Project 02

Problem 1:

- Part A:

```
Arithmetic Returns - Last 5 Rows:
                 SPY      AAPL      EQIX
Date
2024-12-27 -0.011492 -0.014678 -0.006966
2024-12-30 -0.012377 -0.014699 -0.008064
2024-12-31 -0.004603 -0.008493  0.006512
2025-01-02 -0.003422 -0.027671  0.000497
2025-01-03  0.011538 -0.003445  0.015745
Total Standard Deviation (Arithmetic Returns): 0.012679754664908071
```

- Part B:

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Log Returns - Last 5 Rows:
                 SPY      AAPL      EQIX
Date
2024-12-27 -0.011515 -0.014675 -0.006867
2024-12-30 -0.012410 -0.014696 -0.007972
2024-12-31 -0.004577 -0.008427  0.006602
2025-01-02 -0.003392 -0.027930  0.000613
2025-01-03  0.011494 -0.003356  0.015725
Total Standard Deviation (Log Returns): 0.01263076561554221
```

Problem 2:

- Part A:
  To calculate the portfolio value, I extracted the stock prices for SPY, AAPL, and EQIX on the specified date and then used the dot product to multiply these prices by their respective holdings. This gave me the total value of the portfolio for that day.

  Portfolio Value on 1/3/2025: 251862.4969482422

- Part B:
  To calculate the VaR and ES for each stock and the entire portfolio at a 5% alpha level, I implemented three different methods.

  First, I calculated daily returns from the given price data using percentage change and removed any missing values.

  Method 1: Normal Distribution with Exponentially Weighted Covariance (EWMA, λ=0.97): I centered the returns by subtracting the mean (assuming a zero mean return). I then calculated the exponentially weighted covariance matrix using a decay factor (λ=0.97). Using this covariance matrix, I computed the portfolio variance and derived VaR and ES using the normal quantile function.

Method 2: T-Distribution with Gaussian Copula: I fitted a T-distribution to each stock's demeaned returns to account for fat-tailed behavior. I then generated simulated returns using the fitted T-distribution and used them to estimate portfolio returns. From these simulated returns, I extracted VaR as the 5th percentile and ES as the average loss beyond VaR.

Method 3: Historical Simulation: I used past return data and calculated portfolio returns based on historical stock returns and portfolio holdings. Then I estimated VaR from the 5th percentile of historical portfolio returns and ES as the mean of losses beyond VaR.

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Method 1: Normally distributed with exponentially weighted covariance
SPY VaR: -0.013285, SPY ES: -0.016660
AAPL VaR: -0.022177, AAPL ES: -0.027811
EQIX VaR: -0.025266, EQIX ES: -0.031684
Portfolio VaR: -6.576392, Portfolio ES: -8.247061

Method 2: T-distribution using a Gaussian Copula
SPY VaR: -0.014738, SPY ES: -0.018247
AAPL VaR: -0.021925, AAPL ES: -0.029540
EQIX VaR: -0.025244, EQIX ES: -0.032743
Portfolio VaR: -5.504036, Portfolio ES: -7.562423

Method 3: Historic simulation using the full history
SPY VaR: -0.014738, SPY ES: -0.018247
AAPL VaR: -0.021925, AAPL ES: -0.029540
EQIX VaR: -0.025244, EQIX ES: -0.032743
Portfolio VaR: -7.929037, Portfolio ES: -9.909338
```

EWMA (Method 1) gives the lowest Portfolio VaR (-6.58) and ES, likely because it assumes normally distributed returns, which may underestimate tail risks.

T-Copula (Method 2) slightly increases Portfolio VaR (-5.50) and ES, as the T-distribution accounts for fat tails, making it more sensitive to extreme losses.

Historical Simulation (Method 3) gives the highest Portfolio VaR (-7.93) and ES (-9.91), meaning past extreme losses were worse than those predicted by parametric models.

- Part C:

  Normal VaR and ES using Exponentially Weighted Covariance:

  Method:
  - Assumes returns are normally distributed.
  - Uses an exponentially weighted moving average (EWMA) to estimate the covariance matrix, with λ = 0.97 placing more weight on recent data.

  Advantages:
  - Computationally efficient and simple to implement.
  - Adapts to changing market conditions by emphasizing recent volatility.

Disadvantages:

- Assumes normality, which may underestimate risk during market stress.
- Ignores fat tails, making it less effective in capturing extreme events.

T-distribution VaR and ES using Gaussian Copula:
Method:

- Fits a t-distribution to capture heavy tails and excess kurtosis.
- Uses a Gaussian Copula to model dependence between assets.

Advantages:

- Better captures extreme events compared to the normal distribution.
- More realistic risk estimation during market crises.

Disadvantages:

- More complex and computationally intensive to implement.
- The choice of degrees of freedom impacts results and may require careful tuning.

Historical Simulation VaR and ES
Method:

- ○ Uses actual historical returns to calculate risk metrics, without assuming any distribution.

