

# Note for Matrix (by Chen Yiyun)

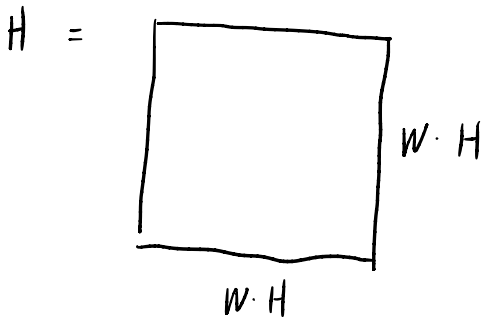
① Real Picture  $\vec{i}$ : input



② Mask 1. Mask 2. ..... Mask  $(W \cdot H)$

$\downarrow$  the Mask Matrix

one row: shape  $(1, W \cdot H)$



During  $\downarrow$  a single scan, we project one of these masks over our object.

③ Sensor result

$$\vec{s} = H \cdot \vec{i} \Rightarrow \text{by } H^{-1} \cdot \vec{s} = \vec{i}, \text{ we get } \vec{i} \text{ from the sensor result}$$

## Analysis of $H$

Part 1. Single Pixel  $\rightarrow$  means sum of rows of  $H$  is 1.

①  $H = I \Rightarrow \vec{s} = \vec{i}$

②  $H_{\text{alt}} = \text{np.random.permutation}(H)$   $\nearrow I$

so masks are in random order, the result will be in random order

$$M = H_{\text{alt}}^{-1} = \text{np.linalg.inv}(H_{\text{alt}})$$

## Part 2 Multi - Pixels

① random  $H$  = generate Random Binary Mask (avg 1s Per Row = 300)



invertible, with approximately 300 pixels illuminated per scan.

## ★ Eigenanalysis & the Robustness of Inverse-Based Reconstruction

$$\vec{S} = H \vec{i} + \underbrace{\vec{w}}_{\text{the random noise}} + \underbrace{\vec{\sigma}}_{\text{a constant offset}}$$

↓

$$\vec{S} = H \vec{i} + \vec{w}$$

$$\vec{i}_{\text{est}} = H^{-1} \vec{S} = \vec{i} + \underbrace{H^{-1} \vec{w}}$$

$$= \alpha_1 H^{-1} \vec{v}_1 + \dots + \alpha_N H^{-1} \vec{v}_N$$

$$= \alpha_1 \frac{1}{\lambda_1} \vec{v}_1 + \dots + \alpha_N \frac{1}{\lambda_N} \vec{v}_N$$

Given  $H$  is diagonalizable

⇒ the  $N \times N$  matrix  $H$  has precisely  $N$  linear-independent eigenvectors.

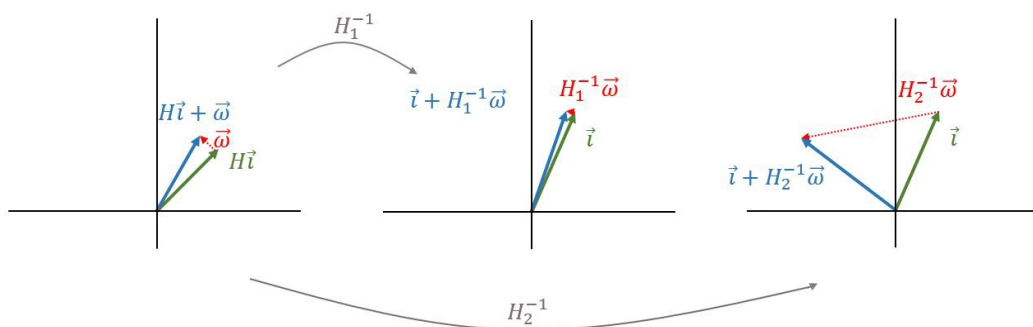
So  $\vec{w}$  can be represented as

$$\vec{w} = \alpha_1 \vec{v}_1 + \dots + \alpha_N \vec{v}_N$$

Conclusion → regardless of the scaling constants  $\alpha$ ,

eigenvalue { large →  $\vec{w}$  attenuated

↓ Graphical Interpretation { small →  $\vec{w}$  amplified



② the hadamard Matrix: <https://mathworld.wolfram.com/HadamardMatrix.html>