Valuation Project 2: Finite Difference

Choice #1 Auto-Callable Contingent Interest Notes

Linked to the Common Stock of Apple Corporation

1. Parameter estimation, discussion of data collection and best estimated value

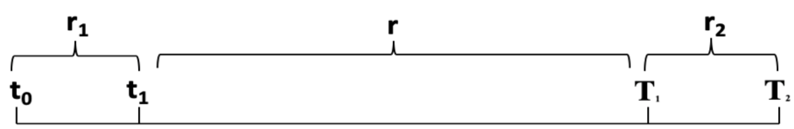
a. Time

The timeline is determined as below:

t_0	t_1	T_1	T_2
Pricing date	Original issue date	Final determination date	Maturity date
March 21,2019	March 26,2019	June 22, 2020	June 25, 2020

b. Risk-free Rates (see Appendix 3. Risk-free Rates for screenshot in Bloomberg)

Maturity Date	Discount	Date of Option	risk-free rate	
6/25/2019	0.993467	**3/26/19**	0.02491891	r1
08/07/2020	0.968815	**6/22/20**	0.02545621	r
08/09/2021	0.963057	**6/25/20**	0.02544762	r2



Here, we used the interpolation method to calculate the continuously compounded annual risk-free rate. We used the continuously compounded annual risk-free rate calculated by the most recent 3-month discount rate (r_1) to calculate the notes price on the issue date. We used r as the discount rate in the period between issue date and the final determination date, and r_2 as the discount rate in the period between final determination date and the maturity date.

c. Dividend Yields (see Appendix 2. Dividends for screenshot in Bloomberg)

Since the project has issued recently in 2019, there is no historical dividend paid. For the future dividends, we used the Bloomberg dividends forecast (BDVD) divided by initial stock price(S_0) as the inputs for dividends from 2019 to maturity date.

Note that the underlying asset Apple common stock pay fixed dividend quarterly, so we used the proportional dividend on every ex-dividend date to approximate the fixed dividend.

Est. Ex-Date	BDVD Forecast	DVD Trend	Day count	Div yield
5/10/19	0.79	0.758	50	0.004258
8/9/19	0.79	0.758	141	0.004258
11/8/19	0.79	0.758	232	0.004258
2/7/20	0.79	0.727	323	0.004258
5/8/20	0.86	0.792	414	0.004628

d. Implied volatilities (see Appendix 1. Volatility for screenshot in Bloomberg)

We searched the implied volatilities in Bloomberg using different conditional prices. We analyzed the valuation results using different volatilities, which can be found in sensitivity analysis.

barrier/trigger	195.09	152.1702
Volatility	27.171%	24.695%

e. Model construction

We used the explicit finite difference model, and the parameters are below:

$$V_j^i = \frac{1}{1+r\Delta t}\left(AV_{j+1}^{i+1} + BV_j^{i+1} + CV_{j-1}^{i+1}\right)$$

$$A = \left(\frac{1}{2}\sigma^2 j^2 + \frac{(r-\delta)j}{2}\right)\Delta t$$

$$B = 1 - \sigma^2 j^2 \Delta t$$

$$C = \left(\frac{1}{2}\sigma^2 j^2 - \frac{(r-\delta)j}{2}\right)\Delta t$$

f. Best Estimated value

By choosing jmax equals to 300, imax equals to 459*28, dividend yield as 0.004 and volalitility equals to 0.24, the best estimated value of option is 958.773. Further discussion of the parameter choosing and the reason of the error between estimated value and the prospectus price will be illustrated in Part 2 and Part 3.

2. Valuation procedure

a. Brief summary of target security

Issuer: JPMorgan Chase Financial Company LLC
Underlying Stock: Common stock of Apple (ticker: AAPL)
Principle amount: $1000 per security
Feature: Auto Callable Contingent Interest Notes
Initial stock price: $195.09 ($S_0$)

Review date and corresponding Interest barrier and trigger value price:

Review Date	Interest Barrier/Trigger Value
June 21, 2019, September 26, 2019, December 23, 2019, March 23, 2020 and June 22,2020	$152.1702 (78%*$S_0$)

b. Parameter choosing

jmax	imax	Risk-free Rates	Dividend Yield	Volatility
300	459*28	$r/r_1/r_2$	0.004 (0.79/S_0, quarterly)	24%

Firstly, when choosing the jmax and imax, we set the imax equals to the 28 times actual days (459) from the pricing date to the valuation date, which ensures that the review dates fall exactly at the grids and that delta t is smaller than sigma^2*j^2.

