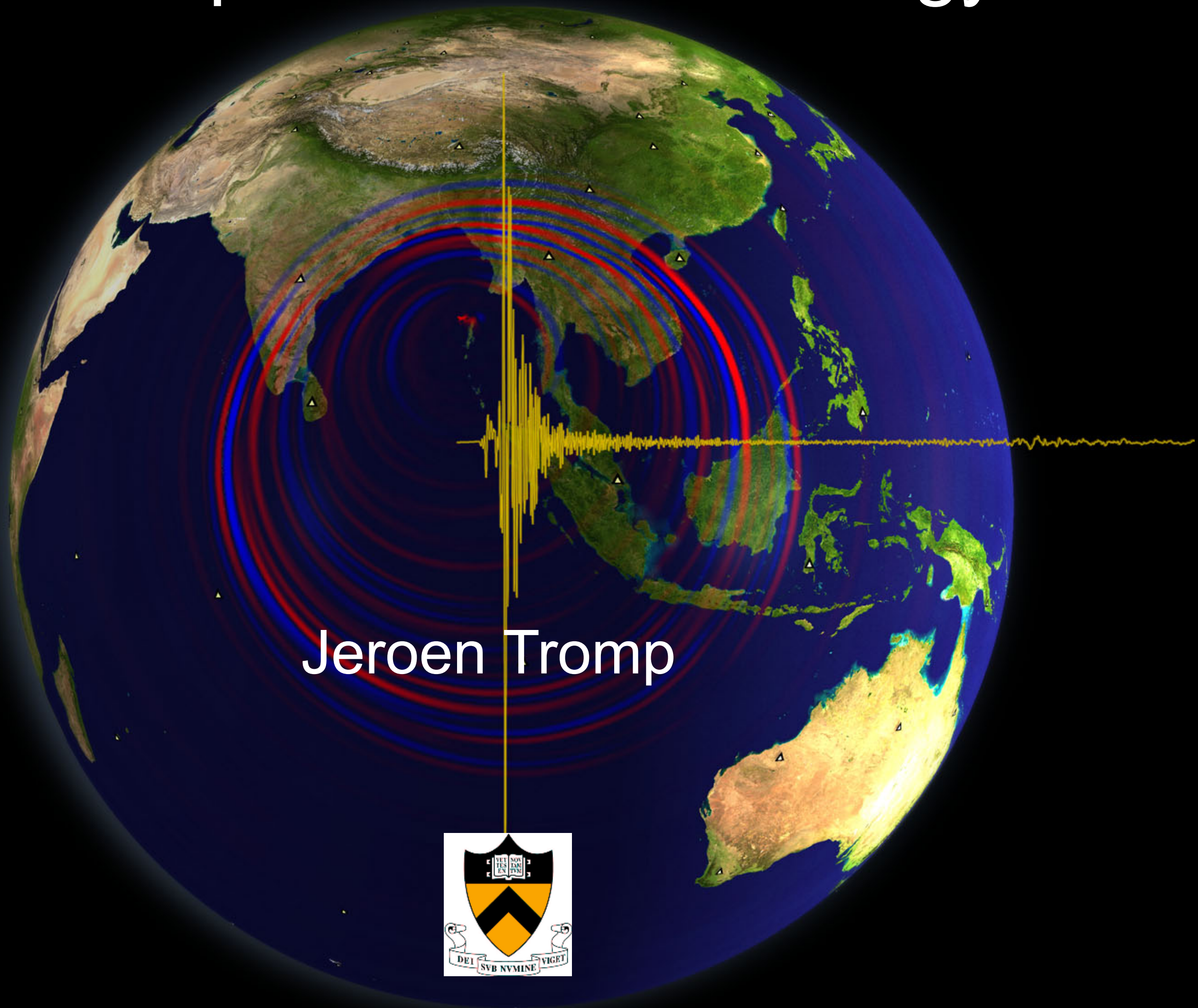
Computational Seismology



Jeroen Tromp

Governing Equations

Equation of motion:

$$\rho\,\partial_t^2 \mathbf{s} - \boldsymbol{\nabla}\cdot\mathbf{T} = \mathbf{f}$$

Constitutive relationship (Hooke’s law):

$$\mathbf{T} = \mathbf{c} : \boldsymbol{\epsilon}$$

Boundary condition:

$$\hat{\mathbf{n}}\cdot\mathbf{T} = \mathbf{0}$$

Initial conditions:

$$\mathbf{s}(\mathbf{x},0) = \mathbf{0}, \qquad \partial_t\mathbf{s}(\mathbf{x},0) = \mathbf{0}$$

Earthquake source:

$$\mathbf{f} = -\mathbf{M}\cdot\boldsymbol{\nabla}\,\delta(\mathbf{x}-\mathbf{x}_{\mathrm{s}})\,S(t)$$

Classic Tools

- Semi-analytical methods
  - Layer cake models: reflectivity/discrete wave number
  - Spherically symmetric models: normal modes
- Asymptotic methods
  - Ray theory
  - WKBJ theory
  - Coupled modes

Strong Form

- Velocity-stress formulation (first-order PDEs, $\mathbf{v} = \partial_t \mathbf{s}$):

$$\partial_t \mathbf{v} = \rho^{-1}(\nabla \cdot \mathbf{T} + \mathbf{f}), \quad \partial_t \mathbf{T} = \mathbf{c} : \nabla \mathbf{v}$$

- Second-order finite-difference method:

$$\frac{\mathrm{d}}{\mathrm{d}x} f(n\Delta x) = (f_{n+1} - f_{n-1})/(2\Delta x),$$
$$n = 1, \ldots, N-2$$

- Fourth-order finite-difference method:

$$\frac{\mathrm{d}}{\mathrm{d}x} f(n\Delta x) = (-f_{n+2} + 8f_{n+1} - 8f_{n-1} + f_{n-2})/(12\Delta x),$$
$$n = 2, \ldots, N-3$$

- Challenges:
  – Boundary conditions
  – Numerical dispersion & anisotropy

Strong Form

- Pseudospectral methods:

$$F(l\Delta k) = \Delta x \sum_{n=0}^{N-1} f(n\Delta x)\exp(-2\pi i n l / N),$$
$$l = 0, \ldots, N-1$$

$$\frac{d}{dx} f(n\Delta x) = \frac{1}{N\Delta x} \sum_{l=0}^{N-1} i(l\Delta k) F(l\Delta k)\exp(2\pi i n l / N),$$
$$n = 0, \ldots, N-1$$

- Challenges:
  - Boundary conditions
  - Parallel implementation

Weak Form

$$\int_\Omega \rho\,\mathbf{w}\cdot\partial_t^2\mathbf{s}\,\mathrm{d}^3\mathbf{x} = -\int_\Omega \boldsymbol{\nabla}\mathbf{w}:\mathbf{T}\,\mathrm{d}^3\mathbf{x} + \mathbf{M}:\boldsymbol{\nabla}\mathbf{w}(\mathbf{x}_\mathrm{s})S(t)$$

- Weak form valid for any test vector
- Boundary conditions automatically included
- Source term explicitly integrated

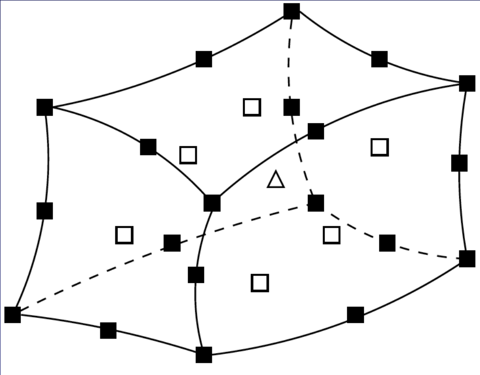
Finite-fault (kinematic) rupture:

$$\mathbf{M}:\boldsymbol{\nabla}\mathbf{w}(\mathbf{x}_\mathrm{S})\,S(t)\rightarrow\int_{S_\mathrm{s}}\mathbf{m}(\mathbf{x}_\mathrm{S},\,t):\boldsymbol{\nabla}\mathbf{w}(\mathbf{x}_\mathrm{S})\,d^2\mathbf{x}_\mathrm{S}$$

Finite Elements

Mapping from reference cube to hexahedral elements:

$$\mathbf{x}(\boldsymbol{\xi}) = \sum_{a=1}^{M} \mathbf{x}_a N_a(\boldsymbol{\xi})$$

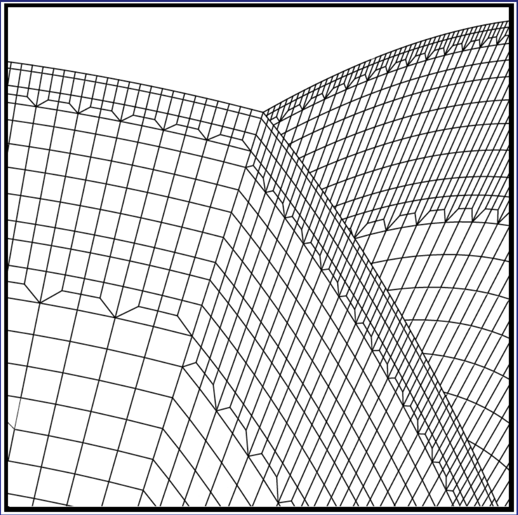


Volume relationship:

$$\mathrm{d}^3\mathbf{x} = \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = J\,\mathrm{d}\xi\,\mathrm{d}\eta\,\mathrm{d}\zeta = J\,\mathrm{d}^3\boldsymbol{\xi}$$

Jacobian of the mapping:

$$J = \left| \frac{\partial(x, y, z)}{\partial(\xi, \eta, \zeta)} \right|$$



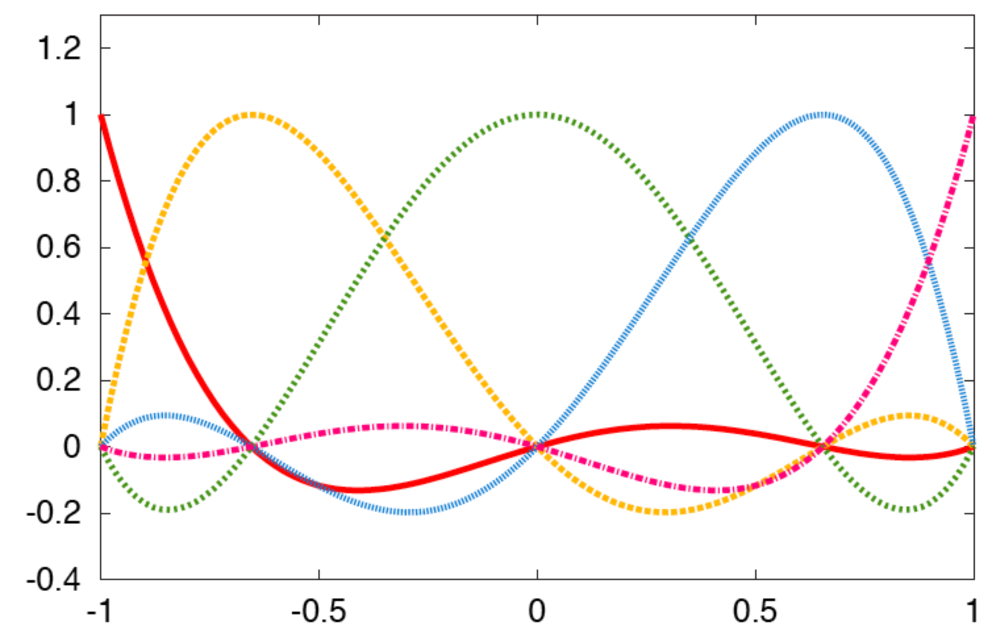
Jacobian matrix:

$$\frac{\partial \mathbf{x}}{\partial \boldsymbol{\xi}} = \sum_{a=1}^{M} \mathbf{x}_a \frac{\partial N_a}{\partial \boldsymbol{\xi}}$$

