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Problem 17.

Generate 5e4 samples and the results is like follows:



The MATLAB code is as follows:

samples = [];

sampleSize = 50000;

for i = 1:sampleSize

u = rand();

if u <= (3 - exp(-2))/3

p = find(mnrnd(1,[1/3,2/3]));

if p == 1

x = -log(1 - u)/2;

if x <= 1

samples = [samples,x];

end

elseif p == 2

x = u;

samples = [samples,x];

end

else

x = -log(3 - 3\*u)/2;

samples = [samples,x];

end

end

hist(samples,200);

Problem 20:

Loop

Step 1: Compute the derivative of , we get ;

Step 2: Denote the vector, where . The vector forms an un-normalized distribution. Sample an integer from the un-normalized multinomial distribution formed by the vector .

Step3 : Use Inverse transform method or acceptance and rejection method to sample from

Until some terminating conditions

Problem 26:

Let

Given that

We have

Where is a Markov chain.

Problem 27.

(a). MATLAB code:

pi0 = [0.1,0.9,0];

A = [0.3,0.3,0.4;0.1,0.9,0;0.1,0.1,0.8];

%% section a

cur\_pi = pi0;

samples = [];

for i = 1:500

samples = [samples,sampleFunc(cur\_pi)];

cur\_pi = cur\_pi\*A;

end

function samp = sampleFunc(cur\_pi)

u = rand();

if u < cur\_pi(1)

samp = 1;

elseif u < (cur\_pi(1) + cur\_pi(2))

samp = 2;

else

samp = 3;

end

end

(b). MATLAB code

%% section b

proposalDist = [1/3,1/3,1/3];

sampleRej = [];

cur\_pi = pi0;

while length(sampleRej) < 500

c = max(cur\_pi)\*3;

x = find(mnrnd(1,proposalDist));

u = rand();

if u < cur\_pi(x)/(c/3)

sampleRej = [sampleRej,x];

end

cur\_pi = cur\_pi\*A;

end

(c). Suppose in the sample path the ordered pair appears times,

Then the transition probability

(d).

Solve the equation and we get .

Verification from numerical simulations:

Samples from Inverse CDF:



Verification from Acceptance-Rejection method:



Problem 28

Suppose

Let

Then .

Consider the eigen decomposition of the matrix

where .

Then we consider

As , then we have

For arbitrary initial distribution , as

Therefore, the distribution converges to the stationary distribution at the rate of , where is the second largest eigenvalue of .

Problem 30.

We denote the row vector where all entries are 0 except the entry is 1.

Then we have , where is the transition matrix.

Also, we have

Problem 33.





MATLAB code:

clc,clear,close all

N = 1000;

Ms = [100,500,1000,2000,5000,8000,10000] ;

L = 100;

A = rand(N);

pik = rand([N,1]);

pik = pik/sum(pik);

%Use rejection sampler

proposalDist = ones(1,N)/N;

c = max(pik)/(proposalDist(1));

At = A';

piKp1\_est = zeros(N,1);

tic

piKp1 = At\*pik;

t1 = toc;

mses = [];

vars = [];

times = [];

for M = Ms

errs = [];

piKp1\_estm = [];

ts = [];

for i = 1:L

num = 0;

tic

while (num < M)

x = find(mnrnd(1,proposalDist));

u = rand();

if u < pik(x)/(c/N)

%accept

piKp1\_est = piKp1\_est + At(:,x);

num = num + 1;

end

end

piKp1\_est = piKp1\_est/M;

t = toc;

ts = [ts,t];

piKp1\_estm = [piKp1\_estm,piKp1\_est];

err = norm(piKp1 - piKp1\_est);

errs = [errs,err];

end

times = [times,mean(ts)];

meanEst = mean(piKp1\_estm,2);

var = trace((piKp1\_estm - meanEst)'\*(piKp1\_estm - meanEst))/L;

mse = sum(errs)/L;

mses = [mses,mse];

vars = [vars,var];

end

figure

plot(Ms,vars,'\*');

xlabel('# of iterations')

ylabel('estimator variance')

figure

plt1 = plot(Ms,times,'b\*');

hold on

plt2 = line([0,10000],[t1,t1],'LineWidth',1.5','Color','red');

legend([plt1,plt2],{'Randomized method','Direct multiplication'})

xlabel('# of iterations')

ylabel('Running time [sec]')

Problem 34.

Collect the daily stock closed price of APPLE INC from Sep 16, 2002 to Sep 16, 2019 and the compute the log return of one day. We have the following histogram:





I used Pearson chi square test.

The continues normal distribution is segmented into 46 bins that have non-zero items in them.

Compute the statistic , which is very large and we can reject the hypothesis that the distribution is normal.