

Quiz 3: Blocked-Based Matrix Multiplication (10 points), 15 minutes

Consider multiplying matrices A and B below using the block-based **1-phase** approach. Note that A and B are each divided into four blocks as shown. Each matrix is stored in a file in a format similar to that in the homework. For example, the file for matrix A has a line “(1,1),[(1,1,1),(1,2,1),(2,2,1)]” for block A¹¹.

$$\begin{array}{cc}
 \begin{array}{c} A^{11} \\ \left[\begin{array}{cc|cc} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{array} \right] \\ A^{21} \end{array} &
 \begin{array}{c} A^{12} \\ \left[\begin{array}{cc|cc} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ \hline 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{array} \right] \\ A^{22} \end{array} &
 \begin{array}{c} B^{11} \\ \left[\begin{array}{cc|cc} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ \hline 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{array} \right] \\ B^{21} \end{array} &
 \begin{array}{c} B^{12} \\ \left[\begin{array}{cc|cc} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ \hline 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{array} \right] \\ B^{22} \end{array}
 \end{array}$$

1. [5 points] State the input and output for each call to the Map function.

A¹¹: (1,1), [(1,1,1),(1,2,1),(2,2,1)] A¹²: (1,2), [(1,2,1),(2,1,1),(2,2,1)]
 A²¹: (2,1), [(1,1,1),(1,2,1),(2,1,1)] A²²: (2,2), [(1,1,1), (2,2,1)]
 B¹¹: (1,1), [(1,1,1),(1,2,1),(2,2,1)] B¹²: (1,2), [(1,2,1),(2,1,1)] B²¹: (2,1), [(1,1,1),(2,2,1)]
 B²²: (2,2), [(2,1,1), (2,2,1)]

A¹¹ => ((1,1)('A', 1, A¹¹), (1,2)('A', 1, A¹¹)) A¹² => ((1,1)('A', 2, A¹²), (1,2)('A', 2, A¹²))
 A²¹ => ((2,1)('A', 1, A²¹), (2,2)('A', 1, A²¹)) A²² => ((2,1)('A', 2, A²²), (2,2)('A', 2, A²²))
 B¹¹ => ((1,1)('B', 1, B¹¹), (2,1)('B', 1, B¹¹)) B¹² => ((1,2)('B', 1, B¹²), (2,2)('B', 1, B¹²))
 B²¹ => ((1,1)('B', 2, B²¹), (2,1)('B', 2, B²¹)) B²² => ((1,2)('B', 2, B²²), (2,2)('B', 2, B²²))

2 points for input, 2 for output.
(key, value-list) format 1 point

2. [5 points] State the input and output for each call to the Reduce function.

Input: ((1,1), [('A', 1, A¹¹), ('B', 1, B¹¹), ('A', 2, A¹²), ('B', 2, B²¹)])
 ((1,2), [('A', 1, A¹¹), ('B', 1, B¹²), ('A', 2, A¹²), ('B', 2, B²²)])
 ((2,1), [('A', 1, A²¹), ('B', 1, B¹¹), ('A', 2, A²²), ('B', 2, B²¹)])
 ((2,2), [('A', 1, A²¹), ('B', 1, B¹²), ('A', 2, A²²), ('B', 2, B²²)])

Output: ((1,1), A¹¹* B¹¹ + A¹²* B²¹) ((1,2), A¹¹* B¹² + A¹²* B²²) ((2,1), A²¹* B¹¹ + A²²* B²¹) ((2,2),
 A²¹* B¹² + A²²* B²²)

2 points for input, 2 points for output (0.5 point each)
(key, value-list) format 1 point