QUIZ SOLUTION - 1

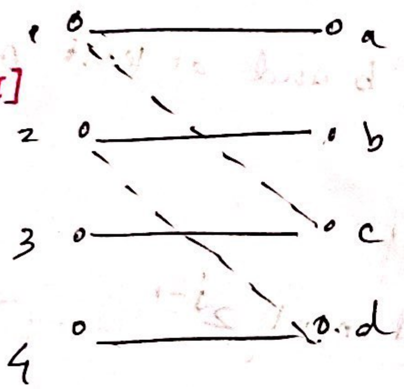
1) - Comparison Ratio measures the worst performance of greedy algorithm over all the input compared to optimal matching. It divides the worst number of matching of matching by the number of matching in optimal algorithm scenario.

$$\text{Competitive Ratio} = \min_{\text{all possible i/p}} \left(|M_{greedy}| \,/\, |M_{optimal}| \right)$$

[1 POINT]

2)

[1 POINT]



worst case = $\{(1, c), (1, d)\}$

optimal case = $\{(1,a)(2,b)(3,c)(4,d)\}$

competitive Ratio = $1/2$

3) Q is the set of queries that are in optimal scenario but matched in current greedy algorithm. A is the set of ads that are linked to queries in set Q.

$|M_{greedy}| \geq |A|$ because ads in A must already be matched otherwise, they will match with a query in Q and that query won't be in Q in the first place

Also $|A| \geq |Q|$ because for each query in Q, there has to be at least one linked ad. Therefore, we have the rule $|M_{greedy}| \geq |A| \geq |Q|$. [1 POINT]

Lets says Q' is the set of queries that are matched then $|M_{greedy}| \geq |Q'|$ because all queries in Q' must be matched in $|M_{greedy}|$. [1 POINT]

Therefore, $|M_{optimal}| = |Q| + |Q'| \leq |Q| + |M_{greedy}|$ and in the worst when $|M_{greedy}|$ is as small as possible, it is equal to $|Q|$, so $|M_{optimal}| \leq 2 \times |M_{greedy}|$, and the competitive ratio $1/2 = 0.5$. [1 POINT]

4)

[1 POINT]

q	(A_1, A_2 or No Ad)	Remaing Budget for A_1	Remaining Budget for A_2
At start	-	40	50
1st query	A_2	40	40
2nd query	A_1	20	40
3rd query	A_2	20	30
4th query	A_2	20	20
5th query	A_1	~~20~~ 0	20
6th query	A_2	0	10
7th query	A_2	0	0
8th query			

5) Suppose we have N advertisers A_1, A_2 --, each with budget B, so is the optimal scenario we exhaust all the budget and the revenue is $N \times B$.

[1 POINT]

In Balance Algorithm, we assign each query to advertisers equally and prefers those with high budget. For round i, we assign the query equally to advertisers A_i, A_{i+1} - - - because A_1, A_2 -- A_{i-1} don't bid for the query anymore

This process stops when $\frac{B}{N} + \frac{B}{N-1} + \frac{B}{N-2} + \cdots + \frac{B}{N-j+1} \geq B$

$$\frac{B}{N} + \frac{B}{N-1} + \frac{B}{N-2} \cdots + \frac{B}{N-j+1}$$

$$= B \times \left(\left(1 + 1/2 + 1/3 + \cdots + 1/N\right) - \left(1 + 1/2 + 1/3 + \cdots + \frac{1}{N-j}\right) \right)$$

[1 POINT]

According to Euler, $1 + 1/2 + 1/3 + \cdots + 1/N = \log_e N$ when N is big enough, so the formula above be converted to $\log_e N - \log_e N-j \geq 1$, and in the worst case when the sign is equal, we can get $j = N - N/e$

[1 POINT]

so the approx. revenue is $B\left(N - \frac{N}{e}\right)$, divided by optimal revenue BN, we can get competitive ratio

$1 - 1/e$.

[1 POINT]