

Modeling Interactions in Artificial Transportation Systems Using Petri Net

Fenghua Zhu, Zhixue Wang, Fei-Yue Wang, *Fellow, IEEE*, Shuming Tang

Abstract—Agents and interactions between them are two key roles in modeling artificial transportation systems. This paper presents a formal theory to describe interactions between agents, which, together with the well developed models to describe components as agents, completes the first step towards establishing the formal structure for artificial transportation systems. By describing individual agents and the interactions between them as independent simple sequence processes and directive communication channels respectively, we model the whole system as one sequential communications system, which is one special type of Petri net. The theory is flexible enough to model diversified interactions between agents, and the resultant model possesses favorable properties. A case study of modeling road network has been conducted for the purpose of illustration.

I. INTRODUCTION

TOUGH ITS technologies have been widely used throughout the world, there is still no effective method to describe the transportation system. This influences all areas in ITS, especially the evaluation of performance and reliability, which are the most important indexes to evaluate one system. Currently, the evaluations are mainly conducted by comparing the status of traffic flow with and without the systems [7], [8]. As the real environments vary in diversified aspects and include many man-made factors, the evaluation result can not reflect the internal properties of the system. What is more, this method can only be carried out after the installation, when lots of money and time has been spent, and even some problems are detected in the evaluation, it is impractical to change to another system.

Manuscript received March 18, 2006. This work was supported in part by a Key Program (60334020) and a General Program (60504025) of the National Natural Science Foundation of China, a Key Sci-Tech Project of Shandong Province (2004GG1104001), a Doctoral Fund (2005006) and the Key Project (2004029) of Shandong Academy of Sciences, and the Open Project Research of the Key Laboratory of Complex Systems and Intelligence Sciences, Chinese Academy of Sciences (20050101).

Fenghua Zhu is with the Key Laboratory of Complex Systems and Intelligence Science, Institute of Automation, Chinese Academy of Sciences, CO 100080 Beijing, P. R. China (corresponding author, phone: 86-10-62554288; fax: 86-10-62548183; e-mail: fenghua.zhu@ia.ac.cn).

Zhixue Wang is with the Key Laboratory of Complex Systems and Intelligence Science, Institute of Automation, Chinese Academy of Sciences, CO 100080 Beijing, P. R. China (corresponding author, phone: 86-10-62554288; fax: 86-10-62548183; e-mail: zhixue_wang@msn.com).

F.-Y. Wang is with the Key Laboratory for Complex Systems and Intelligence Science, Chinese Academy of Sciences, Beijing, China and the Program for Advanced Research in Complex Systems, Department of Systems and Industrial Engineering, University of Arizona, Tucson, AZ 85719, USA. (e-mail: feiyue@sie.arizona.edu).

Shuming Tang is with the Institute of Automation, Shandong Academy of Sciences, Jinan 250014, China(E-mail:sharon@ieee.org)

In order to solve this problem, professor Fei-Yue Wang presented the concept of artificial transportation systems (ATS) [1], [3], [4]. The main idea is to use the theories and methods of artificial society and upgrade traffic simulation method to higher level and wider perspective. According to the definition, we can “develop” and “cultivate” transportation systems, i.e., ATS, using agent-based technologies. Utilizing ATS, we can test one specific traffic management system, and repeat the test conveniently and almost without any cost. Most important, the test can be done before the installation, so it can provide objective, comprehensive, and reliable information for evaluation and improvement in time. Furthermore, coupling ATS with one real management system, we can not only improve and optimize the execution of the real system continuously, but also virtually “train” the managers and users of the real system more effectively.

Agents and the interactions between them are two key roles in modeling ATS. Once every individual component in transportations has been modeled by one agent (this step is feasible since each individual has relatively simple structure), the phenomena of the system, such as congestions and incidents, can “emerge” after a period of interactions among the contained agents. Plenty of work has been done in agent-based modeling [5], [6], and they have been successfully used in ATS [13], [14]. But, to our best known, there is still no formal theory in ATS to describe the interactions between agents. The focus of this paper is on establishing this kind of theory. As the basic character of agent is its autonomy, the interactions are usually implemented in the form of communications between agents. In the sequel, we will use the words *communication* and *interaction* interchangeably.

