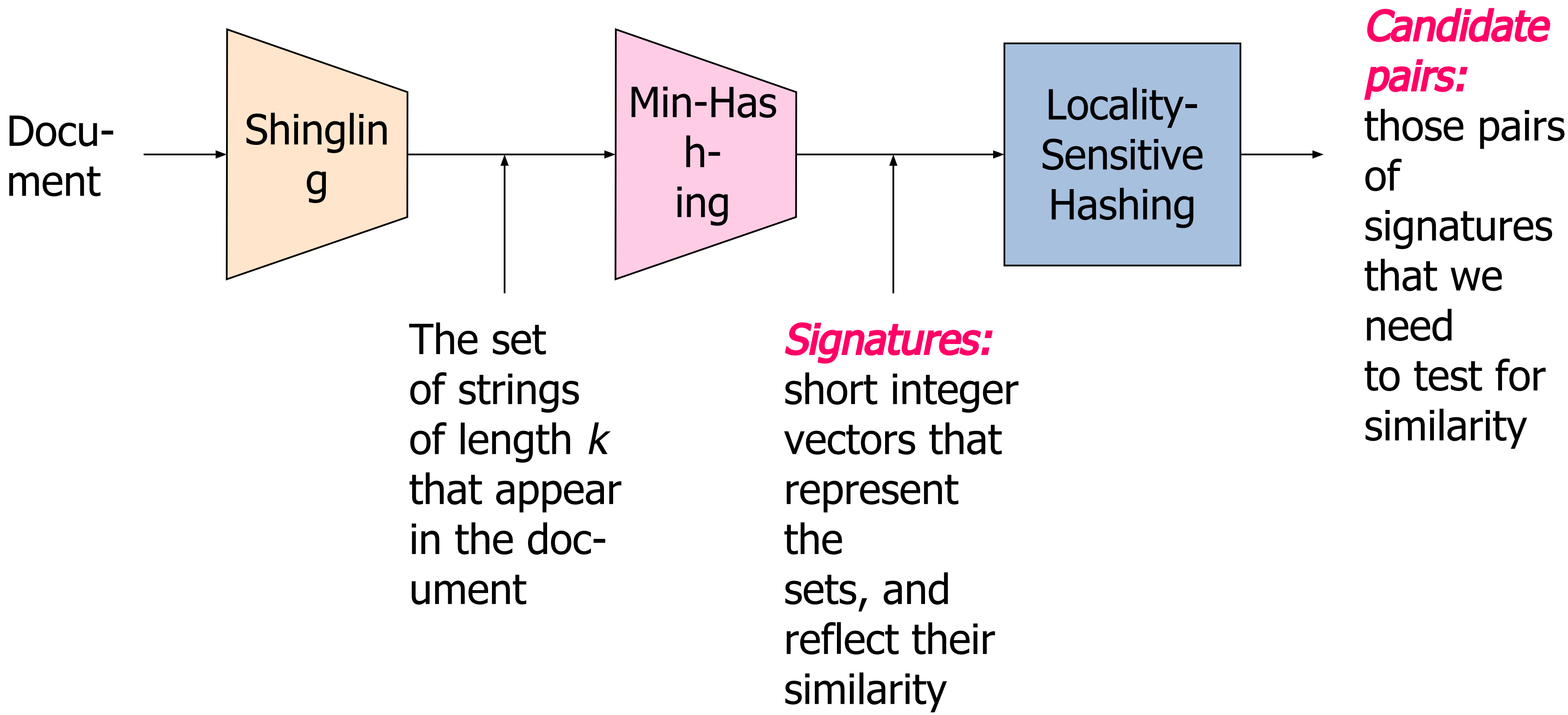
Outline

- Three steps for finding similar items
  1. Shingling: documents sets
  2. Min-hashing: sets signatures
  3. Locality-sensitive-hashing: signatures similarity
- Locality-Sensitive-Hashing (LSH)
- Characteristics of LSH
- Two Applications
  - Finding similar finger-prints
  - Finding similar news articles



Locality Sensitive Hashing

Step 3: ***Locality-Sensitive Hashing:***
Focus on pairs of signatures likely to be from similar documents

Motivation for Locality Sensitive Hashing

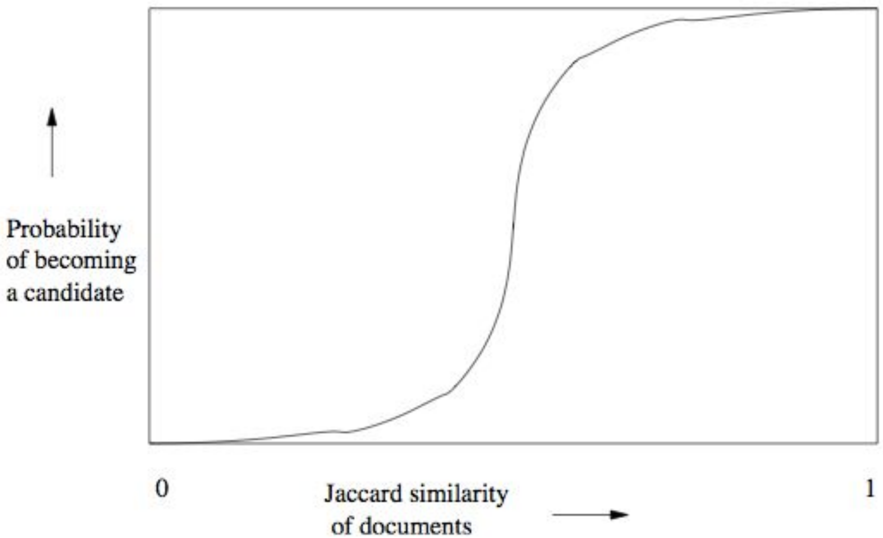
- ***Used k-shingles*** ***to create sets that summarize documents***
- ***Used Minhashing*** ***to generate signatures that represent sets of shingles, reflect their similarity***

- **Suppose we need to find near-duplicate documents among a million 10^6 documents**

- Naïvely, we would have to compute **pairwise Jaccard similarities** for **every pair of signatures**
  - 10^6 choose 2
  - Recall: for large n, $\binom{n}{2}$ is approximately $n^2/2$
  - **≈ $5*10^{11}$** comparisons
  - At 10^5 secs/day and 10^6 comparisons/sec, it would take **6 days**

Locality Sensitive Hashing

- ***Or Near-neighbor search***
- Minhashing is one example of a **family of functions** (the minhash functions) **that can be combined** (by the banding technique) **to distinguish strongly between pairs at a low distance from pairs at a high distance**
- Steepness of the S-curve reflects how effectively we can **avoid false positives and false negatives** among the candidate pairs
- Section 3.6: more general theory of Locality Sensitive Functions



Locality Sensitive Hashing Overview

- **Hash items several times**
  - In a way that **similar items are more likely to be hashed to the same bucket than dissimilar items**
- **Candidate Pair:** Any pair that hashes to the same bucket for **any** of the hashings
- **Check only the candidate pairs for similiarity**
- **False positives:** dissimilar pairs that hash to the same bucket
- **False negatives:** truly similar pairs do not hash to the same bucket for at least one of the hash functions

