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Robust Climate Policies Under Uncertainty: A Comparison of Robust Decision Making and Info-Gap Methods

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This study compares two widely used approaches for robustness analysis of decision problems: the info-gap method originally developed by Ben-Haim and the robust decision making (RDM) approach originally developed by Lempert, Popper, and Bankes. The study uses each approach to evaluate alternative paths for climate-altering greenhouse gas emissions given the potential for nonlinear threshold responses in the climate system, significant uncertainty about such a threshold response and a variety of other key parameters, as well as the ability to learn about any threshold responses over time. Info-gap and RDM share many similarities. Both represent uncertainty as sets of multiple plausible futures, and both seek to identify robust strategies whose performance is insensitive to uncertainties. Yet they also exhibit important differences, as they arrange their analyses in different orders, treat losses and gains in different ways, and take different approaches to imprecise probabilistic information. The study finds that the two approaches reach similar but not identical policy recommendations and that their differing attributes raise important questions about their appropriate roles in decision support applications. The comparison not only improves understanding of these specific methods, it also suggests some broader insights into robustness approaches and a framework for comparing them.

KEY WORDS: Abrupt change; climate change; deep uncertainty; info-gap; robust decision making

1. INTRODUCTION

Recent years have seen an explosion of interest in new tools and methods for helping decisionmakers identify and evaluate robust, as opposed to optimal, decisions. As discussed in more detail later, many definitions of robustness exist, but most capture the idea of satisficing over many plausible future states of the world. Methods for identifying and evaluating robust strategies range from formal analytic approaches such as robust optimization(1) to qualitative scenario(2) and other heuristic methods.(3) Several factors may contribute to this interest in robust strategies, including increased recognition of the fallibility of many forecasts, sensitivity to the importance of unanticipated events,(4) and the need for decision support processes that can engage stakeholders with significantly different expectations about the future.(5)

To date, however, there exist few formal or applied comparisons among the many types of robust

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decision methods. Such comparisons are complicated because these methods often use different definitions of robustness, use different descriptions of uncertainty, and provide different information to decisionmakers at different stages of the decision process. But this wide diversity of approaches also enhances the importance of systematic comparisons that could assist decisionmakers and analysts in choosing among and employing these approaches more effectively.

This article begins to address these issues by applying two robust decision approaches to the same stylized decision challenge and systematically comparing their methods and results. This comparison aims to improve understanding of the two methods and also suggests a template for the type of comparative study that might help bring structure to the emerging field of robust decision methods.

This study compares info-gap as originally developed by Ben-Haim[(6)] and RDM (robust decision making) as developed by Lempert, Popper, and Bankes.[(7)] The two offer an interesting comparison because both provide quantitative decision analytic frameworks designed to evaluate robust strategies under conditions of "deep" or "severe"[6] uncertainty, and because both have been used to inform high-level policy processes. For instance, info-gap has supported flood risk management decisions in the United Kingdom[(8)] and management of invasive species.[(9)] RDM has been used to develop long-range water management plans in the American West,[(10,11)] in an energy policy study[(12)] briefed at the ministerial level to the Israeli government, and in a study of the U.S. Terrorism Risk Insurance Act (TRIA)[(13)] whose results were quoted in debate on the floor of the U.S. Senate.

Both methods have important similarities and differences. Info-gap characterizes uncertainty with nested sets of plausible futures and defines robustness as the range of uncertainty over which a strategy achieves a prescribed level of performance. RDM characterizes uncertainty with sets of plausible futures explicitly chosen to inform the choice among alternative strategies. RDM uses several definitions of robustness, including: (1) trading some optimal performance for less sensitivity to broken assumptions, and (2) performing relatively well compared to the alternatives over a wide range of plausible futures. Neither info-gap nor RDM provide a strict ranking of alternative decisions. Rather, both provide decision support,[7] summarizing trade-offs for decisionmakers to help inform their judgments about the robustness of alternative decision options. RDM also identifies scenarios that describe for decisionmakers vulnerabilities of proposed strategies.

As its test case, this study evaluates alternative policies for reduction of greenhouse gas (GHG) emissions. Specifically, it compares alternative emissions pathways that policymakers might choose to adopt, using a simple but well-established model to evaluate their costs and benefits in terms of reduced adverse impacts, including potentially abrupt changes in the climate system. Many climate-related decisions clearly face conditions of significant uncertainty and recent reports have recommended using robustness criterion to evaluate alternative strategies.[(14)] The potential for abrupt changes, highlighted in many previous studies[(15–19)] increases the salience of robust strategies by injecting considerations of poorly characterized yet consequential uncertainties into decisions about GHG reductions.

This study applies both info-gap and RDM to this test case, using the same models and data, and then compares and contrasts the results. Assessing the value alternative analytic methods provide to decision support is a difficult challenge. A full rendering should consider not only the quantitative results but also how users perceive the credibility, legitimacy, and saliency of the information and the processes that produce it. This current comparison, which does not directly elicit users' views, provides only a partial rendering by focusing on three characteristics of the two methods: their representation of uncertainty, their decision criteria, and the information they provide to users. These characteristics seem important to the ways in which info-gap and RDM contribute to decision support processes.

The next section of this study will describe the decision problem and introduce the simulation model used to address it. The third section will describe and apply info-gap. The fourth section will similarly describe and apply RDM. The last section will discuss what we have learned about these two methods and, more broadly, about robust analysis.

[6] The info-gap and RDM literatures use the phrases severe and deep uncertainty, respectively. We define these terms later and will use both in this article.

