EE 117
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Symmetries in 2D and Polygon-Shaped Cavity Modes

Introduction:

The goal of this project was to explore 2D symmetries in different polygon-shaped cavity modes through simulations. The key topics that were researched were Bravais lattices, waveguides, cavities, eigenfrequencies, and eigenmodes. Simulations were done through COMSOL, a software used to simulate designs, devices, and processes in all areas of engineering. Circle, square, rectangle, triangle, and hexagon cavities were simulated. For each polygon, varying symmetries were investigated including inversion, 2-fold, 3-fold, and 6-fold symmetries. After reviewing the simulations, the results were connected to real-world applications such as waveguides and photonic crystals.

Background:

The exploration of Bravais lattices was essential to the progression of this project. Bravais lattices are infinite arrays of points in space with an arrangement and orientation that appear the same from wherever the array is viewed. They are the simplest lattice arrangements that make up all other lattices. In 2D, there are 5 Bravais lattices part of 4 crystal families. The 5 basic Bravais lattices are square, hexagonal, rectangular, centered rectangular, and rhomboidal. In this project, the simulations centered around 3 of these basic lattices: square, rectangular, and hexagonal. In the square lattice, vectors a and b are equal to each other and are at right angles. In the rectangular lattice, vectors a and b are at right angles to each other, but have different magnitudes. In the hexagonal lattice also referred to as rhombic or triangular, vectors a and b are equal to each other and are at an angle of 120° (or 60°). The hexagonal lattice can be simplified into a rhombus or thought of as triangular since it consists of 2 triangles (one up and one down) to maintain translational symmetry. However, the hexagon is more conventional since it shows true symmetry.

From the 5 basic Bravais lattices there are 14 two dimensional space groups that can be derived showing all of the possible symmetry arrangements. These symmetries can vary for each Bravais lattice depending on the basis. The labeled symmetry elements include mirror planes, glide planes, and axes of rotation. Mirror planes are commonly depicted as a solid black line and represent a reflection symmetry. Glide planes are depicted as a dashed line and represent symmetry defined by a reflection followed by a translation along the axis of the glide plane. Glide planes that are oriented in the same direction as one of the lattice vectors have a translation vector equivalent to ½ the lattice vector. Other glide planes are called diagonal glide planes and in the case of 2D lattices have translation vectors equivalent to ½ a + ½ b or ¼ a + ¼ b. The axes of rotation are points corresponding to rotation symmetries of various degrees. 2 fold rotational symmetry is depicted by an eyelet and all higher degrees of rotational symmetry are shown as polygons with the equivalent number of sides. When studying the different cavity simulations, these various forms of symmetry were present, with many arrangements most distinguishable in the hexagon cavity.

Waveguides were important in examining the simulations of this project. Waveguides are structures that direct and propagate electromagnetic waves with a minimal loss of energy since the transmission of energy is restricted to one direction. There are 5 different types of waveguides: circular, rectangular, elliptical, single-ridged, and double-ridged. Waveguides are often used to transmit high frequency waves and have different cutoff frequencies depending on their dimensions.

There are 2 main types of propagation modes in waveguides: transverse electric (TE) mode and transverse magnetic (TM) mode. TE mode is when only the magnetic field is along the direction of propagation while TM mode is when only the electric field is along the direction of propagation. Transverse electromagnetic (TEM) waves cannot propagate in regular metal waveguides. If there is a TEM wave inside a waveguide, it is required that the magnetic field is a closed curve and completely within the cross section of the waveguide. From Maxwell's first equation, the integral of the magnetic field on a closed curve should be equal to the current in the chain that intersects the curve. Since there is no conduction current in the axial direction of the waveguide, a displacement current in the direction of propagation must be required. This requires the presence of an electric field in the direction of propagation. However, this conclusion contradicts the definition of a TEM wave thus proving that TEM waves cannot be propagated in metal waveguides.