2	1	4	1
1	2	1	2
2	1	2	1

LSH: First Cut

- **Goal: Find documents with Jaccard similarity at least *s*** for some similarity threshold ***s*** (e.g. ***s***=0.8)

- **LSH – General idea:** Use a function ***f(x,y)*** that tells whether ***x*** and ***y*** are a ***candidate pair:*** a pair of elements whose similarity must be evaluated

- **For Min-Hash matrix:**
  - Hash columns of signature matrix ***M*** to many buckets
  - Each pair of documents that hashes into the same bucket is a **candidate pair**

Candidates from Min-Hash

2	1	4	1
1	2	1	2
2	1	2	1

- **Pick a similarity threshold *s* (0 < s < 1)**
- Columns ***x*** and ***y*** of ***M*** are a **candidate pair** if their signatures agree on at least fraction ***s*** of their rows: ***M* (*i*, *x*) = *M* (*i*, *y*)** for at least frac. ***s*** values of ***i***
  - We expect documents ***x*** and ***y*** to have the same (Jaccard) similarity as their signatures

LSH for Min-Hash

2	1	4	1
1	2	1	2
2	1	2	1

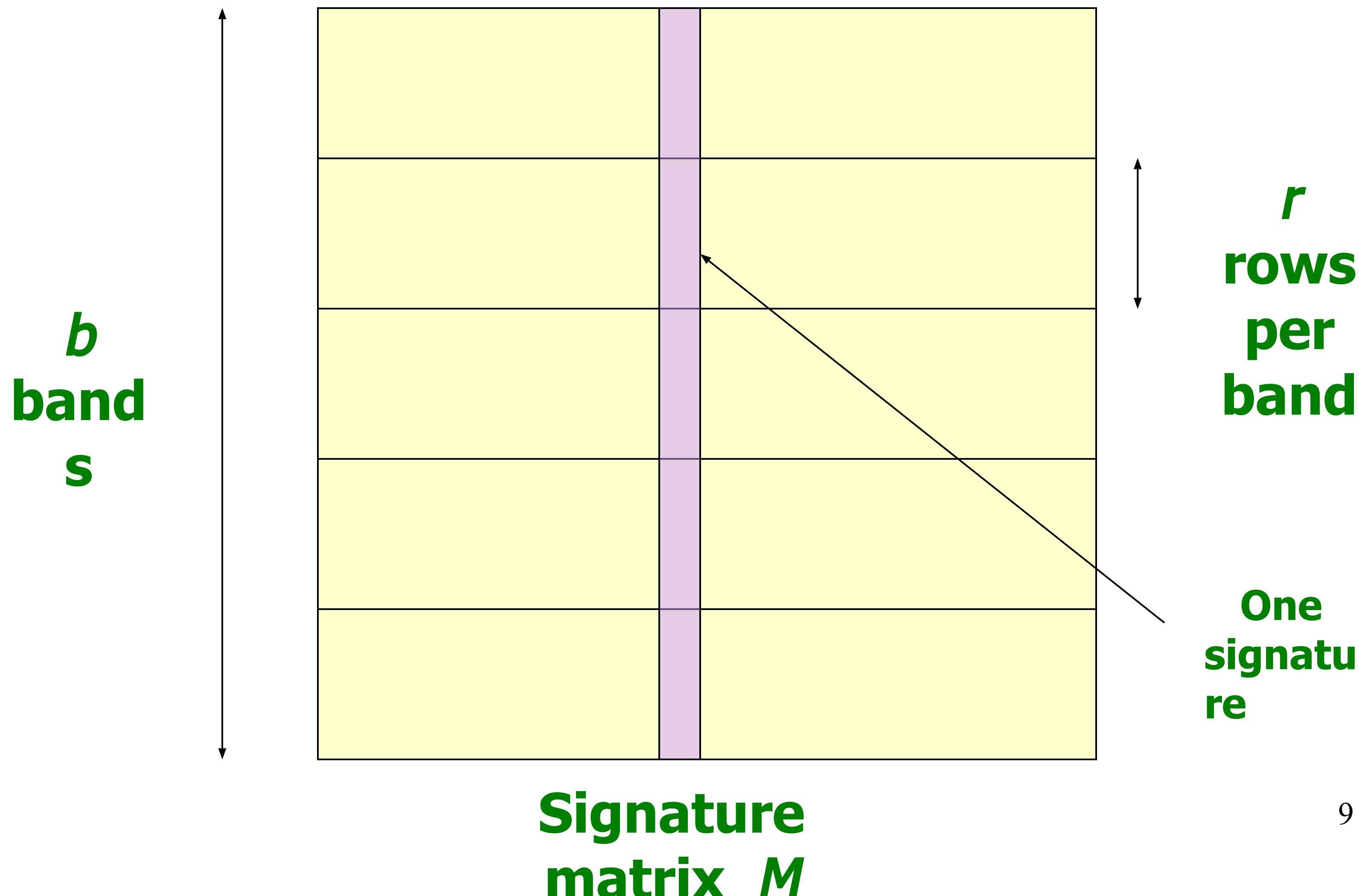
- **Big idea: Hash columns of signature matrix *M* several times**

- Arrange that (only) **similar columns** are likely to **hash to the same bucket**, with high probability

