Stock Portfolio Optimization

MAS 632 FINAL PROJECT

ZHAO CHEN, JACQUELINE REILLY, ELAHEH MAHIANTOOSI

Introduction

For our final project, we decide to collect monthly returns data for the last 12 months for 20 stocks. With this data, we computed the average return and covariance matrix for all stocks. Then, we plotted the efficient frontier of our optimal portfolio of minimum variance. Next, we found the optimal portfolio when the target return is equal to the average return across all stocks. Finally, we changed our method with information ratio to find out the optimal portfolio with a benchmark of Dow Jones Index. And we largely increased our expected return from 2.2% to 28.7%.

This project is important because it allows us to make educated investments. It is necessary to invest in the stock market to compound your savings, but it can be risky. We strategically chose stocks in a variety of industries so that we would have a diverse stock portfolio. The stocks we chose are: TSLA (Tesla), APPL (Apple), GOOG (Google), NVDA (NVIDIA), AMZN (Amazon), MSFT (Microsoft), NFLX (Netflix), BA (Boeing), DAL (Delta Airlines), BYND (Beyond Meat), FB (Facebook), ZM (Zoom), SHOP (Shopify), PTON (Peloton), U (Unity), TGT (Target), HD (Home Depot), BAC (Bank of America), WMT (Walmart), and GS (Goldman Sachs).

Optimal portfolio Using Average Return

For this project, we downloaded monthly returns data for the past 12 months for each of these stocks from Yahoo Finance. Then, we used two ways including Excel and MATLAB to calculate the average annual return for each stock and the covariance matrix (figure 1 and 2). Our set of stocks have returns ranging from -4.97% (PTON) to 7.57% (NVDA).

Next, we used MATLAB to plot the efficient frontier of our portfolio based on minimum variance (figure 3). This plot shows the set of optimal portfolios that offer the highest expected return for a defined level of risk (figure 4). When we solved the optimal portfolio by setting the target return to be equal to the average return across all stocks, we found that we should only invest in 9 stocks using MATLAB scripts (figure 5) from this set. This portfolio are as follows:

AAPL	GOOG	NFLX	DAL	BYND	FB	U	TGT	HD
15%	2.5%	19%	23%	4%	26%	2.5%	20%	9%

Covariance Matrix

	TSLA	AAPL	GOOG	NVDA	AMZN	MSFT	NFLX	BA	DAL	BYND	FB	ZM	SHOP	PTON	U	TGT	HD	BAC	WMT	GS
TSLA	0.0255336	0.00624298	0.00156723	0.00382855	0.00294354	0.00683704	0.00676025	-0.0069146	-0.0093061	0.00409476	-0.0037052	-0.0010936	-0.0011613	0.01442134	0.01077429	0.00197328	0.00388148	0.00140056	0.00243531	0.00159081
AAPL	0.00624298	0.00427188	0.00136018	0.00220882	0.00254535	0.00263707	0.00123461	-0.0023138	-0.0043874	-5.82E-05	0.00152502	-0.0011534	0.00130446	0.00552667	0.00491723	0.00092901	0.00074727	-0.0010505	0.00108045	0.00064969
GOOG	0.00156723	0.00136018	0.00440564	0.00328264	0.00186481	0.00255911	-0.000283	-0.0020489	-0.0004188	0.00027709	0.0016921	0.00251696	0.00299609	-0.0012776	-0.0005336	0.00166868	0.00102392	0.00135081	0.00127518	2.54E-03
NVDA	0.00382855	0.00220882	0.00328264	0.0118271	0.00352586	0.00417685	0.00170496	-0.0021478	-0.0018498	0.00168737	0.00199807	0.00561185	0.00762701	-0.0031059	0.01096312	0.00219651	4.63E-04	0.00056728	0.00234777	0.001604
AMZN	0.00294354	0.00254535	0.00186481	0.00352586	0.0030554	0.00182642	0.00055072	-0.0014402	-0.0016866	0.0004231	0.00205386	-0.000268	0.00193831	-0.0010336	0.00469838	5.68E-05	0.00070093	-4.21E-05	0.00091404	0.00061663
MSFT	0.00683704	0.00263707	0.00255911	0.00417685	0.00182642	0.00347318	0.00140095	-0.002371	-0.0032549	0.00132635	0.00059758	0.00209319	0.0021984	0.00305832	0.00426864	0.00207463	0.00167432	0.000547	0.00168517	0.00144904
NFLX	0.00676025	0.00123461	-0.000283	0.00170496	0.00055072	0.00140095	0.00341268	-0.0014939	-0.000767	-0.0015926	-0.001765	-0.0024617	0.00049324	0.0039054	0.00355992	-0.0009055	-0.0004314	0.00139019	5.49E-05	0.0011868
BA	-0.0069146	-0.0023138	-0.0020489	-0.0021478	-0.0014402	-0.002371	-0.0014939	0.00722258	0.00454814	-0.0061729	0.00228059	-0.003689	-0.0016405	-0.0028328	-0.0041376	0.00010177	0.00100742	0.00258417	-0.0004798	0.00032898
DAL	-0.0093061	-0.0043874	-0.0004188	-0.0018498	-0.0016866	-0.0032549	-0.000767	0.00454814	0.00938168	-0.006545	-0.0004805	-0.0029524	0.00177262	-0.0083723	-0.0073879	-0.0025994	-0.0027799	0.00352634	-0.0026002	0.00233496
BYND	0.00409476	-5.82E-05	0.00027709	0.00168737	0.0004231	0.00132635	-0.0015926	-0.0061729	-0.006545	0.03015913	-0.001953	0.01112354	-0.0014013	0.00796456	0.00114915	0.00118301	-0.0003093	-0.003091	0.00071038	-0.000133
FB	-0.0037052	0.00152502	0.0016921	0.00199807	0.00205386	0.00059758	-0.001765	0.00228059	-0.0004805	-0.001953	0.00492707	-0.0012638	0.00057902	-0.0025636	0.00116437	0.00110489	0.0017098	0.00038107	0.00154922	0.00051495
ZM	-0.0010936	-0.0011534	0.00251696	0.00561185	-0.000268	0.00209319	-0.0024617	-0.003689	-0.0029524	0.01112354	-0.0012638	0.01799458	0.00520399	9.48E-05	-0.0008505	0.00465042	0.00089965	-0.0028734	0.00070158	-0.0017273
SHOP	-0.0011613	0.00130446	0.00299609	0.00762701	0.00193831	0.0021984	0.00049324	-0.0016405	0.00177262	-0.0014013	0.00057902	0.00520399	0.00963081	-0.0011627	0.00308558	0.00119657	-0.0030143	2.26E-05	-0.000539	0.00301893
PTON	0.01442134	0.00552667	-0.0012776	-0.0031059	-0.0010336	0.00305832	0.0039054	-0.0028328	-0.0083723	0.00796456	-0.0025636	9.48E-05	-0.0011627	0.03668049	-0.0060735	0.0037992	-0.0005975	0.00115365	2.05E-05	0.00480515
U	0.01077429	0.00491723	-0.0005336	0.01096312	0.00469838	0.00426864	0.00355992	-0.0041376	-0.0073879	0.00114915	0.00116437	-0.0008505	0.00308558	-0.0060735	0.02423334	-0.000246	0.00147883	-0.0038279	0.00319803	-0.0036165
TGT	0.00197328	0.00092901	0.00166868	0.00219651	5.68E-05	0.00207463	-0.0009055	0.00010177	-0.0025994	0.00118301	0.00110489	0.00465042	0.00119657	0.0037992	-0.000246	0.00383713	0.00244842	0.00020071	0.0016737	0.00045224
HD	0.00388148	0.00074727	0.00102392	4.63E-04	0.00070093	0.00167432	-0.0004314	0.00100742	-0.0027799	-0.0003093	0.0017098	0.00089965	-0.0030143	-0.0005975	0.00147883	0.00244842	0.00478686	0.00101472	0.00232371	-0.0011054
BAC	0.00140056	-0.0010505	0.00135081	0.00056728	-4.21E-05	0.000547	0.00139019	0.00258417	0.0035263	-0.003091	0.00038107	-0.0028734	2.26E-05	0.00115365	-0.0038279	0.00020071	0.00101472	0.00517364	2.52E-04	0.0037556
WMT	0.00243531	0.00108045	0.00127518	0.00234777	0.00091404	0.00168517	5.49E-05	-0.0004798	-0.0026002	0.00071038	0.00154922	0.00070158	-0.000539	2.05E-05	0.00319803	0.0016737	0.00232371	2.52E-04	0.00207601	-1.97E-04
GS	0.00159081	0.00064969	2.54E-03	0.001604	0.00061663	0.00144904	0.0011868	0.00032898	0.00233496	-0.000133	0.00051495	-0.0017273	0.00301893	0.00480515	-0.0036165	0.00045224	-0.0011054	0.0037556	-1.97E-04	0.00530351

