Trinomial Tree Option Pricing: Implementation and Analysis

Yu Chen

Option pricing models play a crucial role in financial engineering, providing methods to estimate the fair value of derivative securities. Among these models, tree-based approaches offer a flexible and intuitive framework for valuing options with varying payoff structures and early exercise features. The binomial tree model, extensively studied and widely used in practice, discretizes time and stock price movements to approximate the evolution of an option's value. However, the model's convergence can be slow, necessitating a large number of time steps to achieve reasonable accuracy.

An alternative to the binomial tree is the trinomial tree model, which improves convergence by introducing an additional middle branch at each node—representing no change in price alongside the traditional up and down paths. This added state results in a more symmetric structure that better captures the lognormal distribution of asset returns and more closely mirrors continuous-time stochastic processes, such as those described by the Black-Scholes framework. At each time step, the model calculates three transition probabilities based on the asset's volatility, time to maturity, risk-free rate, and dividend yield. These probabilities are selected to ensure risk-neutral valuation, with the expected return of the asset aligned to the risk-free rate.

For European options, the model computes the option value at each node by backward induction, starting from the known terminal payoff at maturity. For American options, the model incorporates the possibility of early exercise by comparing the continuation value with the intrinsic value at each step. Dividends are taken into account by adjusting the asset's growth factor accordingly, ensuring the model remains applicable under real-world conditions.

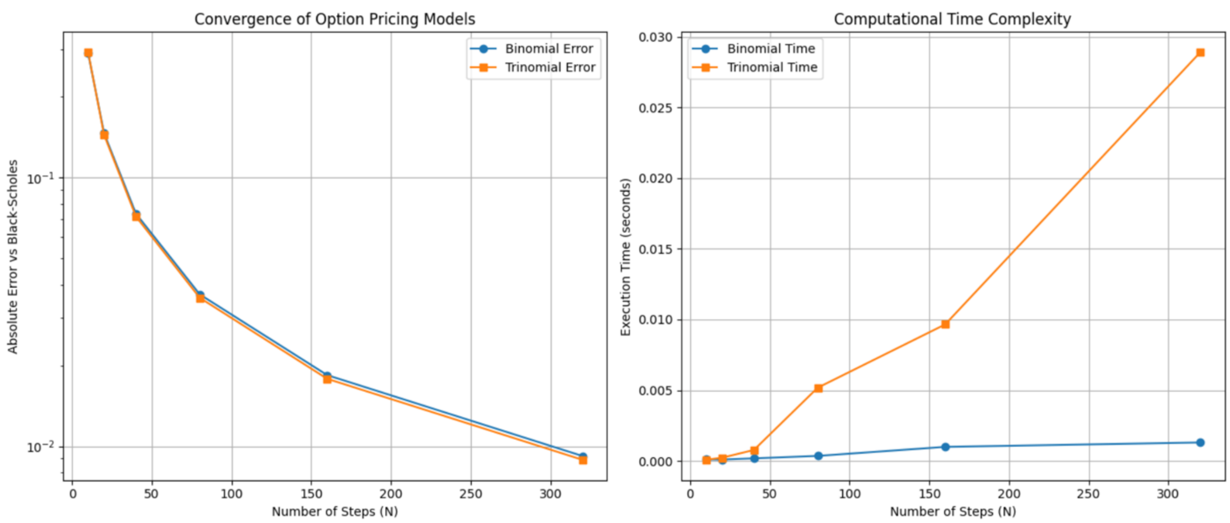
The trinomial tree model is implemented in Python, supporting both European and American options, with or without dividends. The implementation constructs the tree of asset prices, calculates terminal payoffs, and iteratively computes option values backward in time to determine fair pricing.

To evaluate the performance of the trinomial tree model relative to the binomial tree, both convergence behavior and computational cost were examined. The analysis focuses on European call options, which allow for direct error comparison with the Black-Scholes benchmark. Since American options lack a closed-form pricing formula, direct benchmarking is not feasible. Therefore, the convergence analysis focuses on European call options, which allow for comparison with the Black-Scholes model.

To evaluate convergence and computational complexity, a European call option was priced using both models under the following parameters: $\mathrm{S} = 100, \mathrm{K} = 100, \mathrm{T} = 1, \mathrm{r} = 0.05, \sigma = 0.3,$ and $\mathrm{q} = 0$. Option prices were computed under varying numbers of time steps ($N = 10,20,40,80,160,320$). For each case, both the absolute error, as measured relative to the Black-Scholes price, and the execution time were recorded. As shown in figure "Convergence of Option Pricing Models", both models exhibit decreasing error as N increases, indicating convergence. Although both models yield nearly identical errors across all tested step sizes, the trinomial model consistently produces slightly lower absolute error, confirming its marginally faster convergence.

