Report of Project

FIN 567 Financial Risk Management

Introduction

Dispersion trade is a kind of strategy that tries to take advantage of differences in implied volatilities of the index and component stocks. Our portfolio consists of writing a straddle on Dow Jones Index and buying straddles on components of the Index (we use the Index to denote Dow Jones Index for simplicity). We choose Vega-neutral strategy, which is trading volatilities to profit from differences of prices in option markets. If the volatilities of component stocks are higher than that of the comparable index level, the gain from the long position will exceed the loss from the short position and we make a profit. And we need to evaluate our portfolio since it seems profitable.

In the first section of this report, we illustrate the methodology of the system and clarify the assumptions that we need to construct the system. In the second section, we give details of data and our code of constructing the system and examine what specific amounts of options we need to construct such a portfolio. In the third section, we use Monte Carlo Simulation to calculate the Greeks, Value at Risk, and Expected Shortfall using one-day horizon. In the last section, we discuss the results of the simulation and our thoughts regarding the system and discuss the risks that are not captured by this system.

Methodology

To be specific, the way to build a Vega-neutral portfolio is the key equation below,

$$k\sum_{i=1}^{30} w_i * vega\ of\ straddle\ on\ component\ stock_i = vega\ of\ straddle\ on\ the\ Index$$

where k denotes the number of straddles (after multiplied by weight) on component stocks we need and w_i denotes the weights of each component stock in the Index.

By doing this, we can construct a portfolio with Vega equal to zero. But we need more steps to construct this portfolio and evaluate the performance of it.

We can use Filtered Historical Simulation with Dynamic Conditional Correlation. We need a large sample of historical data on the Index and component stocks, for example we may need more than 1,000 days of prices, because we need many times of simulation based on the past historical data. But the components of the Index may have changed during such a long period. It is difficult to consider the changes in the Index, so we can use a similar approach but using smaller samples of data.

Key Steps

- Use GARCH model to estimate sigma of component stocks and the Index
- Standardize the component stock returns with sigma to get Z-return
- Use DCC model to estimate conditional correlations between each component stocks
- Calculate k using the equation of Vega-neutral strategy
- Simulate the possible return on the next day and calculate the profit/loss to compute Value-at-Risk, Expected Shortfall and Greeks

The system above captures the dynamic correlation between component stocks and the difference between this approach with Filtered Historical Simulation with Dynamic Conditional Correlation is that HFS simulates the return based on picking one past day's Z-return after doing de-correlation to the standardized Z-returns ($\hat{z}^u_{t+1-\tau} = \Upsilon^{-0.5}_{t+1-\tau}\hat{z}_{t+1-\tau}$) and the justification of this method is that the correlation is embodied in the Z-return of past days and by randomly picking Z-return of one day many times, the simulated profit/loss can capture the correlation.

And we build this risk-measuring system based on the portfolio that is constructed on Dec.29, 2017, the last trading day of 2017. We can get the data from Bloomberg, including history of component stock prices and the Index prices, the Index Divisor, at-the-money option prices of the Index and component stocks, Vega of these options. We will use past 500 business days (roughly two years) of the historical data on the component stocks and the Index to estimate the parameters of GARCH and DCC models. And we assume that the time to maturity of the options we are going to trade in the system is 3 months for simplicity. From Bloomberg, we can get option prices for different strikes and maturities based on Bloomberg's prediction and valuation, so we can use these option prices in our system. Also, we can get the Greek letters of options from Bloomberg or we can calculate them using Black-Scholes formula.