This rest of this paper is organized as follows. In section II, we represent the definition of sequential communication system to model communications between agents. In the system, which is described using Petri net, every agent is represented by one simple sequence process, and the interactions between each other are represented by directive communication channels. The system is flexible enough to model diversified communication protocols and possesses favorable properties, such as without deadlock and overflow, which make it very suitable to model communications in ATS. In section III, we introduce road network in ATS as one case study, which is composed of vehicle agents, roads agents and intersection agents, and illustrate our theory by modeling the communications in the network. Finally, we conclude in Section IV.

II. FORMAL MODELS TO DESCRIBE COMMUNICATIONS

The following definitions and notations will be used throughout this paper. A Petri net is a 3-tuple $N = \langle P, T, F \rangle$, where P and T are two nonempty disjoint sets called places and transitions. The set $F \subseteq (P \times T) \cup (T \times P)$ is the incident (flow) relation. Given a net $N = \langle P, T, F \rangle$ and a node $x \in P \cup T$, $\bullet x = \{y \in P \cup T \mid (y, x) \in F\}$ is the preset of x , while $x \bullet = \{y \in P \cup T \mid (x, y) \in F\}$ is the post-set of x . This notation is extended to a set of nodes as follows: given $X \subseteq P \cup T$, $\bullet X = \bigcup_{x \in X} \bullet x$, $X \bullet = \bigcup_{x \in X} x \bullet$. Let $N = \langle P, T, F \rangle$ be a net, and let $X \subseteq P \cup T$. Then, X generates the subnet $N_X = \langle P_X, T_X, F_X \rangle$, where $P_X = P \cap X$, $T_X = T \cap X$, $F_X = F \cap (X \times X)$. A string $x_1 \dots x_n$ is called a *path* of N iff $\forall i \in \{1 \dots n-1\} : x_{i+1} \in x_i \bullet$. An *elementary path* is a path whose nodes are all different (except, perhaps, x_1 and x_n). A path $x_1 \dots x_n$ is called a *circuit* iff $x_1 = x_n$. A path is called an *elementary circuit* iff it is an elementary path and $x_1 = x_n$. An elementary path $x_1 \dots x_n$ and an elementary circuit $x_1 \dots x_n$ are denoted as $EP(x_1, x_n)$ and $EC(x_1, x_n)$ respectively.

A. General communication model

Before introduce the formal model to describe communications in ATS, we must design an analytical model for agents.

Definition 1: A simple sequence process (S^2P) is a Petri net $N = \langle P \cup \{p^0\}, T, F \rangle$, where:

- 1) $P \neq \emptyset$, $p^0 \notin P$ is called the process idle place;
- 2) N is a strongly connected state machine, and
- 3) Every circuit of N contains the place p^0 .

Definition 2: Let $N = \langle P \cup \{p^0\}, T, F \rangle$ be an S^2P . An initial marking m_0 is called an acceptable initial marking for N iff: 1) $m_0(p^0) \geq 1$; 2) $m_0(p) = 0$, $\forall p \in P$. The couple $\langle N, m_0 \rangle$ is called a (acceptable) marked S^2P .

S^2P and marked S^2P were firstly introduced in [9] to represent working processes in flexible manufacturing systems, here we use it to model the abstraction of an agent. Note that though this structure is very simple, it is flexible enough in most cases (this is demonstrated in the case study) as we only concern the actions related to communications.

Definition 3: A communication system (CS) is a Petri net $N = \langle \bigcup_{k=1}^n (P_k \cup \{p_k^0\}) \cup H, \bigcup_{k=1}^n T_k, F \rangle$, $n \geq 2$, where:

- 1) $G_k = \langle P_k \cup \{p_k^0\}, T_k, F_k \rangle$, $1 \leq k \leq n$, which are subnets generated by $(P_k \cup \{p_k^0\}) \cup T_k$, are all independent S^2P , i.e., $\forall 1 \leq i, j \leq n, i \neq j$, $(P_i \cup \{p_i^0\}) \cap (P_j \cup \{p_j^0\}) = \emptyset$, $T_i \cap T_j = \emptyset$;
- 2) $H = H_1 \cup H_2 \cup \dots \cup H_m$, $m \geq 1$, $H_i = \{h_{si}, h_{bi}\}$, $H_i \cap H_j = \emptyset$, $1 \leq i, j \leq m, i \neq j$, and $\forall H_i$, $1 \leq i \leq m$, there exist $1 \leq r, s \leq n$, $r \neq s$, satisfying:
 - a) $\forall l \neq r, l \neq s, \bullet h_{bi} \cap T_l = \emptyset, h_{bi} \bullet \cap T_l = \emptyset$, $\bullet h_{si} \cap T_l = \emptyset, h_{si} \bullet \cap T_l = \emptyset$; and
 - b) $\bullet h_{si} \cap T_s = \emptyset, h_{si} \bullet \cap T_r = \emptyset$, $h_{bi} \bullet \cap T_s = \emptyset, \bullet h_{bi} \cap T_r = \emptyset$; and
 - c) There exists one and only one transition $t_1 \in T_s$, such that $h_{si} \bullet \cap T_s = \bullet h_{bi} \cap T_s = \{t_1\}$; and
 - d) there exist transitions $t_2 \in T_r, t_3 \in T_r, t_2 \neq t_3$, such that $h_{bi} \bullet \cap T_r = t_2, \bullet h_{si} \cap T_r = t_3$; and
 - e) for every circuit C in $G_r = \langle P_r \cup \{p_r^0\}, T_r, F_r \rangle$, either
 - i) $h_{bi} \bullet \cap C = \emptyset, \bullet h_{si} \cap C = \emptyset$, or
 - ii) $|h_{bi} \bullet \cap C| = 1, |\bullet h_{si} \cap C| = 1$. Let $\{t'\} = h_{bi} \bullet \cap C, \{t''\} = \bullet h_{si} \cap C$, we can verify that $t' \neq t''$, and there exists one elementary path $EP(t', t'')$ from t' to t'' which does not pass p_r^0 .

In the sequel, we also use $N = G \circ H$, where

$$G = \bigcup_{k=1}^n \langle P_k \cup \{p_k^0\}, T_k, F_k \rangle, n \geq 2, H = \bigcup_{i=1}^m H_i, m \geq 1,$$

to denote the CS $N = \langle \bigcup_{k=1}^n (P_k \cup \{p_k^0\}) \cup H, \bigcup_{k=1}^n T_k, F \rangle$.

Without confusion, we also use G_k denoting the subnet generated by $(P_k \cup \{p_k^0\}) \cup T_k$, $1 \leq k \leq n$, i.e.,

$$G_k = \langle P_k \cup \{p_k^0\}, T_k, F_k \rangle.$$

Definition 4: Let $N = G \circ H$ be a CS, where

$$G = \bigcup_{k=1}^n \langle P_k \cup \{p_k^0\}, T_k, F_k \rangle, n \geq 2, H = \bigcup_{i=1}^m H_i, m \geq 1.$$

$\langle N, m_0 \rangle$ is a (acceptable) marked CS iff

- 1) $\langle G_k, m_0^{G_k} \rangle$, $1 \leq k \leq n$ are all acceptable marked S^2P s. (Here, $m_0^{G_k}$ is the restriction of m_0 on G_k); and

- 2) $\forall H_i = \{h_{si}, h_{bi}\}, 1 \leq i \leq m$, such that
 $m_0(h_{si}) > 0, m_0(h_{bi}) = 0$.

