A Weighted Pattern Recognition Algorithm for Short-Term Traffic Flow Forecasting

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Abstract—The *k*-nearest neighbor (*k*-NN) nonparametric regression is a classic model for single point short-term traffic flow forecasting. The traffic flows of the same clock time of the days are viewed as neighbors to each other, and the neighbors with the most similar values are regarded as nearest neighbors and are used for the prediction. In this method, only the information of the neighbors is considered. However, it is observed that the "trends" in the traffic flows are useful for the prediction. Taking a sequence of consecutive time periods and viewing the a sequence of "increasing", "equal" or "decreasing" of the traffic flows of two consecutive periods as a pattern, it is observed that the patterns can be used for prediction, despite the patterns are not from the same clock time period of the days. Based on this observation, a pattern recognition algorithm is proposed. Moreover, empirically, we find that the patterns from different clock time of the days can have different contributions to the prediction. For example, if both to predict the traffic flow in the morning, the pattern from the morning can lead to better prediction than same patterns from afternoon or evening. In one sentence, we argue that both the pattern and the clock time of the pattern contain useful information for the prediction and we propose the weighted pattern recognition algorithm (WPRA). We give different weights to the same patterns of different clock time for the prediction. In this way, we take both virtues of the *k*-NN method and the PRA method. We use the root mean square error (RMSE) between the actual traffic flows and the predicted traffic flows as the measurement. By applying the results to actual data and the simulated data, about 20% improvement compare with the PRA is obtained.

Keywords-Pattern Recognition Algorithm; Short-term Traffic Flow Forecasting; k-NN

I. INTRODUCTION

Real-time, accurate and efficient traffic flow predicting method is a hot topic in the research of Intelligent Transportation Systems (ITS) [1][2][3][4]. With the predictive capability, ITS can provide proactive control and management. There will be little lag between the collection of traffic data and the implementation of traffic control strategies [5]. The function of short-term traffic flow forecasting, i.e., to provide the predicted traffic flows of next one or several periods of time in future using real-time data is major requirement for providing dynamic traffic control in ITS [6].

A variety of methods and techniques have been developed to predict traffic flow. In general, these methods and techniques can be sorted into two categories. One category is the model-driven methods which include historical average method [7], time-series method [8], local regression method [9] and Kalman filter method [10]. This kind of models uses the analytical mathematical model to describe the traffic state and its trends. The future traffic data and historical traffic data are assumed to have same characteristics [11]. These models are also called parametric algorithms as usually it is assumed that the data to be modeled takes on a structure that can be described by a known mathematical expression with a few parameters [12]. The other category is the data-driven methods which include Nonparametric Regression (NPR) method [5], pattern recognition method [15] and neural network method [13]. It is noted that the prediction performance of this kind of models is good as long as the historical dataset is large enough. It is reported in [14] that the neural network method can provide accurate predicted traffic flow. However, the difficulties of selecting appropriate learning dataset and the complicated computation restrict its usage.

The *k*-NN method, which is a famous non-parametric regression method, only uses the nearest neighbors during the same clock time of different days to forecast the short-term traffic flow. It is argued that only the information of the same clock time of different days is used for *k*-NN method. However, it is observed that the trends of the traffic flows are useful for predicting the short-term traffic flow. The trends of traffic flows during several clock time periods are defined as a pattern. Finding the same pattern as the current pattern of different time of past days and the traffic flow errors of the next time period of these patterns, the future traffic flow can be forecasted by these errors and current clock time traffic flow. Based on this, a pattern recognition algorithm (PRA) is proposed by Taehyung Kim *et al.* [15] who argue that the information of the same pattern of different clock time of different days is used for predicting the short-term traffic flow. Moreover, empirically, we find the patterns from different clock time can have different contributions to the predicted traffic flow. That is to say, it is may be more appropriate to

weight these patterns differently. For example, to predict the traffic flow in the morning, using patterns from the morning can lead to better prediction than same patterns from afternoon or evening. For this case, the weights of the same patterns which belong to the morning should be larger than that of afternoon or evening. In one sentence, we argue that both the pattern and the clock time of the pattern contain useful information for the prediction and we propose the weighted pattern recognition algorithm (WPRA). Compared with the PRA and *k*-NN, the WPRA use more traffic information to predict traffic flow.

