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Search strategies for the feeder bus network design problem

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Abstract

This paper reports on computing solutions for a specific problem arising in public transport systems – the Feeder Bus Network Design Problem (FBDP). The problem requires the design of a set of feeder bus routes and the definition of their service frequencies to satisfy both the resource constraints and the demand for transportation: passengers located at any of the bus stops wish to go to any of the stations of a rail transit line in order to access a common destination identified as the central station. The objective is to minimize a cost function, where both passenger and operator interests are considered. This problem may be formulated as a difficult, nonlinear and nonconvex mixed integer problem, classified as NP-hard. The study focuses on a combined building plus improving heuristic procedure, partially taken from literature. The starting module builds up a solution through a sequential savings or a two-phase method, and for the last module the method includes local search, as well as tabu search heuristics with different strategies. Additionally, computational results from a set of problems simulating real life situations are given. Through this experiment the authors conclude that the simplest short-term version of tabu search is one of most promising heuristics.

Keywords: Heuristics; Tabu search; Transportation; Network design; Routing

1. Introduction

Nowadays, to prevent the increasing use of private transport to enter city centres, efficient alternative means of travel should be proposed. Let us consider a given high capacity, swift transport system, such as hovercraft, suburban rail or even underground. When using this assumption, we should endeavour to set up a second level network of feeder bus routes, whose main purpose is to transport users from the bus stops to the above-specified fixed network. Section 2 of this paper describes the problem of designing such bus routes and defining the respective frequencies to feed a rail transport system.

A similar problem arises when small planes must collect parcels from enterprises, to be dispatched to distant places via scheduled flights made by bigger aircraft operating from major airports [1]. Other issues may occur in varied fields

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such as telecommunication and electric power distribution networks.

The above situations may be tackled by using either analytical models or mathematical programming (also known as network approaches). Although analytical methodologies are easy to implement, their applications are restricted, due to the simplicities introduced into the problem [2]. On the other hand, network approaches are much closer to reality, but prove harder to handle when practical answers are required. They are applied, for instance, in [3] and a survey of network design models may be found in [4].

The formulation given in Section 3 falls into this second category and is taken from [5]. It consists of a large-sized mixed integer nonlinear problem with high computational complexity, which justifies the use of heuristics as being the best way to generate good solutions. In Sections 4 and 5, constructive and local search improvement heuristics (taken from literature or inspired by similar procedures for vehicle routing problems) are described. Section 6 covers tabu search heuristics which were devised for this problem by using the neighbourhood structure of one of the local searches. Diversification and intensification strategies are also introduced into the basic short-term tabu version. Section 7 reports on a computational experiment, performed with a set of test problems, simulating several real situations found in the feeder bus network design.

The favourable results of heuristics, namely those of the basic versions of tabu search, encourage us to subsequently perfect the methodologies applied to this problem by developing different tabu search strategies.

2. Problem description

The purpose of this section is to describe the so-called Feeder Bus Network Design problem (FBDP) in the context of a suburban area, locally served by a bus fleet and connected to the city centre by a rail system.

Let us consider the basic case of a single destination, which corresponds to the standard behaviour of the demand of transportation at the morning peak-period. In this case all passengers share a common destination, identified as the central city station. Passengers gathered at bus stops located in the service area wish to access this destination by first travelling by bus to any of the rail stations and then proceeding to the city centre by train.

The rail network is assumed to be fixed, that is, defined in advance and not subject to changes, whereas the bus network will be determined by the solution to this problem itself. The location of bus stops and the demand of passengers at each stop during a pre-specified period of time are also known. The distance between each pair of stops and stops/stations, and the capacities and operating-speed of the fleet of buses over the planning period, are also given.

The solution to the problem comprises the design of a set of feeder bus routes, as well as the determination of the service frequency of each route for the period of time.

The assumptions of the model are as follows.

Route shape

- Each bus stop must be served by one feeder bus route only;
- each bus route must be linked to just one railway station;
- each bus is assumed to halt at all the stops on its route.

Other constraints

- Buses have fixed operating-speeds and capacities, though they may travel overloaded – within certain specified limits;
- a maximum permissible length is established for each bus route;
- the existing fleet is considered as an aggregated mileage constraint.

From all possibilities compatible with the constraints, the model must choose the one that minimizes the sum of user and operator costs, equating to all costs considered in the system. Passenger costs involve riding and waiting-time costs, both on the bus, as well as the train networks. But operator costs are also considered, and mainly depend on the total length travelled by the vehicles during the planning period. The ticket cost is disregarded, as it corresponds to a flow of money, within the general system, from users to transport operators.

3. Formulation

The above-described situation can, in accordance with Kuah and Perl [5], be formulated as a nonlinear mixed integer problem.

Assuming that the planning period is an hour, previously known data for this formulation are:

- I number of bus stops, $I \geqslant 2$
- J number of train stations, $J \geqslant 2$
- K number of bus routes, $K \geqslant 1$
- g the central station
- v fleet size (vehicles), $v > 0$
- c bus capacity (passengers), $c > 0$
- L_{ih} distance between stop i and stop or station h (miles), $L_{ih} > 0$ for all i and all h
- C_{jg} waiting-time plus riding-time costs per passenger from station j to station g (\$/passenger), $C_{jg} \geqslant 0$ for all j
- D_k maximum length of route k (miles), $D_k \geqslant 0$ for all k
- Q_i average demand per hour at stop i (passengers/hour), $Q_i \geqslant 0$ for all i
- U bus operating-speed (miles/hour), $U > 0$
- ρ bus load factor, $0 < \rho \leqslant c$
- λ_o unit bus operating cost (\$/vehicles-miles), $\lambda_o > 0$
- λ_w value of passenger waiting-time (\$/passenger-hour), $\lambda_w \geqslant 0$
- λ_r value of passenger riding-time (\$/passenger-hour), $\lambda_r \geqslant 0$.

The following notation may be stated:
$A = \{1, \ldots, I\}$ is set of bus stops;
$B = \{I+1, \ldots, I+J\}$ is set containing g and representing train stations;
$N = A \cup B$;
H-subset of N containing B;
$P = \{1, \ldots, K\}$ is set of all bus routes;
$\underline{Q}$-average hourly demand per stop (passengers/hour).