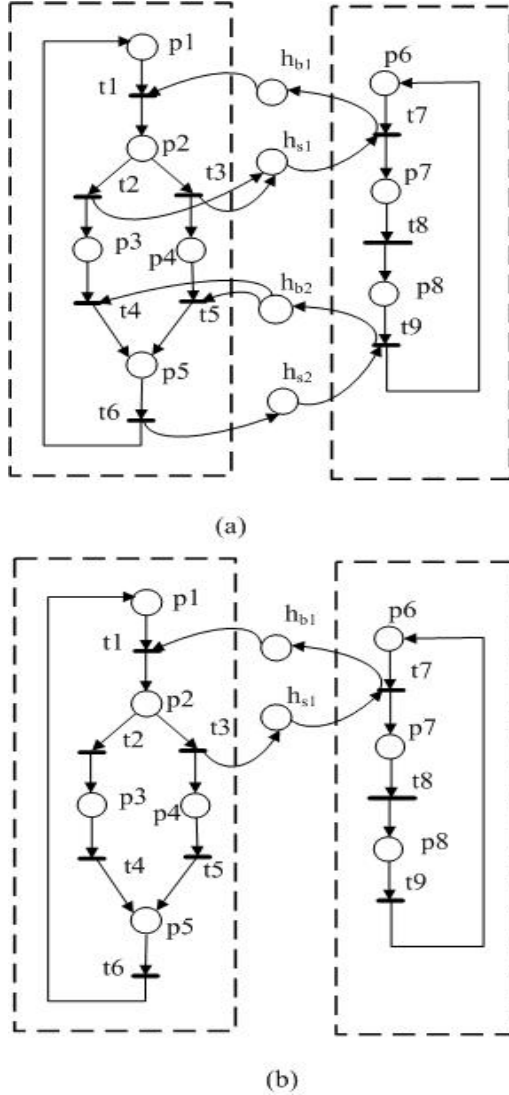


Fig. 1. The Petri nets modeling communications.

Definition 3 and 4 represent the formal model to describe communications in one system. To have better understanding, we will explain definition 3 in details. In definition 3, $\{G_k | 1 \leq k \leq m\}$ model all the agents in the system. H models all the possible communications in the system. $H_i = \{h_{si}, h_{bi}\}, 1 \leq i \leq m$, which is composed of two places, represents one communication channel (denoted by *ComC*). Conditions a)-d) in 2) specify that all *ComCs* are directional and 1-1, for example, the direction of the *ComC* defined in 2) is from G_s (sender) to G_r (receiver). This limitation will not cause problem, as bidirectional and n-n communications can be represented by two and more *ComCs*. Conditions c)-e) in 2) specify the necessary rules in one real communication protocol. Condition c) specifies that before sending message, the sender must check if the channel is

available and there are enough spaces in buffers (through the number of tokens left in the semaphore h_{si}). Condition d) specifies that the receiver must have some mechanisms to confirm the sender that the message has been received successfully. Condition e) is the “key” condition and specifies that the relation between the receiving and its confirmation must be 1-1, i.e., once the receiver successfully received the message, it must reply the corresponding confirmations, and it can not reply the confirmation without corresponding receiving. Note that once the sender put the data in the sending buffer (h_{bi}), it can be fetched by the receiver, i.e., we do not care how it is transmitted in the channel.

Fig. 1 depicts two Petri nets representing communications. The subnets in dashed boxes are all S^2P s. The Petri net in fig. 1(a) is a CS, while in fig. 1(b) is not a CS as condition 2)e) in definition 3 is violated. Note that fig. 1(a) is one very simple example of CS, for example, it includes only 2 S^2P and 1 *ComC*. In fact, according to its definition, there can be arbitrary number of S^2P and *ComC* in one CS.

B. Final communication model

When designing the model for communications in one system, we always expect it possess favorable properties, such as no overflow and deadlock no matter how the system evolves. Though CS is very flexible, it is too simple to satisfy our need. The reason is that the definition only specifies the regulations for a single *ComC*, and does not regulate the relations among *ComCs*, which is more important to assure the system’s properties. Before defining an applicable model, we will first define the following notations.

Let $N = \langle P, T, F \rangle$ be an S^2P .

- Let C be a circuit of N and let x, y be two nodes of C . We will say that x is previous to y in C iff there exists a path in C from x to y the length of which is greater than 1 and does not pass over p^0 . This fact will be denoted as $x <_C y$.
- Let x, y be two nodes of N . We will say that x is previous to y in N iff there exists a circuit C so that $x <_C y$. This fact will be denoted as $x <_N y$.
- Let A, B be two sets of nodes of N . Then $A <_N B$ iff $\forall x \in A, \forall y \in B$, so that $x <_N y$.