The contribution of this paper is that we take into consideration more useful information and make better predictions. PRA uses the information contained in the patterns, and the *k*-NN uses information from the traffic flows of the same clock time of the days. Learning from the *k*-NN method, WPRA improves PRA by setting larger weights for patterns with similar clock time to the traffic flow to be predicted. This is why the WPRA is effective. The remaining parts of the paper are organized as follows. In Section 2 we give a review of related works on data-driven methods. In Section 3 we describe the weighted pattern recognition algorithm (WPRA). In Section 4 we show the experimental results of WPRA and compare it with PRA. In Section 5 we conclude the paper and give a discussion on future work.

II. A REVIEW OF DATA-DRIVEN METHODS

The *k*-NN method [5] searches the nearest neighbors in the selected neighbors and uses the nearest neighbors to predict the future traffic flow. The nearest neighbors are found in the selected neighbors by computing the Euclidean distance between the selected neighbors and the current traffic flow vector. The traffic flow vector is measured by the number of cars passing in consecutive periods. The number of the nearest neighbors is *k*. We define one the nearest neighbor $\vec{x}_i(t)$ (*i*=1,2,…,*k*) which can be written as,

$$\bar{x}_i(t) = \left[V_i(t), V_i(t-1), V_i(t-2), V_{h,i}(t), V_{h,i}(t+1) \right]^T \qquad (1)$$

where *t* is the index of the current clock time period which is the sampling period of the traffic flow, as shown in Fig.1,$V_i(t)$ is the traffic flow during the current clock time period *t*, $V_i(t-1)$ is the traffic flow during the previous clock time period, and so on, $V_{h,i}(t)$ is the historical average traffic flow at the time-of-day associated with current clock time period *t*, $V_{h,i}(t+1)$ is the historical average traffic flow at the time-of-day associated with the next clock time period.

The most straightforward approach to generate the forecast for the dependent variable is to compute a simple average of the dependent variable values of the "neighbors" that have fallen within the nonparametric regression neighborhood [5]. This approach applies equal weight to each of the nearest

neighbors' outputs and it is calculated by the following equation,

$$\tilde{V}(t+1) = \left(\sum_{i=1}^{k} \frac{V_i(t)}{V_{h,i}(t)} V_{h,i}(t+1) \right) \Big/ k \qquad (2)$$

where $\tilde{V}(t+1)$ is the predicted traffic flow, and *k* is the total number of nearest neighbors.

The straightforward approach "weights" the selected neighbors equally. In many cases, a weighted distance metric that the outputs with more "useful" information would be weighed more heavily may be appropriate [5]. So the predicted traffic flow can be written as follow:

$$\tilde{V}(t+1) = \left(\sum_{i=1}^{k} \frac{V_i(t)}{V_{h,i}(t) \cdot d_i} V_{h,i}(t+1) \right) \Big/ \sum_{i=1}^{k} \frac{1}{d_i} \qquad (3)$$

where d_i is the Euclidean distance between vector $\vec{x}_c(t)$ and the selected neighbor vector $\vec{x}_i(t)$. The vector $\vec{x}_c(t)$ is the current traffic flow state.
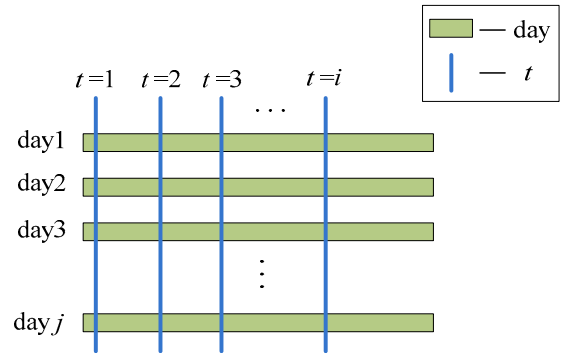

Figure1. The relationship between day and *t*

NPR only uses information from the sample periods with the same clock time periods in different day. Nevertheless, the traffic flow of different clock time periods may also contain useful information for the prediction. Observations have shown that the there exist patterns repeating themselves [14][16].

Taehyung Kim *et al.* [15] propose a pattern recognition algorithm (PRA) and define a pattern as a sequence of "increasing", "equal" or "decreasing" of the traffic flows of two consecutive periods. Their experiments results show that PRA significantly (about 20% improvement) outperforms the *k*-NN nonparametric regression model [16]. In their method, same weights are used for patterns of different clock time periods in any day, i.e., they do not consider the information from the clock time.

In the WPRA method, we improve PRA by considering both the information from the clock time and the patterns. We divide a whole day into several intervals, and the patterns from different intervals are given different weights when making the prediction. The improvement of WPRA to PRA is about 20%, tested on both the actual data and the simulated data.