[7] Decision support represents a set of processes intended to create the conditions for the production and appropriate use of decision-relevant information.[(5)]

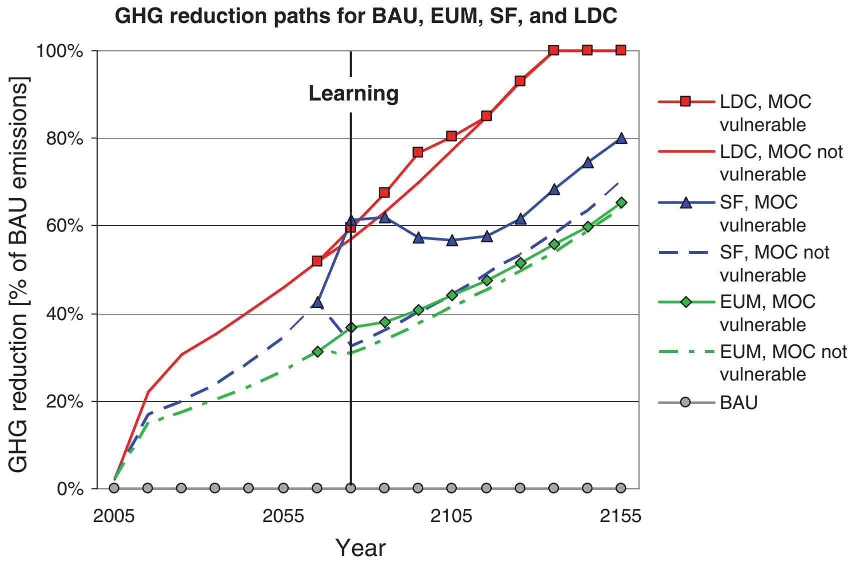

Fig. 1. Greenhouse gas (GHG) BAU, EUM, SF, and LDC emission reduction paths as calculated by DICE 07 for MOC vulnerable and MOC not vulnerable futures. Learning modeled by revealing true value of MOC Vulnerable parameter in year 2075.

Table I. Uncertain Input Parameters to DICE 07 Model Considered in Info-Gap and RDM Analyses

Symbol	Description	Units	Range Sampled in MLK and Used in RDM Analysis	Info-Gap Central Estimate ($\tilde{u}$)
θ_3	Damages from MOC collapse	% GWP	[−0.055, 0.30]	0.015
λ^*	Climate sensitivity	°C	[0.5, 15]	3.4
$g_\sigma(2005)$	Growth rate for carbon intensity	Per decade	[−0.2, −0.02]	−0.073
MOC	Is MOC shutdown possible?	–	[0,1]	0.5

2. EVALUATING ALTERNATIVE GHG EMISSION REDUCTION PATHS

Our test case compares four alternative GHG emission reduction paths of varying degrees of stringency. Decisionmakers must choose among these reduction paths in the face of uncertainty that involves the potential for a large-scale and economically costly collapse of the North Atlantic Meridional Overturning Circulation (MOC).

The four emission reduction paths, shown in Fig. 1 and labeled Business as Usual (BAU), Expected Utility Maximization (EUM), Safety First (SF), and Limited Degree of Confidence (LDC), derive from a previous study that compared alternative decision rules under conditions of well-characterized uncertainty (McInerney, Lempert, and Keller,[20] henceforth "MLK"). Each path prescribes a level of emission reduction through the year 2075. All paths except BAU include future learning, so that in 2075 information regarding an MOC collapse becomes known (specifically, the value of the *MOC Vulnerable* parameter described later is revealed), and in response each path then follows one of two prescribed post-2075 trajectories.

Following MLK, we compare the consequences of these four options using a modified version of the Dynamic Integrated model of Climate and Economy (DICE) model.[21] DICE provides a widely used platform for calculating GHG abatement paths that yield an optimal balance between the uncertain economic costs of abatement and the uncertain impacts of climate change. MLK adds to DICE the possibility of a MOC collapse triggered if and when atmospheric CO_2 levels exceed an uncertain threshold.[16] Using DICE to evaluate the tradeoffs among a small number of prescribed emissions reduction paths proves convenient for this study's comparison because the model is relatively straightforward, yet broadly used in climate policy studies.

Following MLK, we focus on four DICE model input parameters that capture key climate, technology, and economic uncertainties. As shown in Table I, these parameters are:

(1) λ^*, the climate sensitivity, describing the equilibrium increase in mean near-surface air temperatures associated with a doubling of atmospheric CO_2 concentrations from preindustrial levels.

(2) $g_\sigma(2005)$, the initial growth rate of carbon intensity, describing the rate at which the amount of carbon emitted per unit of economic output is decreasing at the model outset in 2005. MLK uses this parameter to approximate key uncertainties related to the cost of reducing GHGs.
(3) θ_3, the economic damages associated with MOC collapse, expressed as a proportion of global economic output, that start to occur immediately in the period of collapse.
(4) *MOC Vulnerable*, a binary parameter indicating whether or not the MOC will actually collapse if the critical CO_2 concentration threshold is exceeded. The threshold depends on the climate sensitivity.[16,22]

MLK calculates the EUM emissions abatement path using probability distributions for each of the four uncertain parameters, derived from the literature and the authors' own best estimates. These distributions were used to identify 11 equally likely intervals for λ^*, $g_\sigma(2005)$, and θ_3, which, in combination with the two possible values for *MOC Vulnerable*, were used to generate a full factorial experimental design of $11^3 \times 2 = 2{,}662$ cases. The DICE solver then finds the EUM solution for these 2,662 equally probable states of the world. Fig. 1 shows EUM reduces GHG emissions in the year 2065 roughly 30% below BAU, which includes no emission reduction.

MLK also uses two other decision criteria to identify emission reduction paths. "SF" maximizes expected utility subject to the constraint that the expected value of the lowest 1% of cases remains at some chosen level. "LDC" maximizes a weighted average of expected utility and the utility of the lowest 1% of cases. For this study, we use a specific parameterization of SF representing moderate caution ($W^* = 55$; constraining the lowest 1% of cases to be 55% between the minimum and maximum BAU utility) and a parameterization of LDC representing extreme risk aversion ($W^* = 0$; expected utility is ignored and only the average utility of the lowest 1% of cases is maximized). We choose the particular parameterizations so that the full set of reduction paths—BAU, EUM, SF, and LDC—offers a wide range of responses to a potential MOC collapse whose robustness can be evaluated by the info-gap and RDM approaches.