Let us now present the objective function, the model constraints and the definition of the variables:

minimize z

$$= \sum_{j=I+1}^{I+J} C_{jg} \sum_{i=1}^{I} Q_i Y_{ij} + \lambda_w \sum_{k=1}^{K} \left(\frac{1}{2F_k}\right) \sum_{i=1}^{I} \sum_{h=1}^{I+J} Q_i X_{ihk}$$

$$+ \left(\frac{\lambda_r}{2U}\right) \sum_{k=1}^{K} \left[\left(\sum_{i=1}^{I} \sum_{h=1}^{I+J} L_{ih} X_{ihk} \right) \left(\underline{Q} + \sum_{i=1}^{I} \sum_{h=1}^{I+J} Q_i X_{ihk} \right) \right] + 2\lambda_o \sum_{k=1}^{K} F_k \sum_{i=1}^{I} \sum_{h=1}^{I+J} L_{ih} X_{ihk} \quad (0)$$

subject to

$$\sum_{k=1}^{K} \sum_{h=1}^{I+J} X_{ihk} = 1 \quad (i = 1, \ldots, I), \quad (1)$$

$$\sum_{i=1}^{I} \sum_{j=I+1}^{I+J} X_{ijk} \leqslant 1 \quad (k = 1, \ldots, K), \quad (2)$$

$$\sum_{h=1}^{I+J} X_{ihk} - \sum_{m=1}^{I} X_{mik} \geqslant 0$$

$$(i = 1, \ldots, I; \quad k = 1, \ldots, K), \quad (3)$$

$$\sum_{i \notin H} \sum_{h \in H} \sum_{k=1}^{K} X_{ihk} \geqslant 1 \quad (\text{for all } H), \quad (4)$$

$$\sum_{h=1}^{I+J} X_{ihk} + \sum_{m=1}^{I} X_{mjk} - Y_{ij} \leqslant 1$$

$$(i = 1, \ldots, I; \quad j = I+1, \ldots, I+J; \quad k = 1, \ldots, K), \quad (5)$$

$$\sum_{i=1}^{I} Q_i \sum_{h=1}^{I+J} X_{ihk} \leqslant F_k c/\rho \quad (k = 1, \ldots, K), \quad (6)$$

$$\sum_{i=1}^{I} \sum_{h=1}^{I+J} L_{ih} X_{ihk} \leqslant D_k \quad (k = 1, \ldots, K), \quad (7)$$

$$\sum_{k=1}^{K} F_k \sum_{i=1}^{I} \sum_{h=1}^{I+J} L_{ih} X_{ihk} \leqslant \frac{1}{2} vU, \quad (8)$$

$$Y_{ij} = 0, 1 \quad (i = 1, \ldots, I; \quad j = I+1, \ldots, I+J), \quad (9)$$

$$X_{ihk} = 0, 1 \quad (i = 1, \ldots, I; \quad h = 1, \ldots, I+J; \quad k = 1, \ldots, K), \quad (10)$$

$$F_k \geqslant 0 \quad (k = 1, \ldots, K). \quad (11)$$

Here, the binary variables Y_{ij} and X_{ihk} stand for the definition of the routes, and the continuous variables F_k represent the route frequencies.

The first set of constraints, Eqs. (1)–(5), together with the definition of the binary variables (9) and (10), determine the feasibility of the bus routes. The last group, Eqs. (6)–(8), with the definition of the route shape and the frequency variables, (10) and (11), imposes the route capacities and lengths and a total fleet mileage.

Turning to the objective function, it can be regarded as four aggregated sums. The first one represents the passenger costs (waiting plus riding) in the rail system. The two intermediate sums provide approximations to the passengers waiting and riding costs in the buses. The last one gives the costs supported by the bus operator, which are proportionate to the total distance travelled by the buses in the planning period. These are difficult expressions, explained in detail, for instance, in [6].

It should be mentioned that this formulation was taken from [5] and only constraints (6) and (8) slightly differ from the ones of that paper. Finally, we should stress the huge number of constraints and variables of this problem, which has been classified as NP-hard (see [6]).

4. Constructive heuristics

Due to the considerable complexity of the FBDP, heuristics may be regarded as natural approaches. We have developed composite heuristics to obtain accurate solutions by combining constructive with improvement procedures.

Let us start by presenting two alternative constructive heuristics.

4.1. Sequential building

The first heuristic, referred to as Sequential Building, is based on a savings procedure taken from previous research work [5]. This procedure generalizes the savings procedure characteristic of the routing problems by incorporating the frequencies.

The procedure begins by calculating, for each bus stop i, the cost of the minimum direct route from i to any railway station.

The cost of a direct linkage between stop i and station j can be calculated from the objective function (0) and is given by

$$C_{jg}Q_i + \lambda_w Q_i/(2f_{ij}) + \lambda_r L_{ij}Q_i/U + 2\lambda_o f_{ij}L_{ij}, \quad (12)$$

where f_{ij} stands for the frequency of the direct route from i to j. Such a frequency will take an optimal value obtained, as in [7], by minimizing the above function of f_{ij}

$$f^*_{ij} = 1/2\sqrt{\lambda_w Q_i/(\lambda_o L_{ij})}. \quad (13)$$

If we replace the value of the frequency in expression (12) by that of (13), it now becomes possible to obtain the value for CT_{ij}

$$\mathrm{CT}_{ij} = C_{jg}Q_i + 2\sqrt{\lambda_o\lambda_w L_{ij}Q_i} + \lambda_r L_{ij}Q_i/U. \quad (14)$$

For each stop i the cheapest station $j^*(i)$ is

$$\mathrm{CT}_{i,j^*(i)} = \min_{j\in B} \mathrm{CT}_{ij} \quad (15)$$

and this cost is known as the direct linking cost for stop i.

Once these calculations have been made, the procedure starts to build the network by choosing from among all the bus stops the one with the greatest direct linking cost to initiate the first route. Such a stop is placed at the beginning of the first bus route and subsequent stops will always be inserted between that stop and the train station (where the route terminates).

