

**CS570**  
Analysis of Algorithms  
Spring 2008  
Exam II

Name: \_\_\_\_\_  
Student ID: \_\_\_\_\_

	Maximum	Received
Problem 1	20	
Problem 2	15	
Problem 3	15	
Problem 4	15	
Problem 5	20	
Problem 6	15	
Total	100	

Note: The exam is closed book closed notes.

1) 20 pts

Mark the following statements as **TRUE**, **FALSE**. No need to provide any justification.

**True [ TRUE/FALSE ]**

If all capacities in a network flow are rational numbers, then the maximum flow will be a rational number, if exist.

**False [ TRUE/FALSE ]**

The Ford-Fulkerson algorithm is based on the greedy approach.

**False [ TRUE/FALSE ]**

The main difference between divide and conquer and dynamic programming is that divide and conquer solves problems in a top-down manner whereas dynamic-programming does this bottom-up.

**False [ TRUE/FALSE ]**

The Ford-Fulkerson algorithm has a polynomial time complexity with respect to the input size.

**True [ TRUE/FALSE ]**

Given the Recurrence,  $T(n) = T(n/2) + \theta(1)$ , the running time would be  $O(\log(n))$

**True [ TRUE/FALSE ]**

If all edge capacities of a flow network are increased by  $k$ , then the maximum flow will be increased by at least  $k$ .

**True [ TRUE/FALSE ]**

A divide and conquer algorithm acting on an input size of  $n$  can have a lower bound less than  $\Omega(n \log n)$ .

**True [ TRUE/FALSE ]**

One can actually prove the correctness of the Master Theorem.

**True [ TRUE/FALSE ]**

In the Ford Fulkerson algorithm, choice of augmenting paths can affect the number of iterations.

**False [ TRUE/FALSE ]**

In the Ford Fulkerson algorithm, choice of augmenting paths can affect the min cut.

2) 15 pts

Present a divide-and-conquer algorithm that determines the minimum difference between any two elements of a sorted array of real numbers.

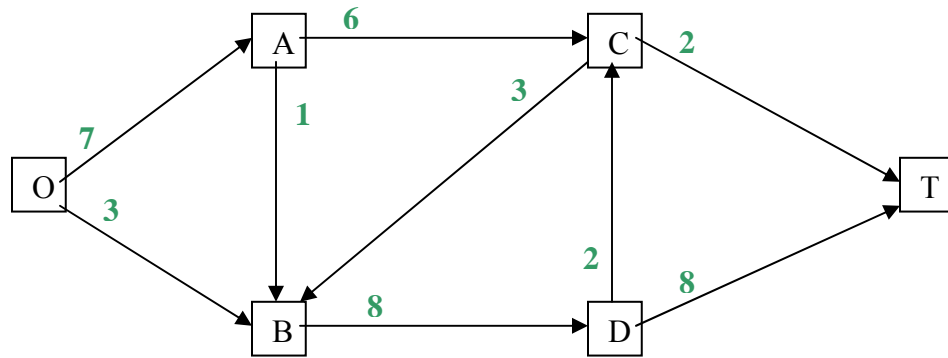
Let us assume the array is  $A[1 \dots n]$

LargestDifference(n)

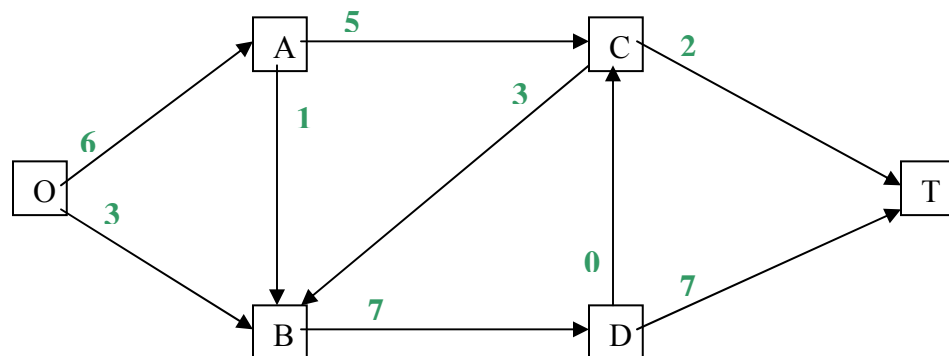
```
{  
    If  $n \leq 1$ , return Infinity;  
    Else, return  $\min(\text{LargestDifference}(n-1), \text{abs}(A[n]-A[n-1]))$ ;  
}
```

3) 15 pts

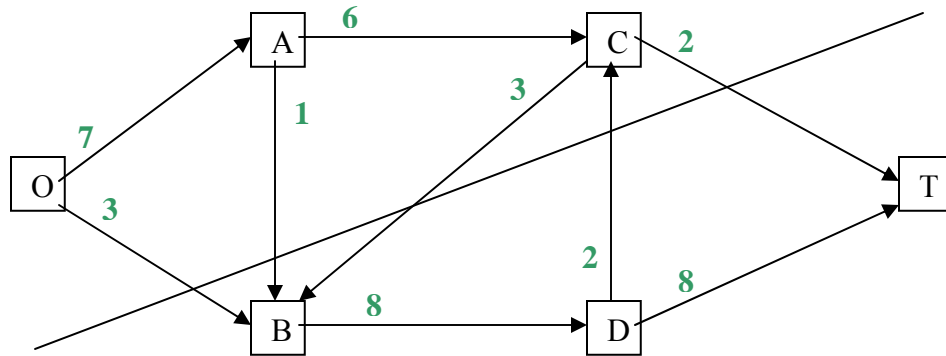
You are given the following directed network with source O and sink T.



a) Find a maximum flow from O to T in the network.



b) Find a minimum cut. What is its capacity?