Average Annual Returns

TSLA	AAPL	GOOG	NVDA	AMZN	MSFT	NFLX	BA	DAL	BYND	FB	ZM	SHOP	PTON	U	TGT	HD	BAC	WMT	GS
6.15%	2.19%	4.71%	7.57%	1.03%	4.08%	2.94%	0.71%	0.79%	-2.80%	1.97%	-4.20%	4.05%	-4.97%	3.29%	3.43%	2.88%	4.75%	-0.05%	5.19%

Figure 1

```
%Final Project

data = xlsread('Final Project v3.xlsx','Returns','B2:U13')
%DATA = xlsread('file_name','sheet_name','range')

[~,names] = xlsread('Final Project v3.xlsx','Returns','B1:U1')

%Part A

C = cov(data) %covariance matrix
R = mean(data) %expected returns
```

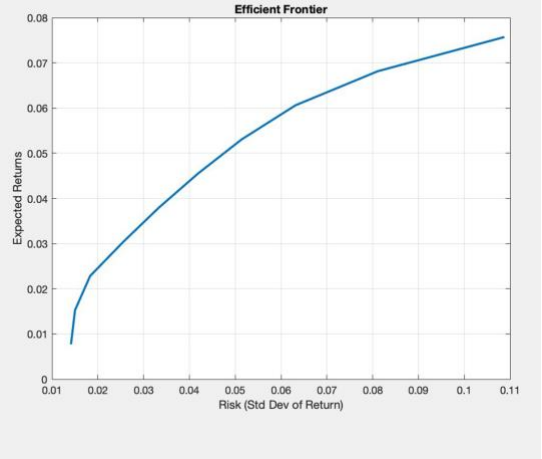
```
%Part B

p = Portfolio;
p = setAssetMoments(p, R, C);
p = setDefaultConstraints(p);


p = setAssetList(p,names);

plotFrontier(p);
xlabel('Risk (Std Dev of Return)');
ylabel('Expected Returns');
```

Figure 2 and 3




```
%Part C

avg_return=mean(R)


opt_portfolio = estimateFrontierByReturn(p, avg_return)
%first column of opt_portfolio matrix gives you the solution for the
%problem of target returns

opt_portfolio
opt_portfolio(:,1)
```

Figure 3 and 5

Optimal Portfolio Using Information Ratio

Considering setting average return as target return is only one of the ways to get optimal portfolio, we also decided to employ more advanced method and compare our portfolio to a benchmark. We consulted students in Finance and find out they usually calculate optimal portfolio with Information Ratio, which is a common financial standard.

The information ratio (IR) is a measurement of portfolio returns above the returns of a benchmark, usually an index such as the S&P 500 or Dow Jones Index, to the volatility of those returns. The information ratio is used to evaluate the skill of a portfolio manager at generating returns more than a given benchmark. A higher IR result implies a better portfolio manager who's achieving a higher return more than the benchmark, given the risk taken. And what we are doing is taking the portfolio manager's role to optimize the portfolio to reach the highest return. The calculations for Information Ratio are shown below. (Figure 6).

$$\text{IR} = \frac{\text{Portfolio Return} - \text{Benchmark Return}}{\text{Tracking Error}}$$

where:

IR = Information ratio

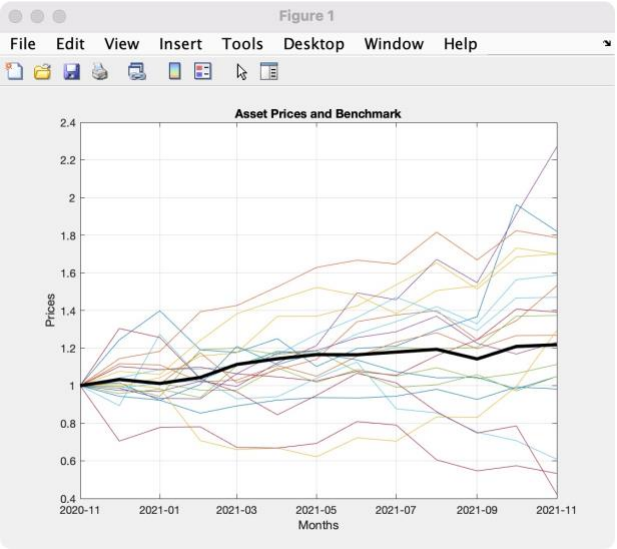
Portfolio Return = Portfolio return for period