Then, we chose risk-free rates which can be referred to table 1b. Risk-free Rates and dividend yield which can be referred to table 1c. dividend yield.

When choosing the sigma, we tested a range of volatilities and found that 24% gaves the result which is closest to the prospectus price from JP Morgan and 24% falls in a reasonable range within what we derived from Bloomberg. For the dividend yield, we did the same thing and chose the most recent Bloomberg forecast dividend yield (0.79) to do our valuation.

c. Valuation Procedure

The main procedure of our valuation is to put the features of our product into our model.

We divided the model into two parts. First, we set the trigger grid (VT[j,i]) which assumes trigger event happened in the grid. This grid is used to find out the trigger boundary in the grid. Then, we set the non-trigger grid (V[j,i]) which assumes trigger event does not happen. When S is between 0 and 0.78S_0, we replaced V[j,i] with VT[j,i] in the non-trigger grid to get the trigger values in the notes.

First, we built the stock grid. We defined the estimated ex-dividend date as “divdate”. When the date falls on the ex-dividend date, the stock price become j*delS*(1-div).

Then, we constructed the trigger grid. There is one thing to emphasize. Since the contingent interest payment dates are three business days after the related review dates, we create a vector “paymentdates” to put all payment dates and the vector “reviewdates” which contains all the review dates. When calculating the interest payment, we use the “paymentdates” vector minus the “reviewdates” vector on each review dates to get the exact days need to be discounted. We set zero as lower boundary and set the discounted payoff and coupon as upper boundary since the notes should definitely be recalled when stock price is that high.
The upper boundary is:

$$VT^{i}_{jmax} = (1000 + 20.375) * e^{-r(TC - i\Delta t)} * e^{-r(paymentdate - reviewdate)/365}, \qquad \text{TC: next reviewdate}$$

The payoff at maturity is:

$$VT_j^{imax} = \begin{cases} (1000 + 20.375) * e^{-r_2(T_2 - T_1)}, & S \geq S_0 \\ (1000 * \dfrac{S_T}{S_0} + 20.375) * e^{-r_2(T_2 - T_1)}, & 0.78S_0 \leq S < S_0 \\ 1000 * \dfrac{S_T}{S_0} * e^{-r_2(T_2 - T_1)}, & S < 0.78S_0 \end{cases}$$

The value at the review dates is:

$$VT_j^i = \begin{cases} (1000+20.375)*e^{-r(paymentdate-reviewdate)/365}, & S \ge S_0 \\ \left(A_j * VT_{j+1}^{i+1} + B_j * VT_j^{i+1} + C_j * VT_{j-1}^{i+1}\right) + 20.375 * e^{-r(paymentdate-reviewdate)/365}, & 0.78S_0 \le S < S_0 \\ A_j * VT_{j+1}^{i+1} + B_j * VT_j^{i+1} + C_j * VT_{j-1}^{i+1}, & S < 0.78S_0 \end{cases}$$

The value in everywhere else is:

$$VT_j^i = A_j * VT_{j+1}^{i+1} + B_j * VT_j^{i+1} + C_j * VT_{j-1}^{i+1}$$

Thirdly, we built our non-trigger grid to calculate our estimated value. We replaced the value of the upper boundary and the value between S_0 and $0.78S_0$ with the value calculated by the trigger grid.

The payoff at maturity is:

$$V_j^{imax} = \begin{cases} (1000+20.375)*e^{-r_2(T_2-T_1)}, & S \ge 0.78S_0 \\ VT_j^{imax}, & S < 0.78S_0 \end{cases}$$

The value at the review dates is:

$$V_j^i = \begin{cases} (1000+20.375)*e^{-r(paymentdate-reviewdate)/365}, & S \ge S_0 \\ \left(A_j * VT_{j+1}^{i+1} + B_j * VT_j^{i+1} + C_j * VT_{j-1}^{i+1}\right) + 20.375 * e^{-r(paymentdate-reviewdate)/365}, & 0.78S_0 \le S < S_0 \\ VT_j^i, & S < 0.78S_0 \end{cases}$$

The value in everywhere else is:

$$V_j^i = \begin{cases} A_j * VT_{j+1}^{i+1} + B_j * VT_j^{i+1} + C_j * VT_{j-1}^{i+1}, & S \ge 0.78S_0 \\ VT_j^i, & S < 0.78S_0 \end{cases}$$

3. Sensitivity analysis

a. Discussion of result, accuracy of valuation

According to our model, the best estimated value of option is 958.773, which is close to but lower than the prospectus price 958.9 from JP Morgan.