Routes are built sequentially by evaluating the savings resulting from the inclusion of a nonassigned bus stop in the current emerging route. The formula for the savings is the modified expression from [8], using the following cost for route k, which is specific for the FBDP:

$$\mathrm{CTP}_k = C_{j(k),g}\mathrm{QP}_k + 2\sqrt{\lambda_o\lambda_w \mathrm{QP}_k} + \frac{1}{2}\lambda_r L_k \mathrm{QP}_k/(2U), \quad (16)$$

where L_k stands for the length of route k, QP_k for the aggregated demand at route k and $j(k)$ for the station to which route k is assigned.

The cost (16) is similarly obtained, as in Eq. (14), through the use of the frequency value

$$F_k = 1/2\sqrt{\lambda_{\mathrm{w}}\mathrm{QP}_k/(\lambda_{\mathrm{o}}L_k)}. \quad (17)$$

The building of a route ends when at least one of the following conditions occurs:

1. it cannot be expanded without violating the capacities or length constraints;
2. the savings resulting from expansion of that route are below a pre-specified value, μ.

Subsequent routes are initiated either by selecting the nonassigned bus stop with the greatest direct linking cost (if the building has ended on account of the first reason), or by choosing the stop with the lowest savings value for the two stops, between which the last noninserted stop was tested (if the second stopping criterion was used). The selected stop is always placed at the beginning of the new route, and the procedure follows as described.

The purpose of these rules is to establish a balance between the number of routes and their lengths (see [9] or [6]). When no more routes can be built, according to the sequential process explained above, some stops may not have been assigned. Then, either the unassigned stops are imposed within the routes built so far, or direct linkage routes are built (in this case, construction is performed irrespective of the capacities and length constraints). The network designed may therefore not be feasible. If this is the case, the improvements to be explained in the next section are not implemented, and the final answer is that of the constructive module.

In addition, on completing this procedure the number of routes designed may not to be equal to K, the desired number of bus routes. In fact, when the total number of routes built is greater than K, the heuristic does not yield a feasible solution for the specific FBDP formulated above.

For illustrative purposes, Fig. 1 presents the solution obtained by the previously described heuristic with a small instance of FBDP, characterized by the data taken from [3,2] – here denoted by problem IL. The problem IL contains $J = 4$ stations and $I = 55$ stops. As may be expected, this solution includes each route's shape and frequency.

Some performance indicators calculated for this solution are found in Table 1.

4.2. Two-phase building

The other constructive procedure that will be described for the FBDP, referred to as Two-Phase Building heuristic, also uses the adapted savings criterion, and was inspired by the two-phase heuristic of Golden et al. [10] for the vehicle routing problem.

The Two-Phase Building takes into particular account the highest dimension problems, by splitting the bus stops into border and nonborder stops. It works separately in these two subgroups of stops, and requires far less calculations.

Let us define, for each stop, the ratio between the direct access cost of the cheapest station, $j^*(i)$, and the same cost for the second, least expensive station, $j^{**}(i)$

$$r(i) = \mathrm{CT}_{i,j^*(i)}/\mathrm{CT}_{i,j^{**}(i)}. \quad (18)$$

After choosing a classifying parameter η, if $r(i) \leqslant \eta$ then stop i is considered as a nonborder stop and is assigned to station j^*. If $r(i) > \eta$ then stop i is classified as a border stop and is not assigned to any station.

In the first phase, border stops are linked to stations by a route-building procedure identical to the one applied in the Sequential Building heuristic. At the end of this phase, all border stops are included in routes constructed to date, and the remaining stops (the nonborder ones) are only assigned to stations.

In a second phase, the problem is divided into J subproblems (one for each station), and the route construction begins by trying to insert the nonborder stops in the existing routes ending at the station to which they were previously assigned. Then, if some stops have still not been included, the procedure creates new routes. This is performed in keeping with the Sequential Building savings procedure, in the particular case of a single station.

Performance indicators, in the case of problem IL, were also calculated for the Two-Phase Building heuristic taken with $\eta = 0.85$ (Table 2). As we can see, the results do not differ much from the figures found in Table 1.

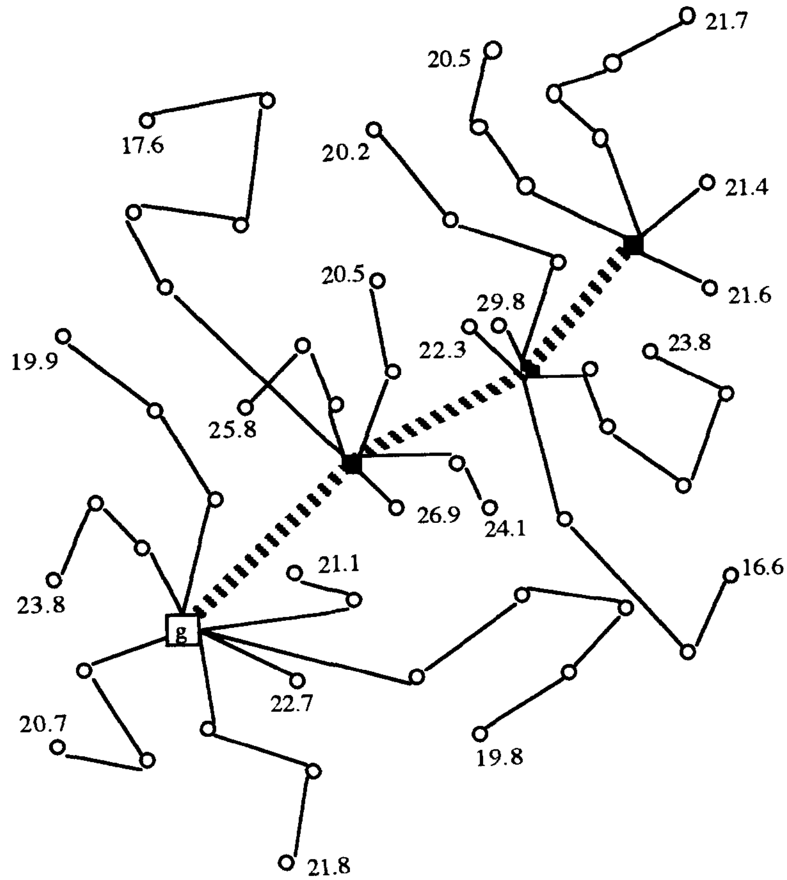

Fig. 1. Network and frequencies for problem IL with the Sequential Building heuristic.