Benchmark Return = Return on fund used as benchmark

Tracking Error = Standard deviation of difference between portfolio and benchmark returns

Figure 4

The reason why we used Dow Jones historical data is because it is representative to compare our stocks with it. And the picture below (figure 7) perfectly displays the even distribution. And then, we compared the benchmark with the 20 portfolio (figure 8). According to industry standard, we should scale the data to annual return and risk and the scale number should be 252. But what we used is the monthly prices and we should scale the size with 21. The MATLAB script and figure are shown below (figure 8). As we can see from the picture, although the return for the benchmark is around average, the risk is the lowest. And that adds value for as to make the comparison between the portfolio and benchmark.




```
%Portfolio Optimization Against a Benchmark
benchPrice = xlsread('Final Project v3.xlsx','Price','B2:b14');
assetPrice = xlsread('Final Project v3.xlsx','Price','c2:w14');
[~,assetNames] = xlsread('Final Project v3.xlsx','Price','c1:w1');
[~,benchName] = xlsread('Final Project v3.xlsx','Price','b1');
Dates=xlsread('Final Project v3.xlsx','Price','a2:a14');

%%convert the dates, they look wierd in matlab

d = datetime(Dates, 'ConvertFrom', 'excel')

assetP = assetPrice./assetPrice(1, :);
benchmarkP = benchPrice / benchPrice(1);


figure;
plot(d,assetP);
hold on;
plot(d,benchmarkP,'LineWidth',3,'Color','k');
hold off;
```

Figure 7

```
%Compute Returns and Risk-Adjusted Returns
benchReturn = tick2ret(benchPrice);
assetReturn = tick2ret(assetPrice);

benchRetn = mean(benchReturn);
benchRisk =  std(benchReturn);
assetRetn = mean(assetReturn);
assetRisk =  std(assetReturn);

%%Calculate historical statistics and plot the annual risk-return. Note that the
%%therefore scaling is performed on the daily returns.

scale = 21;

assetRiskR = sqrt(scale) * assetRisk;
benchRiskR = sqrt(scale) * benchRisk;
assetReturnR = scale * assetRetn;
benchReturnR = scale * benchRetn;

figure;
scatter(assetRiskR, assetReturnR, 6, 'm', 'Filled');
hold on
scatter(benchRiskR, benchReturnR, 6, 'g', 'Filled');
for k = 1:length(assetNames)
    text(assetRiskR(k) + 0.005, assetReturnR(k), assetNames{k}, 'FontSize', 8);
end
text(benchRiskR + 0.005, benchReturnR, 'Benchmark', 'Fontsize', 8);
hold off;

xlabel('Risk (Std Dev of Return)');
ylabel('Expected Annual Return');
grid on;
```

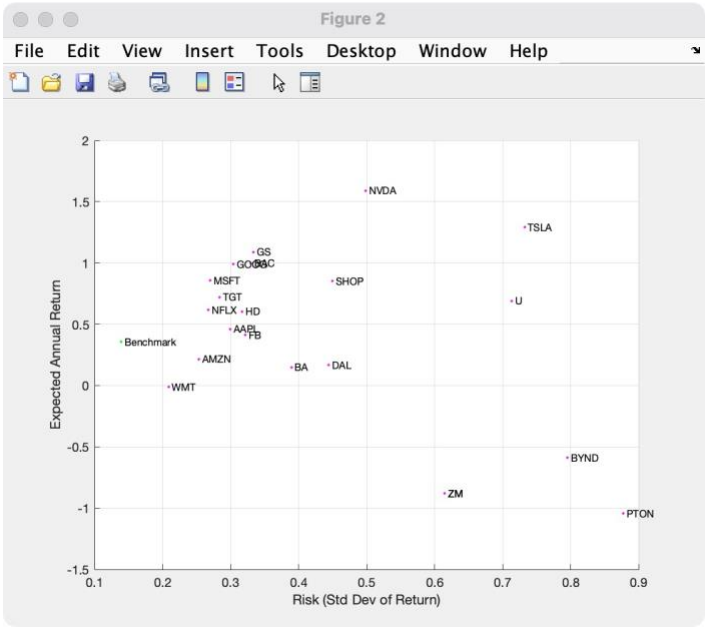


Figure 8

The next step is to compute the efficient frontier using the portfolio object. The detailed steps are as follows:

- Compute the mean-variance efficient frontier of 20 portfolios.
- Visualize the frontier over the risk-return characteristics of the individual assets.
- Furthermore, calculate and visualize the information ratio for each portfolio along the frontier.