MLK provides a database with 2,662 entries for each reduction path, where each entry represents the utility of that path in one of the 2,662 possible future states of the world characterized by one combination of model parameter values. The left-hand side of Fig. 2 shows the distribution of utilities for the four paths. DICE is a Ramsey-style model of intertemporal choice where the utility in a given time period is the product of the population size and the per-capita consumption adjusted by the marginal elasticity of utility. The utilities in each time period are then weighted by a discount factor that approximates the effects of a social rate of time preference and then summed over the entire considered time horizon. The mathematical definitions are given in the appendix of MLK.

BAU has high utility in the highest number of states of the world, but spans the widest range. The difference between BAU's highest and lowest utility is 14,000, corresponding to a loss due to climate impacts of about a third or more. In contrast, LDC has the fewest states with high utility but the shortest low-utility tail. The figure fig. 2 also shows the expected utility for each emissions reduction path (solid lines) and the average value of the lowest 5% of utilities (dashed lines) contingent on the likelihood estimates from MLK. As expected, EUM has the highest expected utility, and LDC has the highest value for its lowest-utility outcomes.

The results show, not surprisingly, that EUM indeed has the highest expected value of the utility. But given the deep/severe uncertainty surrounding any specification of the likelihoods for the four parameters in Table I, decisionmakers might reasonably question which choice is most robust. Info-gap and RDM are both designed to address this question. In the following, we apply each method to the database of cases shown in Fig. 2 and then compare the results.[8]

3. APPLICATION OF INFO-GAP

Info-gap theory has its origins in Ben-Haim's[23] study of the reliability of mechanical systems. Insights into the sensitivity of these systems to uncertainty proved transferable to a broader class of problems. Info-gap has been cultivated into a general method for evaluating robust decisions under conditions of severe uncertainty, which Ben-Haim defines as conditions where the evidence upon which

[8] The database is available from the authors upon request. The DICE model and optimization algorithm are available under an open source GNU license (GNU[24]) from the authors and at http://www.clima.psu.edu/.

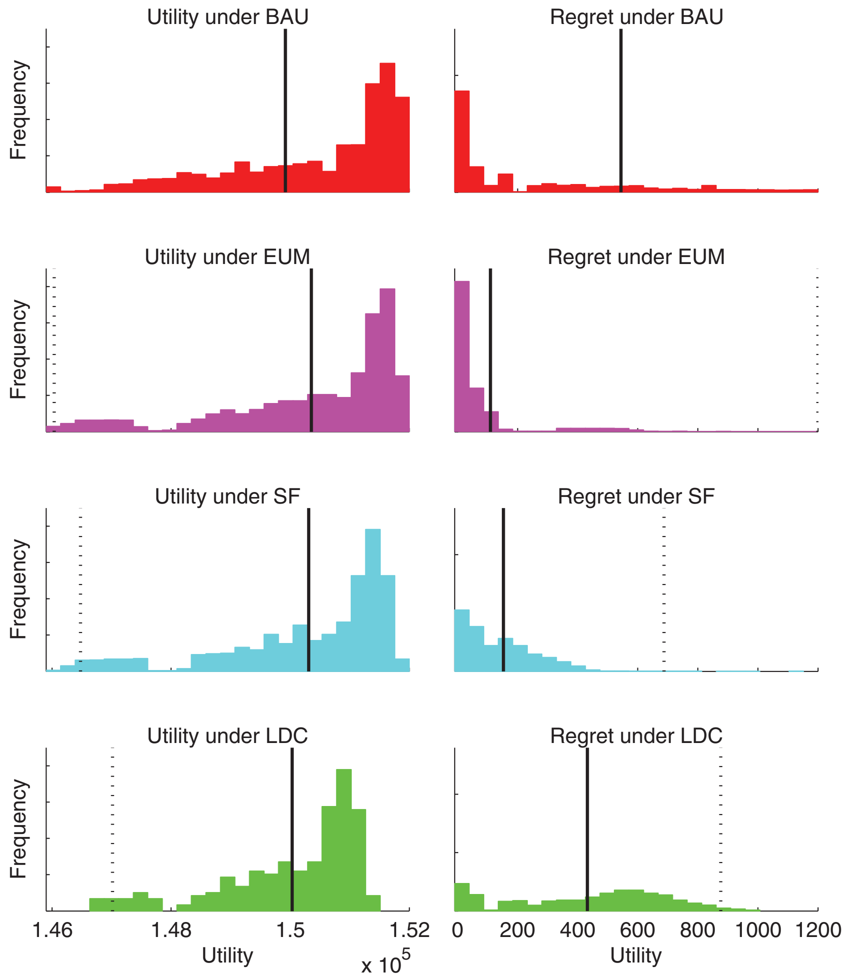

Fig. 2. Utility (left column) and regret as defined following Equation (4) (right column) for BAU, EUM, SF, and LDC reduction paths with learning for 2,662 cases as calculated by DICE 07. Solid lines show average and dashed lines show worst 5% utility and regret using best-estimate probability distributions over model inputs from MLK. Tails of BAU distributions extend further than range of utilities shown.

to base a decision is scarce and only of limited relevance to predicting what may happen in the future. Such uncertainty leads to an information gap—a disparity between what is known and what needs to be known to make a dependable decision. An info-gap analysis employs three elements: a nonprobabilistic, quantified model of uncertainty; a system model that projects the outcome of decisions contingent on the model of uncertainty; and a set of performance requirements that specify the value of the outcomes the decisionmakers require or aspire to achieve.

3.1. Description of the Method

As shown in Fig. 3, info-gap begins by constructing a representation of the severe uncertainty, which it then uses to estimate the consequences of alternative decisions provided exogenously to the analysis. The approach informs decisionmakers by providing them trade-off curves that compare these strategies according to two criteria it calls "robustness" and "opportuneness." We now discuss in detail info-gap's representation of uncertainty, its decision criteria, and the information it provides.

