Valuation Project 1: Binomial Tree

Choice #1 Contingent Coupon Auto-Callable Notes with Step Up redemption feature

Based on the Performance of the Common Stock of Microsoft Corporation, with Step-Up Redemption Threshold Price Feature

1. Parameter estimation and discussion of data collection and tree construction

a. Time

The timeline is determined as below:

t_0	t_1	T_1	T_2
Pricing date	Original issue date	Final determination date	Maturity date
August 3,2018	August 8,2018	August 3, 2021	August 6, 2021

b. Risk-free Rates (see Appendix 3. Risk-free Rates for screenshot in Bloomberg)

Maturity Date	Discount	Days	Date of Option	Days	Interpolated Discount Factor	risk-free rate
11/07/2018	0.994048	96	8/8/18	5	N/A	0.02238679
08/07/2020	0.944615	735	8/3/21	1096	0.91675728	0.02854791
08/09/2021	0.916294	1102	8/6/21	1099	0.91652577	0.02855271

Here, we used the interpolation method to calculate the continuously compounded annual risk-free rate, which will be used in calculating the risk-neutral probabilities in the CRR model. Moreover, we used the continuously compounded annual risk-free rate calculated by the most recent 3-month discount rate to discount option values.

c. Dividend Yields (see Appendix 2. Dividends for screenshot in Bloomberg)

Since the project was issued in 2018, we used the historical dividends from August 15, 2018 to November 14, 2018 for 2018. For the future dividends, we used the Bloomberg dividends forecast (BDVD) divided by initial stock price as dividends from 2019 to maturity date.

Moreover, we counted the day between t_0 and ex-dividend dates and used them for further calculation.

Ex-Date	Historical dividend	Day count	Div yield
8/15/18	0.42	12	0.003887
11/14/18	0.46	101	0.004258
2/20/19	0.46	197	0.004258

Est. Ex-Date	BDVD Forecast	DVD Trend	Imp. Range Low	Imp. Range High	Day count	Div yield
5/15/19	0.46	0.44	0.3495	0.4874	282	0.004258
8/14/19	0.46	0.44	0.4418	0.4909	371	0.004258
11/20/19	0.5	0.48	0.3961	0.3961	467	0.004628
2/19/20	0.5	0.465	0.2377	0.5398	556	0.004628
5/20/20	0.5	0.465	0.2377	0.5398	647	0.004628
8/19/20	0.5	0.465	0.4028	0.4266	736	0.004628

11/18/20	0.53	0.493	0.4028	0.4266	825	0.004906
2/17/21	0.53	0.475	N/A	N/A	914	0.004906
5/19/21	0.53	0.475	N/A	N/A	1006	0.004906
8/18/21	0.53	0.475	N/A	N/A	1095	0.004906

d. Implied volatilities (see Appendix 1. Volatility for screenshot in Bloomberg)

We searched the implied volatilities in Bloomberg using different threshold prices. We analyzed the valuation results using different volatilities, which can be found in sensitivity analysis.

Threshold price	**86.432**	**113.442**	**118.844**	**124.246**
Volatility	28.280%	24.896%	24.542%	24.603%

e. Tree construction

We used the CRR model to build our binomial tree, the parameters are below:

$$u = e^{\sigma\sqrt{\Delta t}}, d = \frac{1}{u}, q_u = \frac{e^{r\Delta t} - d}{u - d}, q_d = 1 - q_u$$

Note that the underlying asset Microsoft Corporation common stock pay fixed dividend quarterly, so we used the proportional dividend on every ex-dividend date to approximate the fixed dividend, so that the stock tree will recombine.