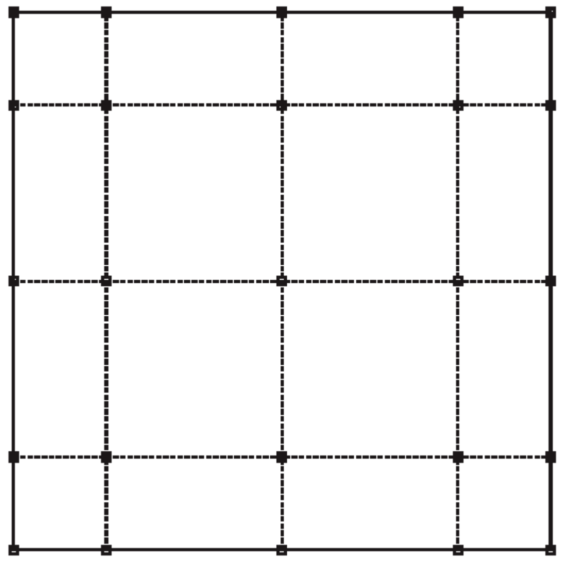
- Challenges:
  - Numerical anisotropy & dispersion
  - Mass lumping/implicit time stepping

Spectral Elements

The 5 degree 4 Lagrange polynomials:



Degree 4 GLL points:



GLL points are n+1 roots of:

$$(1-\xi^2)P_n'(\xi) = 0$$

General definition:

$$h_\alpha(\xi) = \frac{(\xi-\xi_0)\cdots(\xi-\xi_{\alpha-1})(\xi-\xi_{\alpha+1})\cdots(\xi-\xi_n)}{(\xi_\alpha-\xi_0)\cdots(\xi_\alpha-\xi_{\alpha-1})(\xi_\alpha-\xi_{\alpha+1})\cdots(\xi_\alpha-\xi_n)}$$

Note that at a GLL point: $h_\alpha(\xi_\beta) = \delta_{\alpha\beta}$

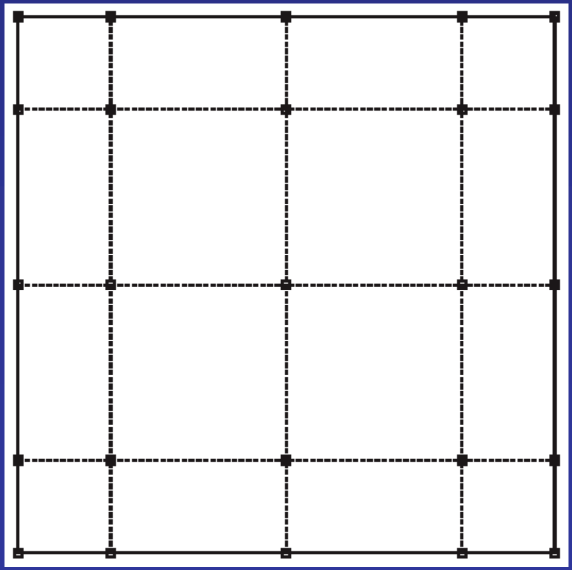
Interpolation

Representation of functions on an element in terms of Lagrange polynomials:

$$f(\mathbf{x}(\xi,\eta,\zeta)) = \sum_{\alpha=0}^{n}\sum_{\beta=0}^{n}\sum_{\gamma=0}^{n} f^{\alpha\beta\gamma} h_\alpha(\xi) h_\beta(\eta) h_\gamma(\zeta)$$

Gradient on an element:

$$\boldsymbol{\nabla} f(\mathbf{x}(\xi,\eta,\zeta)) = \sum_{i=1}^{3}\hat{\mathbf{x}}_i \sum_{\alpha=0}^{n}\sum_{\beta=0}^{n}\sum_{\gamma=0}^{n} f^{\alpha\beta\gamma}\left[h'_\alpha(\xi) h_\beta(\eta) h_\gamma(\zeta)\partial_i\xi + h_\alpha(\xi) h'_\beta(\eta) h_\gamma(\zeta)\partial_i\eta + h_\alpha(\xi) h_\beta(\eta) h'_\gamma(\zeta)\partial_i\zeta\right]$$



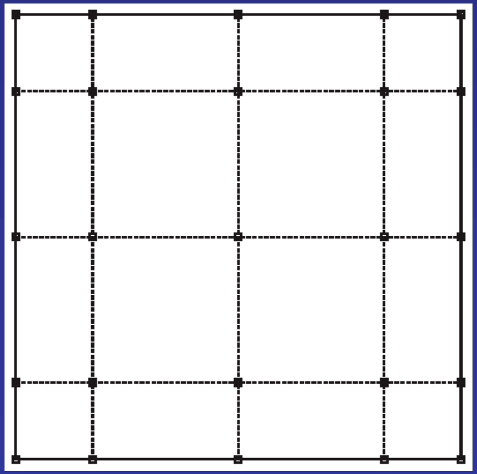
Integration

Integration of functions over an element based upon GLL quadrature:

$$\int_{\Omega_e} f(\mathbf{x})\,\mathrm{d}^3\mathbf{x} = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} f(\mathbf{x}(\xi,\eta,\zeta))\,J(\xi,\eta,\zeta)\,\mathrm{d}\xi\,\mathrm{d}\eta\,\mathrm{d}\zeta = \sum_{\alpha=0}^{n}\sum_{\beta=0}^{n}\sum_{\gamma=0}^{n} \omega_\alpha\omega_\beta\omega_\gamma f^{\alpha\beta\gamma} J^{\alpha\beta\gamma}$$

- Integrations are pulled back to the reference cube
- In the SEM one uses:
    - interpolation on GLL points
    - GLL quadrature

Degree 4 GLL points:



The Diagonal Mass Matrix

Representation of the displacement:

$$\mathbf{s}(\mathbf{x}(\xi,\eta,\zeta),t)=\sum_{i=1}^{3}\hat{\mathbf{x}}_i\sum_{\sigma=0}^{n}\sum_{\tau=0}^{n}\sum_{\nu=0}^{n} s_i^{\sigma\tau\nu}(t)h_\sigma(\xi)h_\tau(\eta)h_\nu(\zeta)$$

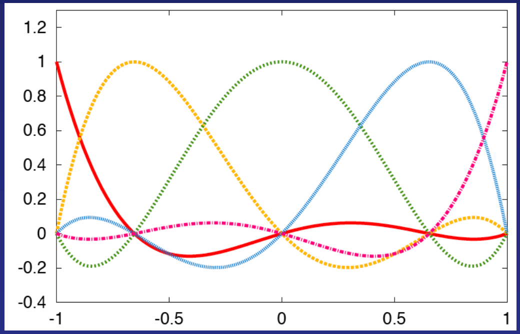
Representation of the test vector:

$$\mathbf{w}(\mathbf{x}(\xi,\eta,\zeta))=\sum_{i=j}^{3}\hat{\mathbf{x}}_j\sum_{\sigma=0}^{n}\sum_{\tau=0}^{n}\sum_{\nu=0}^{n} w_i^{\alpha\beta\gamma}h_\alpha(\xi)h_\beta(\eta)h_\gamma(\zeta)$$

Weak form:

$$\int_\Omega \rho\,\mathbf{w}\cdot\partial_t^2\mathbf{s}\,\mathrm{d}^3\mathbf{x}=-\int_\Omega \boldsymbol{\nabla}\mathbf{w}:\mathbf{T}\,\mathrm{d}^3\mathbf{x}+\mathbf{M}:\boldsymbol{\nabla}\mathbf{w}(\mathbf{x}_s)S(t)$$

Degree 4 Lagrange polynomials:

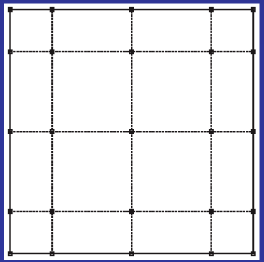


Diagonal mass matrix:

$$\int_{\Omega_e}\rho\,\mathbf{w}\cdot\partial_t^2\mathbf{s}\,\mathrm{d}^3\mathbf{x}=\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}\rho(\mathbf{x}(\boldsymbol{\xi}))\,\mathbf{w}(\mathbf{x}(\boldsymbol{\xi}))\cdot\partial_t^2\mathbf{s}(\mathbf{x}(\boldsymbol{\xi}),t)\,J(\boldsymbol{\xi})\,\mathrm{d}^3\boldsymbol{\xi}=\sum_{\alpha=0}^{n}\sum_{\beta=0}^{n}\sum_{\gamma=0}^{n}\omega_\alpha\omega_\beta\omega_\gamma J^{\alpha\beta\gamma}\rho^{\alpha\beta\gamma}\sum_{i=1}^{3}w_i^{\alpha\beta\gamma}\ddot{s}_i^{\alpha\beta\gamma}$$

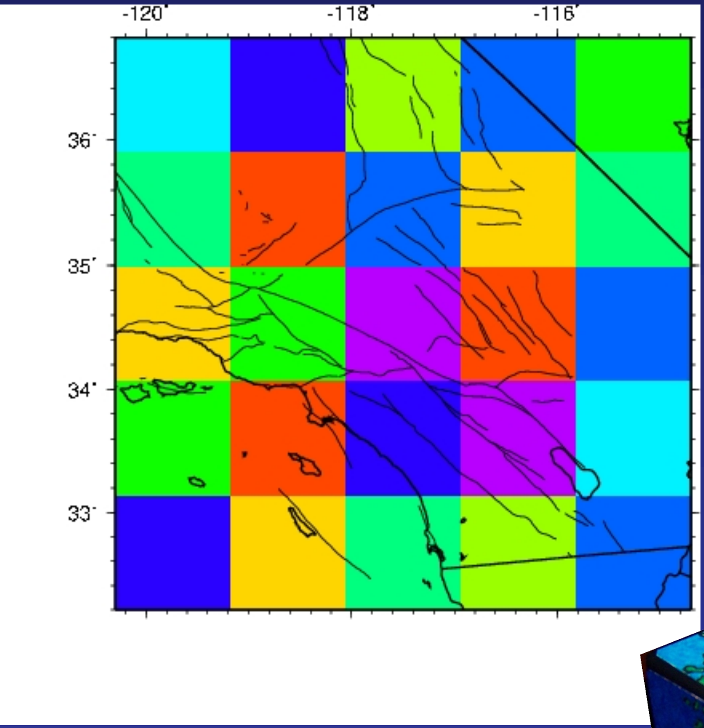
- Integrations are pulled back to the reference cube
- In the SEM one uses:
    - interpolation on GLL points
    - GLL quadrature

Degree 4 GLL points:

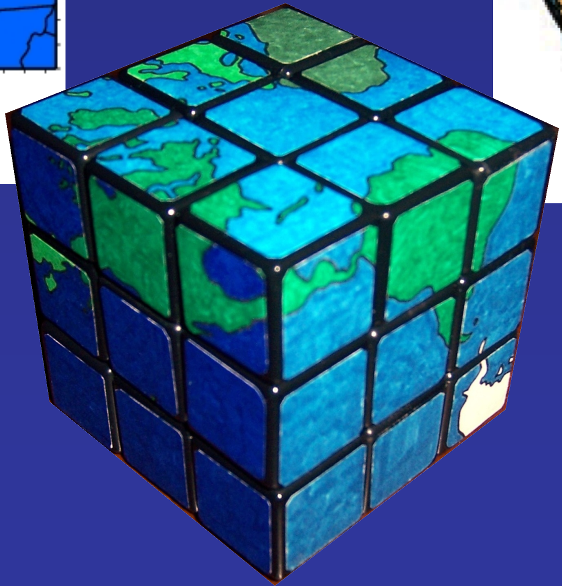


Parallel Implementation

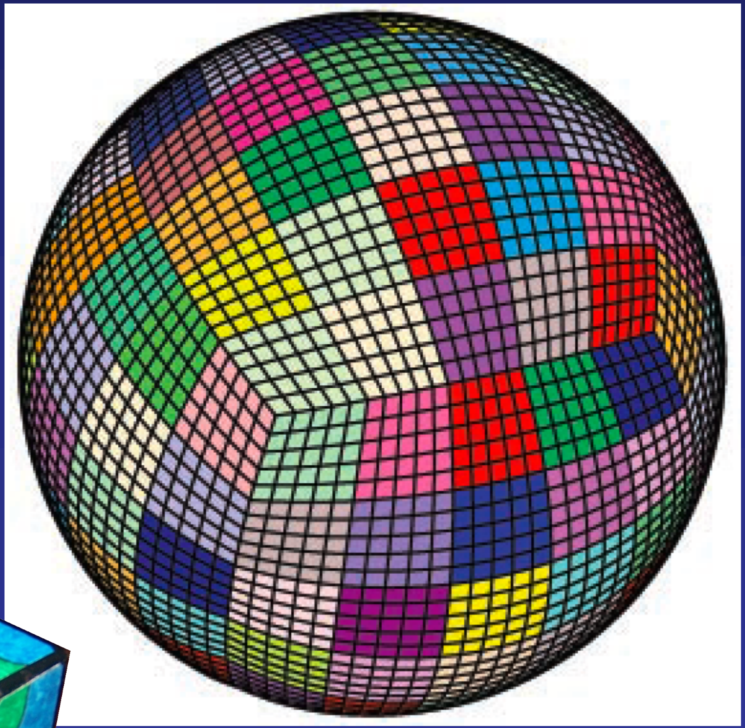
Regional mesh partitioning:



n x *m* mesh slices

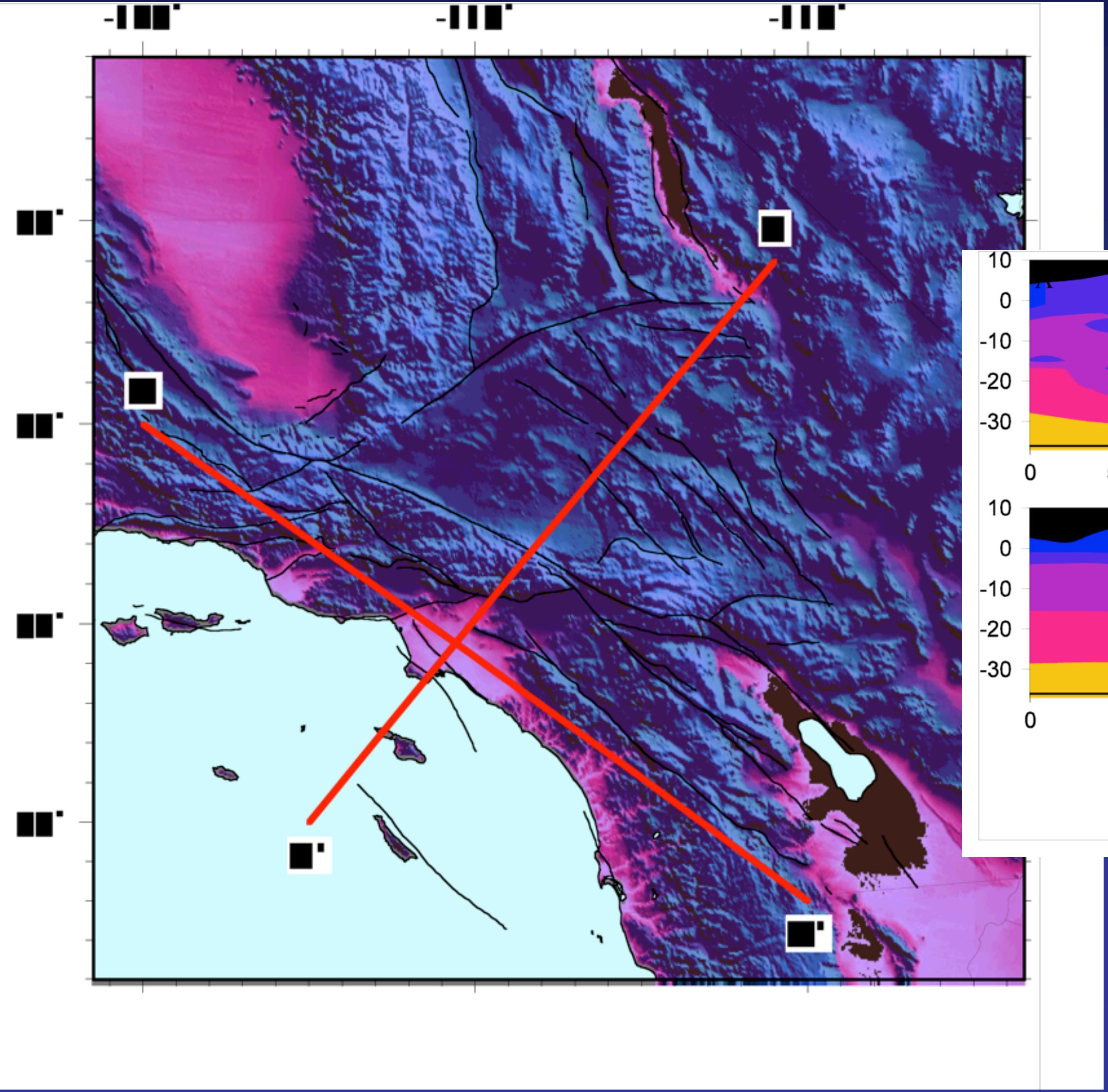


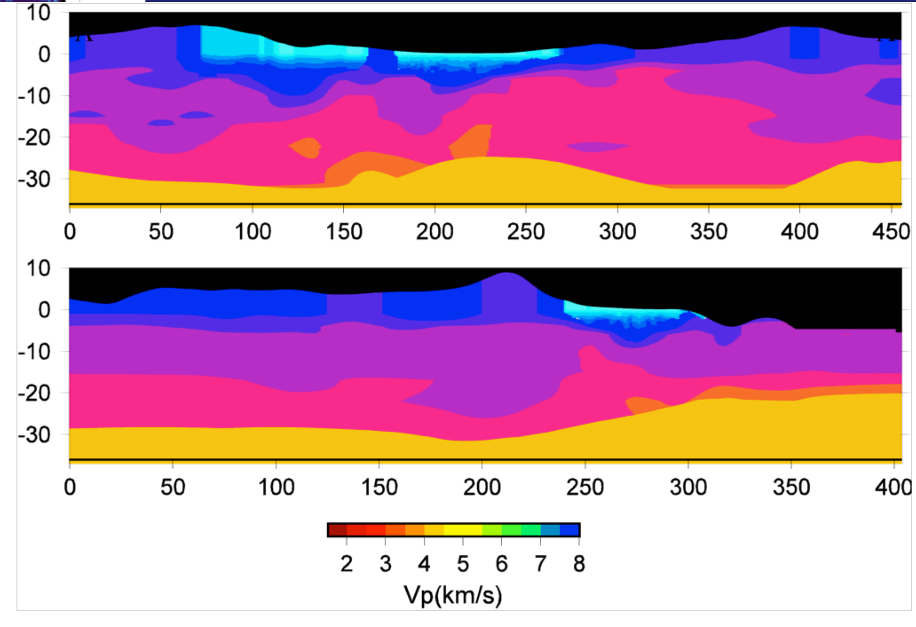
Global mesh partitioning:



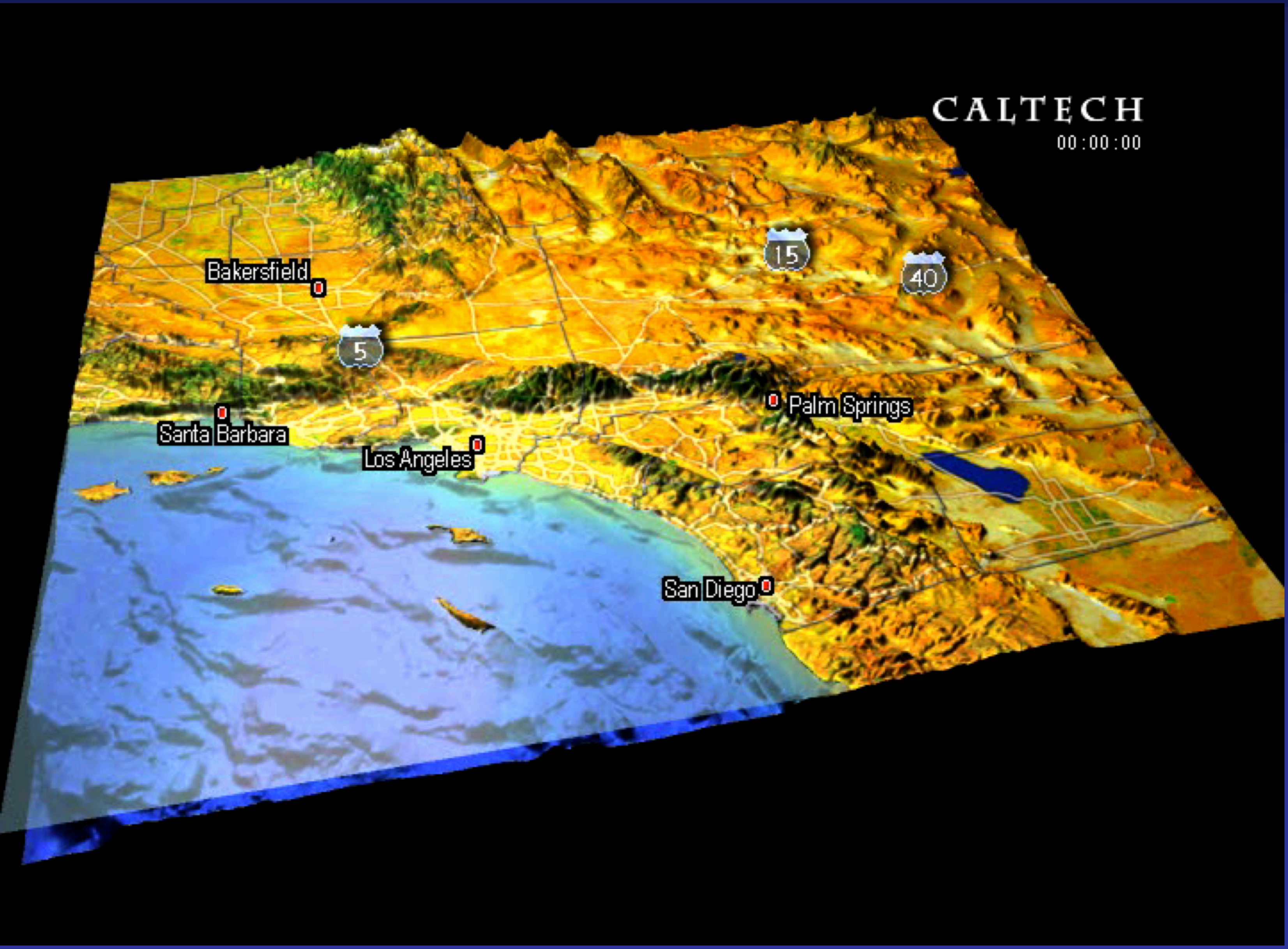
Cubed Sphere: 6 n^2 mesh slices

Southern California Simulations

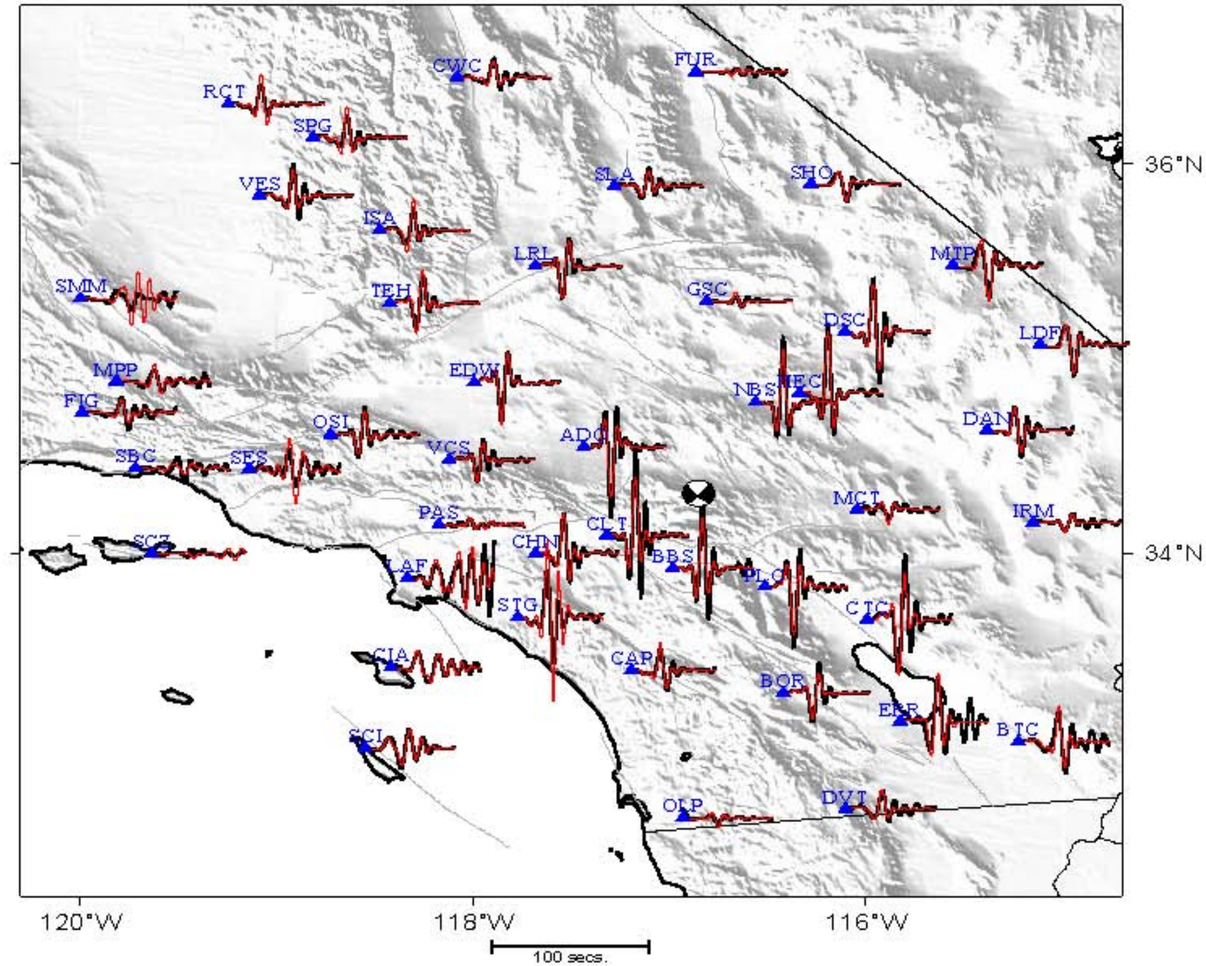




June 12, 2005, M=5.1 Big Bear



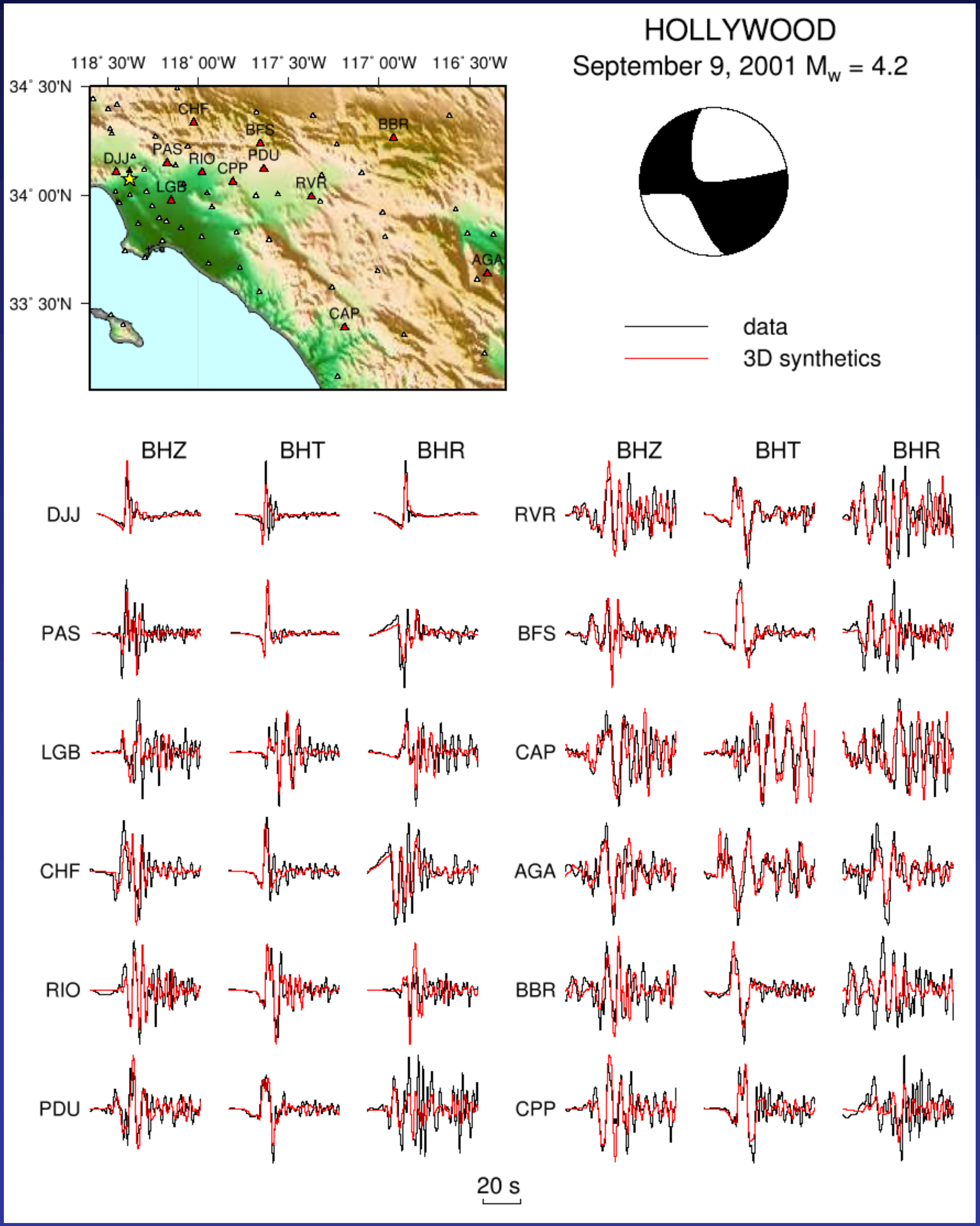
3D Regional Forward Simulations



June 12, 2005, M=5.1 Big Bear

Periods > 6 s

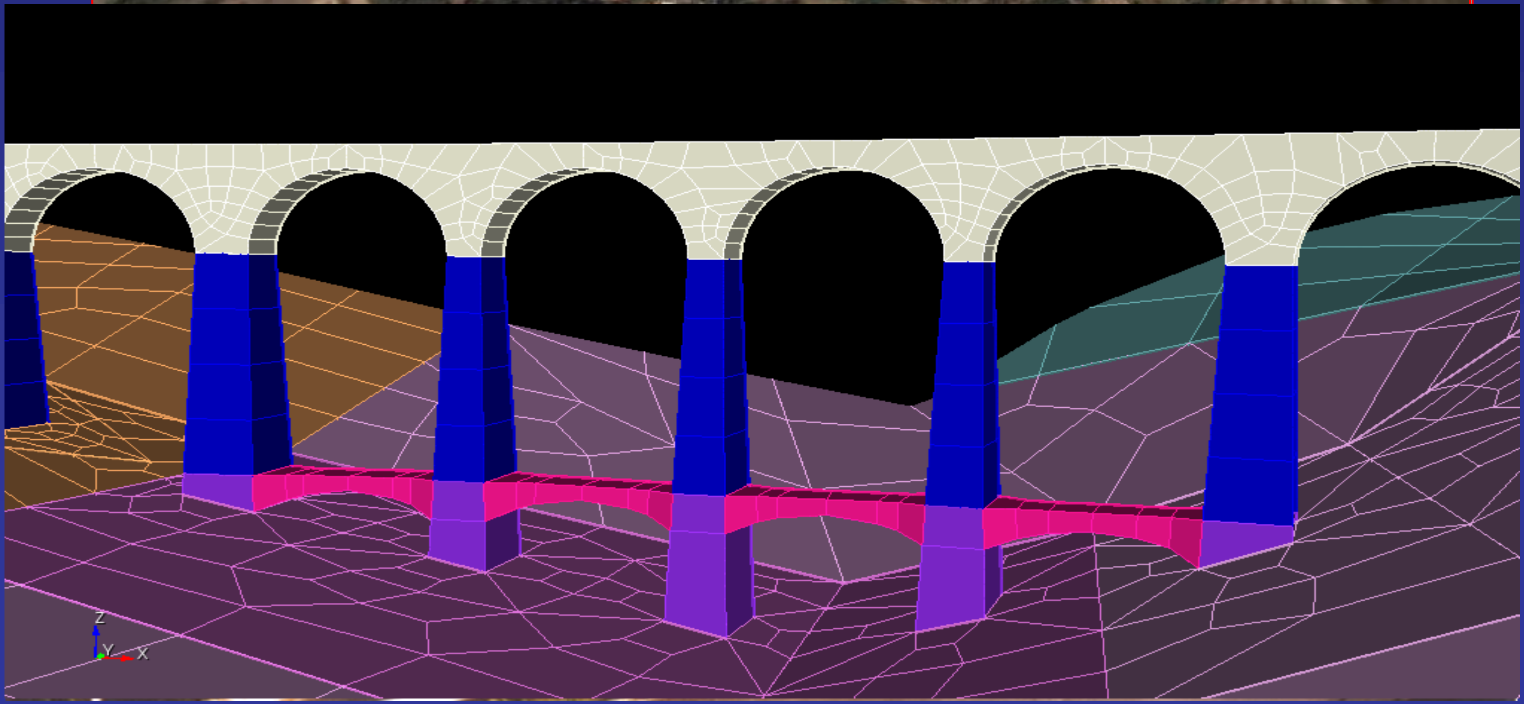
Qinya Liu



Periods > 2 s

Komatitsch et al. 2004

Soil-Structure Interaction

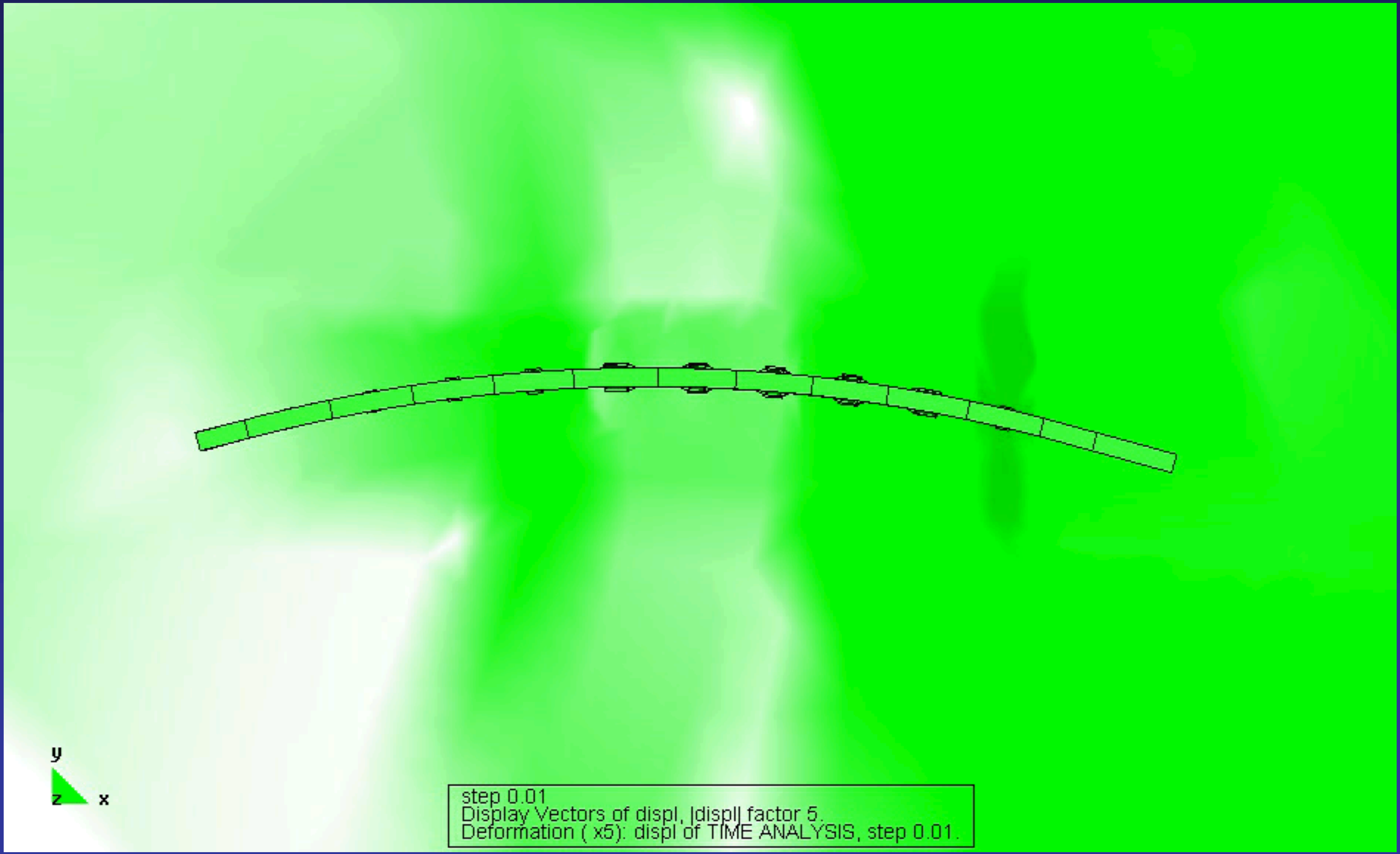


Stupazinni 2007

Soil-Structure Interaction (0.33 Hz)

Stupazini 2007