Let $N = G \circ H$ be a SCS, where

$$G = \bigcup_{k=1}^n \langle P_k \cup \{p_k^0\}, T_k, F_k \rangle, n \geq 2, H = \bigcup_{i=1}^m H_i, m \geq 1.$$

- For the *ComC* $H_i = \{h_{si}, h_{bi}\}, 1 \leq i \leq m$ from G_s to $G_r : r, s \in \{1..n\}, r \neq s$, we use $U(H_i)$ denoting $\bullet H_i \cap G_s$. Clearly, according to definition 3, $U(H_i) = H_i \bullet \cap G_s = \{t\}$, i.e.,

$|U(H_i)| = 1$. We also use $V(H_i)$ denoting the union of all the elementary paths $EP(t', t'')$ in G_r in condition e)ii) in definition 3, i.e., $V(H_i) = \{x | \text{there exists } EP(t', t''), \text{ such that } t' \in h_{bi} \cap G_r, t'' \in h_{si} \cap G_r, p_r^0 \notin EP(t', t''), x \in EP(t', t'')\}$ and $V_p(H_i)$ denoting the set of all places in $V(H_i)$.

- For two ComCs H_i, H_j both from G_s to $G_r : r, s \in \{1..n\}, r \neq s$, we say that H_i is previous to H_j in G_s (G_r), iff $U(H_i) <_{G_s} U(H_j) (V(H_i) <_{G_r} V(H_j))$.
- For two ComCs H_i, H_j , such that H_i is from G_s to $G_r : r, s \in \{1..n\}, r \neq s$, H_j is from G_r to G_s , we say that H_i is previous to H_j in $G_s (G_r)$, iff $U(H_i) <_{G_s} V(H_j) (V(H_i) <_{G_r} U(H_j))$.

Definition 5: Let $N = G \circ H$ be a CS, where $G = \bigcup_{k=1}^n \langle P_k \cup \{p_k^0\}, T_k, F_k \rangle, n \geq 1, H = \bigcup_{i=1}^m H_i, m \geq 1$, H_i and H_j ($1 \leq i, j \leq m, i \neq j$) are two ComCs in N , H_i is from G_{s1} to $G_{r1} : r1, s1 \in \{1..n\}, r \neq s$, H_j is from G_{s2} to $G_{r2} : r2, s2 \in \{1..n\}, r2 \neq s2$, H_i and H_j are defined to be compatible, iff

- 1) $U(H_i) \cap U(H_j) = \emptyset, U(H_i) \cap V(H_j) = \emptyset, V(H_i) \cap U(H_j) = \emptyset, V(H_i) \cap V(H_j) = \emptyset$; and
- 2) If $(r1 = r2 \text{ and } s1 = s2)$ or $(r1 = s2 \text{ and } s1 = r2)$, we can get that H_i is previous to H_j in G_{r1} iff H_i is previous to H_j in G_{s1} .

Definition 5 defines the compatible relation between to 2 ComCs. Note that condition 2) is needed only when they have the same or opposite directions.

Definition 6: Let $N = G \circ H$ be a CS, where

$$G = \bigcup_{k=1}^n \langle P_k \cup \{p_k^0\}, T_k, F_k \rangle, n \geq 2, H = \bigcup_{i=1}^m H_i, m \geq 1.$$

N is a sequential communication system (SCS), iff \forall two ComCs H_i, H_j ($1 \leq i, j \leq m, i \neq j$) in N , H_i and H_j are compatible.

Definition 7: Let $N = G \circ H$ be a SCS, $\langle N, m_0 \rangle$ is an acceptable marked SCS iff $\langle N, m_0 \rangle$ is an acceptable marked CS.

Note that the CS which includes only one ComC is always SCS. The CS in fig. 1(a) is also one SCS, as the only 2 ComCs in it, H_1 and H_2 , are compatible. The 2 CSs in fig. 2(a) and 2(b) are not SCS, as the ComCs in them, H_1 and H_2 , are not

