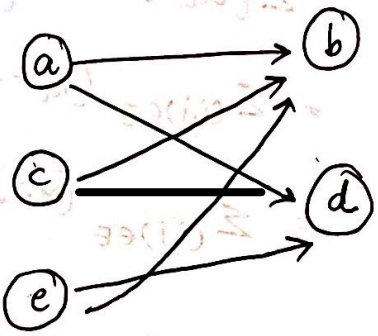
Quiz-9 Solution

1) $a = \{b, c, d\}$
$b = \{c, d, g\}$
$c = \{e, f, d, b\}$
$d = \{e, f, g\}$
$e = \{b, c, d, h\}$
$f = \{h, g\}$
$g = \{h\}$
$h = \{\}$

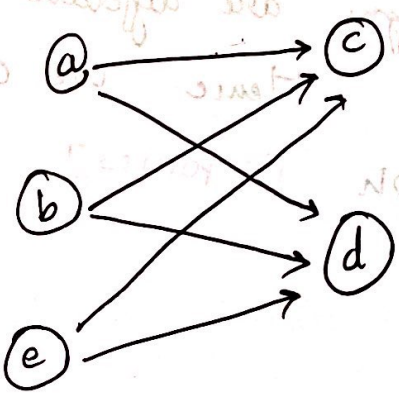
[1.5 POINTS]

Support = 3
$\{b, d\}$ -



OR [1.5 POINTS]

$\{c, d\}$



2) C Green : $\{w, x, y\}$
D Red : $\{w, y\}$
B Blue : $\{y, z\}$

$P_{xy} = P_C$ $\quad P_{xz} = \varepsilon$
$P_{yz} = P_B$ $\quad P_{wz} = \varepsilon$
$P_{wx} = P_C$ $\quad P_{wy} = (1 - (1-P_C)(1-P_D))$

$L = P_{xy} P_{yz} P_{wx} (1 - P_{xz})(1 - P_{wz}) P_{wy}$ [1.5 POINTS]

$= P_C^2 P_B [1 - (1-P_C)(1-P_D)] (1-\varepsilon)^2$

$= P_C^2 P_B (P_C + P_D - P_C P_D)$ [1.5 POINTS]

Take derivative and equate to zero to get the values of P_C and P_D.

3) $X^T L X = \sum_{ij=1}^{n} L_{ij} x_i x_j$

$= \sum_{i,j=1}^{n} (D_{ij} - A_{ij}) x_i x_j$

$= \sum_i d_i x_i^2 - \sum_{(i,j)\in E} 2 x_i x_j$ for each edge (i,j) [1 POINT]

$= \sum_{(i,j)\in E} (x_i^2 + x_j^2 - 2 x_i x_j)$

$= \sum_{(i,j)\in E} (x_i - x_j)^2$ [1 POINT]

To minimize the above equation, if the signs of x_i and x_j are same it will decrease the value of $(x_i - x_j)^2$ but if the signs are different it will increase the value of $(x_i - x_j)^2$. Hence we can use sign of x to split the graph [2 POINTS]