Table 1
Solution indicators for problem IL with the Sequential Building heuristic

No. of routes	Average length per route	Average frequency per route	Approx. no. of buses	Total cost ($\simeq z$)
21	0.79 miles	22.02	44	$6387.7

Table 2
Indicators from the Two-Phase Building heuristic with problem IL

No. of routes	Average length per route	Average frequency per route	Approx. no. of buses	Total cost ($\simeq z$)
22	0.75 miles	21.96	44	$6334.3

In conclusion, one may therefore state that these two constructive heuristics possess the following major limitations:

- the order of the stops in any given route may be suboptimal, largely due to the fact that the savings procedure is sequential;
- once a stop is inserted in a route, there is no re-evaluation of the savings provided by the placement of that stop on routes to be subsequently developed.

5. Local search heuristics

After a feasible solution has been found through a constructive method, use is made of local search improving techniques, designed to overcome the limitations referred to. The Single Route procedure is the initial module of all improvement heuristics. Its purpose is to reorder the bus stop sequence by using a straightforward 2-optimal search. Two alternative ways of completing the local search may ensue, and are presented in the following sections.

5.1. Displacement

The first, taken from [2], is known as Displacement heuristic. In this case the local search goes on with the Internal Displacement procedure, which tries changes from one solution to another, referred to as Internal Displacements. Such changes involve movements of a stop from its current route to another route linked to the same rail node, provided it reduces the total cost – positive displacement – and the new route continues to satisfy problem capacities and length constraints – feasible displacement.

All these Internal Displacements are ranked in a displacement list, and the one providing the highest reduction in the total cost is performed. After updating this list, the Internal Displacement procedure continues until the list is empty.

Once all the Internal Displacements have been tried, the External Displacement procedure begins. This procedure differs from the previous one in so far as a stop can be placed in routes linked to different train stations from the one that serves the current route of that stop. The moves are referred to as External Displacements, and notation of positive and feasible displacements are also used here. Once again the procedure ends when the displacement list is empty.

Fig. 2 and Table 3 show the changes operated by this Displacement heuristic over the network structure given by the Sequential Building heuristic on problem IL, described in Section 4.1 and presented in Table 1 and Fig. 1.

We may observe that some improvements were obtained in the quality of the solution. Comparison of Table 1 with Table 3, indicates that total costs decreased by about \$357 (5.4%), which points to a more favourable route design, both to operator (the number of vehicles fell from 44 to 42) and to passengers (average frequency increased from 22.02 to 22.57). It should also be noted that three routes were scrapped.

Mention should be made that the results presented in [5] for the same case, using a procedure similar to the Displacement heuristic, proved to be equivalent in terms of total cost. In fact, the total cost is \$6033, but the solution is slightly different: the number of routes was 16, the average length per route 1.05 miles, the average frequency 21.1 and the number of buses 54.

5.2. Exchange

It is possible to point out a limitation of the previous local search heuristic. This occurs when a displacement move is cost-effective (positive displacement), but cannot be performed as it fails to satisfy some of the constraints (nonfeasible displacement). In such cases the Displacement heuristic may be too restrictive, as it proposes no alternative. The second local search heuristic, suggested in [9] in the context of a routing problem, was developed to overcome this limitation.

Therefore in this second, so-called Exchange heuristic, the move of a stop from its current route to another route – exchange – is also performed if, in return, a stop from the second route can be placed in the first one with a reduction in total cost – positive exchange – while ensuring that the new routes remain feasible – feasible exchange. The criteria used in the Displacement heuristic to classify the moves as internal and external are disregarded here, and all the exchanges are tested simultaneously.

As this neighbourhood is of an appreciable size, that is, the number of possible real locations for the stops in question is quite considerable, this procedure restricts the possible moves to potential exchanges, defined as follows.

Let i be a stop of route $k1$ that is nearer to p (a stop belonging to another route $k2$) than to its preceding or proceeding stops in $k1$, $\mathrm{PRE}_i = s$ and $\mathrm{POS}_i = t$, respectively. Fig. 3 displays a situation where i is nearer to p than to POS_i.