- **Candidate pairs are those that hash to the same bucket**

Partition *M* into *b* Bands

2	1	4	1
1	2	1	2
2	1	2	1

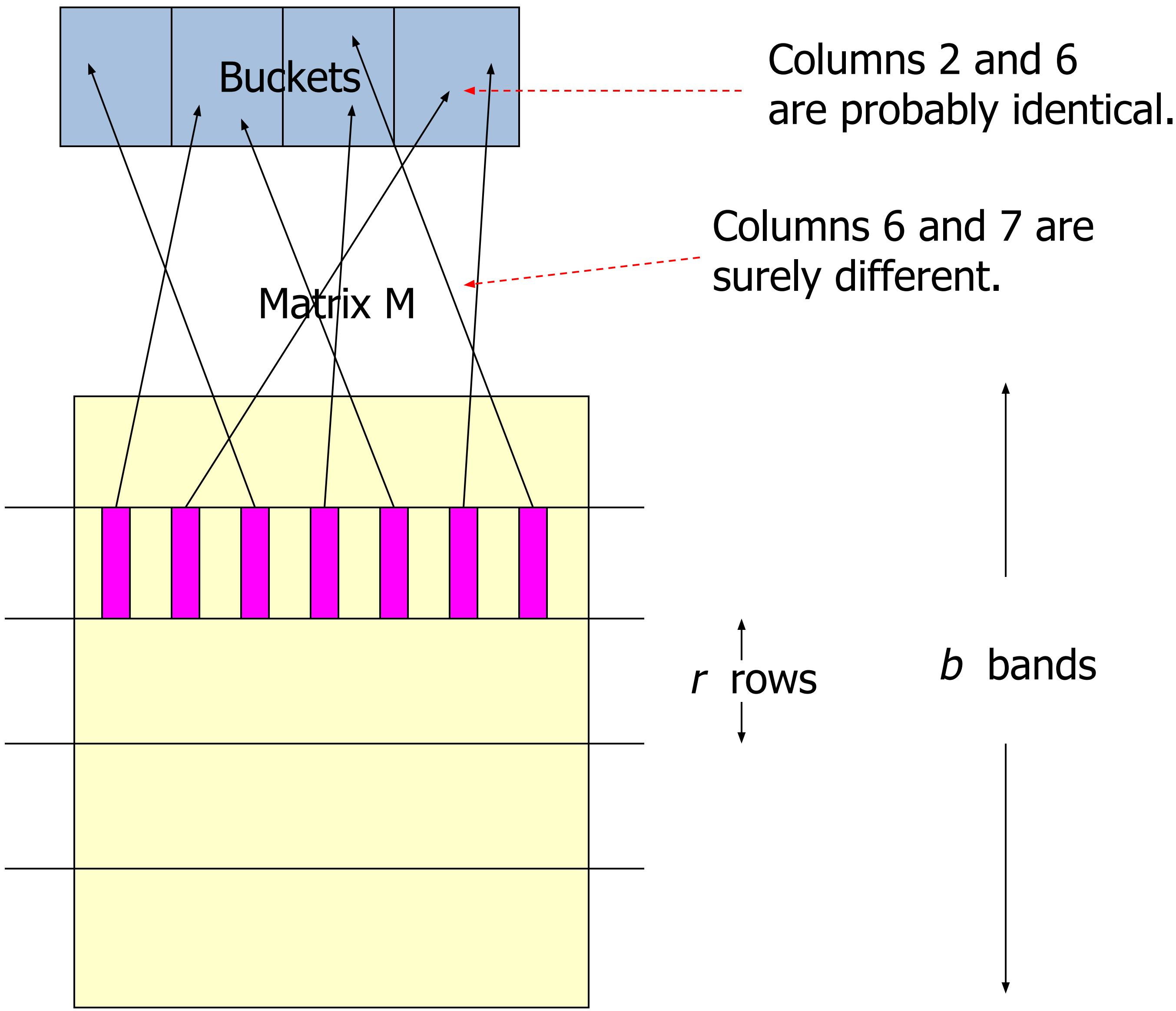


Partition Signatures M into Bands

- Divide matrix ***M*** into ***b*** bands of ***r*** rows
  - Signatures are still too big, so we check band by band
  - So, check "similar signatures" becomes check "similar bands"
- For each band, hash its portion of each column to a hash table with ***k*** buckets
  - Make ***k*** as large as possible
  - Use a separate bucket array for each band so columns with the same vector in different bands don't hash to same bucket

- ***Candidate*** **column pairs** are those that **hash to the same bucket for ≥ 1 band**

- Tune ***b*** and ***r*** to catch most similar pairs, but few non-similar pairs


Buckets
Columns 2 and 6 are probably identical.
Columns 6 and 7 are surely different.
Matrix M
r rows
b bands

Example of Bands

Assume the following case:

- Suppose 100,000 columns of ***M***
  - Correspond to signatures for 100,000 documents
- Signatures of 100 integers (rows)
  - Correspond to 100 hash functions used in minhashing
- 4 bytes per integer
- Therefore, signatures take 40Mb (4 x 100 x 100,000)
- Choose ***b*** = 20 bands of ***r*** = 5 rows of integers/band
- **Goal:** Find pairs of documents that are at least ***s*** *= 0.8* or 80% similar

Recall: Minhashing Example

1	4	3
3	2	4
7	1	7
6	3	6
2	6	1
5	7	2
4	5	5

Input matrix

1	0	1	0
1	0	0	1
0	1	0	1
0	1	0	1
0	1	0	1
1	0	1	0
1	0	1	0

Signature matrix *M*

2	1	2	1
2	1	4	1
1	2	1	2

Analysis of Banding Technique

- Use b bands of r rows each
- **Pair of documents have Jaccard similarity *t***
  - Probability that minhash signatures for the documents agree in any one particular row of the signature matrix is ***t***
- Columns C_1 and C_2 in signature matrix have similarity ***t***
- Pick any band (***r*** rows)
  - **Prob. that all rows in band are equal = t^r**
  - **Prob. that not all *r* rows are equal (some row in band is unequal) = $1 - t^r$**
- **Prob. that no band has rows that are all equal = $(1 - t^r)^b$**
- **Prob. that at least 1 band has rows that are all equal (which is the probability of being a candidate pair) = $1 - (1 - t^r)^b$**

2	1	4	1
1	2	1	2
2	1	2	1

C_1, C_2 are 80% Similar

- **Find pairs of ≥ *s*=**0.8 similarity, set **b=**20, **r=**5
- **Assume:** sim(C_1, C_2) = 0.8
  - Since sim(C_1, C_2) ≥ **s**, we want C_1, C_2 to be a **candidate pair**
  - We want them to hash to at **least 1 common bucket** (at least one band is identical)
- **Probability C_1, C_2 identical in one particular band:**
  $t^r = (0.8)^5 = 0.328$
- Probability C_1, C_2 are ***not*** similar in all of the 20 bands:
  $(1 - t^r)^b = (1-0.328)^{20} = 0.00035$
  - i.e., about .035% of the 80%-similar column pairs are **false negatives** (truly similar pairs that we miss)
  - **We would find 99.965% pairs of truly similar documents**

2	1	4	1
1	2	1	2
2	1	2	1

C_1, C_2 are 30% Similar

- **Find pairs of ≥ *s*=0.3 similarity, set b=20, r=5**
- **Assume:** sim(C_1, C_2) = 0.3
  - Since sim(C_1, C_2) < **s** we want C_1, C_2 to hash to **NO common buckets** (all bands should be different)
  - **Should NOT be a candidate pair!**
- **Probability C_1, C_2 identical in one particular band:** $t^r = (0.3)^5 = 0.00243$
- Will identify C1, C2 as candidate pair if they are **identical in at least one band**
- Probability C_1, C_2 identical in at least 1 of 20 bands: $\mathbf{1 - (1 - t^r)^b} = 1 - (1 - 0.00243)^{20} = 0.0474$
  - Approximately 4.74% pairs of docs with similarity 0.3% end up becoming **candidate pairs**
    - They are **false positives** (dissimilar documents that must be examined as candidate pairs but will have similarity below threshold s)