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III. WPRA FOR TRAFFIC FLOW FORECASTING

A. Traffic flow patterns

A traffic flow pattern is a sequence of "increasing", "equal" or "decreasing" of the traffic flows of two consecutive periods. We use x_t ($t=1, 2, \ldots, n$) to denote the traffic flow in the sample time period t. We take a subsequence of $x_1, x_2, \ldots, x_n$ and denote the subsequence as $(x_i, x_{i+1}, x_{i+2}, \ldots, x_j)$, ($i<j$). We set $s_k=x_{k+1}-x_k$, ($k=i, i+1, \ldots, j-1$). We set:

$$d_i = \begin{cases} -1 & s_i < 0 \\ 0 & s_i = 0 \\ 1 & s_i > 0 \end{cases} \quad (4)$$

The pattern size is defined as the length of the sequence $(s_k, s_{k+1}, \ldots, s_{j-1})$.

We use an example to explain the relationship between traffic flow and traffic flow patterns is shown in Fig.2 and Table I. $(x_t, x_{t+1}, x_{t+2}, x_{t+3}, x_{t+4}, x_{t+5}, x_{t+6})$
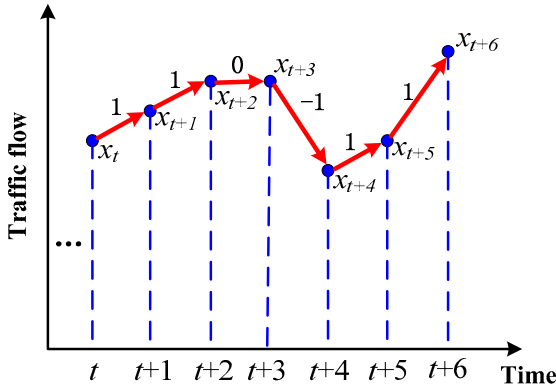

Figure 2. An example of pattern with size 6

TABLE I
PATTERNS AND PATTERN SIZE OF DIFFERENT TRAFFIC FLOW EXAMPLES

Traffic flow	Pattern	Pattern size
(x_t, x_{t+1})	{1}	1
(x_t, x_{t+1}, x_{t+2})	{1,1}	2
$(x_t, x_{t+1}, x_{t+2}, x_{t+3})$	{1,1,0}	3
$(x_t, x_{t+1}, x_{t+2}, x_{t+3}, x_{t+4})$	{1,1,0,-1}	4
$(x_t, x_{t+1}, x_{t+2}, x_{t+3}, x_{t+4}, x_{t+5})$	{1,1,0,-1,1}	5
$(x_t, x_{t+1}, x_{t+2}, x_{t+3}, x_{t+4}, x_{t+5}, x_{t+6})$	{1,1,0,-1,1,1}	6

B. Weights for traffic flow patterns

The PRA "weights" the value of each historical traffic flow patterns equally to predict traffic flow. We argue that the historical traffic flow patterns have different contributions to the predicted traffic flow. So it may be appropriate that a traffic flow pattern with more "useful" information would be weighted more heavily.

The contributions of historical traffic flow patterns to the future traffic flow are not the same. In addition, they change as the time changes. Consider this, we divide daily traffic flow into several time intervals such as $T_1, T_2, \cdots T_m$. The m is the total number of the time intervals in a day. We define the weight of the T_i is w_i. For example, we can divide a whole day into four intervals of "morning", "mid-day", "afternoon" and "night". If a matched pattern is in the same interval with the pattern to be matched, we give a large weight; otherwise we give a small weight.

As the weather conditions affect the traffic a lot, we set different weights when the weather is different, as shown in Fig.3. We define the historical traffic flow in the same weather condition as a group. Accordingly, according to the different weather conditions the historical traffic flow patterns are divided into several groups. Each group has a set of corresponding weights. What's more, the corresponding weights are different between different groups. In this paper, we only consider the historical traffic flow in sunny days.

C. Weighted pattern recognition algorithm

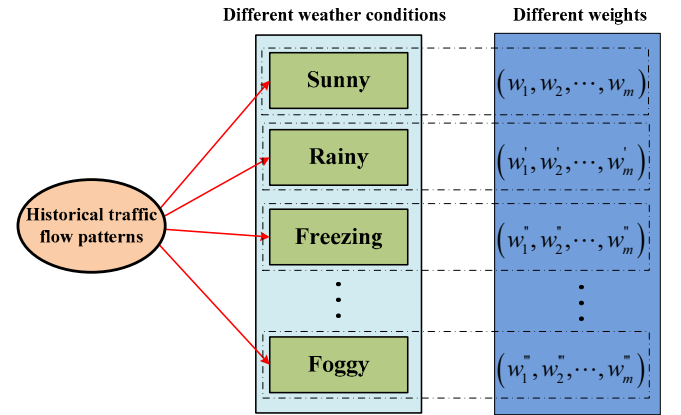The process of WPRA in one weather condition is described in Table II.