2. Valuation procedure

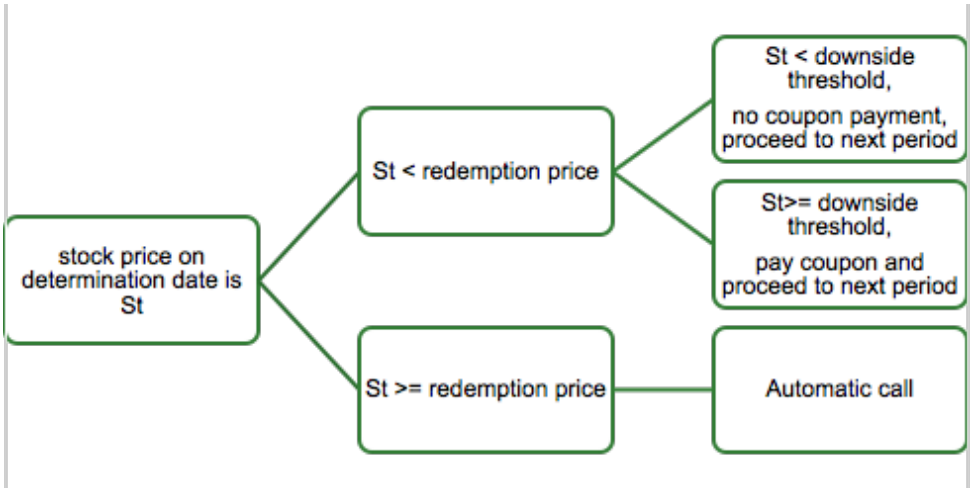
a. Brief summary of target security

Issuer: Morgan Stanley Finance LLC
Underlying Stock: Microsoft Corporation common stock (DASDAQ: MSFT)
Principle amount: $10 per security
Feature: Contingent Income Auto-Callable Securities
Initial stock price: $108.04 ($S_0$)

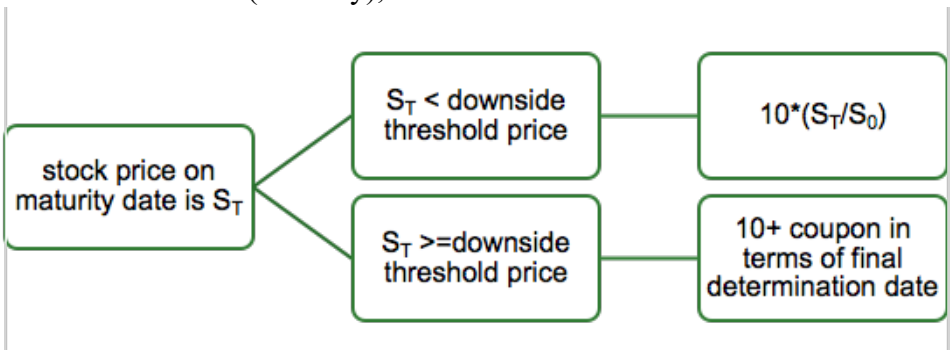
Determination date and corresponding threshold price:

Determination Date	**Threshold Price**
November 5, 2018, February 4, 2019, May 3, 2019, August 5, 2019,	$113.44(105%*$S_0$)
November 4, 2019, February 3, 2020, May 4, 2020, August 3, 2020,	$118.84(110%*$S_0$)
November 3, 2020, February 3, 2021, May 3, 2021	$124.25(115%*$S_0$)
August 3, 2021	$86.432(80%*$S_0$)

The following diagrams illustrate the payoff for the securities depending on (1) the determination closing price(S_t) and (2) the final share price(S_T).
On the first eleven determination dates,



On the last determination date (maturity),



b. Parameter choosing

N	Risk-free Rates	Dividend Yield	Volatility	124.246
1096	0.0223 (discount) 0.0285 (qu, qd)	0.00426 (0.46/S_0, quarterly)	24.542%	24.603%

Firstly, when choosing the time steps, we set the steps equal to the actual days from the pricing date to the valuation date, which ensures that the determination dates fall exactly on the nodes. This is very convenient for us to calculate coupon payment.

We used the short-term risk-free rate as the discount factor for the coupon and option payment; and used the long-term risk-free rate to calculate the risk neutral probability in the CRR model.