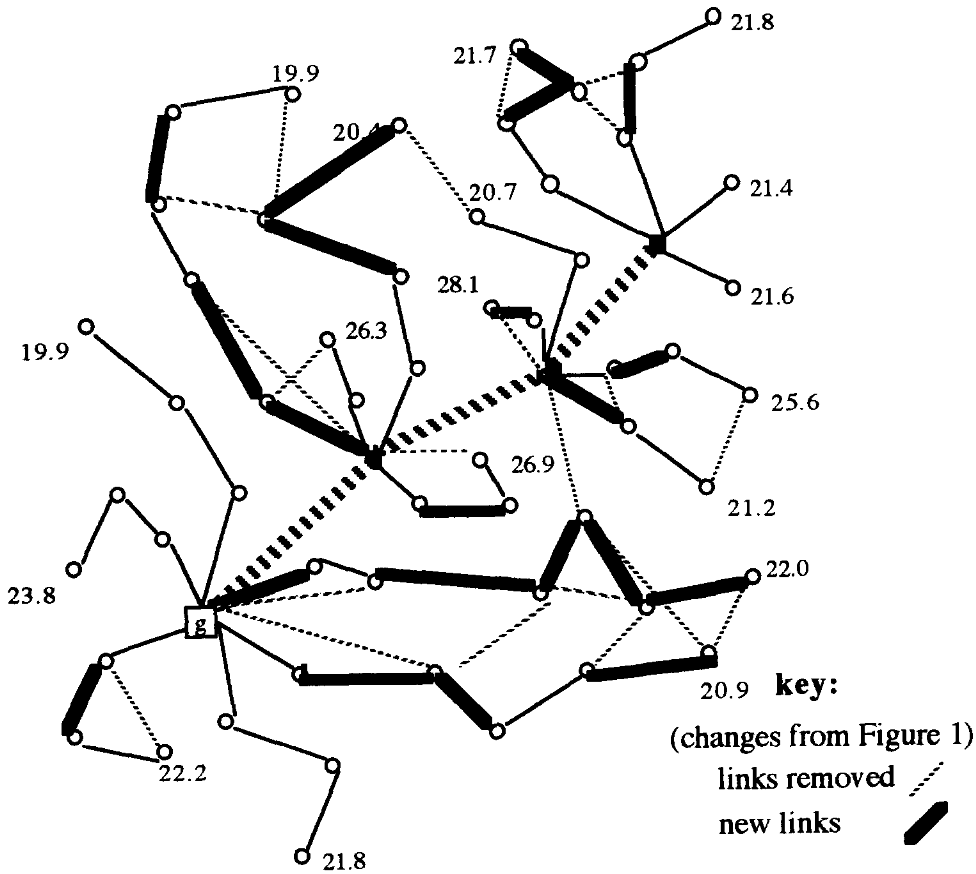

Fig. 2. Network and frequencies for problem IL with the Displacement heuristic.

Table 3
Indicators from the Displacement heuristic with problem IL

No. of routes	Average length per route	Average frequency per route	Approx. no. of buses	Total cost ($\simeq z$)
18	0.84 miles	22.57	42	$6030.7

One potential exchange move is precisely that of i to the position after p on route $k2$ and, at the same time, the inclusion of a stop from $k2$ (except p and $\mathrm{POS}_p = w$ which are the reference stops for i) in any position of route $k1$, to be determined by the savings criterion (as well as between s and t).

As in the Displacement heuristic, a ranked list of the moves is used, and the heuristic terminates when the list is empty.

The Exchange heuristic was developed in two versions characterized by the way in which the exchange of stops is performed, that is, by the precise neighbourhood structure. While in version 1 only potential exchanges are considered, version 2 works both with displacement and potential exchange moves (see [6] for more details). In this case, potential exchanges are tested only if the displacements prove to be nonfeasible. The aim of this second version is once again to overcome a

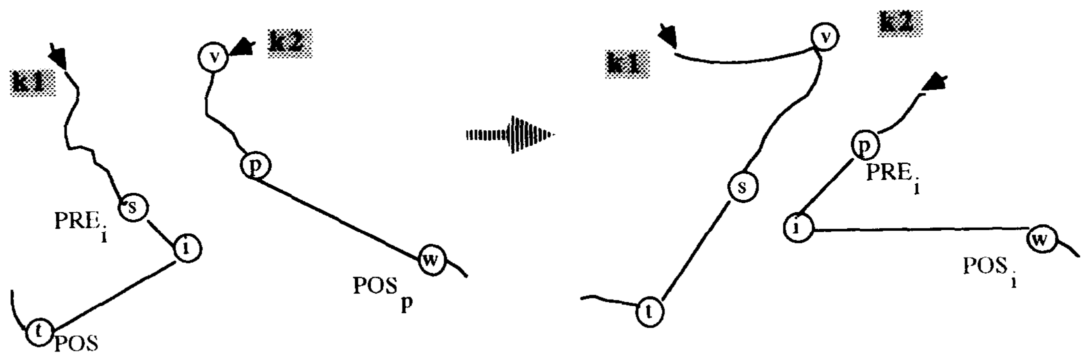

Fig. 3. Potential exchanges.

limitation of the first one, which lies in the compulsory exchange of stops, even when a displacement would be sufficient to improve the solution.

6. Tabu search

As solutions obtained from the various local search heuristics tend to be similar, a local search guided by tabu restrictions was developed, thus inducing the moving away from previously found solutions, to attain solutions never visited before – and possibly better ones. This technique has been successfully applied to multi-depot vehicle routing problems with constraints [11–14] and is explained, for instance, in [15].

Like the previous Improvement heuristics, tabu search also starts with the Single Route procedure.

6.1. *Basic strategies*

Let us begin by presenting all features of a straightforward heuristic which uses only a basic short-term memory, denoted by Basic Tabu. Basic Tabu searches in the neighbourhood, defined through the Exchange heuristic in version 2.

Here, solutions for the FBDP are characterized by the solution attributes stated for each bus stop i: PRE_i is the stop that precedes stop i in the route containing i, and POS_i the stop that proceeds stop i in the same route. The redundancy of these attributes is due to the ease of implementation.

Move attributes consider the changes in the above-defined vectors resulting from a displacement or a potential exchange move. The candidate list of moves corresponds to the complete neighbourhood.

Tabu constraints prevent the replacement of a stop in its previous position for some iterations, the tabu tenure being $\sqrt{I+J}$.

Fig. 4 displays the two kinds of moves (displacements and exchanges). When a displacement takes place, stop i cannot be re-inserted between PRE_i and POS_i (here s and t) for that pre-specified number of iterations ($\sqrt{I+J}$).

In the case of an exchange, there is an additional restriction: stop u cannot be re-inserted between PRE_u and POS_u (here v and w) in the following $\sqrt{I+J}$ iterations.

In other words, in the example of Fig. 4, solutions including attributes $\mathrm{PRE}_i = s$, $\mathrm{POS}_i = t$ ($\mathrm{POS}_s = i$, $\mathrm{PRE}_t = i$) and $\mathrm{PRE}_u = v$, $\mathrm{POS}_u = w$ ($\mathrm{POS}_v = u$, $\mathrm{PRE}_w = u$) are temporarily forbidden.