LSH Involves a Tradeoff

- **Pick:**
  - The number of Min-Hashes (rows of ***M***)
  - The number of bands ***b***, and
  - The number of rows ***r*** per band

  to balance false positives/negatives

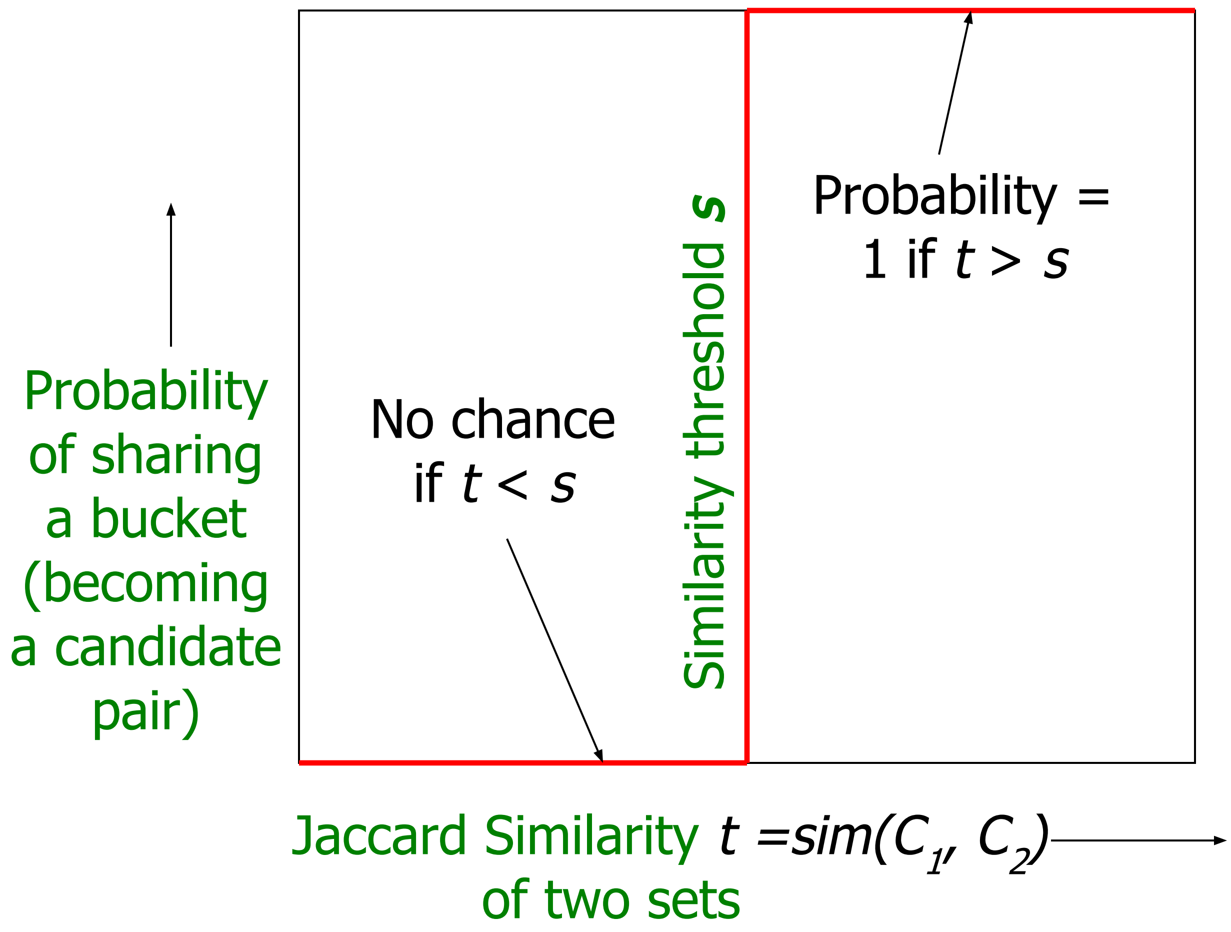
- **Example:** If we had only 15 bands of 5 rows, the number of false positives would go down, but the number of false negatives would go up

Example of Tradeoffs

- Previous example: 20 rows of 5 bands each
  - Probability of false negatives when C1, C2 are 80% similar: 0.00035
  - Probability of false positives when C1, C2 are 30% similar: 0.0474

- What if we use 15 rows of 5 bands each (smaller signature matrix)?
  - **Probability of false negatives higher** when C1, C2 are 80% similar:
    - $(1 - t^r)^b = (1-0.328)^{15} = 0.002573$
  - **Probability of false positives lower** when C1, C2 are 30% similar:
    - $1 - (1 - t^r)^b = 1 - (1 - 0.00243)^{15} = 0.0358$