When choosing the sigma, we had four available volatilities. We used all of them and chose the one that gave the result closest to the estimated value by Morgan Stanley, which is 24.542%. For the dividend yield, we did the same thing and chose the most recent Bloomberg forecast dividend yield (0.46) to do our valuation.

c. Coding

The main procedure of our valuation is to put the features of our product into our model. When building the option tree, we started at the end of our tree. Since our valuation date is three days before maturity, all the payments must be discounted to the valuation date.

Then we go back from the end of the tree to the beginning. We used a judgement statement to filter the steps at the determination date. We put the determination dates into a vector called "determination". If the step is one of the determination dates, then we determine whether the early redemption should be made. We used the "l" parameter to count the time of the determination date and used the "B" parameter to denote the threshold at this particular determination date. And we compute the payment according to product description. If the step isn't one of the determination dates, we used the normal binomial tree equation to calculate the option values at each node.

There is one thing to emphasize. Since the contingent payment dates are three business days after the related determination dates, we create a vector "pay" to put all the actual dates of the coupon payment. When calculating the coupon payment, we use the "pay" vector minus the "determination" vector on each determination dates to get the exact days need to be discounted.

3. Discussion of result, accuracy of valuation

According to our model, the value of option is 9.654 (When step is 1*1096), which is close to but higher than the estimated value 9.646 in product description notes.

There are some reasons that cause this difference.

We chose one small volatility and one small dividend, which will make the valuation a bit higher. Further explanation will be discussed in sensitivity analyses on inputs of the valuation model (volatility, dividend yield, and step N).

Also, the auto-call feature and the coupon payments on different stock price levels at different dates can lead to nonlinearity errors that cause this difference.

4. Sensitivity analysis

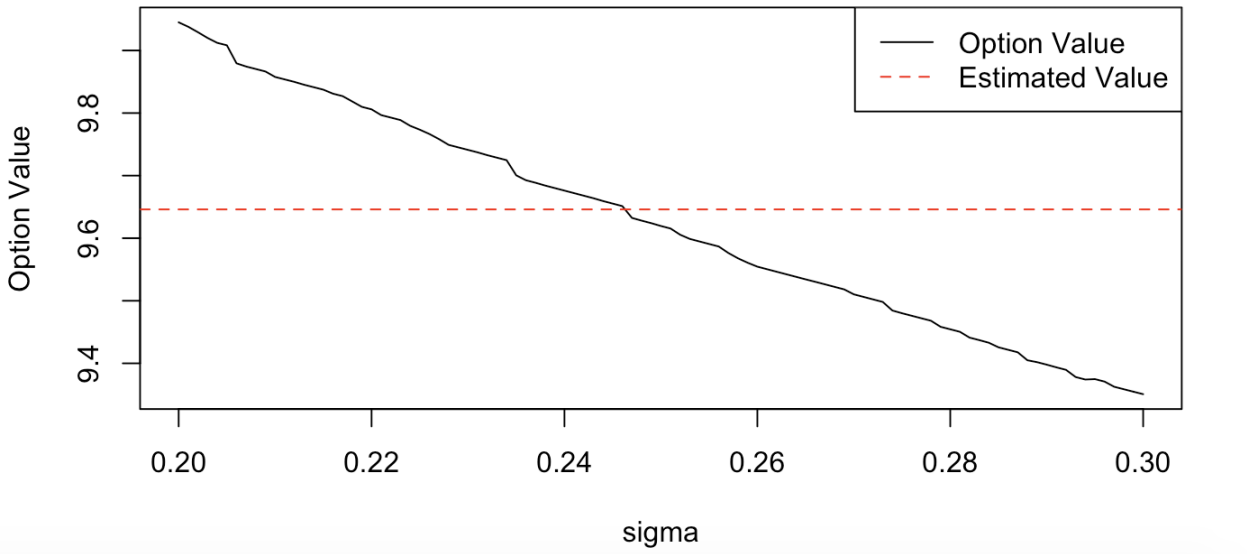
a. Volatility

The graph below shows the option value from our model by changing the parameter sigma (20%-30%, which include all possible volatilities under different threshold price from Bloomberg). The estimated value is from the product description of Morgan Stanley.

