CS570 Spring 2018: Analysis of Algorithms Exam I

	Points
Problem 1	20
Problem 2	10
Problem 3	18
Problem 4	20
Problem 5	15
Problem 6	17
Total	100

Instructions:

1. This is a 2-hr exam. Closed book and notes
2. If a description to an algorithm or a proof is required please limit your description or proof to within 150 words, preferably not exceeding the space allotted for that question.
3. No space other than the pages in the exam booklet will be scanned for grading.
4. If you require an additional page for a question, you can use the extra page provided within this booklet. However please indicate clearly that you are continuing the solution on the additional page.
5. Do not detach any sheets from the booklet. Detached sheets will not be scanned.
6. If using a pencil to write the answers, make sure you apply enough pressure so your answers are readable in the scanned copy of your exam.
7. Do not write your answers in cursive scripts.

1) 20 pts

Mark the following statements as **TRUE** or **FALSE**. No need to provide any justification.

[TRUE/FALSE]
If all the edge weights of a given graph are the same, then every spanning tree of that graph is minimum.

[TRUE/FALSE]
An in-order traversal of a min-heap outputs the values in sorted order.

[TRUE/FALSE]
If all edges in a connected undirected graph have distinct positive weights, the shortest path between any two vertices is unique.

[TRUE/FALSE]
If a connected undirected graph $G(V, E)$ has $n = |V|$ vertices and $n + 10$ edges, we can find the minimum spanning tree of G in $O(n)$ runtime.

[TRUE/FALSE]
If path P is the shortest path from u to v and w is a node on the path, then the part of path P from u to w is also the shortest path.

[TRUE/FALSE]
An amortized cost of insertion into a binomial heap is constant.

[TRUE/FALSE]
Gale-Shapley algorithm is a greedy algorithm.

[TRUE/FALSE]
For all positive functions $f(n)$, $g(n)$ and $h(n)$, if $f(n) = O(g(n))$ and $f(n) = \Omega(h(n))$, then $g(n) + h(n) = \Omega(f(n))$.

[TRUE/FALSE]
Any function which is $\Omega(\log \log n)$ is also $\Omega(\log n)$.

[TRUE/FALSE]
The depths of any two leaves in a binomial heap differ by at most 1.

2) 10 pts.
Consider a list data structure that has the following operations defined on it:

- *Append*(x): Adds the element x to the end of the list

- *DeleteFourth*(): Removes every fourth element in the list i.e. removes the first, fifth, ninth, etc., elements of the list.

Assume that *Append*(x) has a cost 1, and *DeleteFourth*() has a cost equals to the number of elements in the list. What is the amortized cost of *Append* and *DeleteFourth* operations? Consider the worst sequence of operations. Justify your answer using the accounting method.

3) 18 pts.

For each of the following recurrences, give an expression for the runtime $T(n)$ if the recurrence can be solved with the Master Theorem. Otherwise, indicate that the Master Theorem does not apply.

1. $T(n) = 16\, T(n/4) + 5\, n^3$
2. $T(n) = 4\, T(n/2) + n^2 \log n$
3. $T(n) = 4\, T(n/8) - n^2$
4. $T(n) = 2^n\, T(n/2) + n$
5. $T(n) = 0.2\, T(n/2) + n \log n$
6. $T(n) = 4\, T(n/2) + n/\log n$

4) 20 pts

You are given a set $X = \{x_1, x_2, \dots, x_n\}$ of points on the real line. Your task is to design a greedy algorithm that finds a smallest set of intervals, each of length-2 that contains all the given points. Linked list is a data structure consisting of a group of nodes which together represent a sequence. Under the simplest form, each node is composed of data and a reference (in other words, a link) to the next node in the sequence.

Example: Suppose that $X = \{1.5, 2.0, 2.1, 5.7, 8.8, 9.1, 10.2\}$. Then the three intervals [1.5, 3.5], [4, 6], and [8.7, 10.7] are length-2 intervals such that every $x \in X$ is contained in one of the intervals. Note that 3 is the minimum possible number of intervals because points 1.5, 5.7, and 8.8 are far enough from each other that they have to be covered by 3 distinct intervals. Also, note that the above solution is not unique.

a) Describe the steps of your greedy algorithm in plain English. What is its runtime complexity? (10 pts)

b) Argue that your algorithm correctly finds the smallest set of intervals. (10 pts)

5) 15 pts.

Given a $n \times n$ matrix where each of the rows and columns are sorted in ascending order, find the k-th smallest element in the matrix using a heap. You may assume that $k < n$. Your algorithm must run in time strictly better than $O(n^2)$.

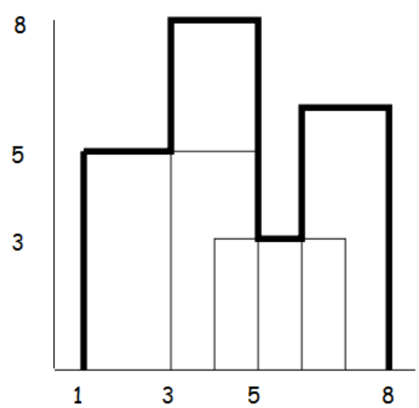
6) 17 pts

Suppose that you are given the exact locations and shapes of several rectangular buildings in a city, and you wish to draw the skyline (in two dimensions) of these buildings, eliminating hidden lines. Assume that the bottoms of all the buildings lie on the x-axis. Each building B_i is represented by a triple (L_i, H_i, R_i), where L_i and R_i denote the left and right x coordinates of the building, respectively, and H_i denotes the building's height. A skyline is a list of x coordinates and the heights connecting them arranged in order from left to right.

For example, the buildings in the figure below correspond to the following input

(1, 5, 5), (4, 3, 7), (3, 8, 5), (6, 6, 8).

The skyline is represented as follows: (1, 5, 3, 8, 5, 3, 6, 6, 8). Notice that in the skyline we alternate the x-coordinates and the heights. Also, the x-coordinates are in sorted order.



a) Given a skyline of n buildings in the form (x_1, h_1, x_2, h_2, …, x_n) and another skyline of m buildings in the form (x'_1, h'_1, x'_2, h'_2, …, x'_m), show how to compute the combined skyline for the $m + n$ buildings in O($m + n$) steps. (5 pts)

b) Assume that we have correctly built a solution to part a), design a divide and conquer algorithm to compute the skyline of a given set of *n* buildings. Your algorithm should run in O(*n* log *n*) steps. (12 pts)