3.1.1. Description of Severe Uncertainty.

Info-gap represents uncertainty with a family of nested sets defined on the space of a decision-relevant variable or variables u. The best estimate of this uncertain quantity u (which can be a scalar or vector) is written $\tilde{u}$. Info-gap assumes that $\tilde{u}$ represents a poor guess at the true values of the parameters, and models the degree of uncertainty regarding this central estimate as a set of expanding nested sets in the parameter space. A larger set of possible values of u represents increased uncertainty. The size of the possible departure of the best estimate $\tilde{u}$ from reality—in other words, the horizon of uncertainty—is parameterized by $\alpha : \alpha \geq 0$. The info-gap

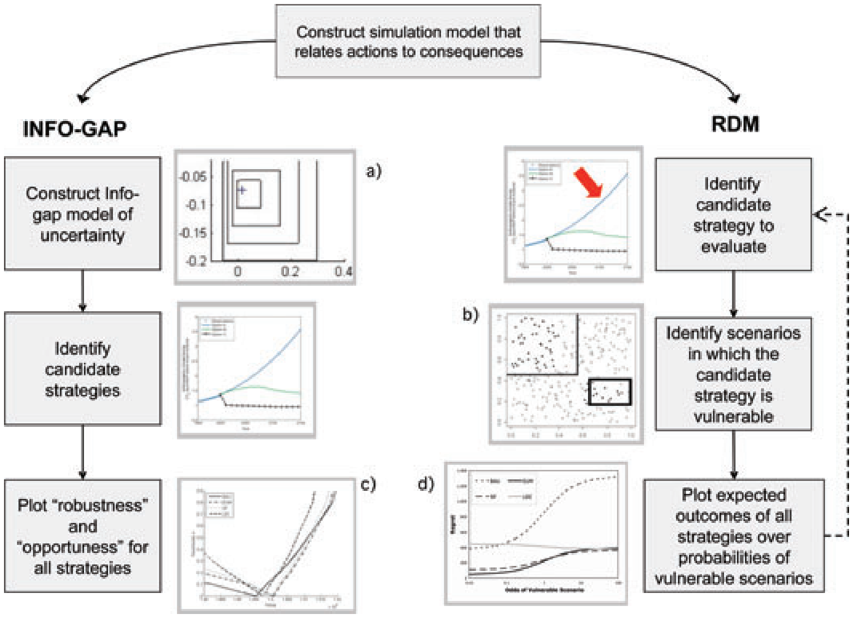

Fig. 3. Comparison of steps in info-gap and RDM analyses.

uncertainty model is therefore written as a nested family of sets $U(\alpha, \tilde{u})$. For small α, searching set $U(\alpha, \tilde{u})$ resembles a local robustness analysis. However, α is allowed to increase so that in the limit the set $U(\alpha, \tilde{u})$ covers the entire parameter space and the analysis becomes one of global robustness. The analysis of a continuum of uncertainty from local to global is one of the novel ways in which info-gap analysis is informative.

The simplest info-gap uncertainty models employ intervals surrounding each uncertain variable. The size of the interval is scaled by α. The approach can easily incorporate additional information about the uncertain quantities, or constraints on their values. For instance, when no information exists about any dependence among variables, the info-gap uncertainty model assumes a cuboid shape. Any dependence relationship among parameters may be represented by excluding less plausible combinations of events, resulting in sets with more elliptical shapes.

3.1.2. Robustness Criteria

Info-gap defines robustness as the maximum uncertainty, measured by the parameter $\alpha : \alpha \geq 0$, over which a strategy achieves a certain level of performance. The method evaluates alternative strategies q_i with a reward function $R(q_i, u)$ that measures the desirability of each option to the decisionmaker for a given point in $U(\alpha, \tilde{u})$. The analysis employs its uncertainty model to calculate the reward $R(q_i, u)$ of decision options q_i at different horizons of uncertainty α. At a given horizon α, there will be a range of possible rewards given by the minimum and maximum levels of $R(q_i, u)$. These levels, which collapse to a singular value $R(q_i, u)$ at $\alpha = 0$, are used to define two criteria:

(1) "Robustness," the minimum reward for each decision option q_i at a given level of uncertainty α.
(2) "Opportuneness," the maximum reward for each decision option q_i at a given level of uncertainty α.

Info-gap combines these two criteria with the concept of robust-satisficing to evaluate the trade-offs among alternative strategies. Robust-satisficing seeks to identify acts that perform acceptably well under a wide range of conditions. The decisionmaker expresses an acceptable level of reward denoted by r_c. The robustness function, $\hat{\alpha}(u)$, measures the maximum uncertainty that can be borne while ensuring r_c:

$$\hat{\alpha}(q, r_c) = \max\left\{\alpha : \min_{u \in U(\alpha, \tilde{u})} R(q, u) \geq r_c\right\} \quad (1)$$

Only at $\alpha = 0$ can the nominal level of reward $R(q_i, u)$ be guaranteed. For any value of $\alpha = 0$, it is trivially true that $\min_{u \in U(\alpha, \tilde{u})} R(q_i, u) \leq R(q_i, \tilde{u})$.

Robustness decreases as the requirement for reward becomes increasingly demanding.

The robustness function reflects the pernicious effects of uncertainty. However, uncertainty can also yield unexpectedly good reward. The opportunity function, $\hat{\beta}(q, r_w)$, which measures the minimum level of uncertainty required to enable a "windfall" level of reward, r_w:

$$\hat{\beta}(q, r_w) = \min \left\{ \alpha : \max_{u \in U(\alpha, \tilde{u})} R(q, u) \geq r_w \right\} \quad (2)$$

If the horizon of uncertainty is as large as $\hat{\beta}$, then reward as large as r_w is possible, but only in the best case. The robustness function expresses immunity against failure so "bigger is better." Conversely, when considering the opportunity function, "big is bad."(6) The different behaviors of these functions illustrate the potential pernicious and propitious consequences of uncertainty.

3.1.3. Information to Decision-Makers.

Decisionmakers To help inform decisionmakers, info-gap presents visualizations showing robustness and opportuneness for each strategy as a function of r_c and r_w. Typically, uncertainty is plotted on the y-axis and target performance values on the x-axis because the latter are considered as exogenously chosen by decisionmakers. In such visualizations, robustness describes the maximum level of uncertainty that can be borne while ensuring a given "critical" (minimum) outcome, and opportuneness describes the minimum level of uncertainty that is necessary to yield the possibility of a given "windfall" (maximum) outcome.