The Capacity is 9.

4) 15 pts

Solve the following recurrences

a)  $T(n) = 2T(n/2) + n \log n$

By the master theorem,  $T(n) = n \log^2 n$

b)  $T(n) = 2T(n/2) + \log n$

c)  $T(n) = 2T(n-1) - T(n-2)$  for  $n \geq 2$ ;  $T(0) = 3$ ;  $T(1) = 3$

$$\begin{aligned} T(n) &= 2T(n-1) - T(n-2) = 2(2T(n-2) - T(n-3)) - T(n-2) = 3T(n-2) - 2T(n-3) \\ &= 4T(n-3) - 3T(n-4) = nT(1) - (n-1)T(0) = 3 \end{aligned}$$

5) 20 pts

You are given a flow network with integer capacity edges. It consists of a directed graph  $G = (V, E)$ , a source  $s$  and a destination  $t$ , both belong to  $V$ . You are also given a parameter  $k$ . The goal is to delete  $k$  edges so as to reduce the maximum flow in  $G$  as much as possible. Give an efficient algorithm to find the edges to be deleted. Prove the correctness of your algorithm and show the running time.



6) 15 pts

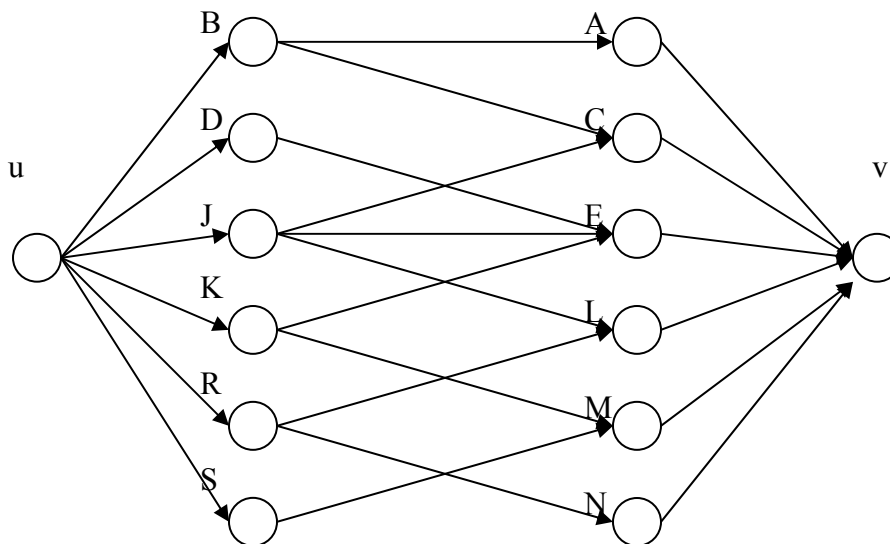
Six men and six women are at a dance. The goal of the matchmaker is to match each woman with a man in a way that maximizes the number of people who are matched with compatible mates. The table below describes the compatibility of the dancers.

	Ann	Cindy	Erin	Liz	Mary	Nancy
Bob	C	C	-	-	-	-
Dave	-	-	C	-	-	-
John	-	C	C	C	-	-
Kevin	-	-	C	-	C	-
Ron	-	-	-	C	-	C
Sam	-	-	-	-	C	-

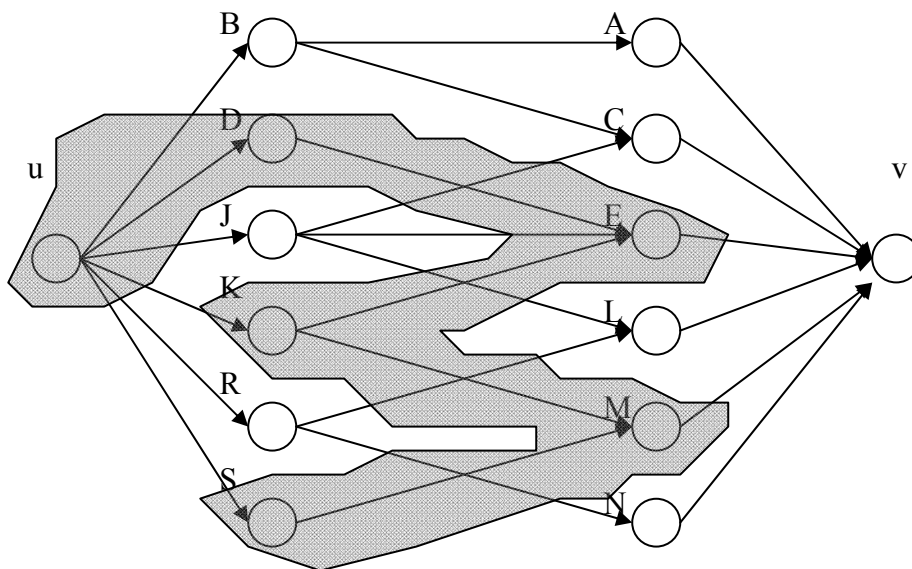
*Note: C indicates compatibility.*

- a) Determine the maximum number of compatible pairs by reducing the problem to a max flow problem.

All edges have capacity of one.



b) Find a minimum cut for the network of part (a).



c) Give the list of pairs in the maximum pairs set.

(B, A), (J, C), (K, E), (R, N), (S, M)