Construction and Calculation

- Acquire the historical data of prices on the Index and component stocks, use MLE to estimate and store coefficients of GARCH model for each stock and the Index
- Use fitted GARCH models to estimate the sigma of component stocks and the Index for the past 500 days and the next day (t+1) of our system and for simulation
- Standardize the component stock returns with sigma to get Z-return and build a three-dimension array to store these values
- Estimate coefficients of DCC models with Z-return of each component stock and use MLE to estimate and store the coefficients of DCC model
- Use fitted DCC models to estimate the conditional correlations between each component stock for the next trading day
- Calculate the estimated covariance matrix of component stocks on the next day for simulation
- Calculate k using the equation of Vega-neutral strategy and the inputs are the Vega of the at-the-money options of component stocks and the Index and weights from Bloomberg
- Calculate the initial value of the portfolio
- Simulate 10,000 returns of the component stocks on the next day with the covariance matrix we get from the previous steps, and calculate simulated stock prices, the Index prices $Index = \frac{\sum_1^{30} p_i}{Divisor}$ using prices on Dec.29, 2017 and option prices for each and get the possible values the portfolio on next day
- Calculate the profit/loss and calculate the Value-at-Risk, Expected Shortfall based on the distribution of the profit/loss and calculate Greek letters of the portfolio

The weights, Vega and quantity of each component stock in the Index on Dec.29 2017

Stock Ticker	Weight	Vega	Quantity ($k \times w_i$)
UTX	3.5534%	0.25	5.579419
MCD	4.7943%	0.34	7.527863
DIS	2.9946%	0.21	4.702072
JNJ	3.8918%	0.28	6.110813
GS	7.0962%	0.5	11.14222

JPM	2.9788%	0.21	4.677142
VZ	1.4743%	0.1	2.314954
HD	5.2793%	0.37	8.289309
GE	0.4861%	0.03	0.763196
PG	2.5593%	0.18	4.018477
KO	1.2780%	0.09	2.006615
CSCO	1.0668%	0.08	1.675095
CAT	4.3893%	0.31	6.89194
MMM	6.5561%	0.46	10.29417
BA	8.2146%	0.58	12.89822
INTC	1.2858%	0.09	2.01886
AAPL	4.7138%	0.33	7.401467
AXP	2.7662%	0.2	4.343435
XOM	2.3298%	0.16	3.658091
V	3.1760%	0.22	4.986795
IBM	4.2735%	0.3	6.709997
PFE	1.0089%	0.07	1.584123
NKE	1.7423%	0.12	2.735696
TRV	3.7782%	0.27	5.932369
MRK	1.5674%	0.11	2.461032
WMT	2.7506%	0.19	4.318944
CVX	3.4871%	0.25	5.475327
UNH	6.1408%	0.43	9.642068
DWDP	1.9838%	0.09	3.114887
MSFT	2.3827%	0.17	3.741189

Source: Bloomberg

The listed Vega are the Vega of the at-the-money option on each stock and the Vega of the straddle on each stock is just twice larger. And the Vega of the option on the Index is approximately 48.67.

$$k = \frac{vega\ of\ straddle\ on\ the\ Index}{\sum_{i=1}^{30} w_i * vega\ of\ straddle\ on\ component\ stock_i} = 157.016$$

Or we can use functions of R package to calculate the Vega and get the slightly smaller value $k = 156.48$. Here we will use $k = 157.016$ for further calculation.