The option price is decreasing when volatility is increasing and the price ranges between 9.8 and 9.3. When sigma is large, the change of stock price is more volatile, so it's more possible for the stock price to reach both the redemption threshold and the downside threshold. When early redeemed, the payoff is $10 plus contingent coupon. But when the stock price falls below the downside threshold, the payment at maturity, which is 10 multiplied by the share performance

factor, can be significantly low. Thus, the option value tends to be undervalued when volatility is high, and vice versa.

Sigma	**0.24542**	**0.24603**	**0.24896**	**0.2828**
Option value	9.653613	9.651147	9.624172	9.437914

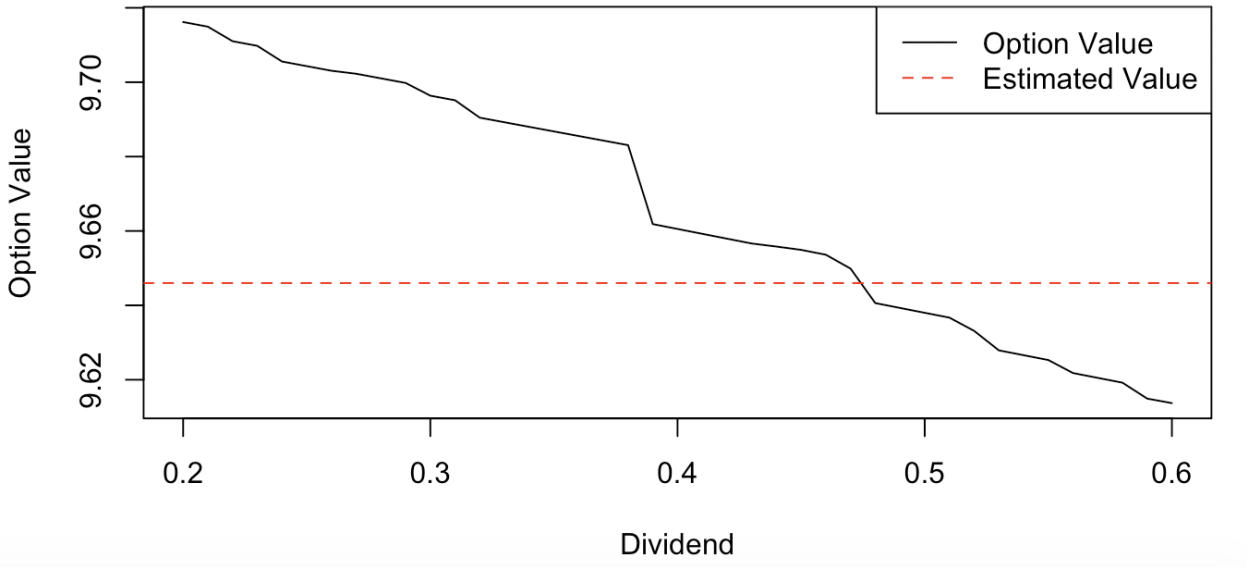


b. Dividend

The graph below shows the option value from our model by changing the parameter dividend (0.2 - 0.6, which include both the low range and the high range of dividend from Bloomberg) . The option value decreases as the dividend increases. Because the larger the dividend is, the further down the stock price tree after ex-dividend date will move. So it's more likely to reach the downside threshold. Therefore, the option value will be lower.

By comparing the volatility sensitivity to dividend sensitivity, the volatility sensitivity is larger. The reason might be that the volatility will influence the value of parameter u and d, which will make the the stock price tree more dispersed. That is to say, the volatility will have much greater influence on the stock price tree than dividend, and thus option value is more sensitive to volatility.

