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Time-varying Universe Based Linguistic Dynamic Analysis of Timing Design for Parallel Traffic Light

Hong Mo
School of Electrical and
Information Engineering
Changsha University of Science and Technology
Changsha, CN 30332–0250
Email: mohong198@163.com

Fei-Yue Wang
the State Key Lab. of Management
and Control for Complex Systems,
Chinese Academy of Science,
Institute of Automation, CN,
Email: feiyue@gmail.com

Fenghua Zhu
the State Key Lab. of Management
and Control for Complex Systems,
Chinese Academy of Science,
Institute of Automation, CN,
Email: fhzhu830@qq.com

***Abstract*—Reasonable timing design for traffic light can induce and maintain the transportation systems in good order. How to allocate the time are the keys. In the paper, the theory of time-varying universe is used to describe the circle time, and corresponding fuzzy sets on the universe are also discussed to modeling the situation of traffic flow, then the parallel traffic management and control methods which are dynamic with the time change are presented. A simulation example are provided to analyze the linguistic dynamic evolution of timing design of traffic light when the traffic flow is change with time-varying for an intersection.**

I. Introduction

With the rapid development of economy and the progress of the society, there is a sharp increase in the number of city vehicle. City traffic congestion, traffic environment deterioration, frequent traffic accidents and energy shortage have become the common problems of the world. Both developed and developing countries are all plagued by the worsening traffic. Merely building or expanding roads can hardly solve the problem because of increasing population and decreasing per capita of city residential area. In the past decades, the method of intelligent vehicle infrastructure cooperative systems is provided to cope with these questions[1], and it have alleviated traffic congestion. At the same time, the conditional control of traffic light is based on fixed timing design. However, the traffic flow always changes with time-variation, and fixed timing design for traffic light can't react to the actual situation of traffic flow, especially the traffic density is very big or very small, and it wastes road resources.

When information technology obtains rapid development, traffic system exhibits the characteristics of dynamics, rapidity, openness, interactive and massive data. At the same time, the problems of traffic management and transportation policy formulation become more and more difficult, social transportation will provide an effective means for the complex and dynamic traffic systems[2].

Social transportation contains not only contains social computing and intelligent transportation systems, but also contains the factor of human[3,4], and management rule and law. The information associated with human behavior are mainly perception information, and it is always represented in the form of nature language. Moreover, traffic management and transportation policy are expressed in language, so how to deal with perception information becomes one of the keys to solve these problems. Linguistic dynamic systems(LDS) is based on computing with words instead of numbers and symbols, and the problems of modeling, analysis, evaluation and control for complex systems can be solved at the level of language[5,6]. In the past, "words" are fuzzy sets or type-2 fuzzy sets on a given universe. Mo and Wang have presented the theory of linguistic dynamic systems on time-varying universe, but the discussed universes are all single-factor[7,8,9].

However, the complex transportation systems contains many factors and every factor plays a key pole in the systems. For example, when the mixed traffic flow is processed, pedestrians, non-motorized vehicle and motor vehicle are three important factors[10,11], so for the description and analysis of mixed traffic flow, the three factors should be considered, meanwhile, as a complex systems, the characteristics of traffic flow are different for different place at the same time, also different for the same place at different time. Thus it is necessary to build linguistic dynamic systems based on time-vary multi-factors universe to analyze and evaluate the mixed traffic flow.

In the paper, fuzzy sets time-vary universe are represented to describe the timing designing of parallel traffic light, and dynamic fuzzy rules on time-vary universe are used to formulate and adjust timing designing, then the strategies of timing design for parallel traffic light to make the traffic flow more smoothly.

The paper is arranged as follows: section 2 introduces some relative concepts and notions; section 3 provides the representation of mixed traffic flow based on time-vary multi-factor universe; the management rule and policy of transportation are in section 4; section 5 uses fuzzy comprehension evaluation analyze traffic management rules and policy; section 6 is the conclusion.

