, = ax.plot(x, y)
l2, = ax.plot(x[-1], y[-1], 'bo')

plt.axhline(0, color = 'red')
plt.axvline(0, color = 'red')
plt.grid(True)
```

```python
# Animation function
def animate(i):
    l.set_data(x[:i], y[:i])
    l2.set_data(x[i], y[i])
    ax.set_title(f't = {t[i]:.2f} s')

# Create animation
ani = FuncAnimation(fig, animate, frames=len(x))

# Show the animation
HTML(ani.to_jshtml())
```

Animation controls



Next steps

- Practice – Lab 5
  - Notebook can be found from OMA assignments
  - Moodle has code check and verification
- Read more
  - Salin, T. Physics lecture notes.
  - What are the kinematic formulas? (article) | Khan Academy
  - Projectile motion - Wikipedia
- Extra
  - matplotlib.animation — Matplotlib documentation