As usual, these tabu constraints are applied in a short-term mode. Other characteristics of Basic Tabu are:
- the aspiration criterion, enabling the heuristic to overcome the tabu constraints, is stipulated by default and by objective;
- the selection criterion for choosing moves is the first best admissible (f.b.a.) or the best

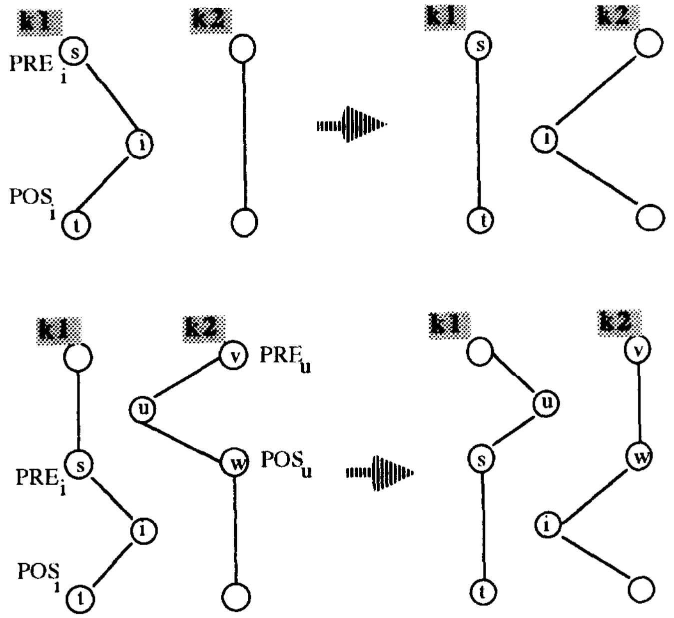

Fig. 4. Displacement and exchange moves.

admissible (b.a.), respectively for the two tabu versions studied (Basic Tabu 1 and Basic Tabu 2);

- the stopping rule for the heuristic acts when the number of iterations (or generated solutions) without changes for the better is equal to $\delta = \sqrt{I+J}\sqrt{K}$.

Results for the illustration problem IL with Basic Tabu 1 are summarized below, in Fig. 5 and Table 4.

In this specific situation the results are similar to the ones obtained by the Displacement heuristic, though slightly worse (compare Table 3 with Table 4). However, the network configuration of Fig. 5 is different from that of Fig. 2: only eight of the eighteen routes are equal, thus ultimately providing a different solution, which was the aim of the tabu search.

6.2. *Diversification and intensification*

In order to improve the performance of tabu search, other versions were designed to include intensification and diversification strategies within the basic short-term tabu searches presented in Section 6.1. These new versions were called Tabu Link and Tabu Node. Intensification strategy is used in the same way in all versions, whereas diversification in Tabu Link is different from that of Tabu Node.

In new versions, intensification is employed on a short-term basis, by forbidding the stops from leaving the routes of the best-so-far solution. It sets out to intensify the search in a region of good solutions before leaving the search to take other directions. The number of iterations where this is imposed is $\sqrt[4]{I+J}$, a value below the tabu tenure $(\sqrt{I+J})$.

Therefore, if the best-so-far solution is found as a result of a displacement of stop i from $k1$ to the place immediately following stop p in $k2$ (supposing that, before the move, POS_p was given by w), all the solutions that do not possess the attributes $\mathrm{PRE}_i = p$, $\mathrm{POS}_i = w$ $(\mathrm{POS}_p = i, \mathrm{PRE}_w = i)$ are forbidden during $\sqrt[4]{I+J}$ iterations. The same applies to cases of exchange moves with the corresponding adaptations.

Two matrixes, similar to those defined for the Basic Tabu constraints, are sufficient to perform this feature.

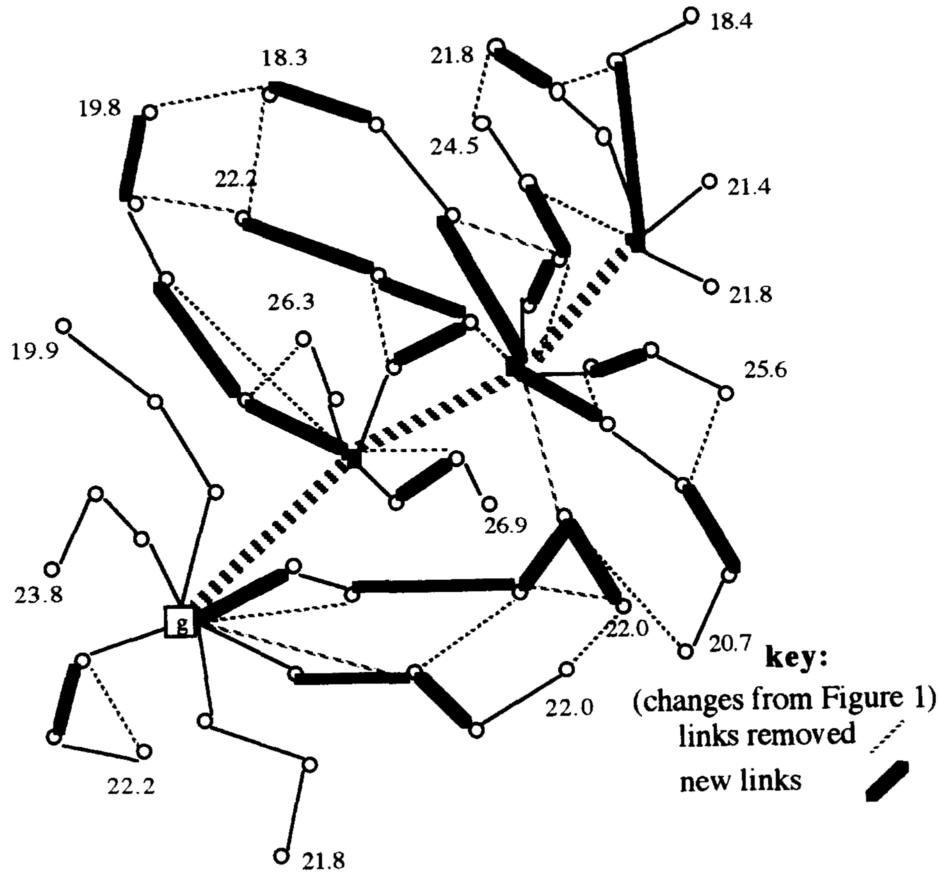

Fig. 5. Network and frequencies for problem IL with the Basic Tabu 1.

