# Quiz Solution - 13

(1)

$$M = \begin{array}{c c} & \begin{array}{ccc} y & a & m \end{array} \\ \begin{array}{c} y \\ a \\ m \end{array} & \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 0 \\ 0 & 1/2 & 1 \end{bmatrix} \end{array} \qquad V^0 = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

$$V' = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 0 \\ 0 & 1/2 & 1 \end{bmatrix} 0.8 \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix} + 0.2 \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$ → [1 POINT]

$$= \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 0 \\ 0 & 1/2 & 1 \end{bmatrix} \begin{bmatrix} 4/15 \\ 4/15 \\ 4/15 \end{bmatrix} + \begin{bmatrix} 1/15 \\ 1/15 \\ 1/15 \end{bmatrix}$$

$$V' = \begin{bmatrix} 5/15 \\ 3/15 \\ 7/15 \end{bmatrix} \Rightarrow \begin{bmatrix} 1/3 \\ 1/5 \\ 7/15 \end{bmatrix}$$ → [1 POINT]

(2) We use $k^2$ map tasks, where $k$ is 4

Thus, we need 16 map tasks

We split M into Vertical strips and V into Corresponding horizontal strips ;

[1 POINT]

<u>Map Task</u> -

m1 : (1, 1×1)
m2 : (1, 2×3)
m3 : (1, 3×2)
m4 : (1, 4×1)
m5 : (2, -1×1)
m6 : (2, 3×2)
m7 : (2, 0)
m8 : (2, 1×5)
m10 : (3, 4×3)
m11 : (3, 2×2)
m12 : (3, 8×1)
m13 : (4, 1×2)
m14 : (4, 3×4)
m15 : (4, 0)
m16 : (4, 6×1)

**Reduce** - For each key, calculate $\sum_{j=1}^{n} m_{ij} \cdot v_j$ — [1 POINT]

for key 1 => [1+6+6+4] = 17

for key 2 => [-1+6+0+5] = 10

for key 3 => [0+12+(-4)+6] = 14

for key 4 => [2+12+0+6] = 20

$\underline{\underline{[17, 10, 14, 20]^T}}$ — [1 POINT]

3. A bucket in DGIM Consists of.

1) Timestep of its end [$O(\log_2 N)$ bits]
2) The number of 1's b/w its begining and end
 = [$O(\log_2 \log_2 N)$)] — [1 POINT]

→ $\log_2 N$ is the maximum # of bits x in a bucket of size N.

→ to store x, we need $\log_2 x$ bits

Hence it is $O(\log_2 \log_2 N)$

Timestamp storage for each bucket would be $O(\log_2 N)$
if N is the window size (0 .... N-1)

Number of 1's :-

$$2^j \le N \Longrightarrow j \le \log_2 N$$

Hence $\log_2(\log_2 N)$ for representing j

Each bucket requires $\approx O(\log_2 N)$ — [1 POINT]

At most $2*j$ buckets of sizes $2^j, 2^{j-1} \ldots 1$

Size of largest bucket $2^j \leq N$

Hence $j \leq \log_2 N$

$2^j \leq 2\log_2 N$

Hence total storage $= O(\log_2 N * \log_2 N)$

$\approx O(\log_2 N)$ —— [1 POINT]

4) Case 1 :- estimate < actual value C

- Worst case : all 1's in bucket b are within range

- To show $C \geq 2^j$

C has atleast one 1 from b and at least one of buckets of lower powers :-

$2^{j-1} + 2^{j-2} + \ldots + 1 = 2^j - 1;$

$C \geq 1 + 2^j - 1$ ; missed at most $2^{j-1}$ —— [1 POINT]

- So estimate missed at most 50% of C.

Case 2 :- estimate > actual value C

- Worst case :- only right most bit of b is within range.

- only one bucket for each smaller power.

$C = 1 + 2^{j-1} + 2^{j-2} + \ldots + 1 = 1 + 2^j - 1 = 2^j$

- Estimate $= 2^{j-1}$ (last bucket) $+ 2^{j-1} + \ldots + 1$

$= 2^j - 1$ (C minus the right most bit) $+ 2^{j-1}$ (last bucket)

$2^{j-1} + 2^{j-2} + \ldots + 1 = 2^j - 1$ —— [1 POINT]

- $\therefore$ estimate is no more than 50% greater than C