Dividend	**0.46**	**0.5**	**0.53**
Dividend Yield	0.004257682	0.004627916	0.004905591
Option Value	9.653613	9.638001	9.627899



c. Time steps

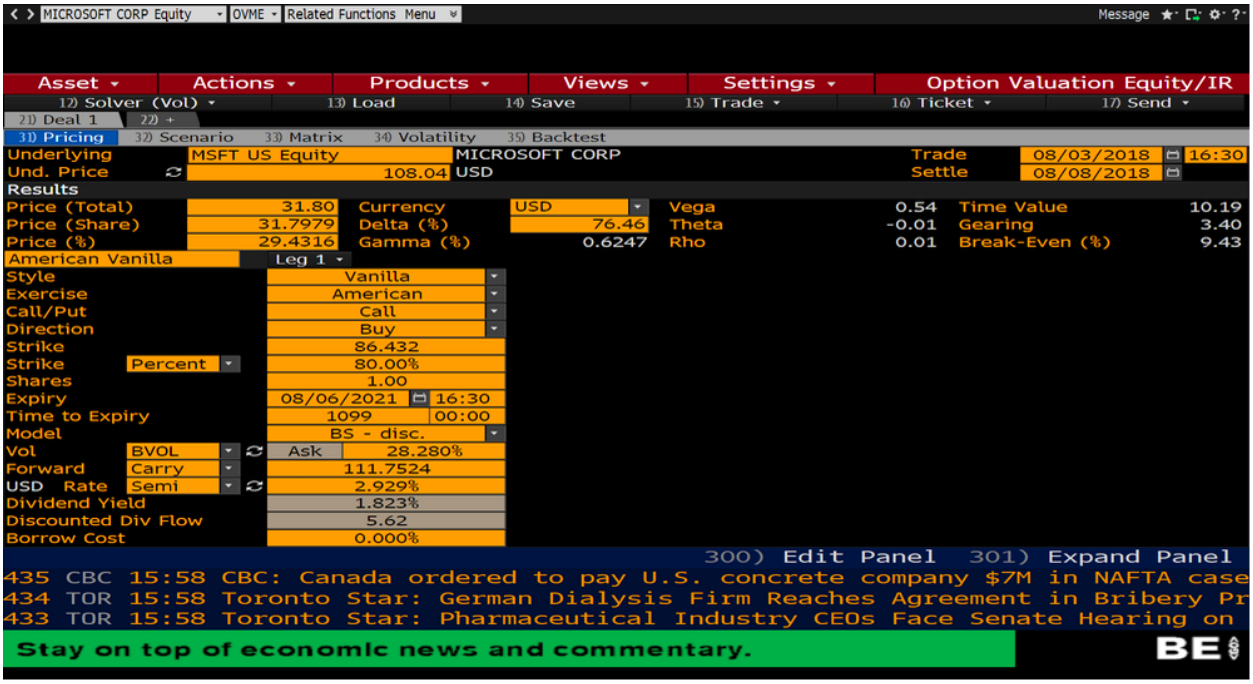
For the time steps, we used the multiple of the days between the pricing date and the valuation date. As shown in the table, time steps have little influence on the valuation. So we use 1*1096 to make the computation fast.