Table 4
Indicators for problem IL with the Basic Tabu 1

No. of routes	Average length per route	Average frequency per route	Approx. no. of buses	Total cost ($\simeq z$)
18	0.85 miles	22.17	43	$6061.3

When this search ends – after δ iterations without improvement – the less frequently changed attributes (of the input solution) are, for the purpose of diversification, penalized within a longer term memory search process. Here the most constant linkages are penalized.

For the purpose of diversification in the Tabu Link heuristic, the most constant linkages are penalized according to

$$L'_{ih} = L_{ih} + (P_{ih}/\mathrm{TIT})\sqrt[4]{\underline{L}}, \qquad (19)$$

where P_{ih} is the number of iterations with the linkage from i to h present, TIT is the total number of iterations from the beginning of the previous run of the basic tabu search, and $\underline{L}$ stands for the average distance between any two points (bus stops or rail stations).

This updated distance matrix is applied to the design of a new network through constructive plus improving procedures. If the resulting solution is better than the previous one (found at the end of tabu search with intensification), the

Table 5
Main differences among tabu versions

	Basic Tabu 1	Basic Tabu 2	Tabu Link 3	Tabu Link 4	Tabu Node 5	Tabu Node 6
Selection criterion	f.b.a.	b.a.	f.b.a.	b.a.	f.b.a.	b.a.
Intensification	No	No	Yes	Yes	Yes	Yes
Diversification	No	No	Yes (links)	Yes (links)	Yes (stops)	Yes (stops)

procedure continues by once more penalizing links with formula (19), until the solution fails to improve.

Tabu Link heuristic was developed in two versions – Tabu Link 3 and Tabu Link 4 – corresponding to Basic Tabu 1 and Basic Tabu 2, respectively, both with intensification and diversification.

The search in versions Tabu Node 5 and Tabu Node 6 is similar to the search in Tabu Link 3 and Tabu Link 4, respectively, but the diversification strategy is performed differently. Here we take advantage of Sequential Building construction methodology. Remember that this constructive process gives priority, at the beginning of the new routes, to stops with high direct linking costs.

Diversification in this case is attempted by increasing the costs of the more frequently moved stops. These are certainly the ones that require more attention and should therefore be the first to consider when building routes. The purpose is again to obtain different solutions from the previously visited ones.

In this search, a new auxiliary matrix is defined and elements p_{ih} are associated with the links between stops and stops/stations. Taking i, a stop involved in a displacement or exchange in the last iteration, this matrix is updated by making $p_{ih} = p_{ih} + 1$ and $p_{hi} = p_{hi} + 1$ with h representing all other stops and stations. In other words, all the elements in column i and row i are increased by one unit. This means the penalization factor is on the node itself and not on the linkages.

When the diversifying option is taken, the distances are recalculated by using Eq. (19) with p_{ih} instead of P_{ih}.

Before closing Section 6, all the tabu schemes studied here are summarized below in Table 5.

7. Computational experiments

7.1. Test problems

The set of test problems was mainly obtained through simulation. In fact some parameters of the FBDP were taken from literature and all the remaining data were established or randomly generated to produce different general network configurations simulating reality.

As in the related literature, the demands (in passengers) were fixed at 200, or randomly generated in the interval [30,300].

Bus stops were randomly located, using a uniform distribution in squares of 3×3 or 5×5 (in miles^2).

The maximum lengths for all routes were 2.5 miles in the case of small squares, and 4.2 miles, in the other cases.

Three groups of FBDP problems were constructed with different rail network configurations: one group with 16 small problems ($I = 50, J = 5$), another with 20 medium ones ($I = 75, J = 7$) and another with 20 large problems ($I = 100, J = 10$). The railway patterns for these three groups were established on the basis of lines (for small problems), junctions (for medium problems) or crossings (for large problems), respectively, as schematically seen in Table 6. The central station adopted different positions.

These options make the number of cases generated for the experiment equal to $4 \times 2 \times 2 + 5 \times 2 \times 2 + 5 \times 2 \times 2$, which totals 56.

7.2. Computing results of the combined heuristic

A computational experiment applying the above methods was carried out on a Pentium run-

Table 6
Railway network configurations

	Shape of the rail line in the rectangle	Place for the central station
16 small problems		
Pattern 1	Diagonal	Central
Pattern 2	Diagonal	Peripheral
Pattern 3	Inverted "V"	Central
Pattern 4	Horizontal	Peripheral
20 medium problems		
Pattern 1	Inverted "Y"	Central
Pattern 2	Inverted "Y"	Peripheral
Pattern 3	"T"	Peripheral
Pattern 4	"T"	Central
Pattern 5	Rotated "T"	Peripheral
20 large problems		
Pattern 1	"X"	Central
Pattern 2	"X"	Peripheral
Pattern 3	"X" plus branch	Central
Pattern 4	"X" plus branch	Peripheral
Pattern 5	"+"	Peripheral

Table 7
The combined heuristic

Sequential Building or Two-Phase Building
+
Single Route
+
Internal + External Displacements or Exchange (versions 1 and 2) or Tabu (versions 1–4 and 6)

Table 8
Average total costs ($) of Sequential and Two-Phase Building heuristics

Problem groups	Sequential Building	Two-Phase Building
16 Small	7407.7	7355.4
20 Medium	10 753.8	10 586.0
20 Large	13 234.7	13 077.5

Table 9
Average computing time (s) of Sequential and Two-Phase Building heuristics

Problem groups	Sequential Building	Two-Phase Building
16 Small	0.07	0.06
20 Medium	0.2	0.1
20 Large	0.4	0.2

ning at 100 MHz and using the Microsoft Fortran Power Station compiler.

Table 7 gives an outline of the combined procedure embedding the heuristics for the FBDP described in this report – Sections 4–6.

Let us first take the constructive heuristics, Sequential and Two-Phase Building, and analyse their computing results.

Tables 8 and 9, respectively, present the average total costs and the average computing time for each group of problems (first – 16 small, second – 20 medium and third – 20 large).

From the above figures it is clear that, in terms of solution quality, the two methods are virtually equal. Although unexpected, Two-Phase Building proved to be slightly better, as well as taking less computing time – the purpose for which it was developed.