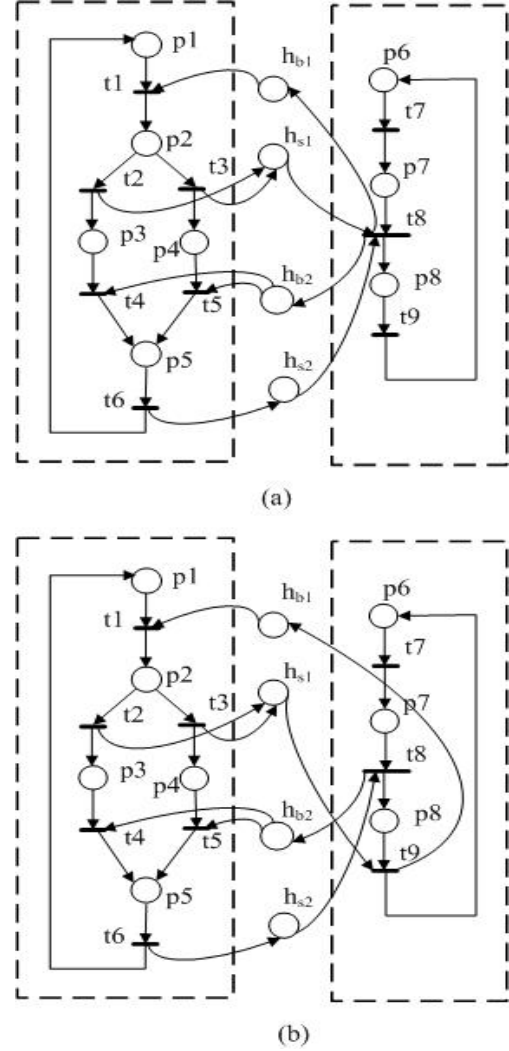


Fig. 2. CSs that are not SCS.

compatible. As mentioned before, the Petri net in fig. 1(a) is one very simple SCS.

We also proved that CS and SCS possess favorable properties and the main results are the following two theorems. As limited by space, the processes to prove them are not presented.

Theorem 1: Let $N = G \circ H$ be a CS, where

$$G = \bigcup_{k=1}^n \langle P_k \cup \{p_k^0\}, T_k, F_k \rangle, n \geq 1, H = \bigcup_{i=1}^m H_i, m \geq 1.$$

Let $H_i \cup U(H_i) \cup V(H_i)$, $1 \leq i \leq m$, denoted by X_i , the subnet N_i generated by X_i is a strongly connected state machine. Furthermore, let M_0 be an acceptable initial

marking of N , $\langle N_i, M_0^{N_i} \rangle$ is bounded, live and reversible, where $M_0^{N_i}$ is the restriction of M_0 on N_i .

Theorem 2: Let $N = G \circ H$ be a SCS, where $G = \bigcup_{k=1}^n \langle P_k \cup \{p_k^0\}, T_k, F_k \rangle, n \geq 1, H = \bigcup_{i=1}^m H_i, m \geq 1$, and M_0 be an acceptable initial marking of N , then $\langle N, M_0 \rangle$ is bounded, live and reversible.

III. CASE STUDY: ROAD NETWORK MODEL OF ATS

To demonstrate the application of SCS, a case study of model communications in road network in ATS has been conducted. Our formal model is based on the road network established by Liu et al in [13] and [14], which is composed of 3 types of agents, i.e., vehicle agent (VA), road agent (LA) and intersection agent (IA).

We have modeled the communications among LA, IA and VA using marked SCS as depicted in fig. 3. According to theorem 2, we can directly conclude that the model is bounded, live and reversible. In fig. 3, the 3 subnets in 3 dashed rectangles are all S^2P and represent the model of IA, LA, and VA respectively.

LA, and VA respectively. $(h_{s_i}, h_{b_i}), 1 \leq i \leq 6$ are all *ComCs*. Note that to make the figure clear, (h_{s_5}, h_{b_5}) and (h_{s_6}, h_{b_6}) both appeared 2 times.

- (h_{s_1}, h_{b_1}) and (h_{s_2}, h_{b_2}) are 2 *ComCs* between IA and LA. The direction of (h_{s_1}, h_{b_1}) is form LA to IA. Through this *ComC*, LA sends the traffic flow information to the connecting IA, while IA use this information to optimize the execution of the signal controller, i.e., adjust the current phase time and phase sequence. In fig. 3, the 3 elementary paths from p_3 to p_{11} in VA represent fixed-time, actuated, and coordinate control method respectively. No matter which control method is effective, the current status of traffic lamps will be send to LA through the *ComC* (h_{s_2}, h_{b_2}) , whose direction is from IA to LA.
- (h_{s_3}, h_{b_3}) and (h_{s_4}, h_{b_4}) are two *ComCs* between LA and VA. The direction of (h_{s_3}, h_{b_3}) is from LA to VA, i.e., from the road to its containing vehicles. LA sends the surrounding traffic status to the running VA through (h_{s_3}, h_{b_3}) . VA then uses the information to choose car-following or lane-changing