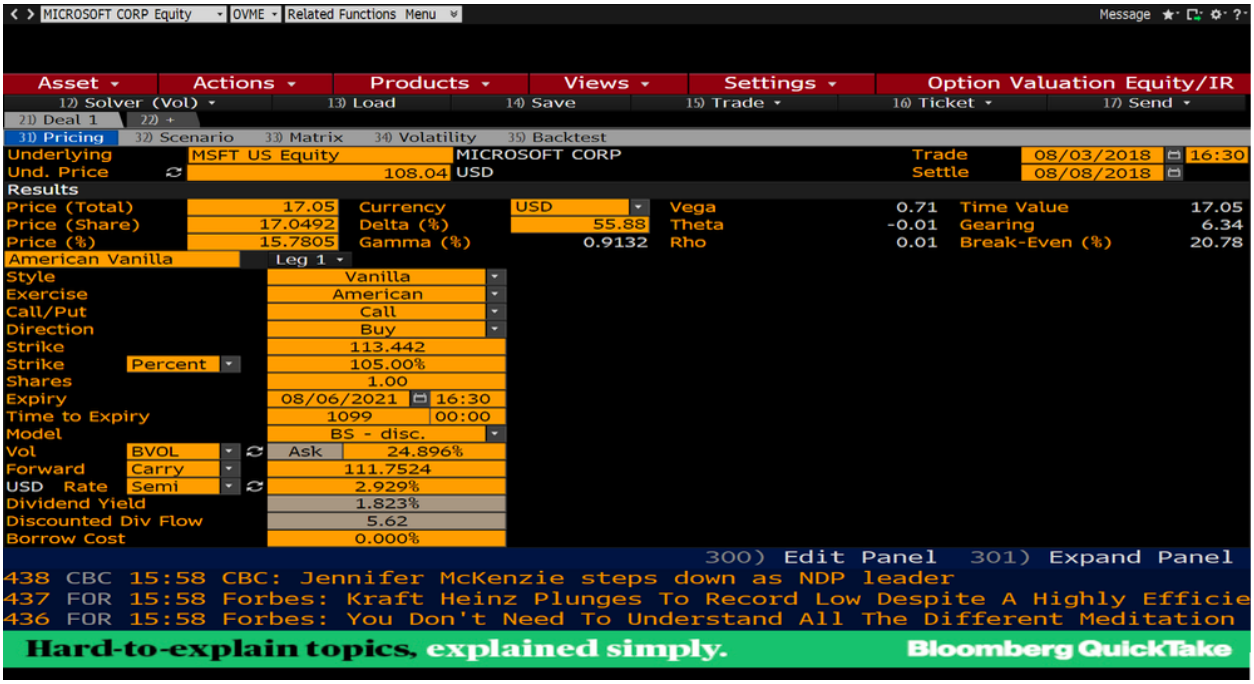
Steps	**1*1096**	**2*1096**	**3*1096**	**4*1096**	**5*1096**
Option value	9.653613	9.647001	9.642326	9.644091	9.635641

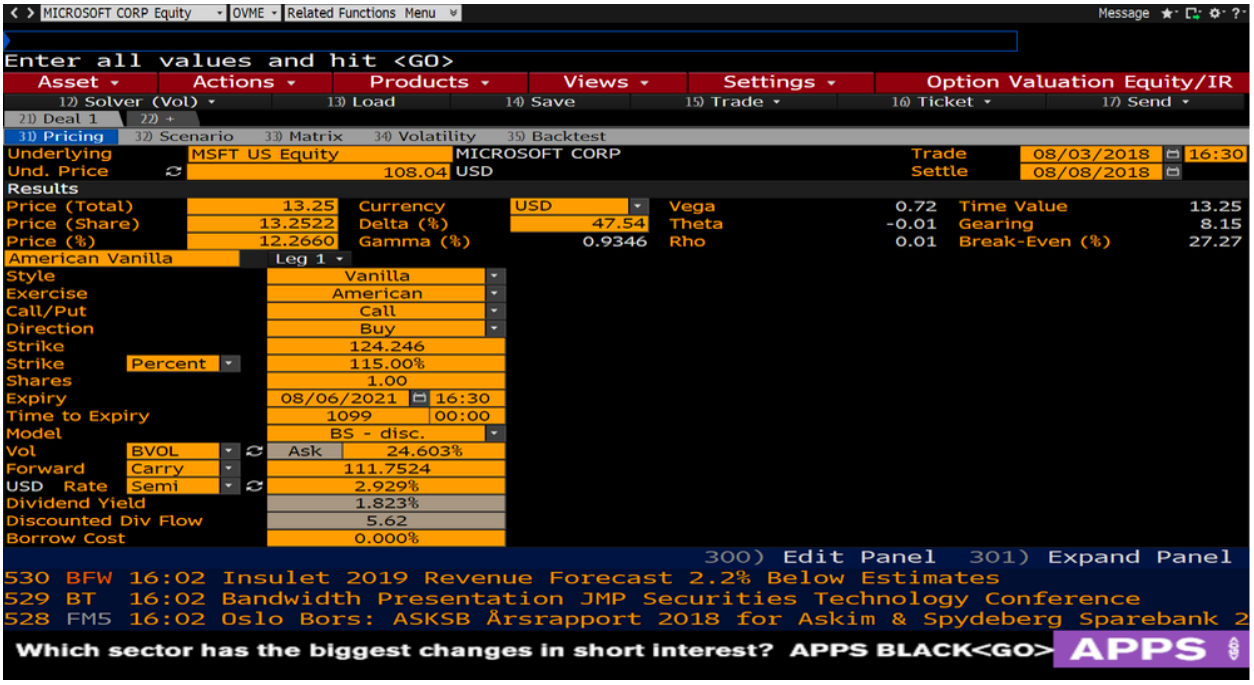
Steps	**6*1096**	**7*1096**	**8*1096**	**9*1096**	**10*1096**
Option value	9.639779	9.640935	9.632543	9.642450	9.631588