A different grouping, on the basis of demand or dimension of service area, did not permit us to draw other conclusions.

The results from Improvement heuristics (Displacement, Exchange 1 and 2, Basic Tabu 1 and 2, Tabu Link 3 and 4, Tabu Node 6) are recorded in the three tables below, Tables 10–12. They contain the following measurements for each problem group: percentage of the best occurrences, the average percentual improvement over the total costs given by constructive heuristic Sequential Building and, lastly, the average computing times. Tabu Node 5 is not included in this summary as its computing results are related to the figures of Tabu Node 6, just as those of Tabu Link 3 are related to Tabu Link 4.

Exchange heuristics offered the poorest quality results (Tables 10 and 11), whilst taking the best computing times (Table 12). Such behaviour is related to the reduction in search effort caused by the definition of the potential moves. As a whole, the Displacement heuristic provided better quality solutions than Exchange heuristics, despite a short increase in computing time. We opted for the Exchange 2 neighbourhood in the tabu heuristics due to its far better computing times as it has to be repeatedly called in these heuristics.

Table 10
Percentage of best occurrences from search heuristics (%)

Problem groups	Displacement	Exchange 1	Exchange 2	Basic Tabu 1	Basic Tabu 2	Tabu Link 3	Tabu Link 4	Tabu Node 6
16 Small	25.0	0.0	6.3	37.5	25.0	25.0	31.3	37.5
20 Medium	25.0	0.0	0.0	15.0	30.0	15.0	30.5	25.0
20 Large	0.0	0.0	0.0	40.0	65.0	0.0	5.0	0.0

Table 11
Average percentual improvements of search heuristics (%)

Problem groups	Displacement	Exchange 1	Exchange 2	Basic Tabu 1	Basic Tabu 2	Tabu Link 3	Tabu Link 4	Tabu Node 6
16 Small	3.7	2.2	3.1	4.1	4.2	4.1	4.1	4.1
20 Medium	3.5	1.9	2.1	3.7	3.9	3.8	3.9	3.9
20 Large	6.6	7.8	5.6	10.3	10.0	6.4	6.6	7.1

Table 12
Average computing times (s) of search heuristics

Problem groups	Displacement	Exchange 1	Exchange 2	Basic Tabu 1	Basic Tabu 2	Tabu Link 3	Tabu Link 4	Tabu Node 6
16 Small	0.1	0.05	0.04	1.6	2.2	1.2	2.7	0.7
20 Medium	0.5	0.09	0.1	6.7	9.6	6.8	12.0	4.3
20 Large	1.5	0.2	0.3	17.2	25.3	24.6	29.7	15.0

The percentual reductions obtained by the tabu searches were, in almost all cases, much more significant for the Basic Tabu (versions 1 and 2) than for the other tabu heuristics, including intensification and diversification strategies (versions 3, 4 and 6), as seen in Tables 10 and 11. This fact may be related to intensification, which prematurely prevents the search from moving to better regions. Experiments under study appear to confirm this idea.

Also, among tabu heuristics we find that the ones with criterion f.b.a. (versions 1 and 3) took less time than versions 2 and 4, no doubt because they select the first best solution instead of the best one – Table 12. The quality of the solution was indeed very similar for both selection criteria – Table 11.

As for solution quality of all the search heuristics, it becomes clear that tabu searches slightly outperform the three other local search heuristics, but at the expense of computing time, which is in fact less favourable, though not dramatically so.

Therefore, in terms of solution quality, comparison of heuristic behaviour leads one to place the Basic Tabu versions first, although the Displacement heuristic showed the best trade-off between solution quality and time consumed. It should, however, be remembered that time criterion is of little importance in a strategic decision problem with a static situation like the one modelled by the FBDP.

8. Conclusions and perspectives for future research

The heuristic methodologies, including building and improving features given in this report, proved to be appropriate for the FBDP. Due to the computing results obtained, particularly from the tabu search in the simplest versions, we are encouraged to improve the tabu search strategies. Whereas the constructive and improvement heuristics are not expected to provide very different results, in tabu, research may be performed in the light of works that have recently appeared on standard routing problems.

We have already tested, with relative lack of success, a randomized diversification/intensification strategy. This was adapted from a powerful iterative method devised by Rochat and Taillard [16] for a vehicle routing problem with time windows. It starts from a set of feasible solutions and in each iteration applies a process alternating between probabilistic diversification and intensification phases.

In addition to the method of Rochat and Taillard, other works have enhanced the ideas of searching with different intelligent strategies. Mention should be made of the paper of Xu and Kelly [14] which also presents a tabu search with intensification and diversification for vehicle routing problems with capacity constraints. Such a method includes oscillation between two neighbourhoods, penalized movements into nonfeasible solutions, destructive and constructive steps and an accurate tuning of all the algorithm parameters according to the past behaviour of the iterative procedure.

Many other ideas have been proposed in literature, and in [15] one may find a careful exposition of tabu features, from the basic to the most elaborate ones.

As for the FBDP, we consider that the candidate list strategies should be further studied for this problem. Our selection of moves is based on the best quality among the candidate ones, but a probabilistic choice from the k-best moves, for instance, could lead to a more favourable behaviour.

The blending of contrasting neighbourhoods is also a promising path. Test performance on two not very different neighbourhoods independently produced similar results. We should find an even more different neighbourhood structure and define a search that oscillates among the vicinities in a controlled fashion.

Another direction for future research would be to ascertain how far the total cost of the heuristic solution lies from the optimum value, for each instance of FBDP. This could be determined by calculating lower bounds for the optimal total cost – the objective value of formulation presented in Section 3. In the case of the FBDP, it seems that lower bounds could only be obtained through radical problem reformulations, while taking into account the presence of route frequencies. At this stage, after testing different search strategies, we now consider this to be the most suitable way to confirm our suspicions that the solutions obtained are not far from the optimum values.

Finally, the problem studied illustrates a very special case of route design, characterized by its nonlinearities, as well as consideration of the frequencies. For these reasons it, no doubt, differs from a lot of real routing problems. Other models should therefore be tried and one could even go as far as to modify the objective function, which is the cause of the major difficulties we have encountered in our research.

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