The waveguide system has 4 requirements. First, the electromagnetic wave must be allowed to exist in the guided wave system with either a traveling wave state or a state dominated by traveling waves. Second, the guided wave system must have all or most of the electromagnetic wave energy confined inside the system with small radiation and transmission losses. Third, the guided wave system must have a certain frequency bandwidth to meet the needs of actual electromagnetic wave signal transmission. Fourth, the guided wave system and the electromagnetic wave signal generation and reception system must easily be able to achieve impedance matching. In the simulations, the 2D cavities represented the cross section of a 3D waveguide. The plots showed which modes could propagate through a certain waveguide. The different modes were observed by using the arrow surface to determine the direction of the electric and magnetic fields.

How to Run the Simulations：

We use COMSOL for the simulation of the electromagnetic cavity mode. Mode analysis is a study of modes that can be propagated through a given waveguide component and its propagation constants. The characteristics of the modes depend on the geometry and frequency of the incident wave. We build our mode in 2D dimension, choose electromagnetic waves, and select frequency domain (emw). In Mode Analysis, set geometry, define material properties, and set boundary conditions. A mesh model was built in order to calculate the field. The mesh model will break up the model into small elements. In this software, the electromagnetic field is described by Poisson or Laplace equations, which are partial differential equations. We can get one equation for each mesh element and finally we can get a system of equations as a matrix that can be solved by the computer. Set the eigenfrequency for what we want the software to simulate. Then, by computing and plotting, we can see the result by the color surface plot and arrow surface plot.

Conditions Used to Simulate：

In the circle simulation, we choose silicon as the circular area material and the outer material as air. The radius of the circle is 1.5 um, which results in the eigenfrequencies of this mode being about an order of magnitude of 10THz or 200THz. For the simulation of square and rectangular, we choose a square with side length 2um, and a rectangular with short side 1um and long side 2um with silicon as the material.

For the simulation of the triangle, we use an equilateral triangle with 1.5um height and an isosceles triangle with 1um height, 2um width. For the simulation of the hexagon, we use the hexagon with 3um height. All of these have eigenfrequencies around 10-200THz because that corresponds to a wavelength of 1.5 to 30 um (Figure 1).

The Eigenfrequency in Simulation:

A circle is not a Bravais lattice, but we can use this case to illustrate what is the eigenfrequency of the cavity mode as it only has the radius as the geometry property. We derive the equation for E and H from Maxwell equations. For electric and magnetic fields separately, we get a second order ordinary differential equation to describe the field distribution in time and space. We do the same for the cavity mode or the cross section of the waveguide. We get the boundary conditions of the cavity mode. In order to satisfy both this differential equation and the boundary condition, our field must satisfy some periodicity. That is the eigenvalue problem in physics. If you feed electromagnetic waves of different frequencies into the cavity, only the electromagnetic waves of the characteristic frequencies are preserved.

Our values and meshes are discrete since some frequencies result in other patterns that would not occur under ideal conditions. Sometimes the cavity mode is not excited by the frequency, but we get the result by the matrix which is not ideal. Finer mesh means more accurate but more computational time.

Simulation and Symmetry Analysis:

Simulations were run, using the described techniques, on a variety of 2D polygons including circles, triangles (both equilateral and isosceles), squares, rectangles, and regular hexagons. In each case we were unable to obtain perfectly real eigenfrequencies to correspond with the modes that we saw due to the simulation parameters we had set but many of them had very small imaginary components and were likely good approximations of the real modes. In order to find these adequate approximations 50-200 modes were generated at a time around a specific search frequency and they were filtered through to find ones in which the vast majority of the E-field was contained within the silicon polygon.