Soil-Structure Interaction (1 Hz)



Stupazini 2007

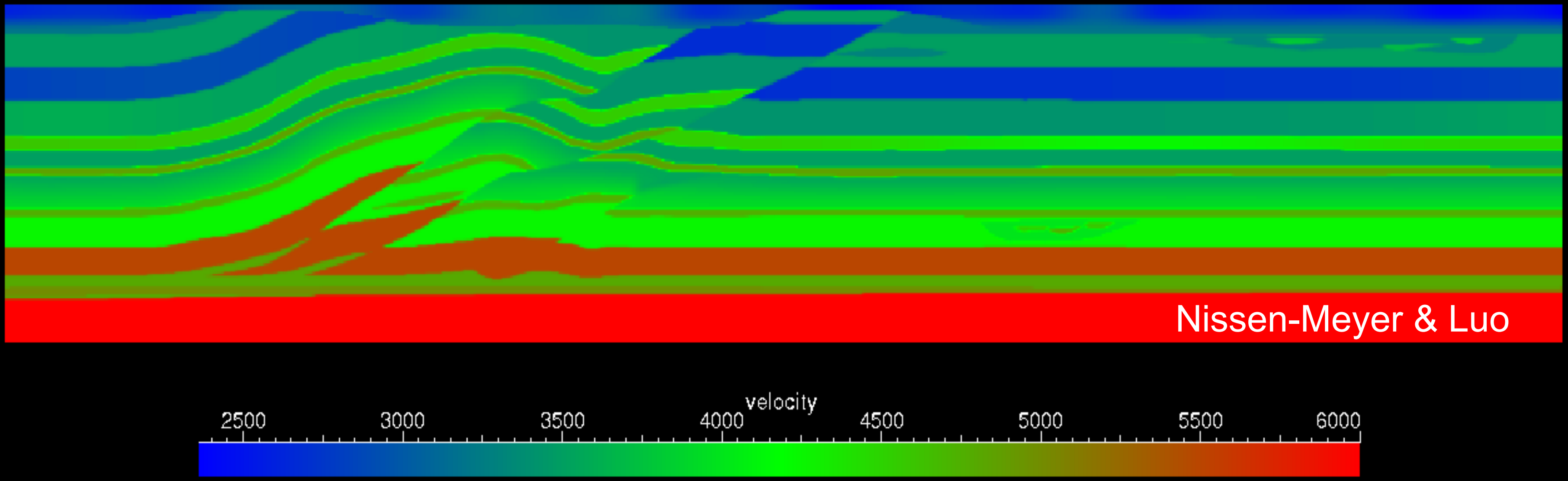
Recent & Current Developments

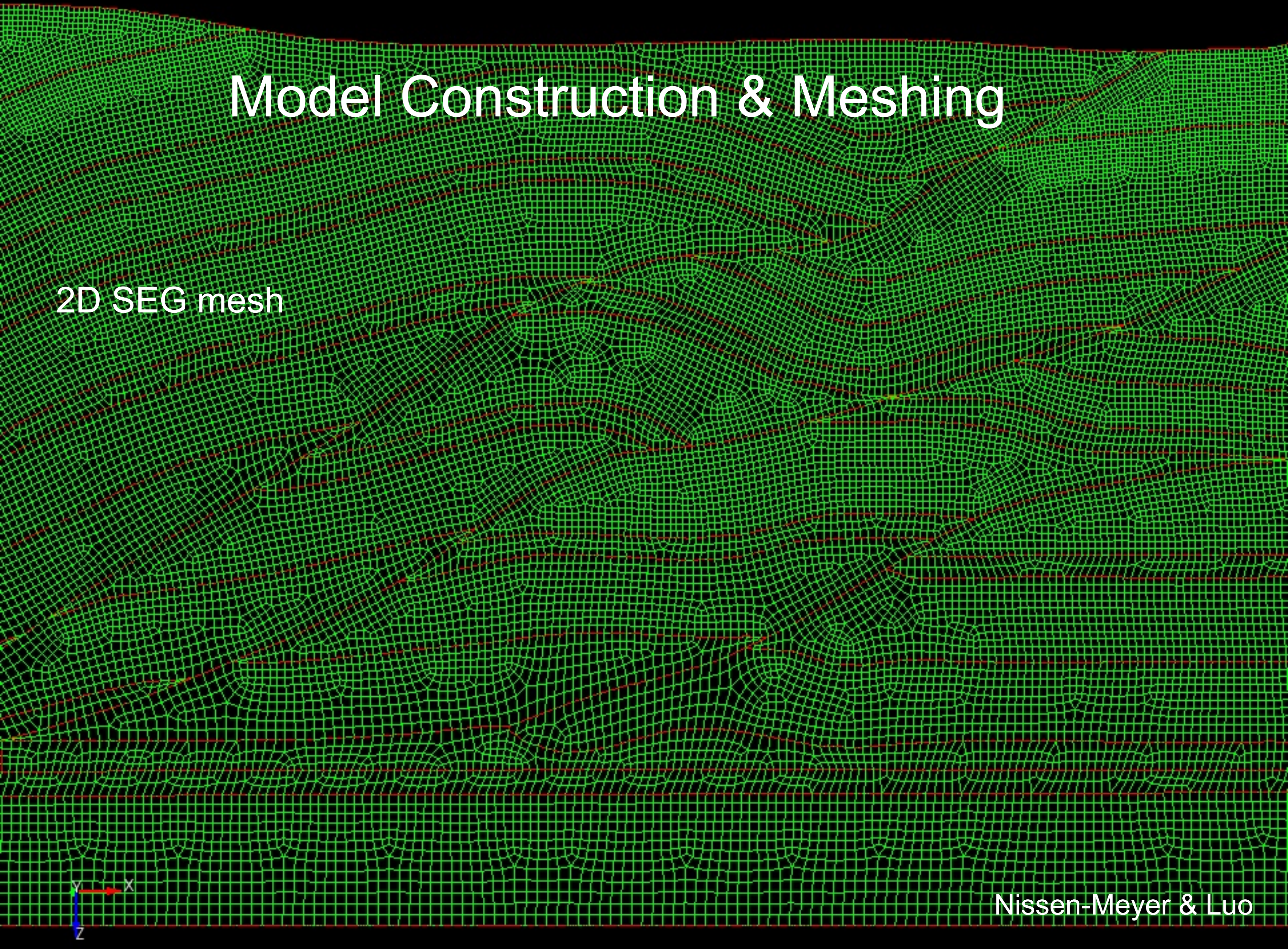
- Switch to a (parallel) “GEO”CUBIT hexahedral finite-element mesher
  - Topography & bathymetry
  - Major geological interfaces
  - Basins
  - Fault surfaces
- Use ParMETIS or SCOTCH for mesh partitioning & load-balancing
- 2D mesher & solver are ready (SPECFEM2D)
- Currently developing SPECFEM3D solver (elastic & poroelastic)
- Add dynamic rupture capabilities