II. Preliminary

A fuzzy set ω on Ω is defined as

$$\mu : \Omega \rightarrow [0, 1]$$

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where, Ω is a single-factor universe. That is to say, for every $x \in \Omega$, there is a definite number $z \in [0,1]$, such that

$$\mu_\omega(x) = z$$

μ is the membership function, and the value of μ at x is membership grade. If there is a subset $E \subseteq \Omega$, and for every $x \in E$, such that $\mu_\omega(x) = 1$, otherwise $\mu_\omega(x) = 0$, then ω is a clear word, written as χ_E. If $E = \{a\}$, then ω is a singleton word, written as χ_a.

Linguistic dynamic systems was provided by Wang in 1995 [17]. In LDS, the state equation, output equation and the feedback equation are converted to the corresponding linguistic forms[18−20]:

The state equation of LDS

$$X(k+1) = F(X(k), U(k), k), F : I^N \times I^M \to I^N \quad (1)$$

The output equation of LDS

$$Y(k) = H(X(k), k), H : I^N \times Z \to I^P \quad (2)$$

The feedback equation of LDS

$$U(k) = R(Y(k), V(k), k), R : I^P \times I^Q \times Z \to I^N \quad (3)$$

where, $Z = \{0, 1, \cdots, K\}$, $X(k) \in I^N$ is the state word of the system, $Y(k) \in I^P$ the output word, $V(k) \in I^Q$ the input word, $U(k) \in I^M$ the control word, k the discrete time series, and F, H, R are all fuzzy logic operators, each defines the system, output and control mapping of LDS.

The universe Ω is the set of all things under consideration during a discussion, examination, or study. However, with the time variation, the universe also change, written as $\Omega(t)$. If for every t, there is $\Omega(t) = \Omega$, then $\Omega(t)$ is a constant universe. In general, we take $t = 1, 2, \cdots, k, \cdots$, then there is a sequence of universe, written as

$$\Omega(1), \Omega(2), \cdots, \Omega(k), \cdots$$

or $\{\Omega(k), k \in Z^+\}$, where Z^+ is the set of all positive integer number.

According to the characteristic of universe, time-varying universes are divided into two classes: one is discrete class, the other is continuous class. And by the characteristic of universe's change, every class of time-varying universe can be further sorted into three types: increasing type, decreasing type and waving type.

III. TIME-VARYING UNIVERSE BASED FUZZY SETS FOR TIMING DESIGN

For a single intersection, there are four phases in general. In practice, the green time of every phase is no less than five minutes and its yellow light time is four minutes. Then the circle time is no less than 36 minutes, at the same time, it is no more than 120 minutes, otherwise drivers or pedestrians might mistake the traffic light being out of order, and don't want to wait for the green light so long as to run red light.

When the time changes, the traffic flow of the single intersection also changes. To control the traffic light effectively can reduce congestion and improve the efficiency of passengers, then to change the circle time and green time is a feasible way. Let $\Omega(t)$ be the time-varying universe corresponding to circle time at t, and T the duration of $\Omega(t)$, also t the variable of time on $\Omega(t)$. Take $t = 0, 1, 2, \cdots$, then form a universes series

$$\Omega(0), \Omega(1), \cdots, \Omega(n), \cdots$$

Let $\Omega_k = [0, 36 + 12k]$, where $k = 0, 1, 2, \cdots, 7$, then $\Omega(t) \in \{\Omega_0, \Omega_1, \cdots, \Omega_7\}$, then

$$\Omega_0 = [0, 36], \Omega_1 = [0, 48]$$

$$\Omega_2 = [0, 60], \Omega_3 = [0, 72]$$

$$\Omega_4 = [0, 84], \Omega_5 = [0, 96]$$

$$\Omega_6 = [0, 108], \Omega_7 = [0, 120]$$

Might as well, let the four phases be D^1, D^2, D^3, D^4, represented the directions for " straight, turn left east and west, straight, turn left north and south " respectively, T^i the allocated time for D^i, then

$$T = T^1 + T^2 + T^3 + T^4$$

where $T^i = T^i_g + T^i_y$, and T^i_g, T^i_y correspond to the durations of green and yellow time for D_i, $i = 1, 2, 3, 4$.