The analysis then calculates robustness and opportuneness curves for each strategy using the same uncertainty model. Decisionmakers may then choose to: (i) minimize worst-case outcomes by using the robustness curve, (ii) maximize best-case outcomes by using the opportuneness curve, or (iii) seek a strategy that provides some attractive tradeoff between robustness and opportuneness. Info-gap does not identify any unique best strategy, although in relatively rare cases some strategies may dominate at all values of α. Rather, it provides decisionmakers information about the tradeoffs between strategies with the best-expected outcomes and those that still perform relatively well when faced with unexpected and harmful circumstances.

3.2. Info-Gap Analysis of Robust GHG Emission Reduction Paths

To implement info-gap for the evaluation of emission abatement policy, we begin by creating an uncertainty model from the four uncertain DICE input parameters and their central estimates as shown in Table I. For convenience, Equation (3) sets $u_1 = \theta_3$, $u_2 = \lambda^*$ and $u_3 = g_\sigma$. Since u_4 represents the probability of a vulnerable MOC, we calculate the utility from a probability-weighted combination of utilities with and without the possibility of an MOC shutdown.

The simplest info-gap uncertainty model is interval bounded, where the uncertain parameters are taken as varying within some interval with bounds scaled by the term α. The interval bounds need not be symmetrical around the central estimate, which is the case for the first three parameters in Table I. A weighting function ψ thus scales the left and right sides of the interval.

This interval-bounded model is appropriate for u_1 and u_3, where there is no known relationship between the parameters. However, we expect some possible dependence between the climate sensitivity and MOC vulnerability.[9] We therefore adopt an ellipsoid uncertainty model, which scales the covariation of u_2 and u_4:

$$U(\alpha, \tilde{u}) = \{u : -\alpha\psi_{l,i} \leq [u_i - \tilde{u}_i] \leq \alpha\psi_{u,i}, i = 1, 3.$$

$$u_4 \in [0, 1],$$

$$\frac{(u_4 - \tilde{u}_4)^2}{\psi_4^2} + \frac{(u_2 - \tilde{u}_2)^2}{\psi_{u,2}^2} h(u_2 - \tilde{u}_2) + \frac{(u_2 - \tilde{u}_2)^2}{\psi_{l,2}^2}$$
$$\times [1 - h(u_2 - \tilde{u}_2)] + \phi(u_2 - \tilde{u}_2)(u_4 - \tilde{u}_4) \leq \alpha^2\}, \quad (3)$$

where ψ_l and ψ_u scale the lower and upper bounds of the set of possibilities: $\psi_l = [0.07, 2.9, 0.127, 0.5]^T$, $\psi_u = [0.285, 11.6, 0.071, 0.5]^T$, $h(x)$ is the step function $h(x) = 0$ if $x < 0$ and $h(x) = 1$ otherwise, and ϕ scales the strength of covariance between u_2 and u_4. The use of the step function means that the ellipse is asymmetric with respect to climate sensitivity.

Fig. 4 shows the six different two-dimensional projections of this four-dimensional info-gap uncertainty model, with contours representing the space

[9] While the MLK analysis assumes uncorrelated parameters, the dependence stipulated here is certainly plausible, and is useful to demonstrate the flexibility of the info-gap to model such relationships.

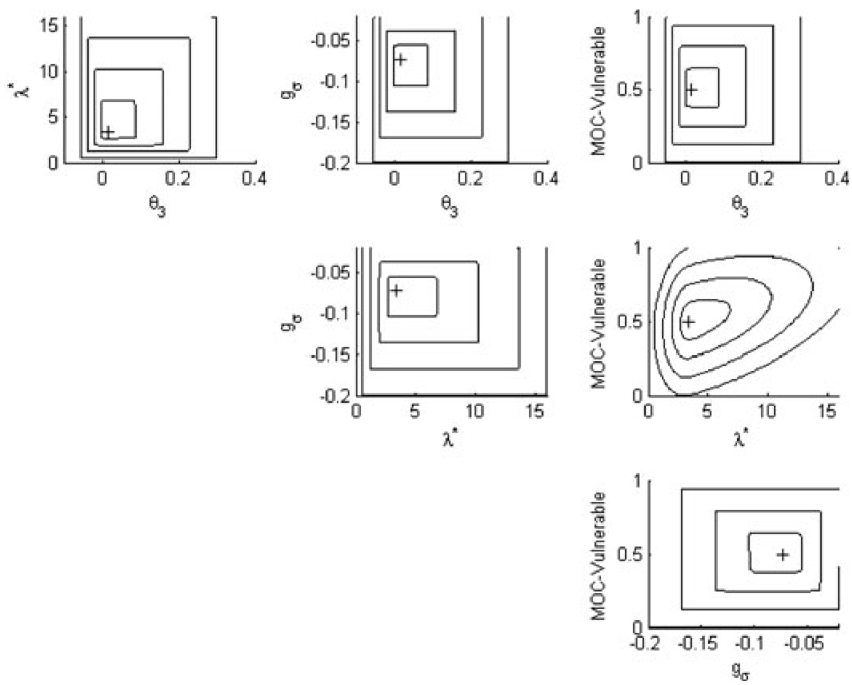

Fig. 4. The six two-dimensional projections of the info-gap uncertainty model for the four uncertain inputs to DICE 07, with contours for illustrative values of the uncertainty bound α.

of possibility at four illustrative values of the uncertainty bound α. The outer bound shown in Fig. 4 encompasses most of the entries in the MLK database shown in Fig. 2, whereas the intermediate values are equally spaced up to this outer bound. The central value $\tilde{u}$ is depicted with a cross. The figure has been constructed from MLK's full-factorial sample of the multidimensional space. Using these results, we construct robustness and opportuneness curves for each of the four emissions reduction paths, identifying for each sample within a set $U(\alpha, \tilde{u})$ the corresponding performance $R(q_i, \tilde{u})$ for each path q.