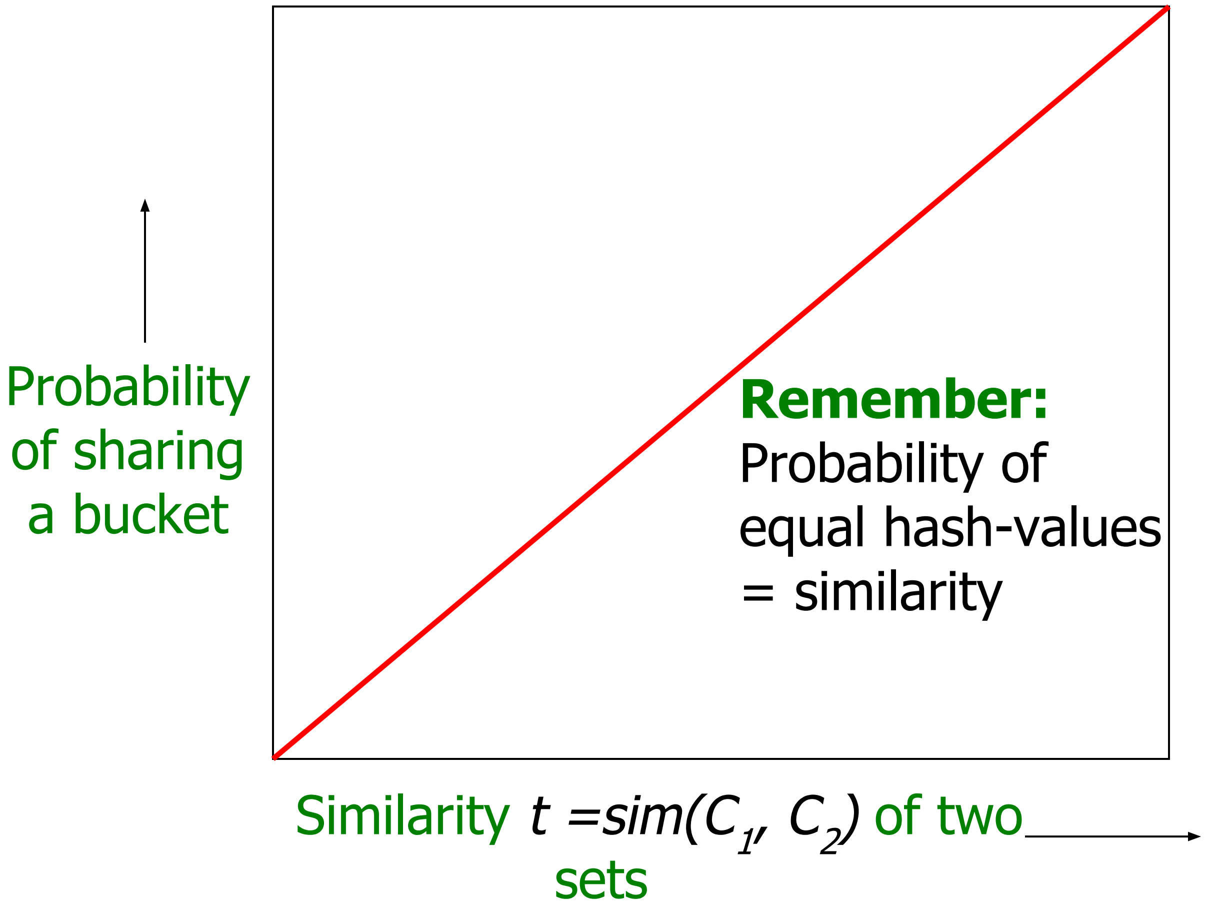
Analysis of LSH – What We Want



What 1 Band (b=1) of 1 Row (r=1) Gives You

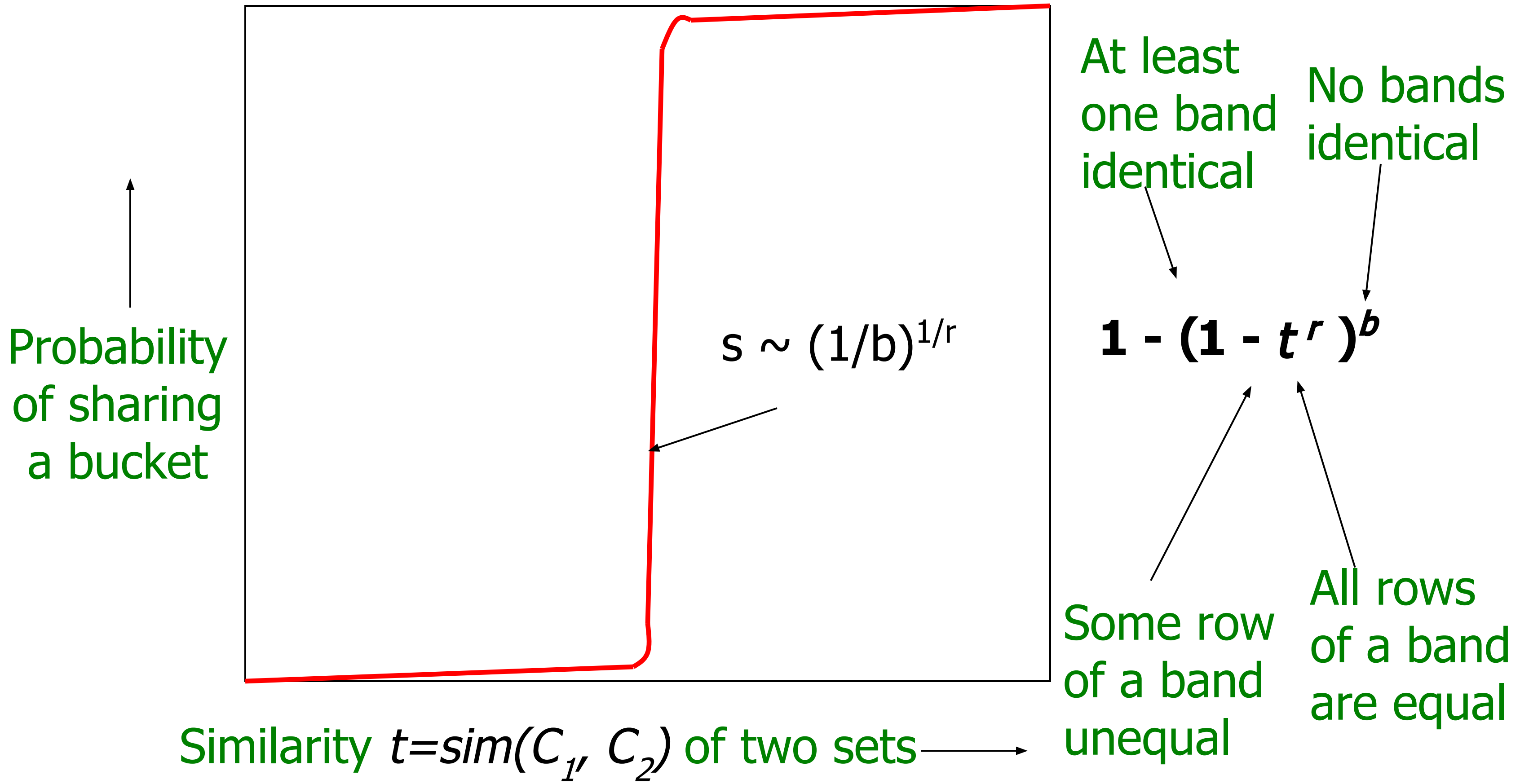
- Compare two values in similarity matrix

  $1-(1-t^{r})^{b} = 1 - (1-t^{1})^{1} = t$



What b Bands of r Rows Gives You: $1 - (1 - t^r)^b$

- Form of an S-curve, regardless of values of b and r
- Threshold s is where rise of curve is steepest: approximately $(1/b)^{1/r}$



r=5, b=20, for t=0.9: $1-(1-t^r)^b$=0.99999; for t=0.1: 0.0000199

Example: b = 20; r = 5

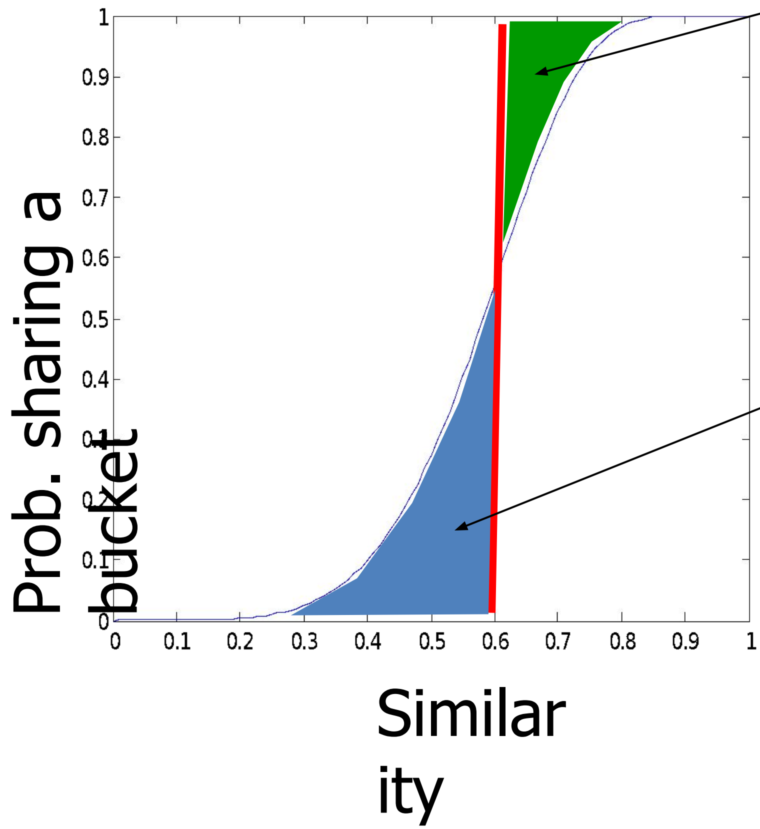
- Similarity ***t*** of two columns
- **Prob. that at least 1 band is identical (so a candidate pair):**

t	$1-(1-t^r)^b$
.2	.006
.3	.047
.4	.186
.5	.470
.6	.802
.7	.975
.8	.9996

- Not an ideal step function
- Probability rises by more than 0.6 going from similarity t = 0.4 to t = 0.6
- Slope in middle > 3.0
  0.6/(0.6-0.4)=3.0