By above discuss, there is

$$[0, 36] \subseteq \Omega(n) \subseteq [0, 120]$$

$$32 \leq T \leq 120$$

where n is non-negative number.

Let $X^i, i = 1, 2, 3, 4$ be the fuzzy variable on $\Omega(n)$ for the phase D^i. Especially, if $\Omega(n) = [0, 36]$, then all the $X^i, i = 1, 2, 3, 4$ are the singleton word χ_9, and if the four phases are all congest, then the corresponding four fuzzy variable are singleton word χ_{30}.

For every $\Omega_{(k)}$, let $\{\omega^j_k, j \in Z^+\}$ be fuzzy sets on $\Omega_{(k)}$. The membership function of every ω^j_k is also variable with the change of the universe.

$$\omega^j_k = \begin{cases} f_0(x), & x \in [c_0, c_1] \\ \vdots & \vdots \\ f_m(x), & x \in (c_m, c_{m+1}] \\ \vdots & \vdots \\ f_{m_0}(x), & x \in (c_{m_0}, c_{m_0+1}] \end{cases}$$

where $c_0 = 0, c_{m_0+1} = 36 + 12k$ and

$$\bigcup_{m=0}^{m_0} [c_m, c_{m+1}] = [0, 36 + 12k]$$

and $f_0(x), \cdots, f_m(x), \cdots, f_{m_0}(x)$ are all continuous functions on the corresponding universes $[c_0, c_1]$,$\cdots$,$(c_m, c_{m+1}]$,$\cdots$,$(c_{m_0}, c_{m_0+1}]$, and their function values are not more than 1.

For example, five base words "very short, short, medium, long, very long (abbr. VS,S,M,L,VL)" are defined on every

Ω_k. A fuzzy set $\omega_k(5)$ is the fuzzy set "very long" on the different universes $[0, 36+12k], k = 0, 1, 2, \cdots, 7$ are defined as

$$\omega_k^5 = \begin{cases} 0, & x \in [0, 30+12k] \\ 0.5x - 15k, & x \in (30+12k, 32+12k] \\ 1, & x \in (32+12k, 36+12k] \end{cases}$$

IV. Parallel traffic management and control strategies based on time-varying universes

For every phase D_i, let L_i be the length of vehicle queue of the phase, and it is covered by three fuzzy sets (also being called the basis words) "$\widetilde{short}, \widetilde{medium}, \widetilde{long}(abbr.\widetilde{S}, \widetilde{M}, \widetilde{L})$", where L_i can be achieved by video detection, $i = 1, 2, 3, 4$. T^i is the allocated time for the phase, five fuzzy sets "very short, short, medium, long, very long (abbr. VS,S,M,L,VL)" are defined on the universe D_i.

In the paper, by the method of time-varying universe, fuzzy rules and parallel systems, parallel traffic management and control strategies(PTMCS) are provided. For PTMCS, the circle time and timing design of every phase are given according to all kinds of situation for traffic flow. However, from the above discuss, there are 81 fuzzy rules should be considered, so it is necessary to simplify the rules. We first suppose that if the left turn traffic flow is congest, then the straight flow is also congest. That is to say, the length of vehicle queue for left turn is no longer than that of straight flow.

The length of vehicle queue for every phase at current moment determines the timing designing of the phase for the next moment.

The following situations are discussed:

1) if one phase is congest, suppose D_1 might as well;

2) if two phases are congest, suppose D_1, D_3 might as well;

3) if three phases are congest, suppose D_1, D_2, D_3 might as well.

Let ω_n^k be fuzzy set on the universe Ω_i for the phase $D(k)$, then the corresponding fuzzy rules can be set as follows:

R_1: if L_2 and L_4 are $\widetilde{S}$,

r_1: and L_1 and L_3 are also $\widetilde{S}$, then Ω is $\Omega(0)$ and $\omega_0^1 = \omega_0^2 = \omega_0^3 = \omega_0^4 = VS$;

r_2: and L_1 is $\widetilde{M}$ or $\widetilde{L}$ and L_3 is $\widetilde{S}$, then Ω is $\Omega(1)$ and $\omega_1^1 = S, \omega_1^2 = \omega_1^3 = \omega_1^4 = VS$;

r_3: and L_3 is $\widetilde{M}$ or $\widetilde{L}$ and L_1 is $\widetilde{S}$, then Ω is $\Omega(1)$ and $\omega_1^3 = S, \omega_1^2 = \omega_1^1 = \omega_1^4 = VS$;

r_4: and L_1 and L_3 are all $\widetilde{M}$, then Ω is $\Omega(2)$, and $\omega_2^1 = \omega_2^3 = M$, and $\omega_2^2 = \omega_2^4 = VS$;

r_5: and L_1 is $\widetilde{L}$ and L_3 is $\widetilde{M}$, then Ω is $\Omega(3)$, and $\omega_3^1 = L, \omega_3^3 = M$, and $\omega_3^2 = \omega_3^4 = VS$;

r_6: and L_1 is $\widetilde{M}$ and L_3 is $\widetilde{L}$, then Ω is $\Omega(3)$, and $\omega_3^1 = M, \omega_3^3 = L$, and $\omega_3^2 = \omega_3^4 = VS$;

r_7: and L_1 and L_3 are all $\widetilde{L}$, then Ω is $\Omega(4)$, and $\omega_4^1 = \omega_4^3 = L$, and $\omega_4^2 = \omega_4^4 = VS$;

R_2: if one of L_2,L_4 is $\widetilde{S}$, then the following two cases are needed:

R_2^1: L_2 is $\widetilde{S}$,

r_8: and L_1, L_3, L_4 are all $\widetilde{M}$, then Ω is $\Omega(4)$, and $\omega_4^1 = \omega_4^3 = \omega_4^4 = M$, and $\omega_4^2 = VS$;

r_9: and L_1, L_4 are all $\widetilde{M}$, and L_3 is $\widetilde{L}$, then Ω is $\Omega(5)$, and $\omega_5^1 = \omega_5^4 = M, \omega_5^3 = L$, and $\omega_5^2 = VS$;

r_{10}: and L_3, L_4 are all $\widetilde{M}$, and L_1 is $\widetilde{L}$, then Ω is $\Omega(5)$, and $\omega_5^1 = \omega_5^3 = \omega_5^4 = M$, and $\omega_5^2 = VS$;

r_{11}: and L_3, L_1 are all $\widetilde{L}$, and L_4 is $\widetilde{M}$, then Ω is $\Omega(6)$, and $\omega_6^1 = \omega_6^3 = \omega_6^4 = M$, and $\omega_6^2 = VS$;

r_{12}: and L_1, L_3, L_4 are all $\widetilde{L}$, then Ω is $\Omega(6)$, and $\omega_6^1 = \omega_6^3 = \omega_6^4 = M$, and $\omega_6^2 = VS$;

R_2^2: L_4 is $\widetilde{S}$,

r_{13}: and L_1, L_3, L_2 are all $\widetilde{M}$, then Ω is $\Omega(4)$, and $\omega_4^1 = \omega_4^3 = \omega_4^2 = M$, and $\omega_4^4 = VS$;

r_{14}: and L_1, L_2 are all $\widetilde{M}$, and L_3 is $\widetilde{L}$, then Ω is $\Omega(5)$, and $\omega_5^1 = \omega_5^2 = M, \omega_5^3 = L$, and $\omega_5^4 = VS$;

r_{15}: and L_3, L_2 are all $\widetilde{M}$, and L_1 is $\widetilde{L}$, then Ω is $\Omega(5)$, and $\omega_5^1 = \omega_5^3 = \omega_5^2 = M$, and $\omega_5^4 = VS$;

r_{16}: and L_3, L_1 are all $\widetilde{L}$, and L_2 is $\widetilde{M}$, then Ω is $\Omega(6)$, and $\omega_6^1 = \omega_6^3 = \omega_6^2 = M$, and $\omega_6^4 = VS$;

r_{17}: and L_1, L_3, L_2 are all $\widetilde{L}$, then Ω is $\Omega(6)$, and $\omega_6^1 = \omega_6^3 = \omega_6^2 = M$, and $\omega_6^4 = VS$;