Appendix

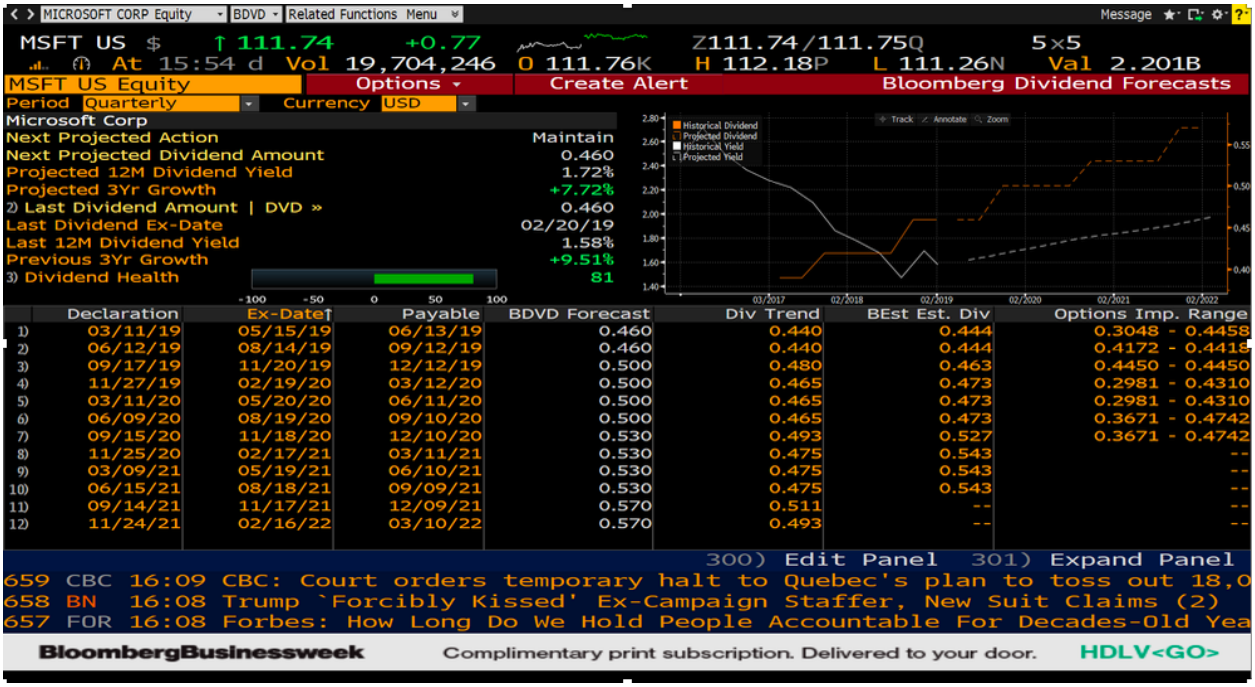
1. Volatilities







2. Dividends Yield



3. Risk-free Rates