There are some reasons that cause this difference.

The parameters we chose might not be accurate. Further explanation will be discussed in sensitivity analyses on inputs of the valuation model (volatility, time steps).

Also, non-linearity errors should also cause this difference. Here are some reasons lead to non-linearity error in our model.

Firstly, we used the proportional dividends to construct the grid. This will force the stock prices move downward on the ex-dividend dates. Thus, we cannot have the stock prices fall exactly at each stock grid.

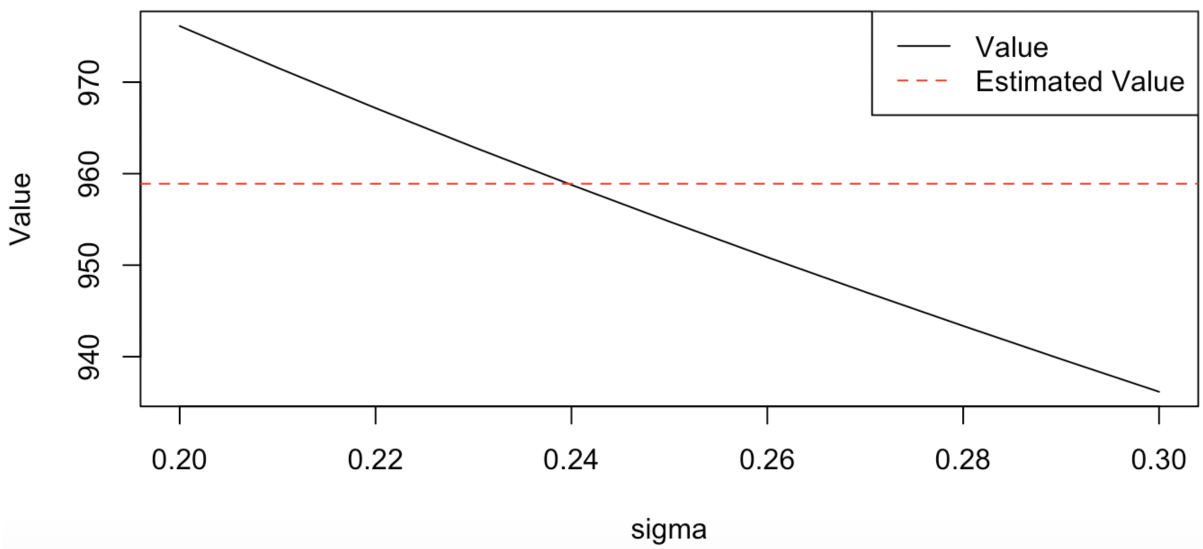
Secondly, since proportional dividends exist, it is hard to make discrete barrier fall exactly between two stock price grid points.

Thirdly, the discontinuous payoff from auto-call features will also cause the non-linearity error.

b. Volatility

The graph below shows the value from our model by changing the parameter sigma (20%-30%, which include several possible volatilities from Bloomberg). The prospectus price is from the product description of J.P.Morgan.

The notes price is decreasing when volatility is increasing and the price ranges between 969.2 and 935.2. When sigma is larger, the change of stock price is more volatile, so it's more possible for the stock price to hit the Trigger Value. And the Auto-call feature limit the influence of larger volatility to a certain extent when price goes up. Thus, the notes value tends to be undervalued when volatility is high, and vice versa.