Info-gap next uses this uncertainty model to calculate the robustness and opportuneness of each reduction path using Equations (1) and (2) and MLK's database of 2,262 cases. Fig. 5 shows the robustness curves ascending to the left and the opportuneness curves ascending to the right. At zero uncertainty EUM yields the greatest, and BAU the lowest, utility. As the horizon of uncertainty increases, the EUM robustness curve increases more slowly than SF and LDC. If decisionmakers are willing to accept 180 less utility, about 1% of the full range of BAU outcomes, SF becomes most robust. If they are willing to accept 980 less utility, about 7% of the full BAU range, then LDC emerges as the most robust. The value of α at which this second robustness curve crossing takes place is not high—an α of 0.15 is well within

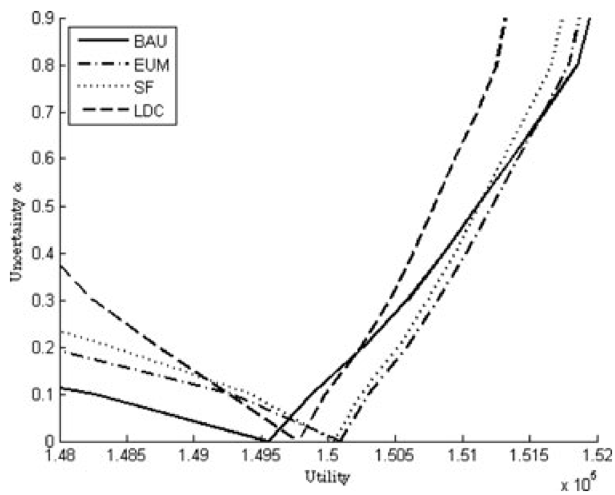

Fig. 5. Robustness and opportuness trade-off curves for info-gap analysis of alternative GHG emission reduction paths.

the bounds of possibility. Thus, of all the paths, LDC can guard most effectively against the potential downsides at this and higher levels of uncertainty. However, this robustness is bought at the price that corresponds to a few percent of global economic product compared to EUM. Although for low uncertainty LDC represents a considerable sacrifice of utility, if broader ranges of uncertain parameters are plausible then LDC is more effective at avoiding the possibility of undesirable outcomes. If, on the other hand, decisionmakers consider this horizon of uncertainty implausible, then SF is the more robust option. BAU is the least robust throughout.

The opportuneness curves of EUM, SF, and LDC are similar in slope and do not cross. These curves therefore provide no further information to help distinguish among these three paths. BAU does show more rapidly increasing utility at high horizons of uncertainty and beyond $\alpha = 0.7$ actually yields the highest possible utility. BAU's possible up-side advantage should be weighed, however, against its lower robustness overall and worse performance at lower horizons of uncertainty.

4. APPLICATION OF RDM

RDM provides an iterative, analytic decision support methodology, often embedded in a process of participatory stakeholder engagement,[10] intended to support decisions under conditions of "deep uncertainty," that is, conditions where the parties to a decision do not know or do not agree on the system model(s) relating actions to consequences or the prior probability distributions for the key input parameters to those model(s).[(7,25)] In addition to informing quantitative tradeoffs among decision options, RDM also employs concepts from the qualitative scenario planning literature[(26)] to facilitate group decision making in contentious situations where parties to the decision have strong disagreements about assumptions and values.[(27,28)] The discussion here focuses on RDM's analytic elements.

4.1. Description of the Method

As shown in Fig. 3, RDM begins by specifying strategies for consideration and then constructs a representation of the deep uncertainty designed to inform the choice among the strategies. The approach informs decisionmakers by providing scenarios that describe future conditions where strategies fail to meet their goals. These scenarios then support comparisons of the robustness of alternative strategies. We now discuss in detail RDM's representation of uncertainty, its decision criterion, and the information it provides.

[10] RDM follows the "deliberation with analysis" process recommended by the National Research Council.[(5)]

4.1.1. Description of Deep Uncertainty.

RDM represents uncertainty with a set of multiple, plausible future states of the world. Bankes[(29)] distinguishes between "consolidative" and "exploratory" models. The former are validated and predictive. The latter provide a mapping of assumptions to consequences, without any judgment regarding the validity of alternative assumptions. RDM employs this "exploratory modeling" concept to create a large database of individual model runs. Each element in the database represents a plausible future, described by a set of assumptions, and the resulting consequences, described by the values of outcome measures of interest. The assumptions can be described in state space $\overline{X}_i$, where $\overline{X}$ is the space of uncertain input parameters to the simulation model and i indexes alternative values of these parameters, or in probability space $\{\rho_i, \overline{X}\}$, where i indexes alternative probability weightings over the space $\overline{X}$.

To make this database of simulation runs useful for decisionmakers, RDM organizes the analysis within a *vulnerability-and-robust-response* framework.[(30)]. As shown in Fig. 3, the analysis begins by specifying one or more proposed strategies and characterizes the uncertainty according to its impact on the choice among options. For instance, given a proposed decision A that aims to achieve some goals, an RDM analysis might, similarly to the policy region analysis of Watson and Buede,[(31)] divide the set of plausible futures $\{\vec{X}_i\}$ into two subsets: $\{\vec{X}_A\}$, those states where A achieves it goals, and $\{X_{\sim A}\}$, those states where A fails to achieve its goals. As described later, RDM might then use statistical analysis and visualizations of the results database to concisely summarize for decisionmakers the combinations of assumptions that best distinguish between the sets $\{\vec{X}_A\}$ and $\{\vec{X}_{\sim A}\}$.

4.1.2. Robustness Criteria.

RDM analyses have employed several definitions of robustness, including trading some optimal performance for less sensitivity to broken assumptions and performing reasonably well compared to the alternatives over a wide range of plausible futures. The first definition is often most appropriate when decisionmakers agree on a best estimate probability distribution over $\vec{X}$. The second definition, related to Starr's domain criteria,[(32)] is often most appropriate when no such distribution exists. Lempert and Collins[25] show that in some cases at least these two definitions lead to similar tradeoffs among strategies. This study uses the first definition.