Figure 3. Different weights of traffic flow patterns at different weather conditions

TABLE II
WEIGHTED PATTERN RECOGNITION ALGORITHM

Give a sequence of traffic flow $x_1, x_2, \ldots, x_n$.
1. Start with a pattern size L. From the pattern size L describing the current traffic flow pattern, i.e., $p_d = [d_{n-L}, d_{n-L+1}, \ldots, d_{n-1}]^T$.
2. Search the historical traffic flow pattern $[d_1, d_2, \ldots, d_{n-L-1}]^T$ to find the same patterns for p_d. If the same patterns can be found in the historical traffic flow pattern, go to step 3. Otherwise, go to step 1 and start with another pattern size.
3. Find the traffic flow difference s_j^l at next clock time period of the lth same pattern in a time interval.
4. Calculate the value x_{n+1} on the basis of the differences for all of the same patterns which belong to different time interval:

$$x_{n+1} = x_n + \sum_{i=1}^{m} w_i c_i \Big/ \sum_{i=1}^{m} w_i \quad \text{where} \quad c_i = \sum_{l=1}^{k} s_j^l \Big/ k \quad (5)$$

5. where c_i is the average difference in the ith time interval. m is the total number of the time intervals. k is the total number of the same patterns the ith time interval.

IV. EXPERIMENTS

In order to evaluate the performance of WPRA, we need traffic data. In our experiments, we test the performance of WPRA by simulation data and real data. And we compared the performance of the WPRA and the PRA by studying the RMSE between the actual and the predicted traffic flow.

A. Weights for traffic flow in experiments

In these experiments, the weather condition of historical traffic flow patterns which we choose is sunny. So we only choose a set of weights for our experiment. Consider this, we divide daily traffic flow into four time intervals: A (5:30, 9:30), B (9:30, 15:30), C (15:30, 18:30) and D (18:30, 5:30). A is the morning peak. B and D are the smooth. C is the evening peak. There are four corresponding variable weights for the four intervals: W_1, W_2, W_3 and W_4. There are four values for these weights: w_1, w_2, w_3 and w_4. Fig.4 has shows how to select weights for the four intervals. For example, If the current time is 07:30, the weight for the traffic flow patterns which belong to A in the historical database is w_1 and the weight for the traffic flow patterns in other intervals (B,C,D) is w_2, w_3 and w_4. The value of w_1 is greater than that of (w_2, w_3 and w_4) in consideration of the contribution of the patterns which belong to A is more than the patterns which belong to other intervals. For our design of experiment, in order to emphasize the different contribution of historical traffic flow patterns for the predicted traffic flow, we choose w_1=0.7, w_2=0.1, w_3=0.1 and w_4=0.1.
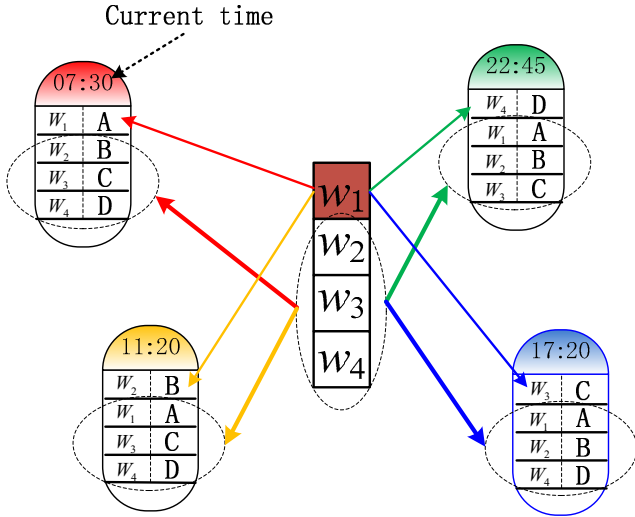

Figure 4. Weights selection for four intervals

B. Test on real data

The traffic flow measurement data were collected at a site monitored by the Center for Intelligent Control and Systems Engineering (CICSE) of Institute of Automation Chinese Academy of Sciences (CASIA). The site chosen within CICSE is located on Suzhou City, Jiangsu Province, China, as shown in Fig. 5 We take the Feihu Intersection for example. From this site, a database of traffic volumes was assembled from 2010. The data set contains a few periods of missing observations, where data is not available for up to 5 minutes. We used traffic flow data in sunny days as a historical data set and the data of May 31 as a test data set.