R_2^3: L_2, L_4 are all $\widetilde{M}$,

r_{18}: and L_1, L_3 are all $\widetilde{M}$, then Ω is $\Omega(6)$, and $\omega_6^1 = \omega_6^3 = \omega_6^2 = \omega_6^4 = M$;

r_{19}: and L_1 is $\widetilde{M}$ and L_3 is $\widetilde{L}$, then Ω is $\Omega(6)$, and $\omega_6^1 = \omega_6^3 = \omega_6^2 = \omega_6^4 = M$;

r_{20}: and L_3 is $\widetilde{M}$ and L_1 is $\widetilde{L}$, then Ω is $\Omega(6)$, and $\omega_6^1 = \omega_6^3 = \omega_6^2 = \omega_6^4 = M$;

R_3: the other cases:

r_{21}: and L_2 is $\widetilde{M}$ and L_1, L_3, L_4 are all $\widetilde{L}$, then Ω is $\Omega(7)$, and $\omega_7^1 = \omega_7^3 = \omega_7^2 = \omega_7^4 = M$;

r_{22}: and L_4 is $\widetilde{M}$ and L_1, L_3, L_2 are all $\widetilde{L}$, then Ω is $\Omega(7)$, and $\omega_7^1 = \omega_7^3 = \omega_7^2 = \omega_7^4 = M$;

r_{23}:and L_1, L_2, L_3, L_5 are all $\widetilde{L}$, then Ω is $\Omega(7)$, and $\omega_7^1 = \omega_7^3 = \omega_7^2 = \omega_7^4 = M$;

By the above discuss, there are

$$R_1 = \bigcup_{l=1}^{7} r_l$$

$$R_2 = R_2^1 \bigcup R_2^2 \bigcup R_2^3 = (\bigcup_{h=8}^{20} r_h)$$

$$R_3 = \bigcup_{s=21}^{23} r_s = r_{21} \bigcup r_{22} \bigcup r_{23}$$

where $R_2^1 = \bigcup_{h_1=8}^{12} r_{h_1}, R_2^2 = \bigcup_{h_2=13}^{17} r_{h_2}, R_2^3 = \bigcup_{h_3=18}^{20} r_{h_3}$.

Let $R = R_1 \bigcup R_2 \bigcup R_3$, then

$$R = \bigcup_{l=1}^{23} r_l$$

Let $\nu_0 = (\nu_0^1, \nu_0^2, \nu_0^3, \nu_0^4)$,where $\nu_0^i, i = 1, 2, 3, 4$ is the fuzzy set defined on the universe of the length of the current vehicle queue for the phase D^i. If there is a fuzzy rule r_{l_0}, such that $r_{l_0}(\nu_0) \neq 0$, then we can say that the fuzzy rule r_{l_0} is activated. By the method of match degree, the circle time and the time design of every phase for the next time can be decided by $R(\nu_0)$.

The fuzzy rules can be seen as the artificial systems of the timing design for the management and control of the actual transportation systems. The actual transportation systems and artificial transportation systems forms a parallel transportation systems. For the parallel transportation systems, the artificial systems can conduct the timing design of the actual transportation systems, at the same time, the actual transportation systems can adjust and modify the artificial systems, and this can improve the development and perfection of the parallel system.

V. Linguistic Dynamic analysis of the timing design for the intersection

Linguistic dynamic systems(LDS) was provided by Wang in 1995[3]. In LDS, computing with numbers or symbols are substituted by the granular computing. By using the method of computing with words and LDS, people can model, analyze, control and evaluate the complex systems at the level of language, and the related concepts and terminologies can be from the references[4−8].

In most of the previous related work, the orbit of linguistic dynamic system is defined on the definite universe, and the fuzzy rule does not change with the time. However, for many complex system, the universe changes with the time-varying, and corresponding fuzzy sets and fuzzy rules also change. The same with the timing design of intersection.

In most Chinese cities, the traffic flow is not the same at different. The early and evening peak is very obvious, but after midnight, the number of traffic vehicle is always too few. So the timing design can't be the same in a long day. The following is an example to describe the variation of the timing design.