The first simulation ran was for circles and as was expected many symmetries shown had radial nodes which increased in number with higher frequencies. These modes contained infinite rotational and mirror symmetries matching that which would be seen with a circle alone. Interestingly, there were also angular nodes that formed which reduced the number of mirror symmetries to be equivalent to the number of angular nodes. These also introduced a new type of symmetry, similar to mirror symmetry, in which a reflection followed by a phase inversion reproduces the same image. These inversion lines appeared along the angular nodes which makes sense as each of these nodes represents a change in phase between adjacent lobes. The rotational symmetry was also reduced to n-fold where n corresponded to the number of angular nodes (Figure 2). It is important to note that the fold symmetry is half what is expected from the magnitude plot due to change of phase.

The eigenmodes of the triangle simulation also showed symmetries reflecting that of the shapes themself. The equilateral triangle mostly showed three fold symmetries accompanied by either a set of either three mirror lines if lobes were oriented toward the corners or three inversion lines if angular nodes were oriented toward the center. All of these lines fell along the bisectors and the 3-fold symmetry was on the circumcenter of the triangle. In both the equilateral and isosceles triangles we also saw modes with a single mirror or inversion line connecting the vertex angle and base. This makes sense as a single reflection symmetry is the only symmetry found in isosceles triangles.

The square and rectangle simulations produced modes that resembled typical TE and TM modes present in rectangular waveguides. These consisted of a grid-like structure of nodes with two fold symmetries. Again, when 2-fold symmetries appeared their axis was about the intersection of either two mirror or inversion symmetry lines. In the case of the square cavity some higher eigenfrequencies gave rise to four fold symmetries which had 4 lines of symmetry intersecting. Two from opposite sides and two from opposite corners.

The last simulation ran was for the regular hexagon which had the highest degree of symmetry of the polygons observed. For this shape many of the same things were observed as before with the important addition of 6-fold symmetries. These 6 fold symmetries, as expected, occurred at the intersection of either six mirror or inversion lines of symmetry. All of the same 1-fold, 2-fold, and 3-fold symmetries were observed as described in the previous shape. Interestingly at high eigenfrequencies certain modes produced would share similar components in one direction but not in the others leading to different overall symmetries in the magnitude of the electric field (Figure 3). Interestingly, the eigenfrequencies where this was seen were very similar in value.

Applications in Waveguides:

Waveguides can be constructed to carry waves over a wide portion of the electromagnetic spectrum, but are especially useful in the microwave and optical frequencies ranges. Depending on the frequency, they can be constructed from either conductive or dielectric materials. Waveguides are used for transferring both power and communication signals. Waveguides used at optical frequencies are typically dielectric waveguides, structures in which a dielectric material with high permittivity, and thus high index of refraction, is surrounded by a material with lower permittivity. The structure guides optical waves by total internal reflection. An example of an optical waveguide is optical fiber.

Applications in Photonic Crystals:

A photonic crystal is an optical nanostructure in which the refractive index changes periodically. Photonic crystals are attractive optical materials for controlling and manipulating light flow. One dimensional photonic crystals are already in widespread use, in the form of thin-film optics, with applications from low and high reflection coatings on lenses and mirrors to color changing paints and inks. Higher-dimensional photonic crystals are of great interest for both fundamental and applied research, and the two dimensional ones are beginning to find commercial applications. In addition, photonic crystals have been proposed as platforms for the development of solar cells and optical sensors.

Conclusion:

In future experiments, other aspects of these simulations could be altered to produce 2D symmetries in polygon-shaped cavity modes. For one, other polygons could be studied such as octagons or decagons. These cavities would most likely exhibit symmetries similar to the hexagon cavity, but would provide more insight for real world applications. Other simulations could look at the variability of different materials. In these simulations, air and silicon were used, but aluminum, silver, copper, or any material with low bulk resistivity could also be investigated. Additionally, these results could be expanded to 3D simulations since 2D cavity modes are cross sections of 3D waveguides. By observing 3D symmetries, it would be a more accurate representation of real world waveguides. In actuality, optical cavities with polygonal structures have been displayed in laser gyroscopes for geophysical studies such as measuring the changes in rotation rate of the Earth. They have been used in ring resonators in folded laser systems and processing devices. Polygonal resonant cavities show promise in microdisk laser configurations and even in optical tests for the theory of relativity.