Picking *r* and *b*: The S-curve

- **Picking *r* and *b* to get the best S-curve**
  - 50 hash-functions (r=5, b=10)

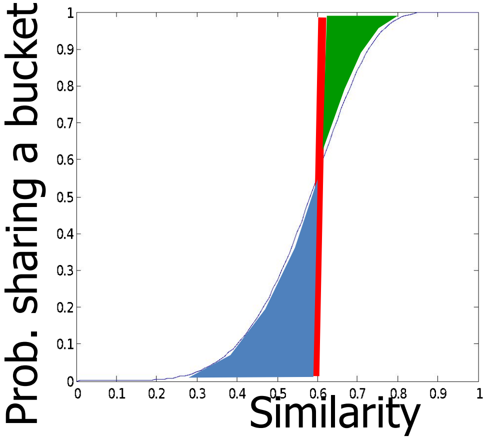


Green area:
False Negative rate
Similar documents that are not identified as candidate pairs

Blue area:
False Positive rate
Dissimilar documents that are identified as candidate pairs

Picking *b* and *r*

- **Threshold *s* defines how similar documents have to be for them to be regarded as a similar pair** (e.g., s = 0.8)
- **Length *n* for minhash signatures**
- Pick number of bands ***b*** and number of rows ***r*** such that ***br = n*** and threshold ***s*** is approximately $(1/b)^{1/r}$
- **To avoid false negatives (green area):**
  - **Select *b* and *r* to produce a threshold lower than *s***
- **To avoid false positives (blue area):**
  - **Select *b* and *r* to produce a higher threshold than *s***



Example: n=100

b	r	$(1/b)^{1/r}$
50	2	0.1414
20	5	0.5493
10	10	0.7943
5	20	0.9227

Example

- ***(1/b)***$^{1/r}$ **represents the threshold of the S curve for function $1 - (1 - t^r)^b$, the probability of being a candidate pair**
- If **s=0.6** (similarity of documents to be a candidate pair) what values should you choose for b and r to reduce the number of **false negatives**?

- **To avoid false negatives: Select *b* and *r* to produce a threshold lower than *s***
- **To avoid false positives: Select *b* and *r* to produce a higher threshold than *s***

- **Could choose (b=20, r=5) or (b=50, r=2): both give threshold lower than s**

- **Better answer probably b=20, r=5**
- **Because b=50, r=20 will have a higher rate of false positives: TRADEOFFS**

Example:
n=100

b	r	$(1/b)^{1/r}$
50	2	0.1414
20	5	0.5493
10	10	0.7943
5	20	0.9227

LSH Summary

- Tune ***M, b, r*** to identify **almost all candidate pairs with similar signatures**, but **eliminate most pairs that do not have similar signatures**
- Then **check in main memory that candidate pairs really do have similar signatures**
- **Optional:** In another 2nd pass through data, check that **the remaining candidate pairs really represent similar documents**

Summary: 3 Steps

- **Shingling:** Convert documents to sets
  - We used hashing to assign each shingle an ID
- **Min-Hashing:** Convert large sets to short signatures, while preserving similarity
  - We used **similarity preserving hashing** to generate signatures with property $\mathbf{Pr}[h_\pi(C_1) = h_\pi(C_2)] = \mathbf{\mathit{sim}(C_1, C_2)}$
  - We used hashing to get around generating random permutations
- **Locality-Sensitive Hashing:** Focus on pairs of signatures likely to be from similar documents
  - We used hashing to find **candidate pairs** of similarity $\geq$ **s**

Combining the techniques (1)

1. Pick a value of *k* and **construct from each document the set of *k*-shingles**
    - Optionally **hash the k-shingles** to shorter bucket numbers
2. **Sort the document-shingle pairs** to order them by shingle
    - Which sets contain which elements (shingles)
3. Pick a length *n* for minhash signatures corresponding to *n* minhash functions and **compute the minhash signatures** for all the documents

Combining the techniques (2)

4. **Choose threshold s** that defines how similar documents have to be for them to be regarded as a "similar pair"
   - **Pick number of bands b and number of rows r** such that br = n
   - **Adjust b and r to limit false positives or negatives**

5. **Construct candidate pairs with LSH** technique

6. **Examine candidate pair signatures** and determine whether fraction of components where they agree is at least s

7. **Optionally,** if signatures are sufficiently similar, **compare documents** to check they are truly similar

CHARACTERISTICS OF LSH

Locality Sensitive Hashing

- ***Or Near-neighbor search***
- Minhashing is one example of a **family of functions** (the minhash functions) **that can be combined** (by the banding technique) **to distinguish strongly between pairs at a low distance from pairs at a high distance**
- Steepness of the S-curve reflects how effectively we can **avoid false positives and false negati**ves among the candidate pairs
- Section 3.6: more general theory of Locality Sensitive Functions

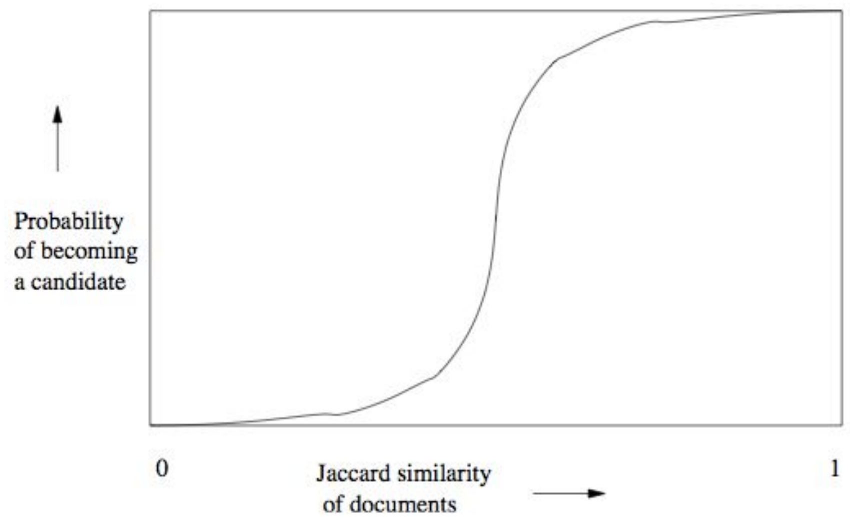


Figure 3.7: The S-curve

Families of Functions for LSH

- **Families of functions** (including minhash functions) that can serve to **produce candidate pairs efficiently**
  - Space of sets and Jaccard distance OR other space and/or distance measure
- **Three conditions for family of functions:**