For vector $X=[x_1, x_2, …, x_n]^T$ and its estimation $Y=[y_1, y_2,…,y_n]^T$, the performance measure RMSE can be given in the following form [19]:

$$RMSE(X,Y) = \left[\sum_{i=1}^{n} \left(x_i - y_i \right)^2 \bigg/ n \right]^{\frac{1}{2}} \qquad (6)$$

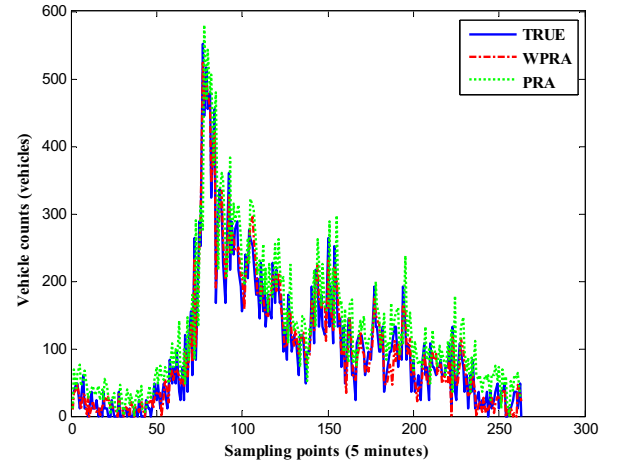Figure 5. Spatial location of study site

Figure 6. Traffic flow profiles of two algorithms with actual data

In this experiment, the RMSE between the actual traffic flow and the traffic flow which is predicted by WPRA is almost twenty percent less than that of the actual traffic flow and the traffic flow calculated by PRA, as shown in Table III. Fig.6 shows the actual and the predicted traffic flow computed by WPRA and PRA for sampling points of the test data set (May 31, Feihu intersection). The Table III also implies that the pattern size of the WPRA and PRA is not linked with the RMSE. According to the experiments, the WPRA method is more successful in predicting the future state of traffic flow than the PRA.

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TABLE III
RMSE OF WPRA AND PRA WITH DIFFERENT PATTERN SIZES

Patten size(t)	WPRA(RMSE) R_1	PRA(RMSE) R_2	Improvement $(R_2 - R_1)/ R_2 \times 100\%$
2	23.72	28.02	15.35
3	23.30	28.62	18.59
4	23.00	29.00	20.69
5	23.21	31.08	25.32
6	24.52	30.60	19.87

C. Test on simulation data

In order to generate the simulation traffic data, we use the historical traffic flow data of Feihu intersection in Fig.5. The simulation data were generated by using the exponential distribution. In traffic flow theory, the exponential distribution has been used since Adams [18]. The exponential distribution is the inter-arrival time distribution of the Poisson process. The probability distribution function of the exponential distribution is as shown in (7).

$$P(t) = \begin{cases} 1 - e^{-\lambda t} & t \geq 0 \\ 0 & t < 0 \end{cases} \quad (7)$$

where λ is the scale parameter of the exponential distribution.

Variable t can be used to describe the time interval of two cars. The expectation of this distribution is the reciprocal of the scale parameter. The maximum likelihood estimator (MLE) for the scale parameter is the reciprocal of the sample mean:

$$\tilde{\lambda} = \frac{1}{\bar{t}} \quad (8)$$

The over procedure of generating the simulation data using the historical traffic flow data is shown as follows.

• Step 1: Compute the average traffic flow $\bar{x}_i$ of the ith sampling period using historical data:

$$\bar{x}_i = \frac{\sum_{j=0}^{D} x_{ij}}{D} \quad (9)$$

where the D is the total days of the selected traffic flow data, j is the jth day of the selected traffic flow data, x_{ij} is the value of the traffic flow during the ith sampling period in the jth day.

• Step 2: Calculate $\bar{t}_i$ of every sampling point. The sampling period is T minutes :

$$\bar{t}_i = \frac{60 \cdot T}{\bar{x}_i} + u_i \quad (10)$$

where u_i is the perturbation of $\bar{t}_i$ which is between -0.5 and 0.5.