To formalize this first definition, Lempert and Collins[25] assume a set of strategies $s \in \overrightarrow{S}$ with performance $P_s(x)$ in each of a set of plausible states of the world $x \in \overrightarrow{F}$ and a set of probability distributions $\rho_i(x) \in \overrightarrow{D}$ over these states of the world. The expected regret of strategy s contingent on distribution i is given by

$$\overline{R}_{s,i} = \int_x R_s(x)\,\rho_i(x)\,dx, \tag{4}$$

where $R_s(x) = \max_{s'}[P_{s'}(x)] - P_s(x)$ is the regret[33] of strategy s in state x.

The optimum strategy b for the decisionmakers' best estimate distribution $\rho_{\text{best}}(x)$ is the strategy that minimizes the expected regret $\overline{R}_{b,\text{best}}$. The worst case for this strategy is given by $\overline{R}_{b,\text{worst}}$, where $\rho_{\text{worst}}(x) \in \overrightarrow{D}$ is the distribution that yields the largest expected regret for strategy s. A robust strategy exists when decisionmakers can trade some optimal performance for less sensitivity to broken assumptions. That is, compared to the optimal strategy a robust strategy r will have a smaller value of the weighted average, V_r, of the best and worst expected regret,

$$V_r = (1-z)\bar{R}_{r,\text{best}} + z\bar{R}_{r,\text{worst}} < (1-z)\bar{R}_{b,\text{best}} + z\bar{R}_{b,\text{worst}}, \tag{5}$$

for some range of z on the interval $0 < z \leq 1$. As described later, the parameter z will be a function of the decisionmakers' preferences, risk aversion, and level of uncertainty about the distribution $\rho_{\text{best}}(x)$.

A regret-based measure of performance is often useful for RDM, although not necessary, for several reasons. First, Equation (5) interpolates between the optimum and minimax decision criteria. For $z = 0$, the equation yields the ordering of strategies produced by an expected utility calculation. For $z = 1$, the equation yields the minimax decision criteria if $\overrightarrow{D}$ includes distributions that put all their weight on a single state of the world (e.g., delta functions). The regret measure can also focus decisionmakers' attention on those future states of the world most important to their decision. In some futures, outcomes will be desirable or undesirable largely independent of the decision taken, whereas in some futures the desirability of outcomes may depend strongly on decisionmakers' choices. The regret criteria can help focus attention on these later cases.

4.1.3. Information to Decisionmakers.

The decision sciences literature suggests that decision support tools can provide information to support two distinct types of tasks: a *choice task* that involves choosing among a menu of available options and a *decision structuring task* that involves defining the scope of the problem, goals, and the options under consideration. RDM aims to support both types of tasks.

RDM's third step supports the choice task with various visualizations that describe, similarly to info-gap, the tradeoffs among alternative strategies. When defining robustness as trading some optimal performance for less sensitivity to broken assumptions, RDM analyses often present visualizations showing the expected regret of alternative strategies as a function of the probability assigned to the scenarios where the optimal strategy fails to meet its goals (see, for example, fig. 7 in Ref. 25 and fig. 4.12 in Ref. 34). Such visualizations often help define a probability threshold, that is, a value above which the likelihood of failure is sufficiently high that decisionmakers ought to consider abandoning the optimal or proposed strategy for some alternative. Such probability thresholds provide a means to utilize imprecise probabilistic information in situations where decisionmakers may have very different expectations. For instance, Dixon *et al.*[13] used such thresholds to help reduce the salience of disagreements about the likelihood of large terrorist attacks in the public debates regarding congressional reauthorization of the U.S. TRIA.

RDM's second step supports the decision-structuring task. As shown in Fig. 3, this stage of the analysis characterizes uncertainty by concisely summarizing the future conditions where a proposed strategy would fail to meet its goals.[28,35] These clusters of cases share key attributes with scenarios as described in the scenario planning literature, specifically the concept of presenting multiple plausible futures each with a sense of plausibility rather than prediction. This approach can expand the range of cases considered[36] by making unexpected or inconvenient futures psychologically less threatening to those holding different worldviews.[37,38] The scenario literature[39] and our experience with RDM suggests that considering a multiple, plausible scenarios can also assist decisionmakers in thinking more expansively about policy options and, in particular, ways in which options intended for one future might by augmented to improve the ability to adapt if another future comes to pass.

4.2. RDM Analysis of Robust GHG Emission Reduction Paths

The RDM analysis of emissions abatement policy begins by choosing an initial candidate emissions reduction path. To assist in this choice, the right-hand side of Fig. 2 plots the distribution of regret $R_s(x)$ (see Equation (4)) for the four paths $s \in \{BAU, EUM, SF, LDC\}$ in each of the 2,662 states of the world analyzed by MLK. The expected regret (solid lines) and the expected value of the highest 5% of regrets (dashed lines) are also shown for each path. Note that the ranking of paths by expected regret is identical to the ranking by expected utility in Fig. 2 (left-hand side).

In many applications one might choose as the initial candidate strategy that with the best expected utility, in this case EUM. Here, however, we choose SF as the initial candidate because the distribution of regrets in Fig. 2 suggests this path sacrifices only a small amount of expected regret relative to EUM in return for a large improvement in performance for the worst 5% of cases.

The next step is to characterize the vulnerabilities of the SF path. This step aims to help decisionmakers understand the tradeoffs involved with choosing SF and to encourage them to think about potential modifications to the path that might reduce vulnerabilities.

To characterize a path's vulnerabilities requires establishing a criterion for acceptable and unacceptable performance. Some applications might provide a natural choice for such a criterion, for instance, a budget constraint, an organization's requirements for return on investment, or well-defined stakeholder preference. In this test case, however, we infer a threshold that might approximate decisionmakers' preferences. We define SF to have unacceptable performance in any future state of the world that yields a regret greater than 272, which characterizes the worst 15% (402 of 2,662) SF cases in the database.