SPECFEM3D Users Map

Model Construction & Meshing

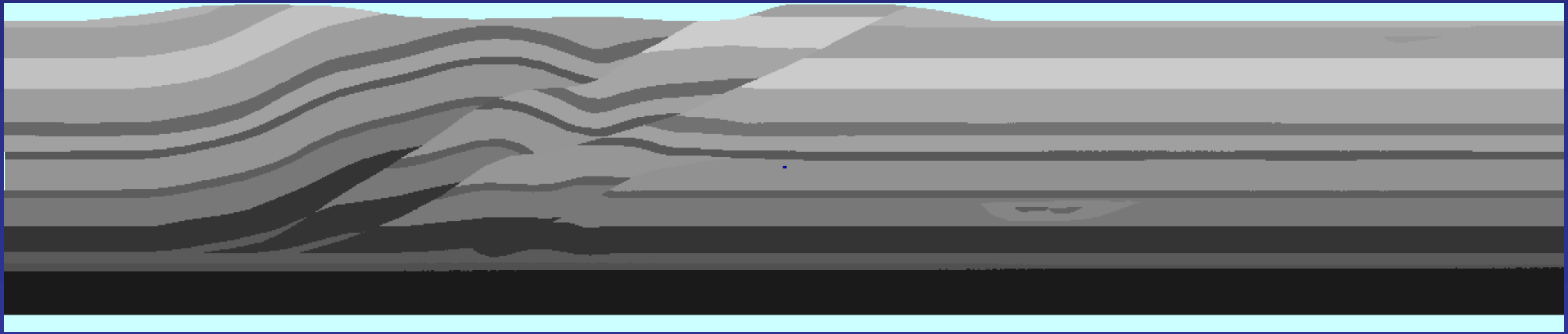
2D SEG model




Model Construction & Meshing
2D SEG mesh
Y
X
Z
Nissen-Meyer & Luo

SEM Simulation

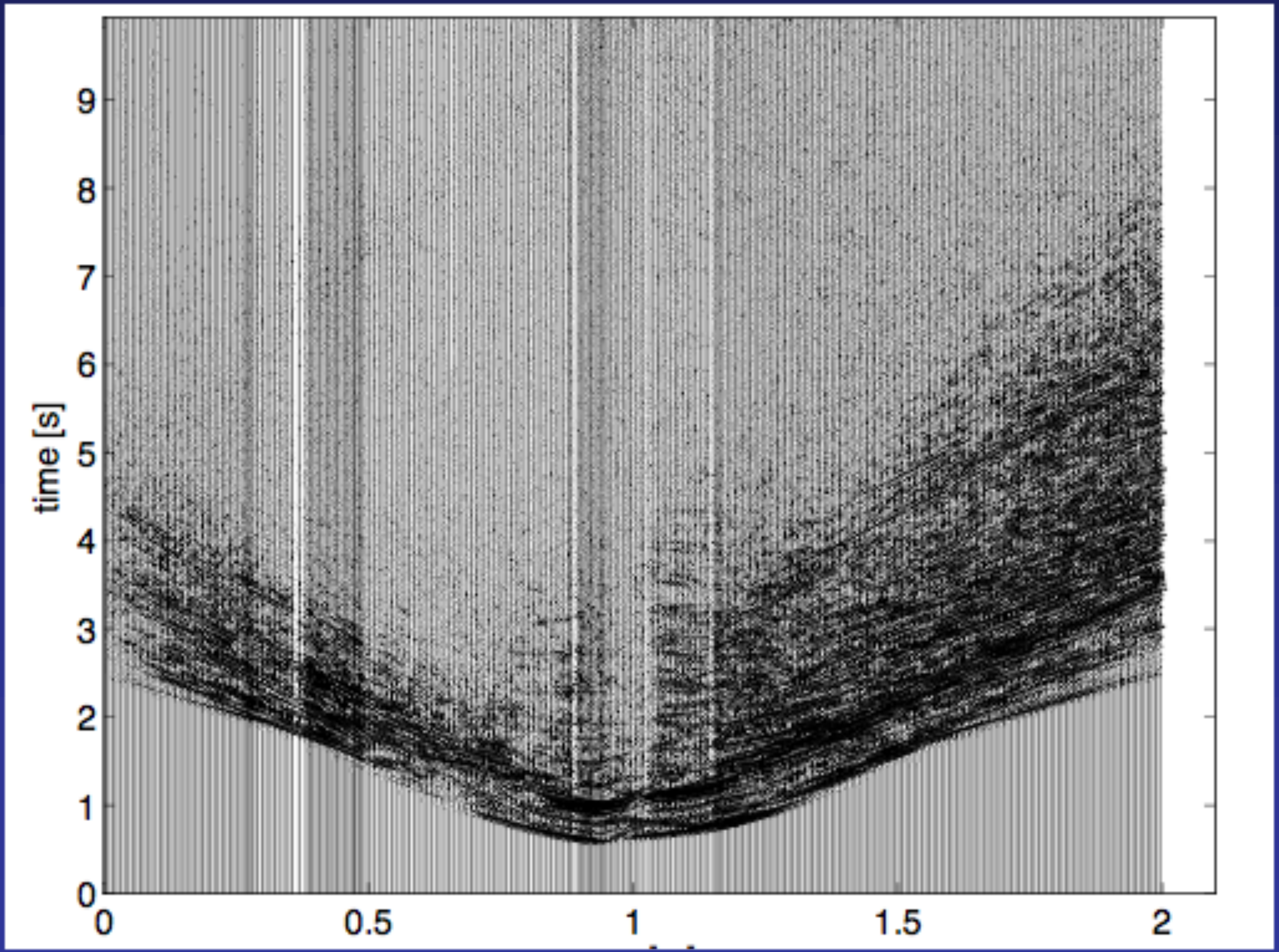
2D SEG model (deep explosive source)



Komatitsch & Le Goff

Nissen-Meyer & Luo

SEM Seismograms

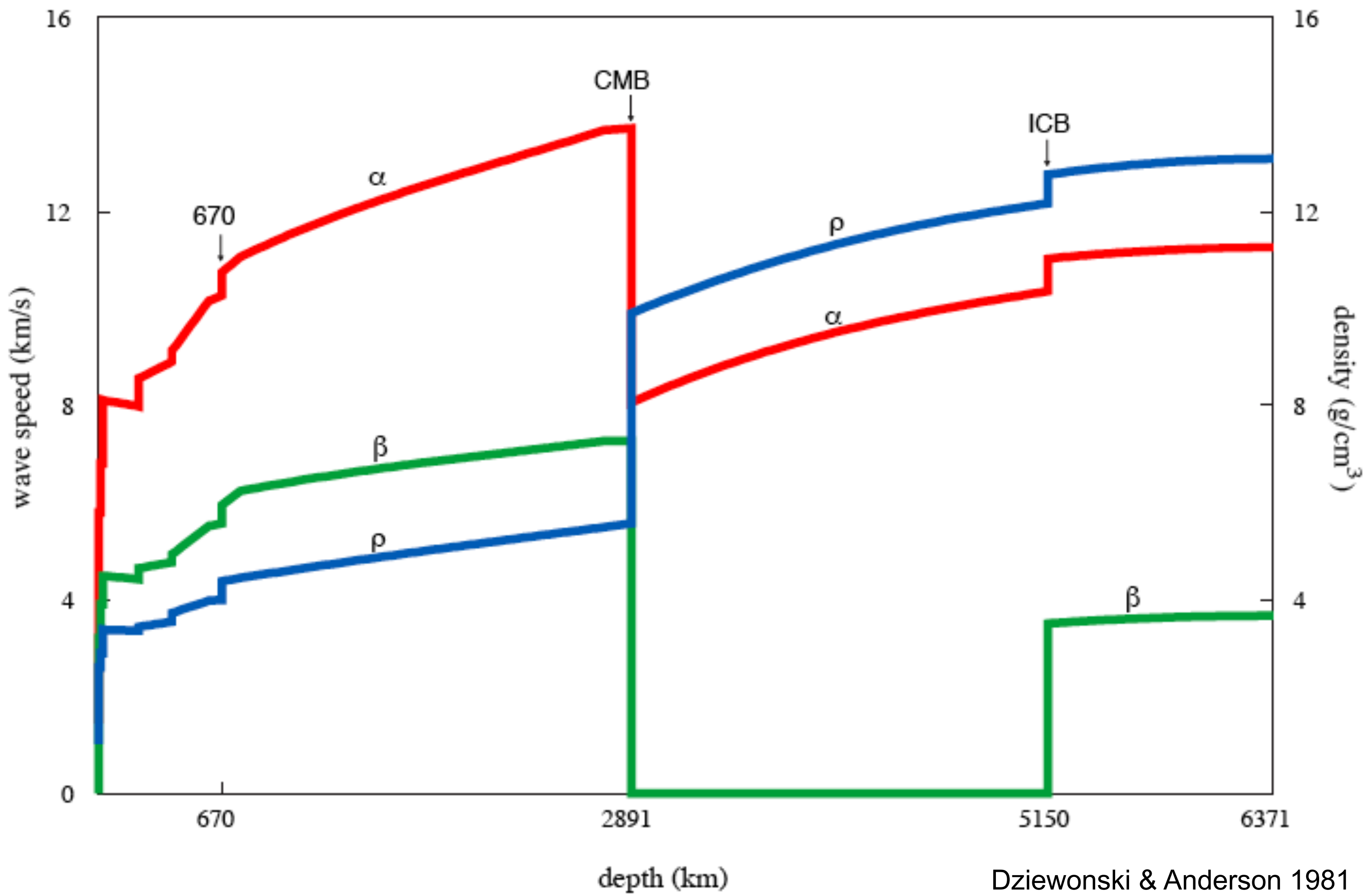


Komatitsch & Le Goff

Nissen-Meyer & Luo

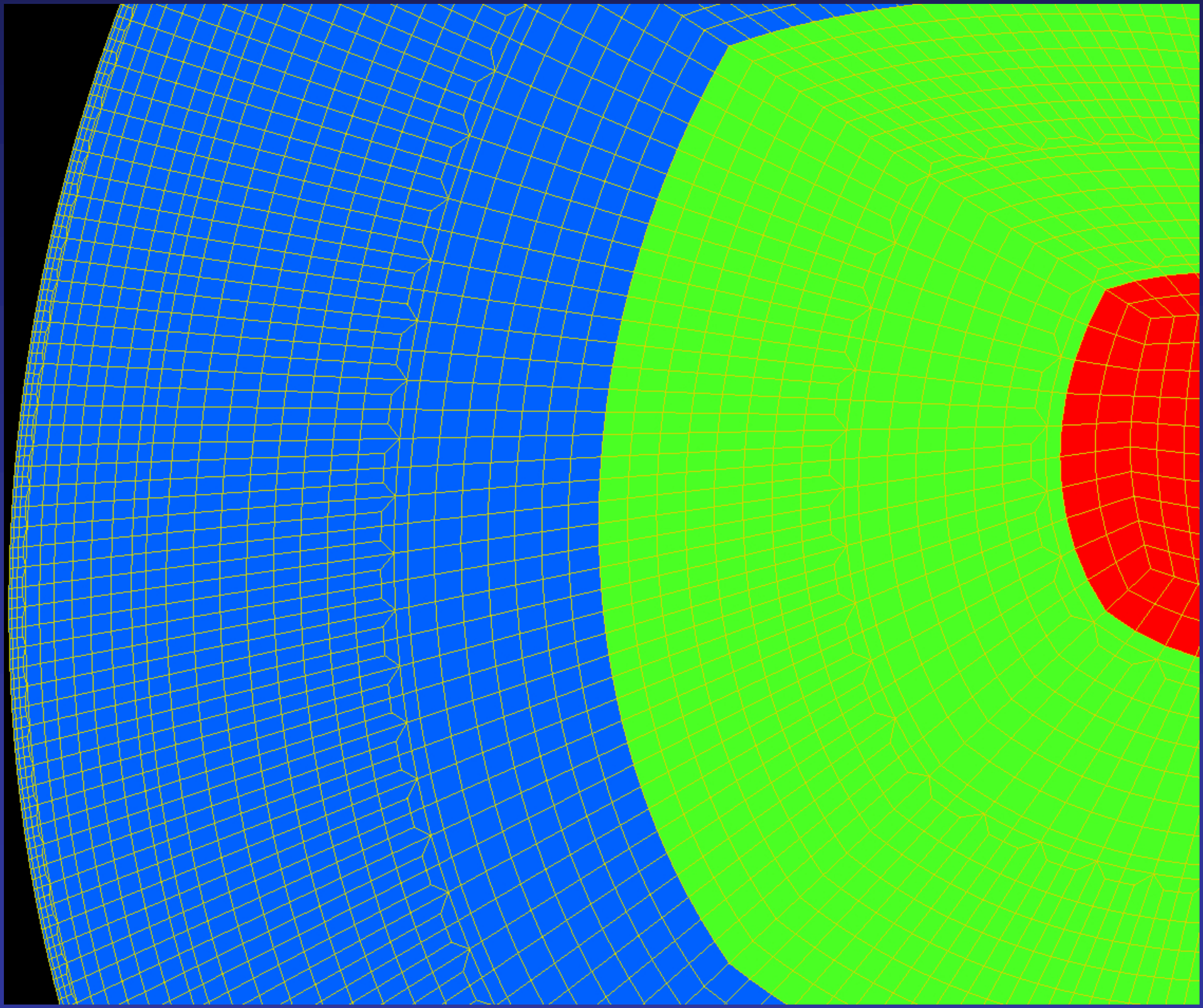
Global Simulations

PREM benchmarks



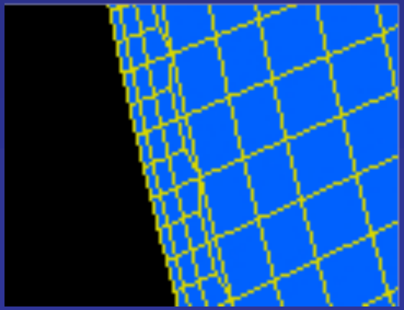
Dziewonski & Anderson 1981

New V4.0 Mesh



Four doublings:
- below the crust
- in the mid mantle
- two in the outer core

Note: two-layer crust



Michea & Komatitsch

SEM Implementation of Attenuation

Anelastic, anisotropic constitutive relationship:

$$\mathbf{T}(t) = \int_{-\infty}^{\infty} \partial_t \mathbf{c}(t-t') : \boldsymbol{\nabla}\mathbf{s}(t')\, dt'$$