• Step 3: According to the (7) and (8), we can calculate the time interval t_{ki} in the ith sampling point:

$$t_{ki} = -\bar{t}_i \cdot \ln(1 - P_k) \quad (11)$$

where P_k is the uniform distribution between 0 and 1.
• Step 4: Compute the traffic flow q_i of the ith sampling period:

$$q_i = \left\{ l \left| \sum_{k=1}^{l} t_{ki} \leq 60 \cdot T \text{ and } \sum_{k=1}^{l+1} t_{ki} < 60 \cdot T \right. \right\} \quad (12)$$

• Step 5：Repeat step 2 to 4 to generate the simulation traffic flow data of a day.

The simulation data was divided into two parts: a historical data set and a test data set. The historical data set contains the data of two months and the test data set is the data of one day. In our experiment, we test the WPRA and PRA with different pattern sizes, from 2 to 8. What's more, we test the two algorithms with simulation data added the perturbation u_i in (10) and simulation data which doesn't add the perturbation, respectively.

TABLE IV
RMSE OF WPRA AND PRA WITH DIFFERENT PATTERN SIZES USING
SIMULATION DATA ADDED THE PERTURBATION

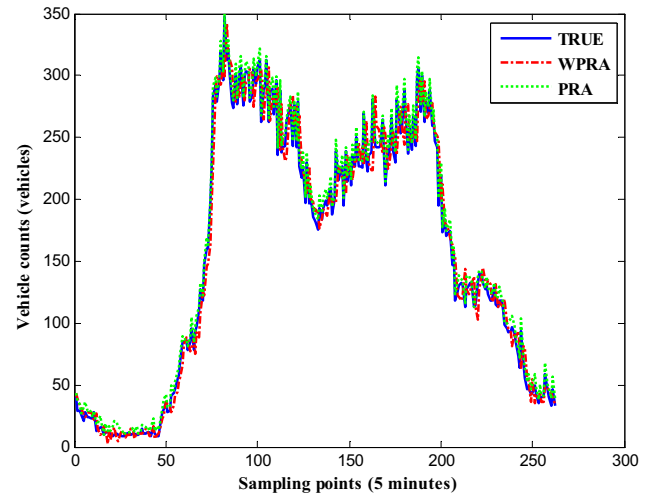| Patten size(t)	WPRA(RMSE) R_1	PRA(RMSE) R_2	Improvement $(R_2 - R_1)/ R_2 \times 100\%$
2	18.50	23.65	21.78
3	18.61	23.52	20.88
4	18.45	23.59	21.79
5	18.52	23.84	22.32
6	18.63	23.75	21.56
7	18.61	24.12	22.84
8	18.34	24.26	24.40

Figure 7. Traffic flow profiles of two algorithms with simulation data added the perturbation

The RMSE between the actual traffic flow and the traffic flow which is predicted by WPRA is almost twenty percent

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less than that of the actual traffic flow and the traffic flow calculated by PRA, as shown in Table IV. Table IV also implies that the pattern size of the WPRA and PRA is not linked with the RMSE. Fig.7 shows that the WPRA is more successful in predicting the future state of traffic flow than the PRA. Table V shows the RMSE between predicted traffic flow and the actual traffic flow with the simulation data which doesn't add the perturbation.

TABLE V
RMSE OF WPRA AND PRA WITH DIFFERENT PATTERN SIZES USING SIMULATION DATA WITHOUT THE PERTURBATION

Patten size(t)	WPRA(RMSE) R_1	PRA(RMSE) R_2	Improvement $(R_2 - R_1)/ R_2 \times 100\%$
2	16.27	21.38	23.90
3	16.26	21.58	24.65
4	16.65	21.41	22.23
5	17.08	21.83	21.76
6	17.94	22.21	19.23
7	16.60	22.78	27.13
8	17.00	22.83	25.54

V. CONCLUSIONS AND FUTURE RESEARCH

In this paper, we take virtues from both of the k-NN and PRA methods. We take use of clock time information and the pattern information. And good prediction results are obtained.

There are two things which we should do in the future research. On the one hand, we have used one month traffic data to test the WPRA in our experiment. If the data sets are not large enough, the performance of the WPRA may be not good when traffic flow is fluctuating. That is to say, larger databases conceptually will lead to better results. The reason for this is that more historical traffic flow patterns can be used to predict the future state of traffic flow. On the other hand, we only choose the traffic data in sunny days. We should collect the traffic data in the different weather condition to test the performance of the WPRA in the future work.

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