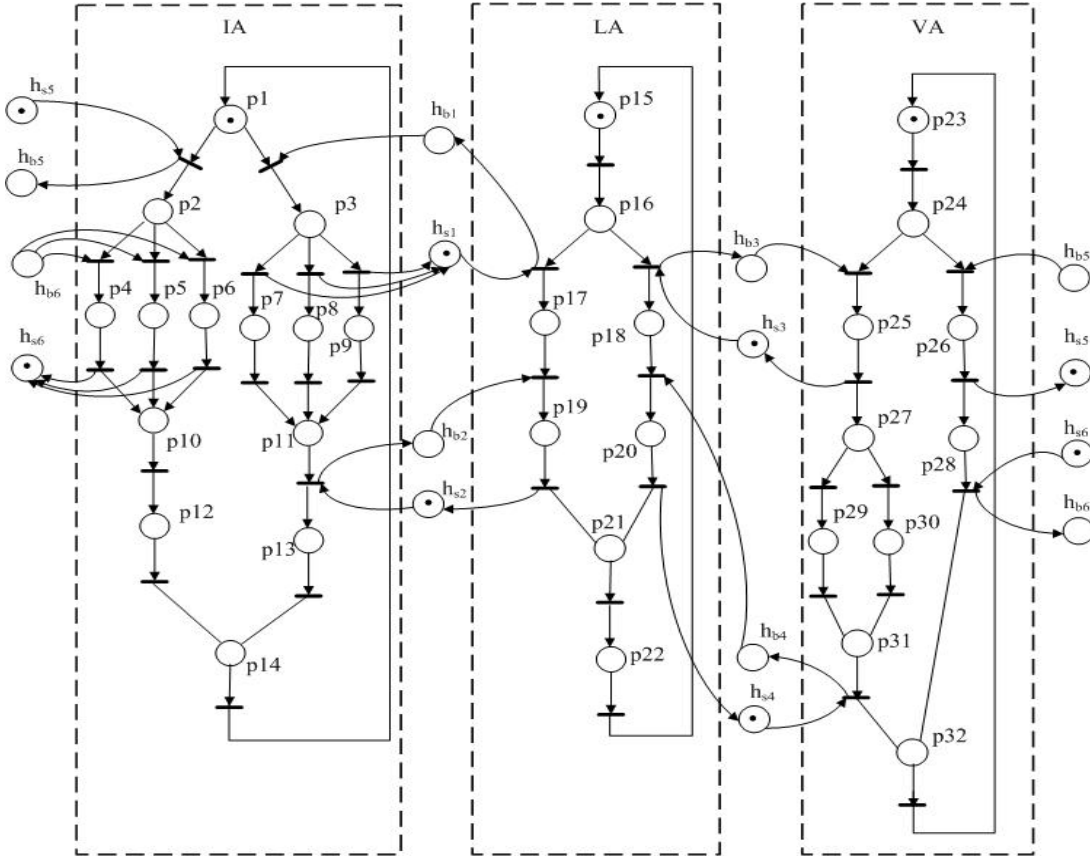


Fig. 3. The models of communications among IA, LA and VA.

in the next step. The choosing process is depicted in fig. 3 by the two elementary paths from $p27$ to $p31$ in VA. Once the *position*, *speed*, or *acceleration* of VA changed, the current status will be send to LA through (h_{s4}, h_{b4}) .

- (h_{s5}, h_{b5}) and (h_{s6}, h_{b6}) are two *ComCs* between IA and VA. The directions of them are from IA to VA and vice versa respectively. The interactions between IA and VA demonstrate the process that vehicles pass one intersection. VA receives the status of traffic lamps through (h_{s5}, h_{b5}) from IA and return its position to IA through (h_{s6}, h_{b6}) . The returned information can be used as the source of actuated signals for IA.

IV. CONCLUSION

An analytical theory for communications in ATS has been established in this paper by establishing a formal model describing the communications between individual agents. We also illustrate the theory using road network in ATS as one case study. Obviously, the resultant model of the case study can be extended to describe one city's transportation network conveniently.