Sigma	0.20	0.21	0.22	0.23	0.24	0.25
Value	976.1324	971.5753	967.1677	962.9034	958.7730	954.7651

Sigma	0.26	0.27	0.28	0.29	0.30
Value	950.8680	947.0698	943.3595	939.7269	936.1627

c. Implied volatility

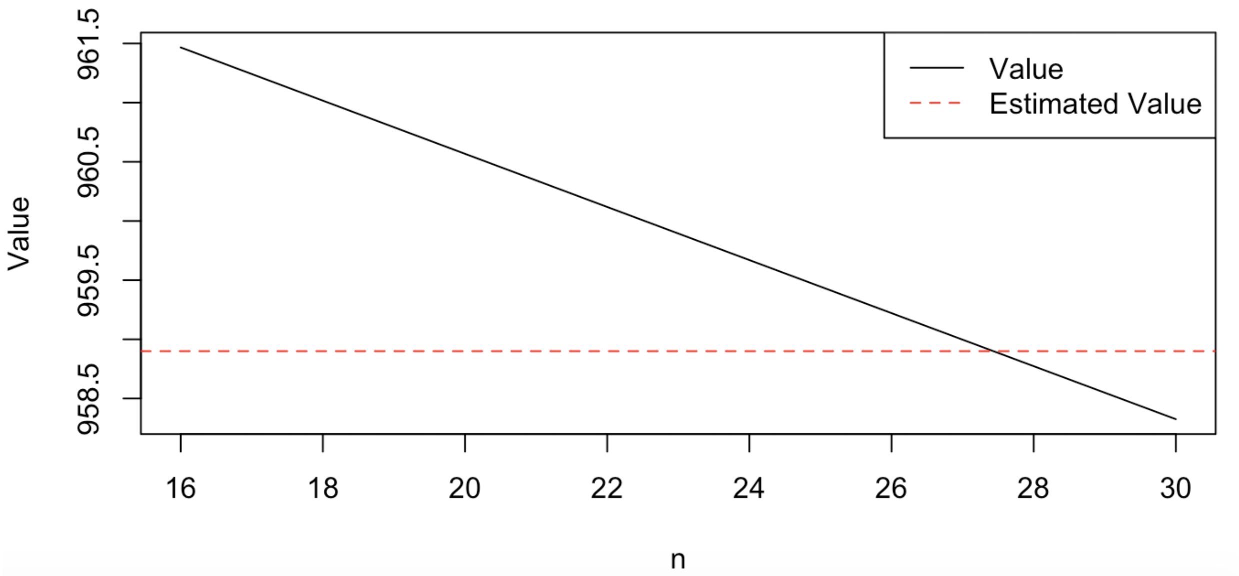
According to the sensitivity analysis above, the implied volatility for the prospectus price is 0.24, accurate to 2 decimal places.

d. Time steps

For the time steps, we used the multiple of the days between the pricing date and the valuation date, which is n*459. In order to achieve stability, n has to be relatively large, which is greater than 16 in our case, to make sure B are larger than 0. As shown in the table, time steps have relatively small influence on the valuation. We choose 28*459 which gives a close value to the prospectus price.

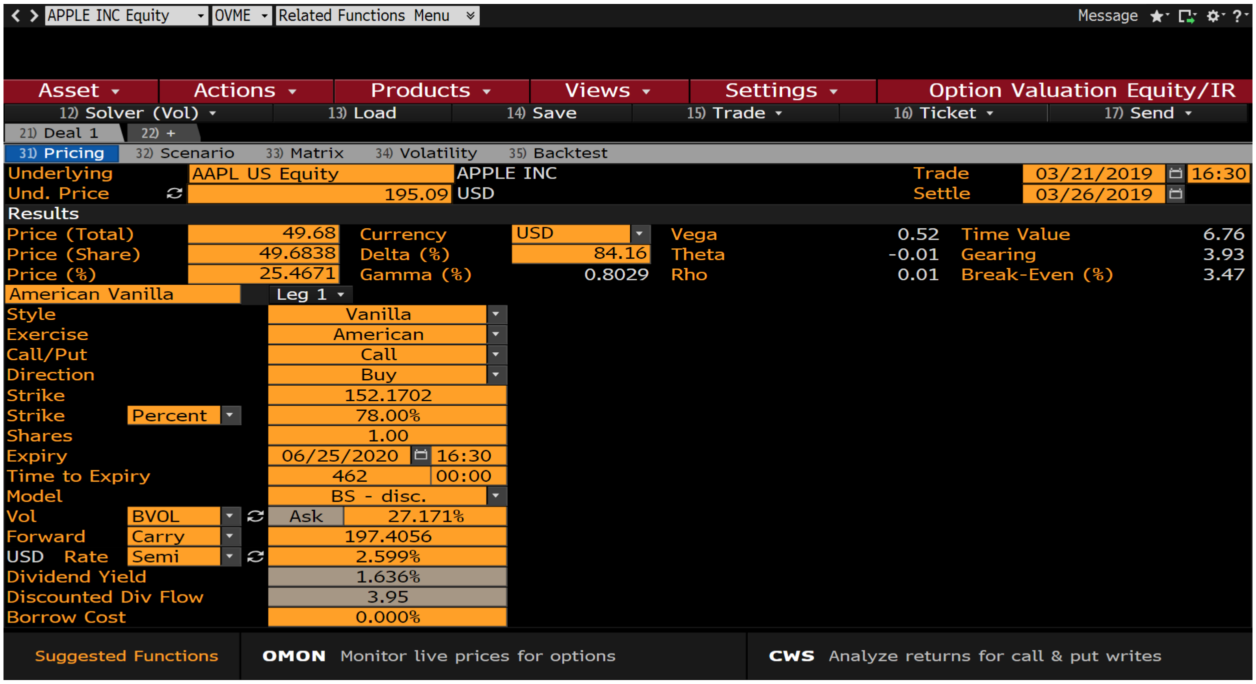
n	16	17	18	19	20	21	22	23
Value	961.4664	961.2416	961.0168	960.7921	960.5675	960.3430	960.1185	959.8941

