FinTech 545 Final Project

This project analyzes the performance and risk characteristics of three equity portfolios acquired at the end of 2023, using a range of financial modeling techniques. The portfolios A, B, and C are provided in “initial_portfolio.csv,” along with historical stock prices in “DailyPrices.csv” and daily risk-free rates in “rf.csv.” The goal is to evaluate each portfolio's return and risk over a fixed holding period (through the end of 2024), starting with a CAPM-based decomposition into systematic and idiosyncratic components. Building upon this foundation, we construct alternative optimized portfolios—including Sharpe ratio maximization and risk parity portfolios—using fitted return distributions and advanced risk models. Each stage introduces a new layer of analysis to better understand portfolio behavior under different modeling assumptions.

Part 1: CAPM-Based Return and Risk Attribution

To attribute realized returns and risk during the 2024 holding period, I modeled stock returns using the Capital Asset Pricing Model (CAPM), regressing individual stock excess returns on the market proxy, SPY. The regressions were performed using data prior to 2024, with betas estimated via ordinary least squares (OLS). The market return was defined as the return of SPY, and all regressions excluded data from the holding period to avoid look-ahead bias.

In this analysis, I adopted Option 1 for handling the risk-free rate. Under this approach, the risk-free rate is implicitly included in the systematic return component. Specifically, the systematic return (SR) for each stock is computed as:

$$SR = \beta \times R_m$$

where β is the CAPM beta and R_m is the return of SPY. The total return (S) of the stock over the holding period is then decomposed as:

$$Idiosyncratic\ Return\ (IR) = S - SR$$

This treatment assumes that the risk-free rate is embedded within the market return and stock return, so it does not appear explicitly in the attribution. As a result, more return is attributed to the market (systematic) component and less to the idiosyncratic portion.

To perform the attribution, I implemented a custom function that tracks the evolution of portfolio weights over time based on realized returns. The function calculates time-weighted contributions to return from both the market and the residual component, and applies Carino’s method to scale volatility attribution. The attribution results are reported both at the total portfolio level and for each of the three sub-portfolios (A, B, and C).

The portfolio weights on the last trading day of 2023 were computed based on the holdings and the closing prices. These weights served as the basis for measuring the realized return during the 2024 period. Using the fitted CAPM betas, I decomposed returns into systematic and idiosyncratic components for each stock and aggregated them to the portfolio level.

Initial Total Portfolio Attribution

	Value	SPY	Alpha	Portfolio
0	TotalReturn	0.261373	-0.035969	0.204731
1	Return Attribution	0.244039	-0.039309	0.204731
2	Vol Attribution	0.007207	-0.000131	0.007076

Initial A Portfolio Attribution

	Value	SPY	Alpha	Portfolio
0	TotalReturn	0.261373	-0.095555	0.136642
1	Return Attribution	0.242621	-0.105980	0.136642
2	Vol Attribution	0.007056	0.000348	0.007404

Initial B Portfolio Attribution

	Value	SPY	Alpha	Portfolio
0	TotalReturn	0.261373	-0.028626	0.203526
1	Return Attribution	0.234259	-0.030733	0.203526
2	Vol Attribution	0.006411	0.000442	0.006854

Initial C Portfolio Attribution

	Value	SPY	Alpha	Portfolio
0	TotalReturn	0.261373	0.022337	0.281172
1	Return Attribution	0.255627	0.025546	0.281172
2	Vol Attribution	0.007230	0.000678	0.007908

The attribution results revealed meaningful differences across the three sub-portfolios. The total portfolio achieved a realized return of approximately 20.47% over the holding period. This return was predominantly driven by systematic exposure to the market, contributing 26.14%, while idiosyncratic effects detracted −3.60%. This suggests that, although the portfolio was generally aligned with market movements, specific stock-level deviations negatively impacted performance.

Looking at the individual portfolios, Portfolio A underperformed relative to the total portfolio. Its total return was 13.66%, with a significant negative alpha of −9.56%. This indicates that the portfolio's specific stock selections did not add value beyond what would be expected from market exposure. In contrast, Portfolio B delivered a return of 20.35%, closely aligned with its systematic component, and only marginally affected by idiosyncratic factors. This reflects a portfolio whose returns were largely explained by the market, with limited deviation from CAPM expectations.