The formal theory for modeling communications between agents, presented here, together with the well developed theory for describing individuals as agents, completes the first step toward the formal structure of ATS. Work is in progress and will be one focus in our next step to establish the specific ATS of Beijing, the holder of Olympic Games in 2008, using the formal structure [17]. Besides participants of transportation system, the environmental factors will be modeled as agents in the specific ATS as many as possible, and comprehensive evaluations will be conducted by directly observing the phenomena emerging after a period of interactions between individual agents.

REFERENCES

- [1] F.Y. Wang, S. Tang, "A Framework for Artificial Transportation Systems: From Computer Simulations to Computational Experiments", *Proceedings of 8th IEEE International Conference on Intelligent Transportation Systems (ITSC'05)*, pp. 1130-1134, Vienna, Austria, Sep. 13-16, 2005.
- [2] F.Y. Wang, S. Tang, "Artificial Societies for Integrated and Sustainable Development of Metropolitan Systems", *IEEE Intelligent Systems*, 2004, Vol.19, No.4, pp.82-87.
- [3] F.Y. Wang and S. Tang, "Concept and Framework of Artificial Transportation System", *J. Complex Systems and Complexity Science*, vol.1, no. 2, 2004, pp. 52-59.
- [4] F.Y. Wang and S.J. Lansing, "From Artificial Life to Artificial Societies: New Methods in Studying Social Complex Systems", *J. Complex Systems and Complexity Science*, vol.1, no. 1, 2004, pp. 33-41.
- [5] N.R. Jennings, K. Sycara, and M. Woodbridge, "A Roadmap of Agent Research and Development", *Autonomous Agents and Multi-Agent Systems*, vol. 1, no. 1, 1998, pp. 7-38.
- [6] Gerhard Weiss, *Multiagent Systems-A modern Aproach to Distributed Artificial Intelligence*, The MIT Press, Cambridge, Massachusetts, 2000.

- [7] Park B, Messser C J, T Urbanik II. "Estimating Queue Lengths by Using SIGNAL94, SYNCHRO3, TRANSYT-7F, PASSER II-90, and CORSIM," *Transportation research Record 1683*, 1999, pp. 110-117.
- [8] E. Niver, K. C. Mouskos, T. Batz, and P. Dwyer, "Evaluation of the TRANSCOM's System for Managing Incident and Traffic(TRANSMIT)," *IEEE Trans. Intell. Transport. Syst.*, vol. 1, Mar. 2000, pp. 15-31.
- [9] J. Ezpeleta, J. M. Colom, and J. Martinez, "A petri net based deadlock prevention policy for flexible manufacturing systems," *IEEE Trans. Robot. Automat.*, vol. 11, pp. 173-184, Apr. 1995.
- [10] T. Murata, "Petri Nets: Properties, analysis, and applications," *Proc. IEEE*, vol. 77, pp. 541-580, Dec. 1989.
- [11] J. Peterson, *Petri Net Theory and the Modeling of Systems*. Englewood Cliffs, NJ: Prentice-Hall, 1981.
- [12] M. C. Zhou, F. DiCesare and A. A. Desrochers, "A hybrid methodology for synthesis of Petri net for manufacturing systems," *IEEE Trans. Robotics Automat.*, vol.8, no.3, 1992.
- [13] Xiaoming Liu, Xiaoyan Gong. "Study of Intersection Traffic Flow Control on the Basis of Agents", *Proceedings of 6th IEEE International Conference on Intelligent Transportation Systems (ITSC'01)*, pp. 1755-1758, Oct. 12-15, 2003.
- [14] Xiaoming Liu, F.Y. Wang, "Study of City Traffic Coordination Control based on Coordinated Hierarchical and Multi-Agent Theory", Ph.D. thesis, CASIA, BJ, China, Jul. 2004.
- [15] Zhiyong Liu, *The Theories and Applications of Intelligent Transportation Systems*, Beijing: Science Publishing Company, 2003.
- [16] Zhi-jun Zou, Dong-yuan Yang, "A Comprehensive Review of Road Traffic Simulation Research", *J. Traffic and Transportation Engineering*, Vol. 1, no. 2, pp. 88-91, Jun. 2001.
- [17] F.Y. Wang et al., "Toward Intelligent Transportation System for the 2008 Olympics," *IEEE Intelligent Systems*, vol.18, no. 6, 2003, pp8-11.