And we can see the plot by MATLAB (figure 9) below to clearly find out the optimal point. Based on the definition of the Information Ratio, the highest point implies the best portfolio. And this point shows that the return is 28.7% and the risk is 0.031.

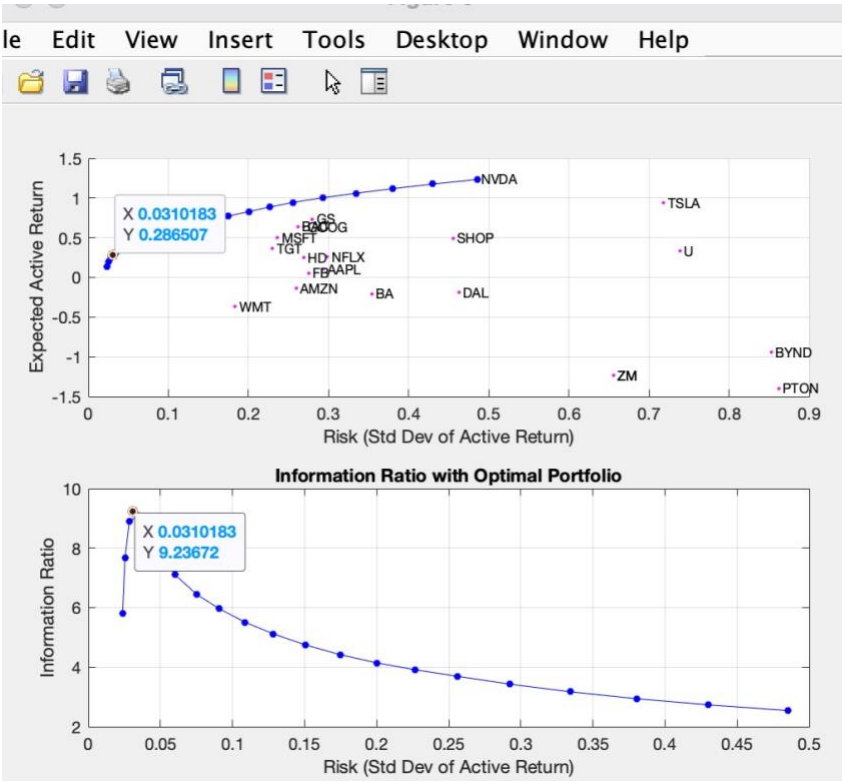


Figure 9

What we used here is to import a function called infoRatioTargetReturn (figure 10). The infoRatioTargetReturn local function is called as an objective function in an optimization routine (fminbnd) that seeks to find the target return that maximizes the information ratio and minimizes a negative information ratio. And we can also find out weights of the portfolio from this function.

```
function [infoRatio,wts] = infoRatioTargetReturn(targetReturn,portObj)
% Calculate information ratio for a target-return portfolio along the
% efficient frontier.
wts = estimateFrontierByReturn(portObj,targetReturn);
portRiskAct = estimatePortRisk(portObj,wts);
infoRatio = targetReturn/portRiskAct;
end
```

Figure 10

And finally, we compared the portfolio results between the information ratio and average return as target correspondently. (Figure 11). All the stocks chosen are listed below, along with the weights on each side.

Maximum information ratio Portfolio

GS	21.472
HD	18.048
BA	15.717
TGT	15.468
AAPL	11.877
GOOG	6.3005
DAL	5.0351
NFLX	3.8527
U	2.2302

Expected return: **28.7%**
Expected risk: **3.1%**

Target return: average return

DAL	23.74
TGT	20.4
NFLX	19.28
AAPL	15.43
HD	9.06
BYND	4.38
FB	2.68
U	2.52
GOOG	2.51

Expected return: **2.2%**
Expected risk: **1.76%**

Conclusion

Each of these portfolio optimizations resulted in a selection of 9 stocks; however, the stocks were not the same in each case. Notably, the average return portfolio told us to invest 4.38% into BYND and 2.68% in FB, when these were not included in the maximum information ratio portfolio. Similarly, the maximum information ratio portfolio told us to invest 21.47% in GS and 15.72% in BA, when these were not included in the average return portfolio. The maximum information ratio portfolio has an expected return of 28.7% and an expected risk of 3.1%. In comparison, the average return portfolio has an expected return of 2.2% and an expected risk of 1.76%. In this case, it appears that the maximum information ratio portfolio is far superior to the average return portfolio. The expected risk is only 1.25% higher for the maximum information ratio portfolio; however, the expected return is 26.5% higher.