Equivalent Standard Linear Solid (SLS) formulation:

$$\mathbf{T} = \mathbf{c}^U : \boldsymbol{\nabla}\mathbf{s} - \sum_{\ell=1}^{L} \mathbf{R}^\ell$$

Memory variable equation:

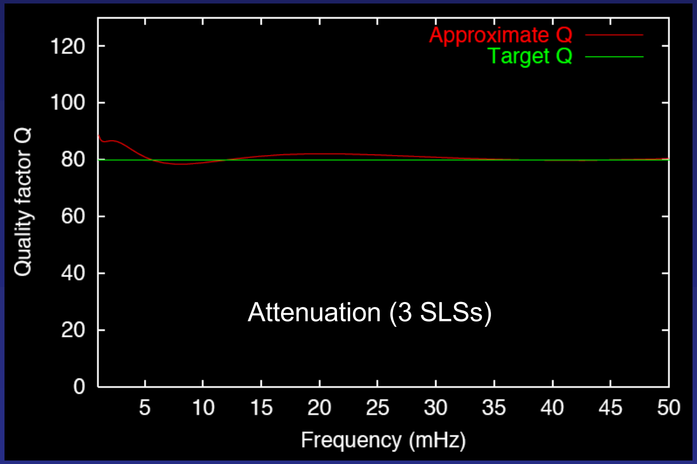
$$\partial_t \mathbf{R}^\ell = -\mathbf{R}^\ell/\tau^{\sigma\ell} + \boldsymbol{\delta}\mathbf{c}^\ell : \boldsymbol{\nabla}\mathbf{s}/\tau^{\sigma\ell}$$

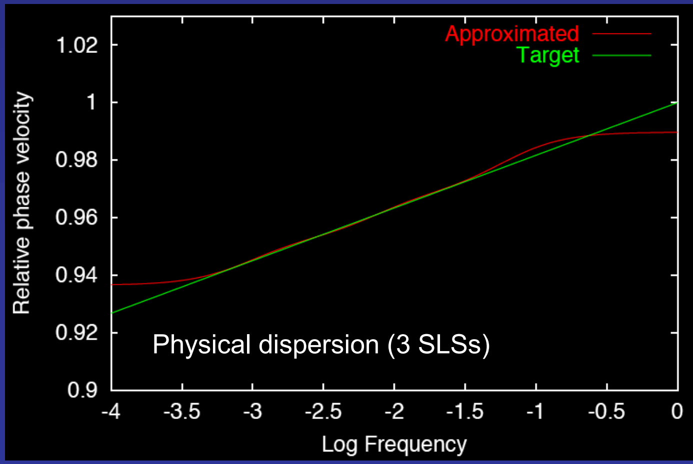
Unrelaxed modulus:

$$c_{ijkl}^U = c_{ijkl}^R \left[1 - \sum_{\ell=1}^{L} (1 - \tau_{ijkl}^{\epsilon\ell}/\tau^{\sigma\ell})\right]$$

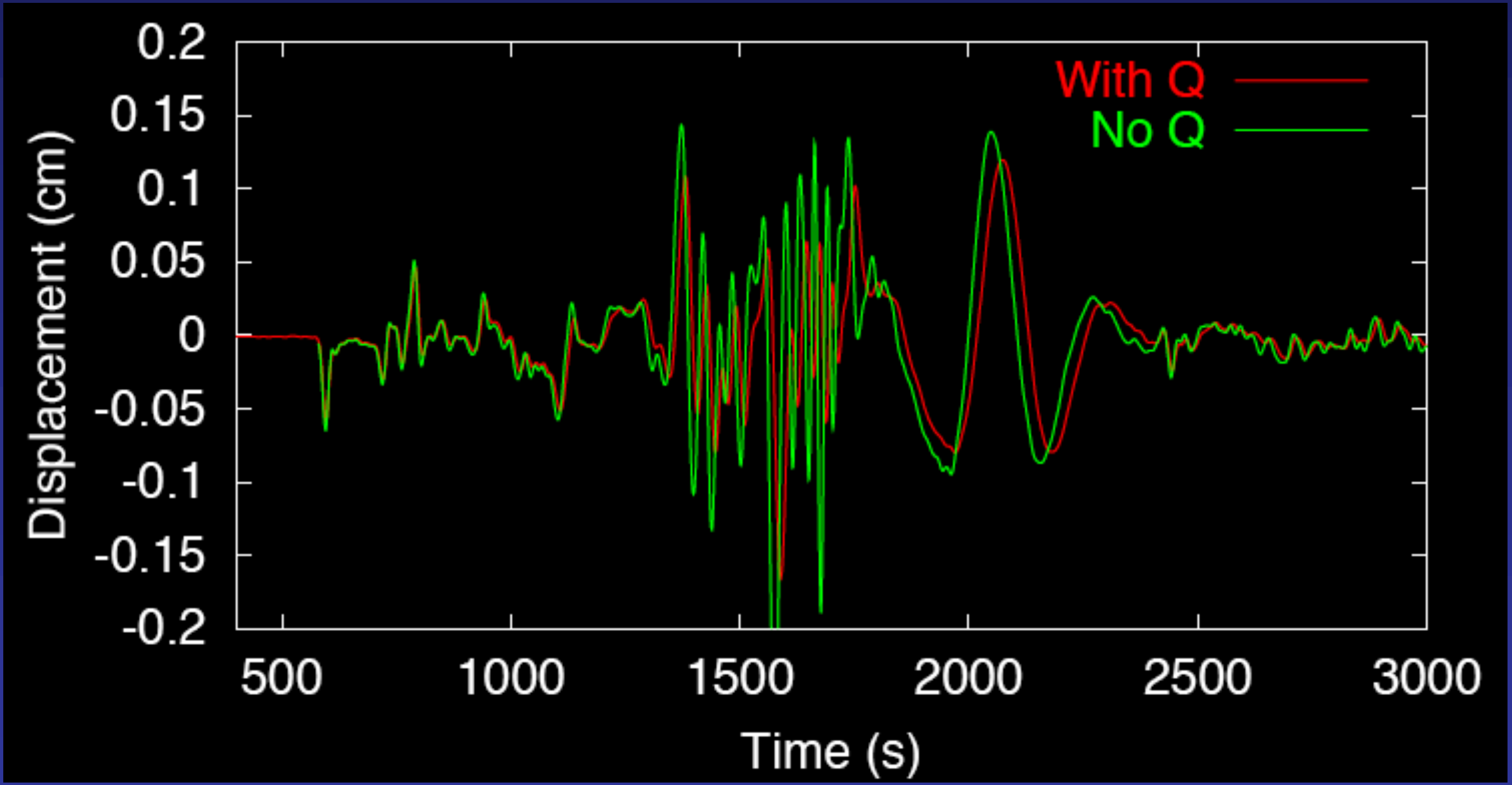
Modulus defect:

$$\delta c_{ijkl}^\ell = -c_{ijkl}^R (1 - \tau_{ijkl}^{\epsilon\ell}/\tau^{\sigma\ell})$$