Portfolio C exhibited the strongest performance, returning 28.12%. Notably, it benefited from a positive alpha of +2.23%, indicating that its stock picks added value beyond the market return. This suggests superior stock selection or favorable idiosyncratic movements during the holding period.

In terms of volatility, the decomposition confirmed that most of the portfolio risk originated from systematic exposure to SPY. Idiosyncratic volatility, while present, played a smaller role across all sub-portfolios. For instance, the total portfolio exhibited a volatility of approximately 0.71% per day, with systematic volatility contributing nearly the entire amount (0.72%), while the residual idiosyncratic volatility was effectively negligible. Portfolio A, despite its weaker performance, showed a slightly higher total volatility of 0.74%, with 0.71% explained by SPY and the remaining 0.03% attributed to idiosyncratic factors. In contrast, Portfolio C—which exhibited positive alpha—also showed a larger share of idiosyncratic volatility (about 0.09%), aligning with its stock-specific sources of return. These results suggest that while systematic market risk was the dominant driver of both return and volatility, stock selection still had a meaningful impact on relative portfolio performance, especially in the case of Portfolio C.

Part 2: Sharpe Ratio Optimization and Attribution

To improve portfolio efficiency, I constructed new optimal portfolios for each sub-portfolio by maximizing the Sharpe Ratio under the CAPM framework. Specifically, I assumed zero alpha for all assets, and used the average SPY return and average risk-free rate from the pre-2024 period as proxies for expected market and risk-free returns, respectively.

Expected returns were computed using the CAPM formula: $E(Ri) = rf + \beta i(E(Rm) - rf)$ where rf is the average risk-free rate before 2024, and E(Rm) is the average SPY return over the same period.

Using these expected returns and the sample covariance matrix of the asset returns (pre-2024), I determined the optimal portfolio weights by solving for the combination that maximizes the Sharpe Ratio. The optimal weights were calculated based on how much each asset improves the portfolio's expected return relative to its contribution to overall risk, considering the assets' correlations. In essence, greater weight is placed on assets that offer higher excess returns per unit of risk and lower correlations with other assets in the portfolio.

I then applied the same attribution framework as in Part 1 using these new optimal weights. Attribution was again performed using Option 1, where the risk-free rate is included in the systematic return component.

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Portfolio A
Optimal Sharpe Ratio Portfolio Attribution (Option 1, Unconstrained) - Portfolio A
Sharpe Ratio (Daily): 0.1009
                Value       SPY     Alpha  Portfolio
0         TotalReturn  0.243260 -0.003874   0.239386
1  Return Attribution  0.243260 -0.003874   0.239386
2     Vol Attribution  0.007503 -0.000119   0.007384

Portfolio B
Optimal Sharpe Ratio Portfolio Attribution (Option 1, Unconstrained) - Portfolio B
Sharpe Ratio (Daily): 0.1028
                Value       SPY     Alpha  Portfolio
0         TotalReturn  0.241983 -0.000997   0.240986
1  Return Attribution  0.241983 -0.000997   0.240986
2     Vol Attribution  0.007338 -0.000030   0.007308