Appendix

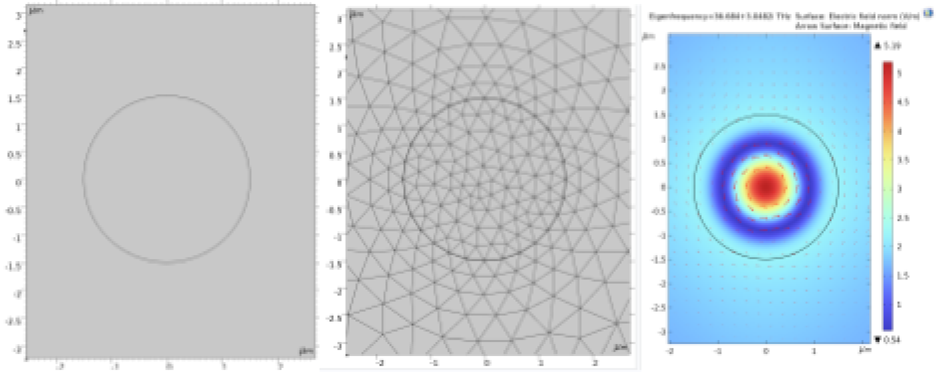

Figure 1: The steps of 2D EM mode simulation in COMSOL.

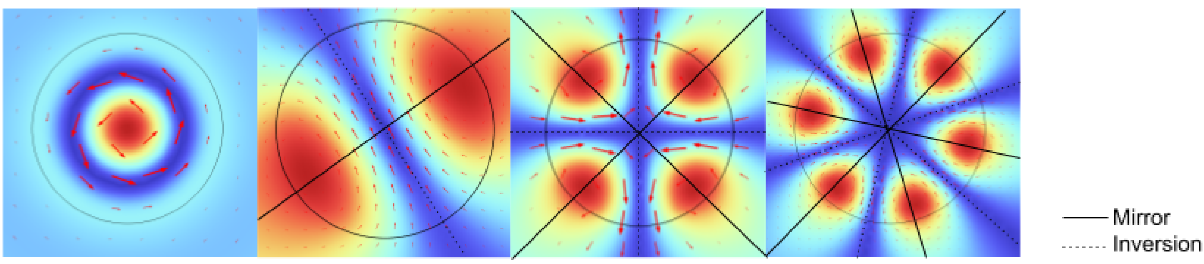

Figure 2: E-field magnitude plots of the eigenmodes of circular cavity with labeled symmetries.

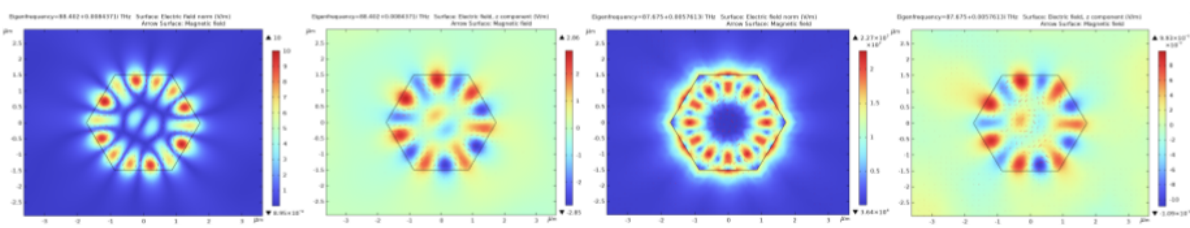

Figure 3: Simulations of hexagons with eigenfrequencies 88.4 THz (left two) and 87.7 THz (right two). The first and third images are of the E-field magnitude and the second and fourth are the z components.

Works Cited

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