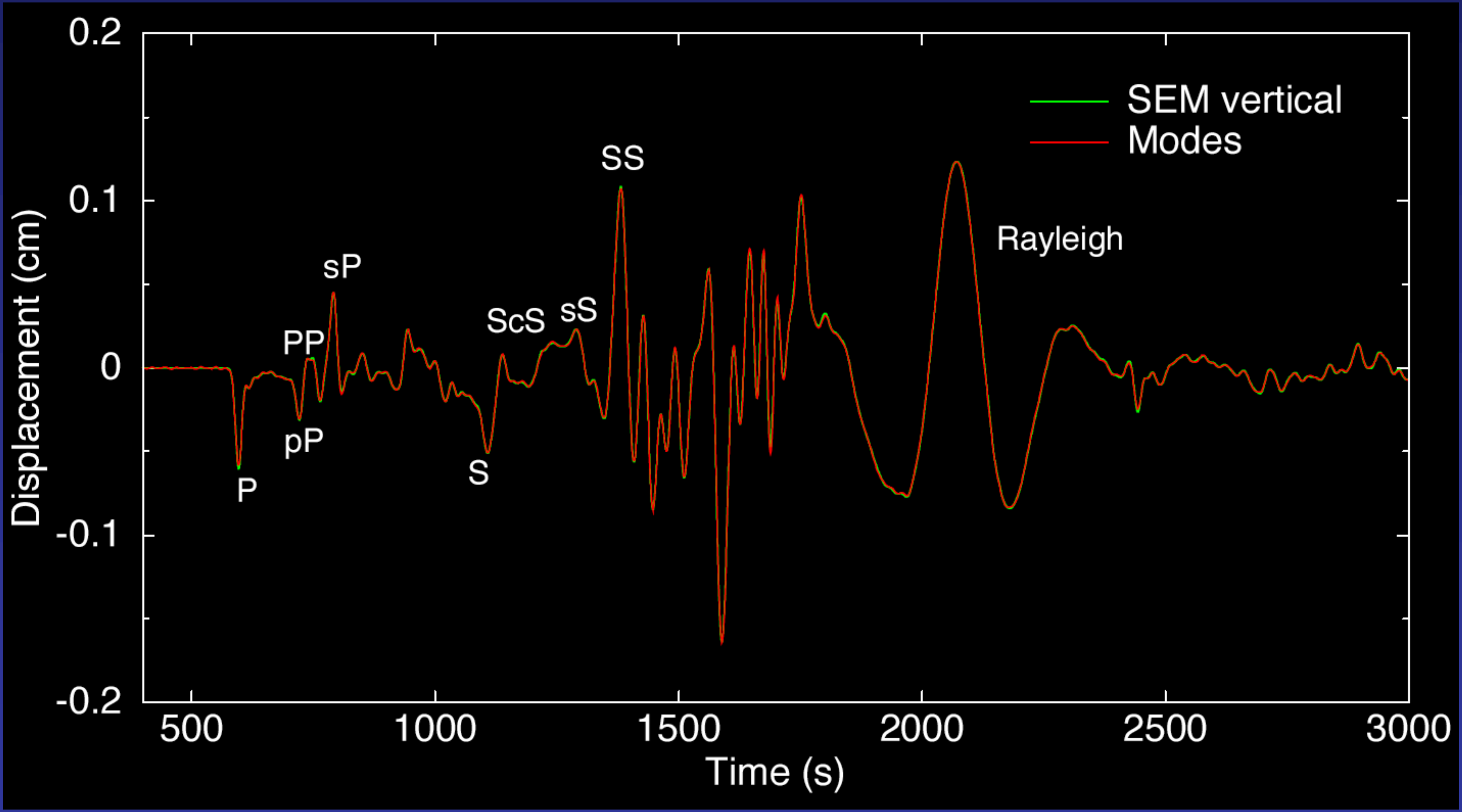




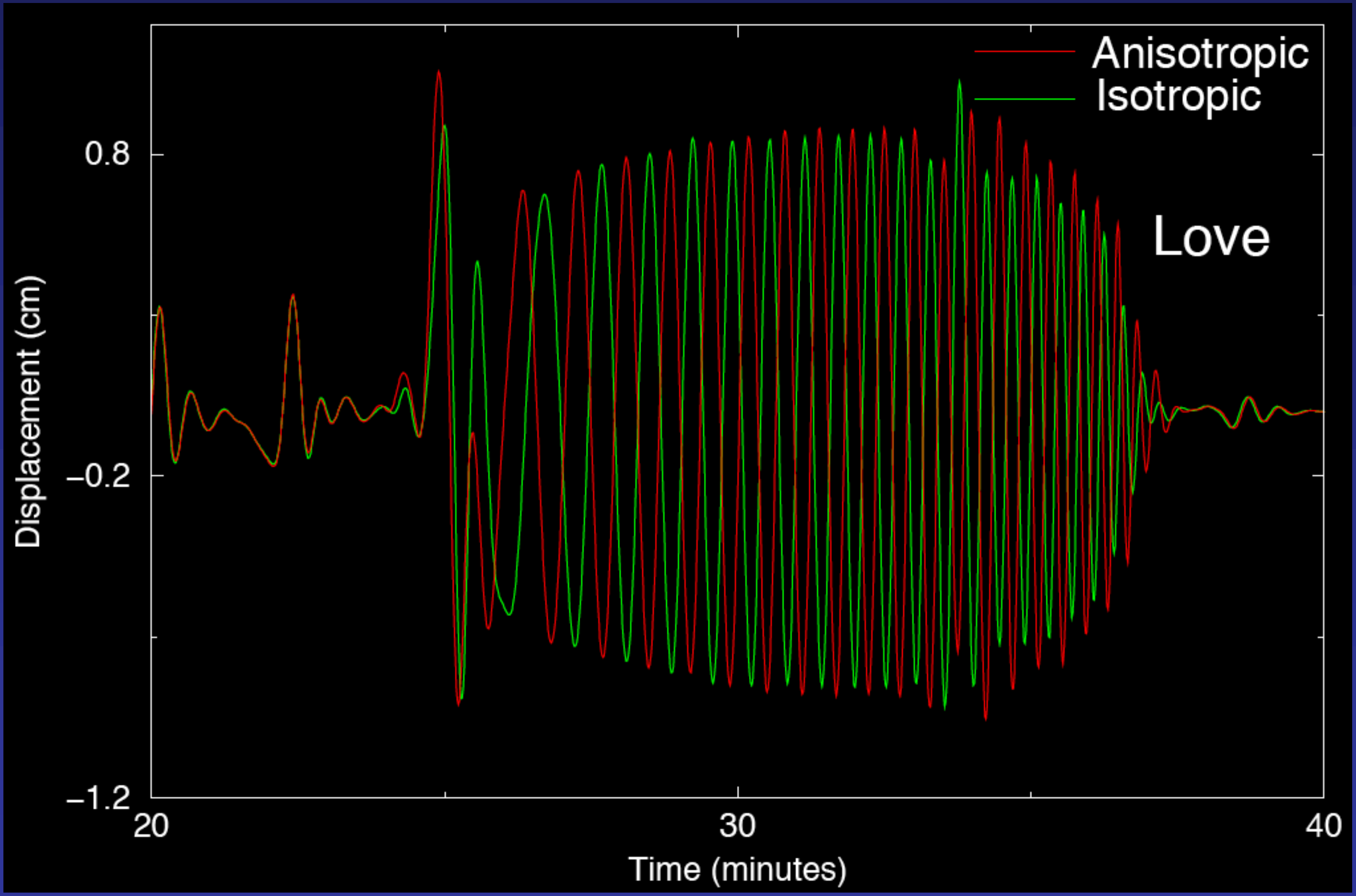
Effect of Attenuation



Attenuation

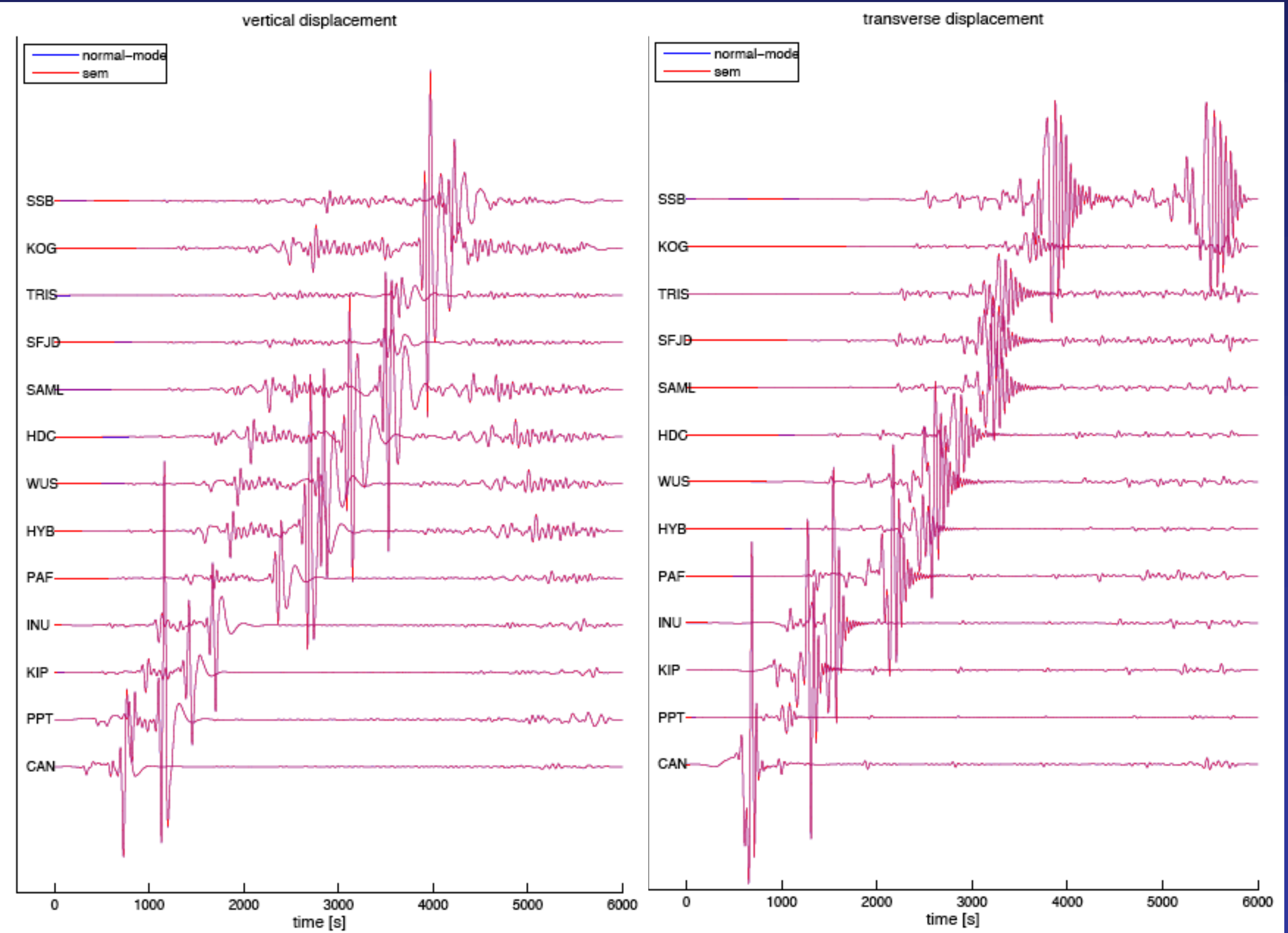


Effect of Anisotropy



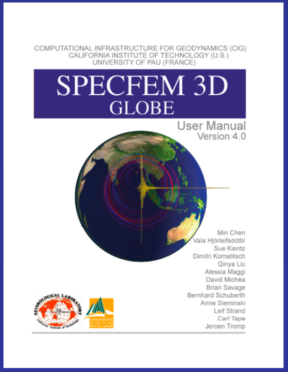
11/26/1999 Vanuatu shallow event

50 s - 500 s benchmark



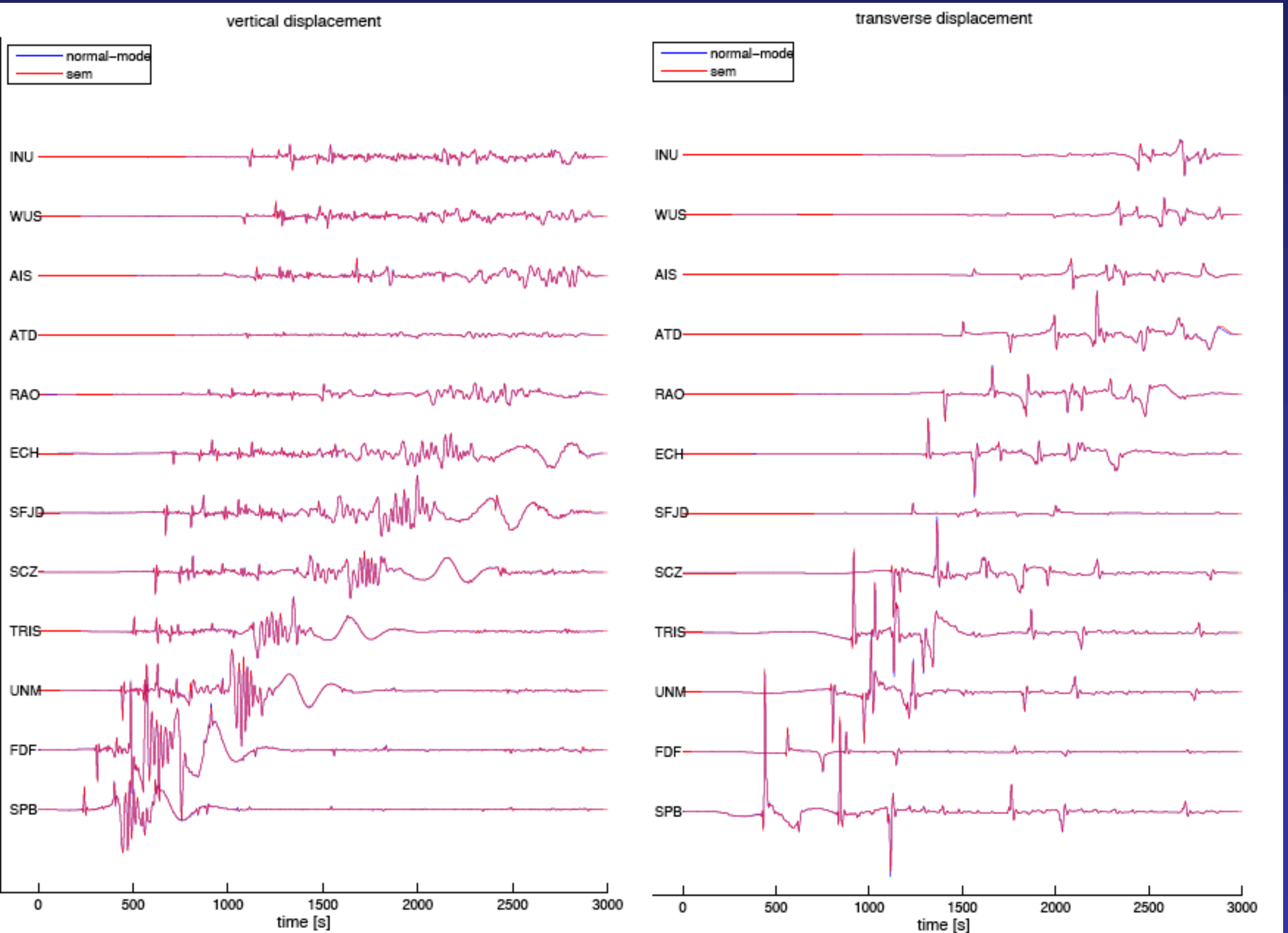
PREM benchmarks include:

- Attenuation
- Transverse isotropy
- Self-gravitation (Cowling)
- Two-layer crust



6/4/1994 Bolivia Deep Event

10 s - 500 s benchmark



PREM benchmarks include:
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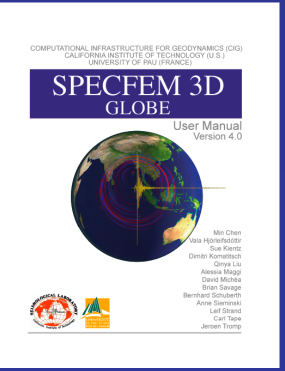

PKP Phases

15 s - 500 s