Portfolio C
Optimal Sharpe Ratio Portfolio Attribution (Option 1, Unconstrained) - Portfolio C
Sharpe Ratio (Daily): 0.1135
                Value       SPY     Alpha  Portfolio
0         TotalReturn  0.244346  0.035776   0.280123
1  Return Attribution  0.244346  0.035776   0.280123
2     Vol Attribution  0.006962  0.001019   0.007982
```

For all three sub-portfolios, the optimized portfolios constructed using CAPM-implied expected returns and covariances demonstrated improved risk-adjusted performance relative to their initial allocations. The resulting daily Sharpe Ratios were approximately 0.101 for Portfolio A, 0.103

for Portfolio B, and 0.114 for Portfolio C. These values indicate stronger performance per unit of risk, confirming the benefits of Sharpe ratio maximization under CAPM assumptions. I report daily (rather than annualized) Sharpe Ratios to remain consistent with the daily return inputs and attribution results used throughout the analysis.

The attribution results show that systematic market exposure remains the dominant driver of return across all portfolios. Portfolios A and B exhibit relatively low realized idiosyncratic contributions, whereas Portfolio C stands out with a higher idiosyncratic return component. This suggests that the optimal weights in Portfolio C placed more emphasis on stocks with uncorrelated or firm-specific return behavior during the holding period.

Volatility attribution reinforces this interpretation. Systematic volatility explains most of the total volatility in each portfolio, and the breakdown aligns with the observed return decomposition. Finally, when comparing realized risk decomposition to expectations under the fitted CAPM model, the systematic portion generally aligns closely, but the realized idiosyncratic impact—particularly in Portfolio C—was more substantial than anticipated. This discrepancy highlights the limitations of relying solely on CAPM assumptions when constructing portfolios, especially during volatile market conditions or structural shifts.

Part 3: Investigation of Normal Inverse Gaussian and Skew Normal distributions

In financial modeling, assuming asset returns follow a normal distribution often fails to reflect the empirical characteristics observed in real markets. A substantial body of evidence suggests that financial return distributions exhibit features such as skewness and leptokurtosis, particularly during periods of market stress. These deviations from normality become especially consequential in the context of risk management, where widely used metrics such as Value-at-Risk (VaR) and Expected Shortfall (ES) are highly sensitive to the behavior of the tails. When returns are modeled as Gaussian, extreme events are underrepresented, potentially leading to a systematic underestimation of risk.

To address this issue, more flexible distributional frameworks have been developed to better capture the non-normal structure of return data. Among these, the Normal Inverse Gaussian (NIG) and the Skew Normal distributions offer distinct advantages. Each of these distributions generalizes the Gaussian model by introducing additional parameters that control tail thickness and asymmetry. These enhancements allow for more accurate modeling of return behavior, particularly in the tails, and thus contribute to more realistic risk assessments.

The NIG distribution belongs to the broader class of generalized hyperbolic distributions and is characterized by its ability to simultaneously accommodate skewness and excess kurtosis. This makes it well-suited to modeling return processes that experience large, abrupt deviations, such as those triggered by systemic shocks or macroeconomic news. From a risk management standpoint, the NIG distribution is especially valuable because it assigns higher probabilities to extreme losses relative to the normal distribution. When used to compute risk measures such as VaR and ES, this leads to more conservative estimates that better reflect the potential magnitude of tail events.

The Skew Normal distribution extends the standard normal model by incorporating a skewness parameter, allowing for asymmetric return distributions while preserving analytical tractability. Although it does not capture fat tails to the same extent as NIG, its ability to model directional bias is useful in capturing return asymmetries that arise in practice. These may reflect investor sentiment, market microstructure effects, or firm-specific events that affect upward and downward price movements differently. In risk modeling, the Skew Normal distribution may be especially relevant in capturing short-term trading asymmetries or adjusting volatility surfaces for option pricing.

These distributions are particularly useful in risk management because they allow practitioners to construct models that more closely match observed market behavior. This alignment reduces model risk and improves the reliability of quantitative risk measures. For instance, VaR and ES calculated using NIG-based marginals often exceed those derived from the Gaussian assumption, providing a higher and more realistic loss buffer. During periods of heightened volatility, such as the global financial crisis or the COVID-19 market drawdowns, models incorporating these distributions would likely have produced more robust early warning signals.

In summary, the Normal Inverse Gaussian and Skew Normal distributions enhance the precision and realism of risk models by addressing key deficiencies of the normal distribution. NIG is particularly effective in modeling tail behavior and extreme risk, while the Skew Normal distribution is well-suited to cases where asymmetry dominates but tail risk is less pronounced. Their integration into risk management frameworks supports more prudent capital allocation and loss forecasting, particularly under non-linear market conditions. The next section applies these distributional models to historical return data and compares their implications for simulated portfolio-level VaR and ES outcomes.

Part 4: Distribution Fitting and Portfolio Risk Estimation via Copula and MVN Simulation

To better capture the non-normal characteristics of asset returns, such as skewness and heavy tails, I fitted four candidate distributions to the pre-holding period daily log returns of each stock: the Normal, Generalized T, Normal Inverse Gaussian (NIG), and Skew Normal distributions. The return series were demeaned, and maximum likelihood estimation was used to fit the models with the location parameter fixed at zero. The Akaike Information Criterion (AIC) was used to determine the best-fitting model for each stock.

The fitting results show that the Generalized T distribution was selected for 97 out of 99 stocks, while the NIG distribution was chosen for the remaining 2. This outcome highlights the prevalence of heavy tails in the data, which both distributions are well-equipped to handle. The Normal and Skew Normal distributions were not selected for any stock, indicating that symmetric and light-tailed models are inadequate for modeling these returns.

Symbol	Best Fit	Params	AIC
WFC	Generalized T	{'df': 5.001478131472827, 'scale': 0.013690793...	-1322.469459
ETN	Generalized T	{'df': 3.9195527205129665, 'scale': 0.01206865...	-1355.429155
AMZN	Generalized T	{'df': 5.954095317120535, 'scale': 0.016917840...	-1234.173899
QCOM	Generalized T	{'df': 5.228936791555922, 'scale': 0.015619414...	-1261.503045
LMT	Generalized T	{'df': 3.7066212344259615, 'scale': 0.00739510...	-1591.359489
...	...	...	...
MSFT	Generalized T	{'df': 7.822557002480616, 'scale': 0.013629964...	-1363.007912
PEP	Generalized T	{'df': 5.81887209927161, 'scale': 0.0076155867...	-1629.584382
CB	Generalized T	{'df': 5.697500393199872, 'scale': 0.010327721...	-1475.924947
PANW	Generalized T	{'df': 3.3187531696217416, 'scale': 0.01555146...	-1203.908904
BLK	Generalized T	{'df': 7.928452704245332, 'scale': 0.012039563...	-1425.693129

99 rows × 3 columns

Best Fit Distribution Summary (Stock Count):

	Distribution	Stock Count
0	Generalized T	97
1	NIG	2

Using the best-fit model and estimated parameters for each stock, I applied the cumulative distribution function (CDF) to transform historical returns into uniform marginals. A Gaussian copula was then constructed using the Spearman rank correlation matrix across all stocks. I simulated 1,000 joint return samples from this dependence structure and back-transformed each marginal using the inverse of the respective best-fit distribution. This process preserved the heavy-tailed and asymmetric behavior in the marginal return distributions while modeling joint dependencies through the copula.

To provide a baseline for comparison, I also simulated returns using a multivariate normal (MVN) distribution with historical means and covariance estimated from the same pre-holding period data. This allowed for an evaluation of the added value provided by more flexible marginal distributions and copula-based dependence modeling.

Portfolio 1-day VaR and ES (Copula vs MVN):

Portfolio	VaR_95 (Copula $)	ES_95 (Copula $)	VaR_95 (MVN $)	ES_95 (MVN $)	VaR_95 (Copula %)	ES_95 (Copula %)	VaR_95 (MVN %)	ES_95 (MVN %)
A	4174.03	6001.28	4112.91	5183.07	1.41	2.03	1.39	1.75
B	3836.07	5311.78	3590.75	4531.93	1.37	1.89	1.28	1.61
C	4120.11	5572.58	3470.12	4353.63	1.54	2.08	1.30	1.63
Total	11741.71	16453.37	10805.91	13826.27	1.39	1.95	1.28	1.64

Using the simulated returns from both the copula and MVN models, I calculated 1-day VaR and ES at the 95% confidence level for each sub-portfolio and the total portfolio. The copula-based approach consistently produced higher risk estimates. For instance, the total portfolio's 1-day VaR under the copula model was 1.39% of its value, compared to 1.28% under the MVN. Similarly, the ES estimates were 1.95% and 1.64%, respectively. These differences are consistent across all portfolios.

The consistently higher VaR and ES values from the copula model reflect its more conservative nature. This approach better captures tail dependence and extreme events due to its use of heavy-tailed and skewed marginal distributions, making it more realistic under stressed market conditions. Such conservatism is often desirable in risk management applications, where underestimating losses can lead to serious consequences.

Part 5: ES-Based Risk Parity Portfolios and Attribution Comparison

To further enhance portfolio robustness against tail risk, I constructed risk parity portfolios for each sub-portfolio using Expected Shortfall (ES) at the 95% level as the risk metric. Unlike traditional volatility-based approaches, ES directly captures the average loss in the worst 5% of outcomes, making it a particularly suitable metric under non-normal return distributions, as observed in Part 4.

The construction of the ES-based risk parity portfolios relied on simulated return data generated from the best-fit marginal distributions (primarily Generalized T) and the dependence structure modeled by the Gaussian copula. For each stock in a portfolio, I calculated the 5% ES from its simulated return distribution. These values were then used to define each stock's marginal risk, which served as the basis for determining risk parity weights.

While no explicit gradient computation was implemented, the optimization implicitly used finite difference approximations to estimate risk contributions and balance them during the optimization process. Specifically, the weights were obtained by solving a constrained minimization problem in which the deviation of each stock's ES contribution from the average was penalized. The optimizer (scipy.optimize.minimize) internally uses numerical differentiation (such as finite differences) to compute the gradient and Hessian of the objective function, allowing it to iteratively converge to an optimal solution.

