Homework 2

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Problem 2

There's some connection problem to 'login.xsede.org' on Sunday, 29 March. So I ran sample sort on Mercer, a cluster of NYU. I ran the program on 4 nodes with 16 cores on each node.

N	Time (s)
500	1.398149
1000	1.307633
2000	1.360792
4000	1.338509
8000	1.298755
16000	1.339113
32000	1.410995
64000	1.394691
128000	1.442345
256000	1.503083
512000	1.613325
1024000	1.917241
2048000	2.384213
4096000	2.755875
8192000	3.972705
16384000	5.897198

Final Project Summary

I would like to work with Yifei Sun. The following is the summary.

1 Background

This project will study a parallel implementation of Galerkin finite element method for evolutionary stochastic partial differential equation

$$du = [-Au + f(u)]dt + G(u)dW(t), u(0) = u_0$$

on a bounded domain D, where A is a linear differential operator, $W(x,t)$ is a Q-Wiener process or Cylindrical Wiener process (also called space-time white noise). Here we suppress the dependence on space (and time) and write $u(t)$ for $u(t,x)$, $f(u)$ for $f(u,t,x)$, $G(u)$ for $G(u,t,x)$ and $W(t)$ for $W(t,x)$.

A general setting for u, A, f, G is following:
$u_0 \in H$, where H is a Hilbert space (e.g. $L^2(D)$). W is a U-valued Q-Wiener process or Cylindrical Wiener process, where U is another Hilbert space and Q is its covariance operator. u is viewed as a H-valued stochastic process. $A : \mathcal{D}(A) \subset H \to H$ satisfies that it has a complete set of orthonormal eigenfunction with all positive eigenvalues. $f : H \to H$. $G : H \to L_0^2$, where L_0^2 is set of linear operators $B : U_0 \to H$ such that: $\|B\|_{L_0^2} := \|BQ^{\frac{1}{2}}\|_{HS(U,H)} < \infty$ (here $U_0 = \{Q^{\frac{1}{2}}u : u \in U\}$).

2 Numerical Scheme

2.1 Semi-implicit Euler-Maruyama and Galerkin finite element approximation

Now we consider a finite dimensional subspace V_h of H, say, the finite element space, and seek the finite element approximation of the solution $u_h(t) \in V_h$ to the SPDE:

$$\begin{aligned} < du_h, v > &=< [-Au + f(u)]dt, v > + < G(u)dW(t), v > \\ &=< -Au, v > dt+ < f(u)], v > dt+ < G(u)dW(t), v > . \forall v \in V_h \end{aligned}$$

The term $< -Au, v >$ is rewritten (usually by integral by part) to a bi-linear form, which satisfies: 1. Continuity, 2. Garding's inequality (To have well-posedness). And we denote by A_h the matrix for the bilinear form on space V_h. The stochastic field $W(t)$ has the expansion

$$W(t) = \sum_{j=1}^{\infty} \sqrt{q_j}\xi_j\beta_j(t),$$

where q_j are eigenvalues of covariance operator Q, ξ_j are eigenfunctions of covariance operator Q. $\beta_j(t)$ are independent Brownian motions. Notice that for space-time white noise, $q_j = 1$ for all j.
We make a truncation for $W(t)$, only take the first J terms:

$$W_J(t) = \sum_{j=1}^{J} \sqrt{q_j}\xi_j\beta_j(t),$$

and use semi-implicit Euler-Maruyama in time discretization, i.e., take implicit in $-Au$ part, but explicit in $f(u)$ and $G(u)$ part:

$$< u_{h,n+1}, v > - < u_{h,n}, v >=< -Au_{h,n+1}, v > \Delta t+ < f(u_{h,n}), v > \Delta t+ < G(u_{h,n})\Delta W_{J,n}, v > .$$

After these preparation, we now have the vector form for this numerical scheme:

$$(I + \Delta t A_h)\mathbf{u}_{h,n+1} = \mathbf{u}_{h,n} + \mathbf{f}(\mathbf{u}_{h,n})+ < NOISE\ TERM >, \tag{1}$$

where assuming V_h has basis $\phi_i,\ i = 1, 2, \cdots, m$, the i-th entry of the noise term is given by matrix $(< G(u_{h,n})\sqrt{q_j}\xi_j, \phi_i >)$ times the increment of Brownian motion $(\Delta\beta_j)$ during Δt.

2.2 Parallelization

Two parts of this scheme can be parallelized:
1. Parallely assemble the noise term $(< G(u_{h,n})\sqrt{q_j}\xi_j, \phi_i >)(\Delta\beta_j)$
2. Use parallel algorithm to solve linear system (1).