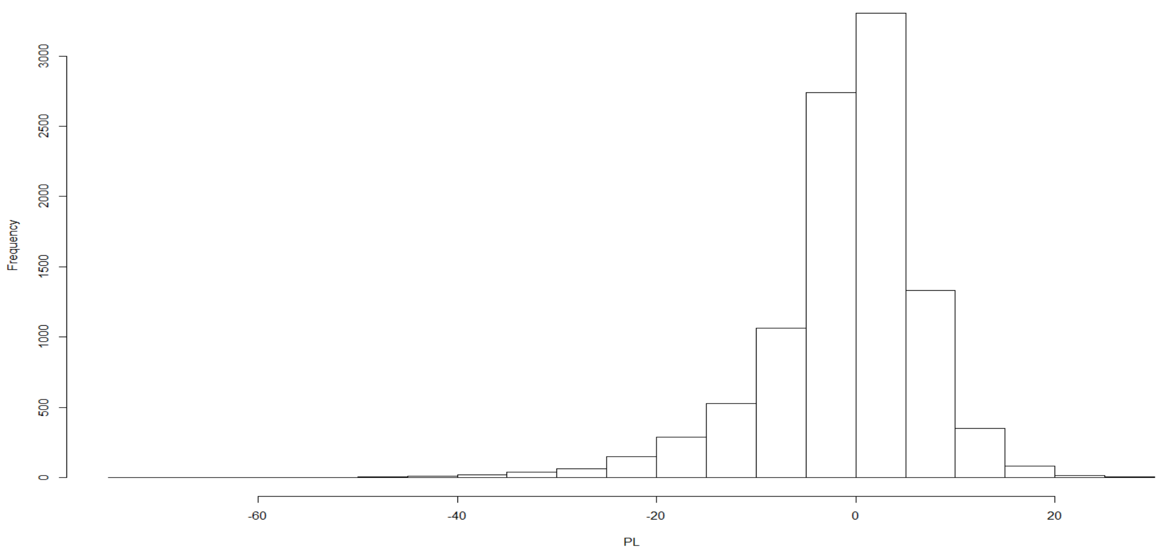
So, the number of straddles we need to trade is listed as the fourth column. Using the price of each option on Dec.29, 2017, we get the initial value of our portfolio, which is about $840.37.

The risk-free rate for 3 months quoted on Dec.29, 2017 is about 1.69428%.

Then we simulate 10,000 returns based on the covariance matrix above and calculate the simulated stocks prices and the Index levels. Using GBSOption function in R, we use the simulated prices and calculate 10,000 possible prices of options with same strike price on the component stocks and the Index for the next day. Then we calculate 10,000 profit/loss. The Value-at-Risk is calculated by function quantile under 5% and 1% and calculate Expected Shortfall under 5% and 1% using Value-at-Risk. The Greeks are calculated by GBSGreeks function in R. Or we can use the Greeks quoted on Bloomberg to calculate the Greeks of the portfolio.

Results

The distribution of the profit/loss



Since we choose the Vega-neutral strategy to construct our portfolio, the Vega should be equal to zero. Theoretically, our portfolio is constructed by the straddles only, the Delta should be equal to zero either.

Greeks

Greek	Value
Delta	0
Gamma	5.38218
Vega	0
Theta	-1774.4554
Rho	3.4822

Value-at-Risk and Expected Shortfall at 5% and 1% level

$VaR_{95\%}$	16.5
$VaR_{99\%}$	28.8
$ES_{95\%}$	24.42
$ES_{99\%}$	38.15

Discussion

1. From the results in the previous section, we can see that the distribution of one-day possible profit/loss has long left tail. And Value-at-Risk of the portfolio is small relative to the value of the portfolio.
2. The Delta is theoretically zero, which means the portfolio is not exposed to change in the prices of the Index and component stocks. The Vega is constructed to be zero, so the portfolio is not exposed to volatility risk. The Gamma is 5.38218, which means the value of the portfolio will increase when the underlying value changes. The Theta is -1774.4554, which means the value of the portfolio will decrease a lot when time passes. The Rho is 3.4822, which means the value of the portfolio will increase if the interest rate changes.
3. This portfolio is constructed to profit from the difference between volatilities of Index and components stocks but if the components of the Index change, i.e. a stock with high volatility is added into the Index or the weights of components change a lot, the portfolio will no longer holds the Vega-neutral strategy and we will be exposed to volatility risk.
4. In the paper on Risk.net, investors think Vega-neutral strategy is more favorable when the volatility is low. In other words, if the volatility of the market increases, Theta-neutral strategy may be favorable, so our portfolio may seem less profitable when volatility increases.
5. The value of options above are all calculated using Black-Scholes formula, which is not the true market value of options on that date and may bring some error in valuation.
6. We make many assumptions when doing the valuation, i.e. we assume the risk-free rate is constant, and volatilities of the three-months options are constant, and the daily expected return of component stocks is zero. These assumptions seem reasonable but may also bring some error and risk to our portfolio performance evaluation.