Example. The traffic flow Y of a intersection for Changsha city is: 0:00-5:30, the traffic flows of all phase for the section are very low. 5:31-6:30, $L_i, i = 1,2,3,4$ is S, 6:31-6:50, $L_i, i = 1,2,3,4$ is M, 6:51-6:59, $L_i, i = 1,2,3,4$ is L, 7:00-9:00, $L_i, i = 1,2,3,4$ is VL. According to the situation of traffic flow, the traffic lights are set as yellow during 0:00-5:30, and the circle time of the next time slots are $\Omega(0)$,Ω_1 ,Ω_2,Ω_4, and timing design of every phase is as follows:

$$\omega_0^1 = \omega_0^2 = \omega_0^3 = \omega_0^4 = \chi_9$$

$$\omega_1^1 = \omega_1^3 = M, \omega_1^2 = \omega_1^4 = \chi_9$$

$$\omega_2^1 = \omega_2^3 = L, \omega_2^2 = \omega_2^4 = M$$

$$\omega_3^1 = \omega_3^3 = \omega_3^2 = \omega_3^4 = M$$

$$\omega_4^1 = \omega_4^3 = \omega_4^2 = \omega_4^4 = M$$

Then there is a linguistic dynamic orbit of timing design for all the phase of an intersection.

VI. Conclusion

In the paper, the theory of time-varying universe is used to describe the circle time, and corresponding fuzzy sets on the universe are also discussed to modeling the situation of traffic flow, then the parallel traffic management and control methods which are dynamic with the time change are presented. At last, a simulation example are provided to analyze the linguistic dynamic evolution of timing design of traffic light when the traffic flow is change with time-varying for an intersection.

In future, the parallel management and control for the traffic flow of all kinds of intersection will be studied, and then the strategies of real time will also be provided for every case. Finally, we wish these new ways can ease the traffic and reduce the travel time.

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References

[1] Xubin Sun, Hairong Dong, Bing Ning and etc. *ACP-based Emergency Evacuation Systems*. *Acta Automatica Sinica*, 40(1):16-24, 2014.(in Chinese)

[2] F.-Y Wang, Dajun Zeng, Wenji Mao. Social computing: Its significance, development and research status. *e-Science Technology and Application*, 1(2)(2010):3-15.

[3] F.-Y Wang. Modeling, analysis and synthesis of linguistic dynamic systems: a computational theory, *Proc. of IEEE International Workshop on Architeture for Semiotic Modeling and Situation Control in Large Complex Systems*, Monterey, CA, August, 1995, **27-30**: 173−178.

[4] F.-Y Wang. Computing with words and a framework for computational linguistic dynanic systems.*Pattern Recognition and Artificial Intelligence*, 2001, **14**(4):377−384.(in Chinese)

[5] F.-Y Wang. On the abstraction of conventional dynamic systems: from numberical analysis to linguistc analysis. *Information Science*,2005, **171**:233−259.

[6] H. Mo, F.-Y Wang. Linguistic dynamic systems based on computing with words and their stabilities. *Science China, F-series: Information Science*, 2009, **52**(5): 780−796.

[7] H. Mo, F.-Y Wang. Linguistic dynamic systems and type-2 fuzzy logic. *Beijing :China Science and Technology Press*, 2013.(in Chinese)

[8] H. Mo. Linguistic dynamic orbits in the time varying universe of discourse, *Acta Automatica Sinica*, 38(10)(2012):1585-1594.(in Chinese)

[9] Peizhuang Wang. Fuzzy set and its application. *Shanghai Press of Science and Techonology*, 1983, Shanghai.

[10] Chengdong Li, Jianqiang Yi, Guiqing Zhang. On the monotonicity of interval type-2 fuzzy logic systems, *IEEE Transactions on Fuzzy Systems*. 2014, **22**(5):1197−1212

[11] Chengdong Li, Guiqing Zhang, Huidong Wang, and Weina Ren. Properties and data-driven design of perceptual reasoning method based linguistic dynamic systems. *ACTA AUTOMATICA SINICA*, 2014 **40**(10):2221-2232.