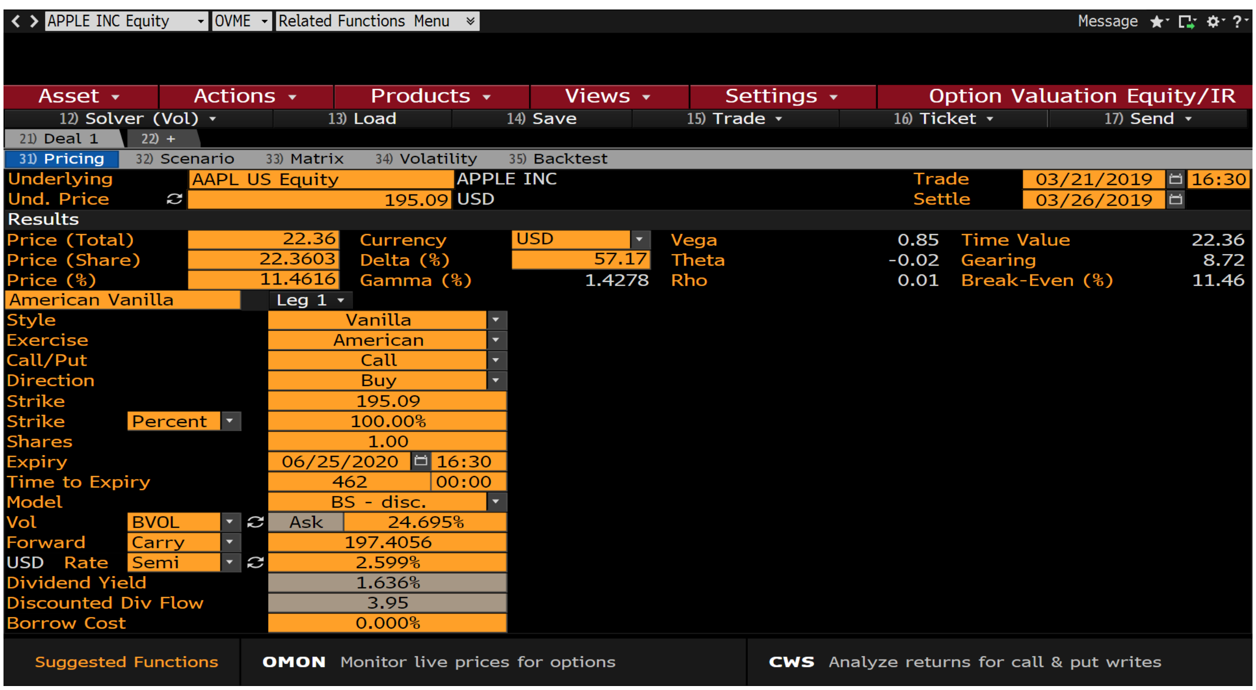
n	24	25	26	27	28	29	30
Value	959.6697	959.4454	959.2212	958.9971	958.7730	958.5490	958.3251