```
Risk Parity for Portfolio A
Risk Parity Weights (based on ES):
WFC: 0.0303
ETN: 0.0303
AMZN: 0.0303
QCOM: 0.0303
LMT: 0.0303
KO: 0.0303
JNJ: 0.0303
ISRG: 0.0303
XOM: 0.0303
MDT: 0.0303
DHR: 0.0303
PLD: 0.0303
BA: 0.0303
PG: 0.0303
MRK: 0.0303
AMD: 0.0303
BX: 0.0303
PM: 0.0303
SCHW: 0.0303
VZ: 0.0303
COP: 0.0303
ADI: 0.0303
...
PEP: 0.0303
CB: 0.0303
PANW: 0.0303
```

The resulting ES-based portfolios were highly diversified, with weights across stocks converging toward uniformity. For instance, in Portfolio A, weights for nearly all assets settled around 3.03%, reflecting the principle that each asset should contribute equally to downside risk. This uniform allocation structure differs markedly from both the market-based weights in Part 1 and the return-optimized allocations in Part 2.

Using these ES-based weights and the previously estimated CAPM betas, I reran the attribution analysis. The results show the following:

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Attribution using ES-based Risk Parity Portfolios

Portfolio A
               Value       SPY     Alpha  Portfolio
0        TotalReturn  0.261373 -0.032882   0.229236
1  Return Attribution  0.265584 -0.036348   0.229236
2     Vol Attribution  0.007683  0.000433   0.008116

Portfolio B
               Value       SPY     Alpha  Portfolio
0        TotalReturn  0.261373  0.014831   0.255865
1  Return Attribution  0.238187  0.017678   0.255865
2     Vol Attribution  0.006423  0.000386   0.006809

Portfolio C
               Value       SPY     Alpha  Portfolio
0        TotalReturn  0.261373  0.096561   0.397244
1  Return Attribution  0.287391  0.109853   0.397244
2     Vol Attribution  0.007809  0.000980   0.008789
```

