CS570 Fall 2018: Analysis of Algorithms Exam I

	Points		Points
Problem 1	20	Problem 5	16
Problem 2	16	Problem 6	10
Problem 3	16	Problem 7	12
Problem 4	10		
	Total	**100**	

Instructions:
1. This is a 2-hr exam. Closed book and notes
2. If a description to an algorithm or a proof is required please limit your description or proof to within 150 words, preferably not exceeding the space allotted for that question.
3. No space other than the pages in the exam booklet will be scanned for grading.
4. If you require an additional page for a question, you can use the extra page provided within this booklet. However please indicate clearly that you are continuing the solution on the additional page.
5. Do not detach any sheets from the booklet. Detached sheets will not be scanned.
6. If using a pencil to write the answers, make sure you apply enough pressure so your answers are readable in the scanned copy of your exam.
7. Do not write your answers in cursive scripts.

1) 20 pts

Mark the following statements as **TRUE** or **FALSE**. No need to provide any justification.

[TRUE/FALSE]

Assume that no two men have the same highest-ranking woman. If the women carried out the proposal to men, then the Gale-Shapley algorithm will contain a matching set where every man gets their highest-ranking woman.

[TRUE/FALSE]

Consider a binary heap with n nodes stored as an array. The parent, left child and right child of the node with index 11 are at indices 6, 22, and 23 respectively.

[TRUE/FALSE]

Inserting an element into a binary min-heap takes O(1) time if the new element is greater than all the existing elements in the min heap.

[TRUE/FALSE]

Using the master's theorem the asymptotic bounds for the recurrence 2T(n/4) + n is Θ(n).

[TRUE/FALSE]

Consider a version of the interval scheduling problem where all intervals are of the same size. A greedy algorithm based on earliest start time will always select the maximum number of non-overlapping intervals.

[TRUE/FALSE]

For any tree with *n* vertices and *m* edges we can say that O(m+n) = O(m).

[TRUE/FALSE]

A directed graph G is strongly connected if and only if G with its edge directions reversed is strongly connected.

[TRUE/FALSE]

The number of binomial trees in a binomial heap with n elements is at most O(log n).

[TRUE/FALSE]

A minimum spanning tree of a bipartite graph is not necessarily a bipartite graph.

[TRUE/FALSE]

In Fibonacci heaps, the decrease-key operation has an amortized cost of $O(1)$.

2) 16 pts.

A Maximum Spanning Tree is a spanning tree with maximum total weight.

a) Present an efficient algorithm to find a Maximum Spanning Tree of an undirected graph G. (8 pts)

b) Prove the correctness of your solution in part a. (8 pts)

3) 16 pts.

In an instance of the Stable Matching problem, man m has claimed “I’m gonna marry w, anyway!” and by that, he means in every stable matching, m must marry w — or equivalently, if in a matching, m and w do not pair up, then the matching is definitely unstable. It is a strong claim and you want to figure out if it is true. For example, if w is the topmost choice of m and m is also the topmost choice of w, they must pair up in every stable matching and thus, the claim is true in this case. Design an $O(n^2)$ algorithm that, given an instance of Stable Matching problem as well as man m and woman w, checks whether m marries w in every stable matching or not.

4) 10 pts

Rank the following functions in order from smallest asymptotic complexity to largest. No justification needed.

(lg is log base 2 by convention)

$lg\ n^{10}$, 3^n, $lg\ n^{2n}$, $3n^2$, $lg\ n^{lg\ n}$, $10^{lg\ n}$, $n^{lg\ n}$, $n\ lg\ n$

5) 16 pts

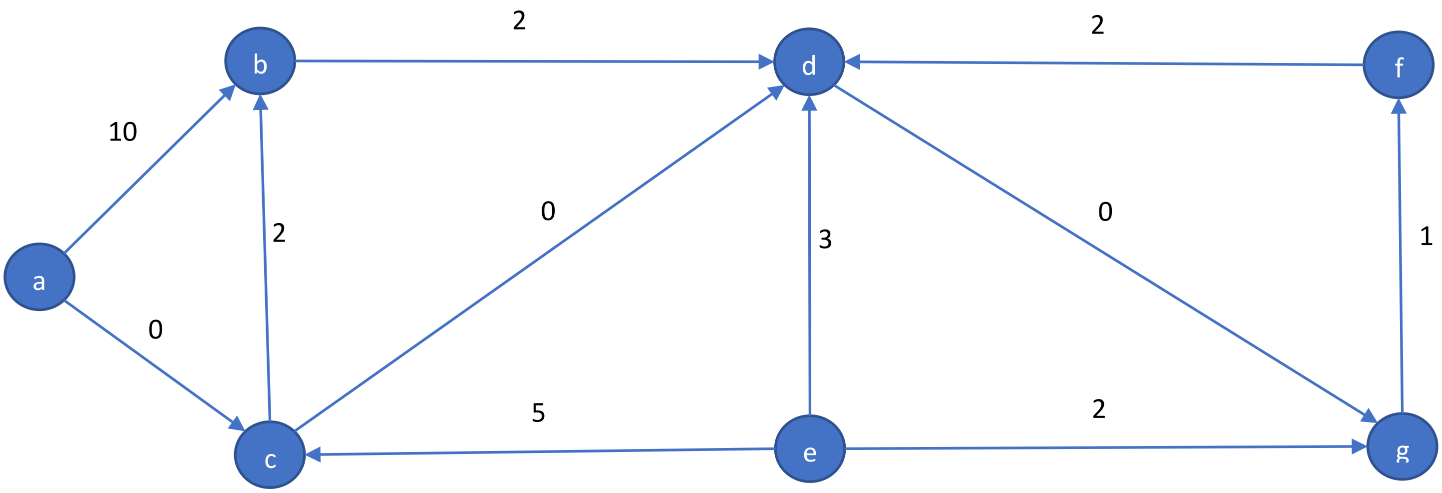
The Fractional Knapsack problem is described as follows. We have a resource (knapsack) with capacity W, i.e. it can hold items of total weight at most W. There are n items, each of weight $w_1, w_2, \ldots, w_n$. Each item also has a value $v_1, v_2, \ldots, v_n$. You are allowed to put all or any fraction of an item (say, 1/3 of an item) into the knapsack. The goal is to select a set of fractions $p_1, p_2, \ldots, p_n$ for all items in order to maximize the total value $p_1v_1 + p_2v_2 + \ldots p_nv_n$ subject to the capacity constraint $p_1w_1 + p_2w_2 + \ldots p_nw_n \leq W$.

a) Present an efficient solution to solve the Fractional Knapsack problem

b) Prove that the solution provided in part a is correct.

6) 10 pts

Below figure shows graph G along with its edge costs marked on each edge. Use Dijkstra's algorithm to find the shortest paths from node *a* to all nodes that can be reached from *a*. You should list all shortest paths in the order they are found.



7) 12 pts

Given an array of n distinct integers sorted in ascending order, we are interested in finding out if there is a Fixed Point in the array. Fixed Point in an array is an index i such that arr[i] is equal to i. Note that integers in array can be negative.

Example:

Input: arr[] = {-10, -5, 0, 3, 7}

Output: 3 // since arr[3] == 3

a) Present an algorithm that returns a Fixed Point if there are any present in array, else returns -1. Your algorithm should run in O(log n) in the worst case. (6 pts)

b) Use the Master Method to verify that your solutions to part a) runs in O(log n) time. (3 pts)

c) Let's say you have found a Fixed Point P. Provide an algorithm that determines whether P is a unique Fixed Point. Your algorithm should run in O(1) in the worst case. (3 pts)