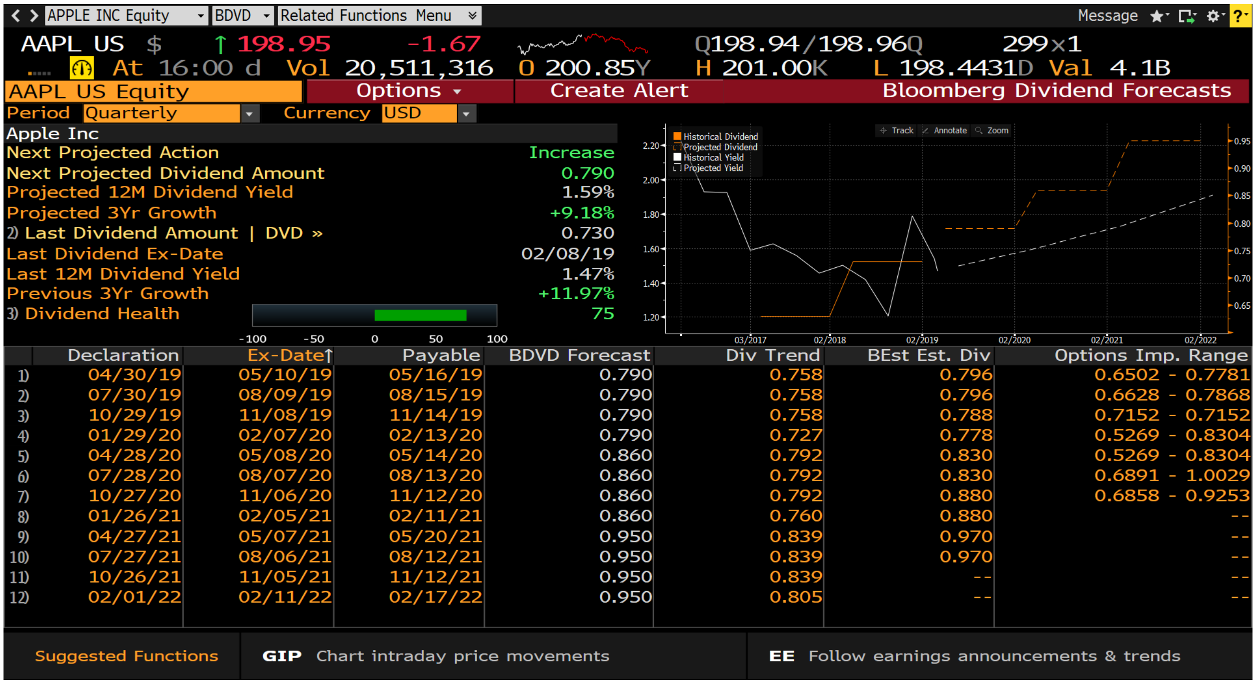
Appendix

1. Volatilities



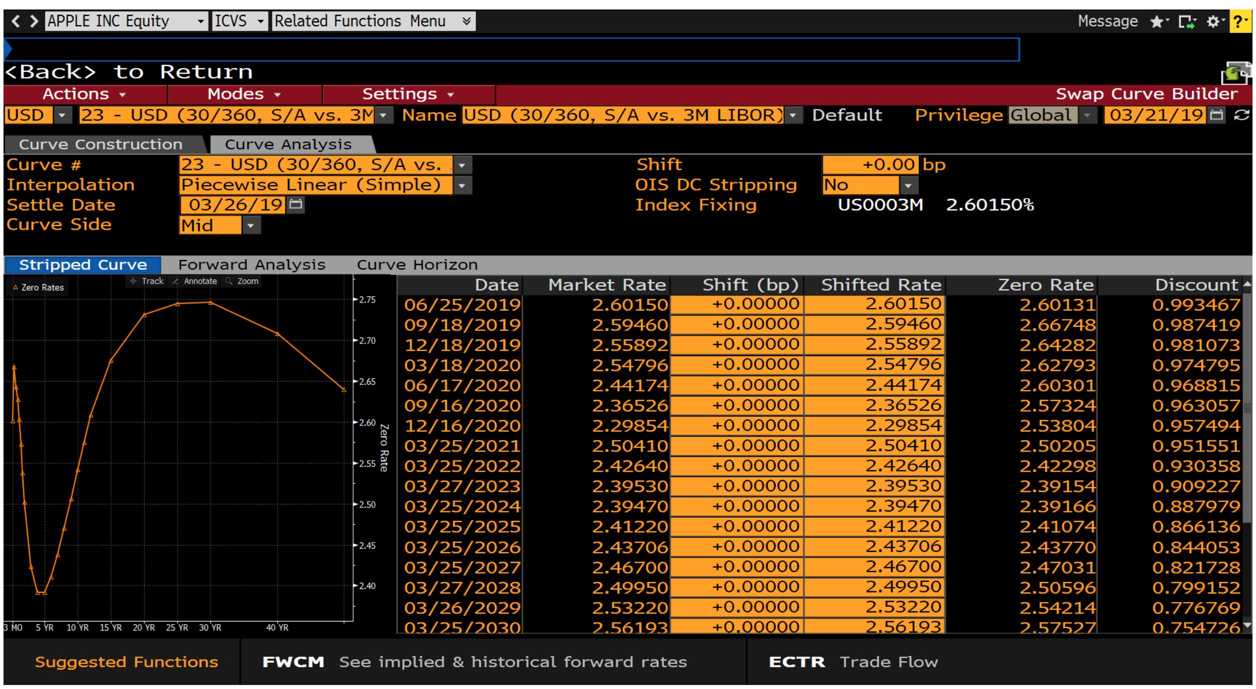


2. Dividend Yield



	Declaration	Ex-Date	Payable	BDVD Forecast	Div Trend	BEst Est. Div	Options Imp. Range
1)	04/30/19	05/10/19	05/16/19	0.790	0.758	0.796	0.6502 - 0.7781
2)	07/30/19	08/09/19	08/15/19	0.790	0.758	0.796	0.6628 - 0.7868
3)	10/29/19	11/08/19	11/14/19	0.790	0.758	0.788	0.7152 - 0.7152
4)	01/29/20	02/07/20	02/13/20	0.790	0.727	0.778	0.5269 - 0.8304
5)	04/28/20	05/08/20	05/14/20	0.860	0.792	0.830	0.5269 - 0.8304
6)	07/28/20	08/07/20	08/13/20	0.860	0.792	0.830	0.6891 - 1.0029
7)	10/27/20	11/06/20	11/12/20	0.860	0.792	0.880	0.6858 - 0.9253
8)	01/26/21	02/05/21	02/11/21	0.860	0.760	0.880	--
9)	04/27/21	05/07/21	05/20/21	0.950	0.839	0.970	--
10)	07/27/21	08/06/21	08/12/21	0.950	0.839	0.970	--
11)	10/26/21	11/05/21	11/12/21	0.950	0.839	--	--
12)	02/01/22	02/11/22	02/17/22	0.950	0.805	--	--

3. Risk-free Rates