1. **More likely to make close pairs be candidate pairs than distant pairs**
2. **Statistically independent**
3. **Efficient** in two ways
   1. **Be able to identify candidate pairs in time much less than time to look at all pairs**
   2. **Combinable to build functions better at avoiding false positives and negatives** (e.g., banding techique takes single minhash functions, combines them to produce S-curve shape we want)

Locality-Sensitive Functions

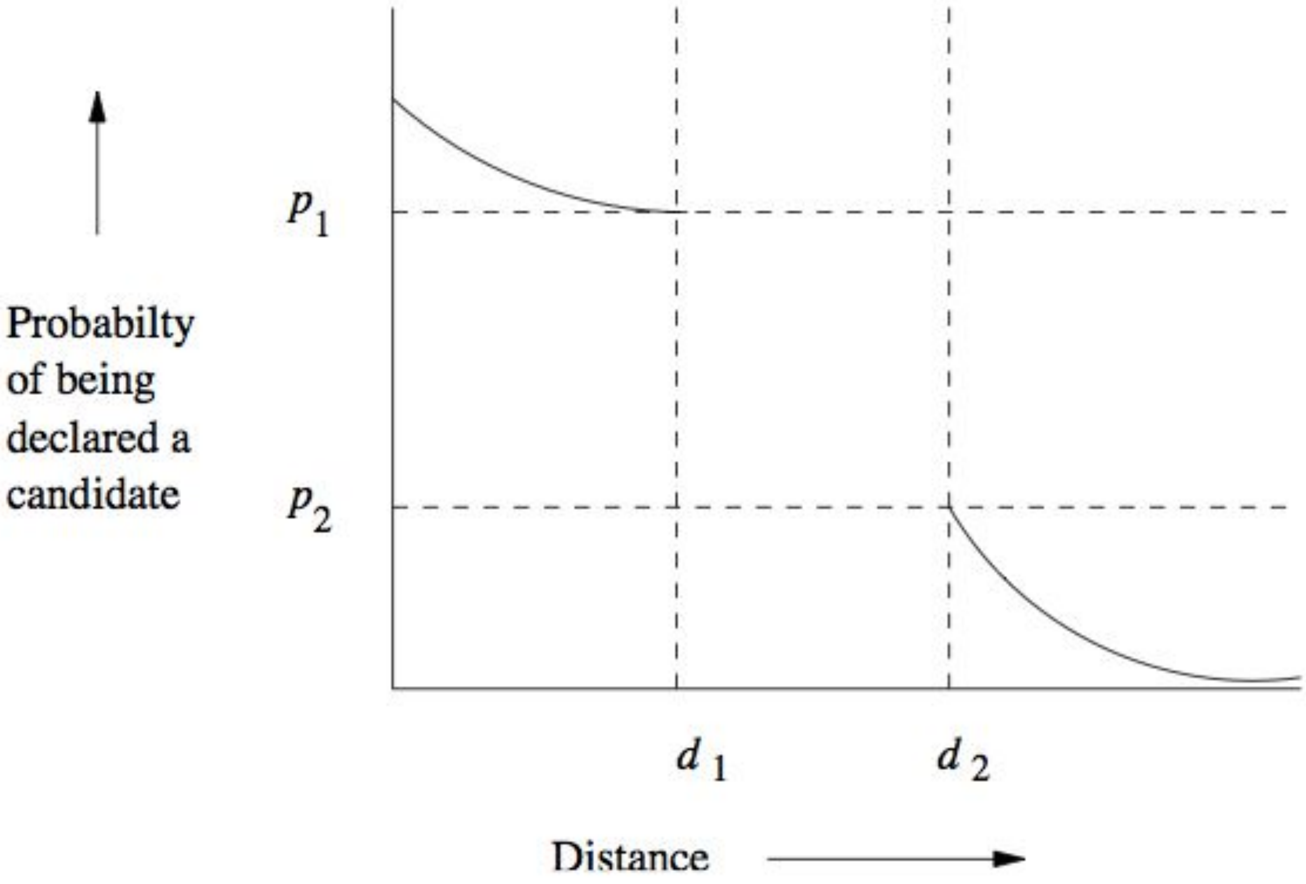


Figure 3.9: Behavior of a (d_1, d_2, p_1, p_2)-sensitive function

LS Families of Hash Functions

- Suppose we have a **space S of points** with a **distance measure d**
- A **family H of hash functions** is said to be ***(d_1,d_2,p_1,p_2)-sensitive*** if for any x and y in S:
  1. **If $d(x,y) \leq d_1$, then prob. over all h in H, that $h(x) = h(y)$ is at least p_1**
  2. **If $d(x,y) \geq d_2$, then prob. over all h in H, that $h(x) = h(y)$ is at most p_2**
- Note: we say nothing about what happens when the distance between items is between d1 and d2
  - But can make d1 and d2 as close as we wish
  - Can drive p1 and p2 apart while keeping d1 and d2 fixed

Locality Sensitive Hashing for Other Distance Measures

- We focused on **minhashing**, a locality sensitive hashing family that uses **Jaccard distance**
  - Based on sets representing documents and their Jaccard similarity
- Book covers LSH families for **other distance measures**:
  - **Euclidean distance:** based on the locations of points in a *Euclidean space* with some number of real-valued dimensions
  - **Cosine distance:** angle between vectors from the origin to the points in question
  - **Edit distance:** number of inserts and deletes to change one string into another
  - **Hamming Distance:** number of positions in which bit vectors differ