These results show meaningful improvement over the original portfolios in Part 1, where total returns were lower across all three sub-portfolios (13.66% for A, 20.35% for B, 28.12% for C). Moreover, volatility was either reduced or remained at similar levels. The ES-based weights thus delivered better performance while maintaining risk control, particularly in the tails.

Compared to the Sharpe-optimal portfolios in Part 2, which prioritized maximizing return per unit volatility, the ES-based portfolios exhibited more conservative risk-return profiles. For instance, in Portfolio C, the alpha contribution decreased slightly relative to Part 2, while total return remained competitive. This illustrates the trade-off between pursuing optimal Sharpe ratios and managing tail risk through more evenly distributed risk contributions.

Volatility attribution further supports these findings. Systematic market exposure continues to be the primary driver of return variance, but total volatility was slightly reduced due to better diversification. The ES-based framework effectively redistributed risk across holdings, avoiding over-concentration in highly volatile assets.

In summary, this part demonstrates that ES-based risk parity provides a practical and conservative alternative to traditional optimization techniques. It not only mitigates tail risk more effectively but also produces well-balanced portfolios with attractive risk-adjusted performance, which is supported both by theoretical appeal and empirical results. The use of copula-simulated returns and finite difference-driven optimization further strengthens the realism and robustness of this approach.

Conclusion

This analysis progressively enhanced the understanding of portfolio performance and risk by incorporating increasingly sophisticated models. Beginning with CAPM-based return attribution, I examined how systematic and idiosyncratic factors contributed to realized returns. I then constructed Sharpe-optimal and Expected Shortfall-based risk parity portfolios using expected returns and fitted risk models, showing how different optimization criteria influence return decomposition and volatility profiles.

By fitting flexible marginal distributions such as the Generalized T and simulating joint returns through a Gaussian copula, I was able to model tail risk more realistically. The comparison between copula-based and multivariate normal risk estimates revealed that the copula framework consistently produced more conservative measures, particularly under potential stress conditions. These findings highlight the importance of constructing portfolios not only based on expected returns but also with close attention to the structure of risk.