The execution time, shown in figure "Computational Time Complexity", was recorded to empirically assess the computational cost of each model. While the trinomial model provides higher pricing accuracy, its computational complexity increases due to the three-way branching structure at each node. The additional runtime remains moderate and becomes more noticeable only as N grows large. In most practical scenarios, this trade-off is acceptable given the gains in convergence and accuracy.

The convergence behavior for put options is similar and is omitted here for brevity.



To further evaluate the practical performance of the trinomial tree model, additional pricing experiments were conducted using more volatile market conditions and a lower number of time steps, where convergence differences are expected to be more visible. The parameters were set to $\mathrm{S} = 100, \mathrm{K} = 100, \mathrm{T} = 1, \mathrm{r} = 0.05, \sigma = 0.3,$ and $\mathrm{N} = 20$, with dividend yields of $q = 0$ and $q = 0.02$.

As shown in Table 1, under these conditions the trinomial model continues to produce option prices that are very close to the theoretical Black-Scholes values, even at relatively low time steps. For instance, in the no-dividend European call scenario, the trinomial price is 14.0870, deviating by only 0.1443 from the Black-Scholes value of 14.2313, while the binomial result is 14.0849 with a deviation of 0.1464. Similar levels of accuracy are observed in European puts and options with dividend adjustments. For example, with a 2% dividend yield, the trinomial European call price is 12.8752, differing by 0.1451 from the Black-Scholes benchmark of 13.0203, slightly more accurate than the binomial result of 12.8766.

These findings support the observation that the trinomial tree model maintains reliable performance even at lower step counts, offering slightly improved precision due to its refined three-branch structure. The differences between models are small but consistent, and become more apparent when higher accuracy is needed with limited computational steps.

Table 1: Option Prices under $N = 20$

Option Type	Dividend	Style	Black-Scholes	Binomial Tree	Trinomial Tree
Call	No	European	14.2313	14.0849	14.0870
Put	No	European	9.3542	9.2079	9.2100
Call	No	American	N/A	14.0849	14.0870
Put	No	American	N/A	9.7980	9.7246
Call	Yes	European	13.0203	12.8766	12.8752
Put	Yes	European	10.1234	9.9797	9.9788
Call	Yes	American	N/A	12.8766	12.8753
Put	Yes	American	N/A	10.3872	10.3321

To evaluate model performance at higher levels of discretization, the same experiments were repeated using $N = 200$. As shown in Table 2, both the binomial and trinomial models produced option prices that closely approximate the theoretical Black-Scholes values across all European cases. For example, in the no-dividend European call scenario, the trinomial model yields a price of 14.2170, compared to 14.2165 from the binomial model, while the Black-Scholes benchmark is 14.2313, demonstrating an absolute deviation of less than 0.015 for both methods.

Similar levels of accuracy were observed for European put options and for options with dividend adjustments. The differences between the binomial and trinomial models become negligible at this level of granularity, indicating that both models achieve strong convergence when sufficient steps are used. Nonetheless, the trinomial model continues to offer marginal improvements in precision, reinforcing its utility, especially when computational steps are constrained.

Table 2: Option Prices under $N = 200$

Option Type	Dividend	Style	Black-Scholes	Binomial Tree	Trinomial Tree
Call	No	European	14.2313	14.2165	14.2170
Put	No	European	9.3542	9.3395	9.3399
Call	No	American	N/A	14.2165	14.2170
Put	No	American	N/A	9.8632	9.8564
Call	Yes	European	13.0203	13.0059	13.0060
Put	Yes	European	10.1234	10.1090	10.1091
Call	Yes	American	N/A	13.0059	13.0060
Put	Yes	American	N/A	10.4822	10.4790

For American options, both models produce consistent and reasonable results across all settings. In particular, American put options show higher prices than their European counterparts, reflecting the added value of early exercise. American options offer the flexibility of early exercise, allowing for the realization of intrinsic value before maturity, especially in falling markets. This ability to realize intrinsic value early—rather than waiting until maturity—explains the premium often observed in American put prices. The close agreement between binomial and trinomial prices for American options further confirms the correctness and stability of both implementations.

This study examined the implementation and empirical performance of the trinomial tree model for option pricing, highlighting its advantages over the traditional binomial approach. Results show that the trinomial model consistently achieves slightly faster convergence to the Black-Scholes benchmark for European options and effectively incorporates early exercise features in American options. Additionally, it remains robust under dividend-paying conditions. While it incurs moderately higher computational cost, this trade-off is acceptable in practical applications where precision is essential. Overall, the trinomial tree offers a reliable and flexible framework for valuing a wide range of options when analytical solutions are unavailable.