Date	Market Rate	Shift (bp)	Shifted Rate	Zero Rate	Discount
06/25/2019	2.60150	+0.00000	2.60150	2.60131	0.993467
09/18/2019	2.59460	+0.00000	2.59460	2.66748	0.987419
12/18/2019	2.55892	+0.00000	2.55892	2.64282	0.981073
03/18/2020	2.54796	+0.00000	2.54796	2.62793	0.974795
06/17/2020	2.44174	+0.00000	2.44174	2.60301	0.968815
09/16/2020	2.36526	+0.00000	2.36526	2.57324	0.963057
12/16/2020	2.29854	+0.00000	2.29854	2.53804	0.957494
03/25/2021	2.50410	+0.00000	2.50410	2.50205	0.951551
03/25/2022	2.42640	+0.00000	2.42640	2.42298	0.930358
03/27/2023	2.39530	+0.00000	2.39530	2.39154	0.909227
03/25/2024	2.39470	+0.00000	2.39470	2.39166	0.887979
03/25/2025	2.41220	+0.00000	2.41220	2.41074	0.866136
03/25/2026	2.43706	+0.00000	2.43706	2.43770	0.844053
03/25/2027	2.46700	+0.00000	2.46700	2.47031	0.821728
03/27/2028	2.49950	+0.00000	2.49950	2.50596	0.799152
03/26/2029	2.53220	+0.00000	2.53220	2.54214	0.776769
03/25/2030	2.56193	+0.00000	2.56193	2.57527	0.754726