Advantages:

- Easy to understand and implement, directly reflecting past market behavior.
- No reliance on distributional assumptions, making it more flexible.

Disadvantages:

- Dependent on past data, which may not capture future shocks.
- Requires a long dataset to provide accurate risk estimates.

Problem 3:

- Part A:
  Implied Volatility: 0.3350803924787905
  To calculate the implied volatility, I used the Brent method (root_scalar) to solve for the sigma that makes the Black-Scholes call option price equal to the market price (C_market). I defined a function that calculates the difference between the Black-Scholes price and the market price, and then used root_scalar to find the value of sigma within the range of 0.01 to 2.0. If the solver converges, I return the implied volatility; otherwise, I return NaN.

- Part B:
  Delta: 0.6659296527386921, Vega: 5.640705439230117, Theta: -5.544561508358898
  Change in Call Price due to 1% IV increase: 0.056498427517343686

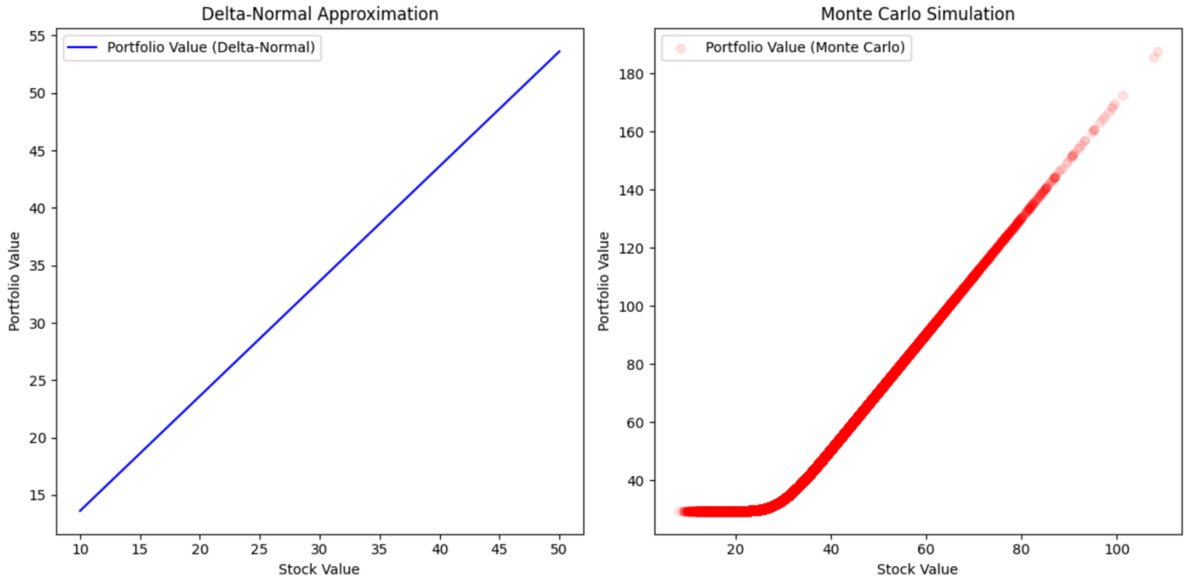
  Proof: By multiplying the Vega by the change in volatility, Vega×0.01=5.6407×0.01=0.0564, which matches the change in the call price computed using the Black-Scholes formula with the new volatility.

- Part C:
  Put Price: 1.259297360849974
  Put-Call Parity Difference: 3.552713678800501e-15
  Put-Call Parity difference is 3.55e-15, which is effectively zero, indicating that Put-Call Parity holds within numerical precision.

- Part D:
  To calculate the VaR and ES for a portfolio consisting of 1 call option, 1 put option, and 1 share of stock, I used two methods: the Delta-Normal Approximation and Monte Carlo Simulation. The code takes into account option value decay over a 20-day holding period and assumes normally distributed returns with a 0% expected return and 25% annual volatility. The 5% confidence level is used to estimate the potential risk for both methods, where the Delta-Normal Approximation uses volatility and portfolio sensitivities, and the Monte Carlo Simulation generates simulated returns assuming a normal distribution.

  VaR Delta-Normal Approximation: -3.987
  ES Delta-Normal Approximation: -5.0
  VaR Monte Carlo Simulation: 0.148
  ES Monte Carlo Simulation: 0.148

- Part E:



The graph compares portfolio value vs. stock price under Delta-Normal Approximation (first plot) and Monte Carlo Simulation (second plot). The Delta-Normal method shows a smooth, linear increase in portfolio value as stock price rises, reflecting the assumption of a proportional relationship. In the Monte Carlo plot, the portfolio value increases in a similar pattern but is based on simulated stock price paths, reflecting real market conditions. While both methods provide similar trends, Monte Carlo can better capture non-linear behaviors and risks in more complex portfolios compared to the linear assumption of Delta-Normal.