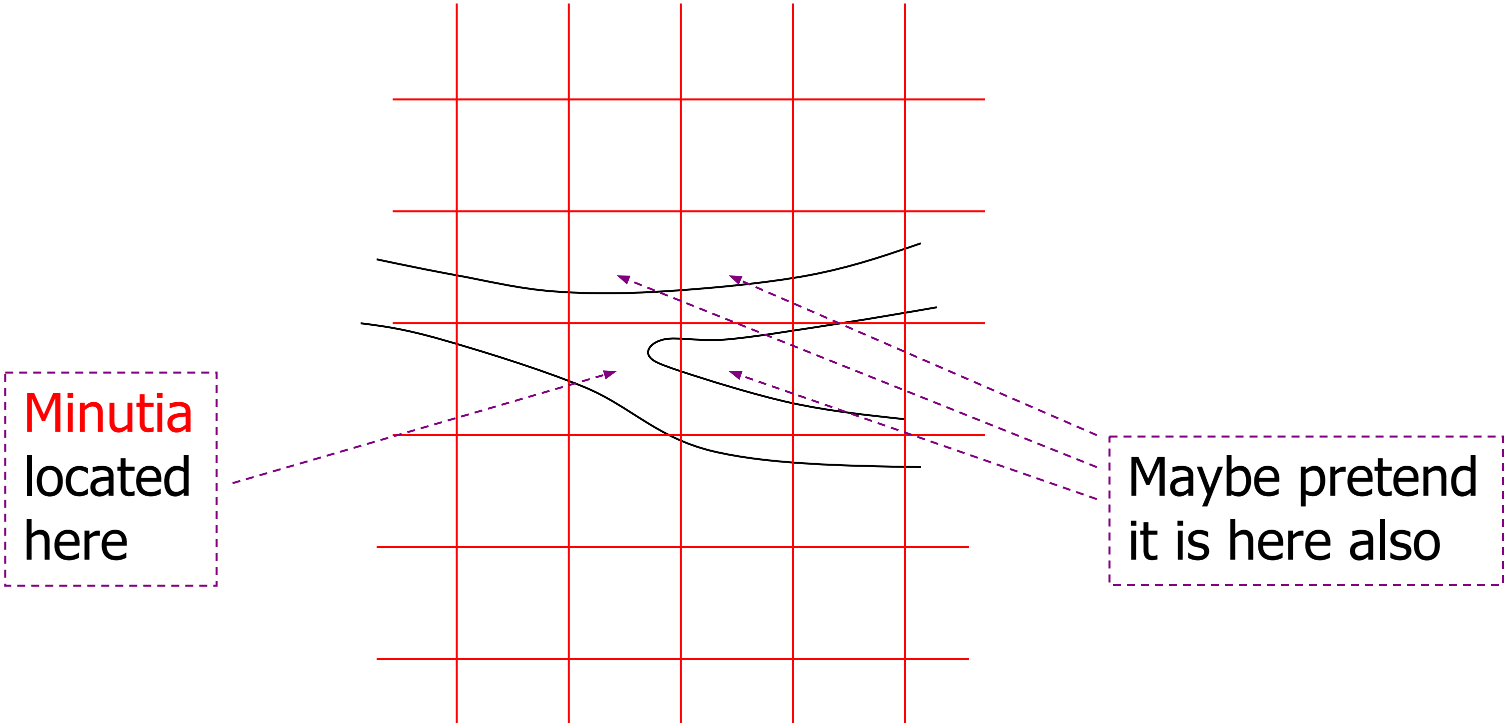
LSH and Shingling Application Examples

- Matching fingerprints
- Identifying similar news articles

LSH for Fingerprints

- Typical representation is not an image, but **set of locations in which minutiae are located**
  - **Place where something unusual happens**: two ridges merging or a ridge ending
- Place a grid over a fingerprint
  - Normalize for size and orientation so that identical prints will overlap
- Represent fingerprint by set of grid points where minutiae are located
  - Possibly, treat minutiae near a grid boundary as if also present in adjacent grid points

Discretizing Minutiae



Place a minutia in several adjacent grid squares if it lies close to the border of the squares

Applying LSH to Fingerprints

- **Make a bit vector for each fingerprint's set of grid points with minutiae**
  - Similar to set representing a document: 1 if the shingle is in the document, 0 otherwise
- We could minhash the bit vectors to obtain signatures
  - But since there probably aren't too many grid points, we can work from the bit-vectors directly

Matching Fingerprints with LSH: Many-to-many problem

- **Many-to-many version of fingerprint matching:** take an entire database of fingerprints and identify if there are any pairs that represent the same individual
  - **Analogous to finding similar documents among millions of documents**
- **Define a locality-sensitive family of hash functions:**
  - **Each function f in the family F is defined by 3 grid squares**
  - **Function f says "yes" for two fingerprints if both have minutiae in all three grid squares, otherwise, f says "no"**
  - **"Yes" means the two fingerprints are candidate pairs**
- **Sort of "bucketization"**
  - Each set of three points creates one bucket
  - **Function f sends fingerprints to its bucket that have minutae in all three grid points of f**
- **Compare all fingerprints in each of the buckets**

Matching Fingerprints with LSH: Many-to-One Problem

- **Many-to-one version :** A fingerprint has been found at a crime scene, and we want to **compare it with all fingerprints in a large database to see if there is a match**
- Could use many functions f from family F
- **Precompute their buckets of fingerprints to which they answer "yes" on the large database**
- **For a new fingerprint:**
  - **Determine which buckets it belongs to**
  - **Compare it with all fingerprints found in any of those buckets**

Example 3.22

- 1024 functions chosen randomly from F
  - **Each function f says “yes” for two fingerprints if both have minutiae in all three grid squares, otherwise, f says “no”**
- Suppose **typical fingerprints have minutiae in 20% of the grid points**
- Suppose **fingerprints from the same finger agree in at least 80% of their points**
- **Probability two random fingerprints each have 1 in all three points = $(0.2)^6$ = .000064**
  - 2 fingerprints, 3 points each, all independent events

Example: Continued

First image has 1 in a point

Second image of same finger also has 1

- Probability **two fingerprints from the same finger each have 1's in three given points** = $((0.2)(\boxed{0.8}))^3 = \boxed{.004}096$ (Analogy: t^r)
- Prob. for **at least one of 1024 sets of three points** = $1-(1-.004096)^{1024} = .985$ (Analogy: $1 - (1 - t^r)^b$)
- But for **random fingerprints**: $1-(1-.000064)^{1024} = .063$

1.5% false negatives

6.3% false positives

Choosing the number of functions from F

- **Want to use many functions from F, but not too many**
- **Want a good probability of matching fingerprints from the same finger while not having too many false positives**
- Previous example: only 1.5% chance we fail to identify a print on the gun (false negative), but have to look at 6.3% of entire database (due to false positives)
- **Increasing number of functions from F increases number of false positives**
  - Only a small benefit in reducing false negatives below 1.5%
- **Can use constructions/combinations of functions**
  - Several examples in the chapter

Finding Same/Similar News Articles

- **Want to organize large repository of on-line news articles**
  - **Group together web pages derived from same basic text**
- **Scenario:** the same article, say from the Associated Press, appears on the Web site of many newspapers, but looks quite different
- **Each newspaper surrounds the text of the article with:**
  - Its own logo and text
  - Ads
  - Perhaps links to other articles
- **A newspaper may also "crop" the article (delete parts)**

Variation on Shingling

- **Looks like earlier problem:** find documents whose shingles have high Jaccard similarity
- **But: Shingling treats all parts of document equally**
- For this application, **we want to ignore certain parts of the documents** (e.g., ads, links to other articles, etc.)
- **There is a difference between text that appears in prose and text in ads or headlines/links**
  - Prose contains greater frequency of ***stop_words***
    - E.g., common words like “and” or “the”
  - Common to use list of several hundred most frequent words

New Shingling Technique

- **News articles have a lot of stop words, while ads do not**
  - "Buy Sudzo" vs. "I recommend that you buy Sudzo for your laundry."
- **Define a *shingle* to be a stop word plus the next two following words**
  - Shingles are: "I recommend that", "that you buy", "you buy Sudzo", "for your laundry", "your laundry <nextword>"
- **Then compare the similarity of the sets of shingles that represent each document**
  - Don't use minhashing or LSH in this example

Why it Works

- By requiring each shingle to have a stop word**: bias the mapping from documents to shingles** so it picked more shingles from the article than from the ads

- **Pages with the same article, but different ads, have higher Jaccard similarity** than those with the same ads, but different articles