We then use a statistical "scenario discovery" process[28] to provide a concise description of those cases where SF fails to meet its goals. Here we use a modified version of the PRIM (patient rule induction method)[40] to identify hyperrectangular regions in the space defined by the four uncertain model input parameters in Table I that are highly predictive of high regret cases for the SF path. In particular, PRIM seeks to identify one or more such "scenarios" that maximize three measures: *coverage*, the percentage of high-regret cases contained within the scenarios; *density*, the percentage of cases within the scenarios that have high regret; and *interpretability*, the ability of the scenarios to provide insight to decisionmakers. Following the experience of the qualitative scenario planning literature (Schwartz[42]), we assume that hyperrectangular regions defined by two or three parameters are more interpretable, and that interpretability drops as more parameters are used to define the region. In general, these three measures are in tension with one another (e.g., increasing density decreases coverage) so that PRIM generates a set of scenarios along an "efficient frontier" (e.g., see fig. 3 in Bryant and Lempert[28]) that allows the users to choose the scenarios that they find represent the best tradeoff among the measures for their application.

Table II. Scenarios in which SF emission reduction path performs poorly as suggested by RDM "Scenario Discovery" process. Density measures the fraction of cases within the scenario where SH has high regret. Coverage measures the fraction of all cases with high regret that the scenario contains. Combined, the two scenarios yield 86% coverage with 72% density.

Scenario	Definition	Density	Coverage
Catastrophic Climate Sensitivity	$\lambda^* \geq 9.2$	62%	37%
Overreaction	$\lambda^* \leq 1.20$ and MOC = 1	82%	49%

We implement this process using a "scenario discovery toolkit" that combines the PRIM code with a variety of useful graphical and diagnostic routines for scenario discovery.[11] The results are shown in Table II. Two of the four uncertain input parameters—the climate sensitivity and the possibility of MOC collapse—dominate in explaining the high-regret cases for the SF path.

The first scenario suggests that SF performs poorly in those futures with extremely high climate sensitivity, $\lambda^* \geq 9.2$.[12] This *Catastrophic Climate Sensitivity* scenario describes 149 of the 402 high-regret cases for SF (37% coverage). SF has high-regret in about two-thirds of the cases within this

[11] This toolkit, implemented in the *R* statistical computing environment, is available at http://cran.r-project.org/web/packages/sdtoolkit/index.html.

[12] Our experimental design contains no values for λ^* between 5.9 and 12.4—values leading to acceptable and unacceptable regret, respectively. We choose the midpoint $\lambda^* = 9.2$ as our scenario boundary, noting that a denser sampling in this region might help refine this choice.

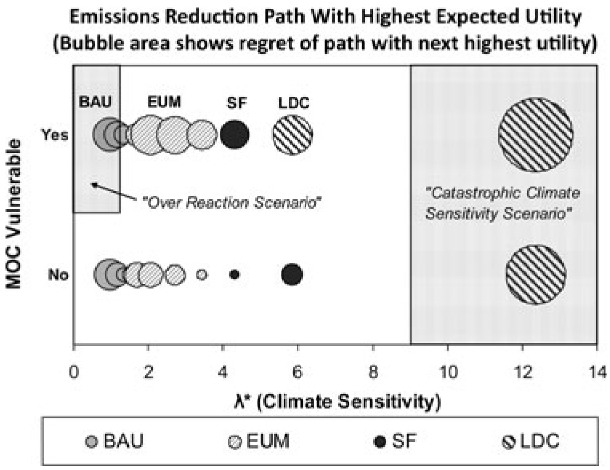

Fig. 6. Optimum reduction path in each of 2,662 cases as a function of climate sensitivity and MOC vulnerability. Area of bubble is regret of next best path. Shaded regions show "Overreaction" and "Catastrophic Climate Change Sensitivity" scenarios.

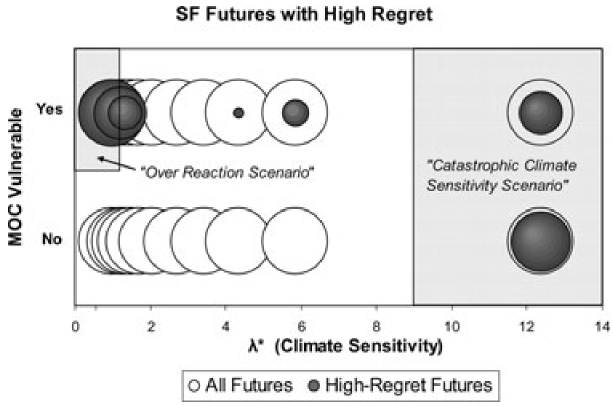

Fig. 7. Number of cases where SF reduction path has high regret (dark bubbles) compared to number of all cases (white bubbles) as a function of climate sensitivity and MOC vulnerability. Shaded regions show "Overreaction" and "Catastrophic Climate Change Sensitivity" scenarios.

scenario (62% density). The SF path fails to meet its goals in this scenario because its near-term emissions reductions are too low compared to those of LDC, which of the four paths considered does the best job of limiting climate impacts in these high sensitivity cases.

The second scenario in Table II suggests that SF also performs poorly in futures with very low climate sensitivity, $\lambda^* \leq 1.20$, and the MOC vulnerable to shutdown. This scenario describes 198 of the 402 high-regret cases for SF (49% coverage) and SF has high regret in about four-fifths of its cases (82% density). At first glance, this scenario might seem counterintuitive because it mixes favorable conditions—a low climate sensitivity—with adverse conditions—a possible MOC collapse.

To explain why SF fails to meet its goals in this scenario consider which path performs best. Fig. 6 shows the optimum path as a function of the parameters describing climate sensitivity and the potential for an MOC collapse. The size of each dot shows the regret of the second-best path in each case. BAU performs best in this scenario in the upper left-hand corner of the figure. SF fails in these futures because it reduces emissions too much, an interpretation consistent with the scenario's low values of λ^*, but in apparent conflict with the scenario's vulnerable MOC.

This apparent conflict is explained by considering how this study implements learning. SF is designed to begin with moderate near-term emissions reductions and then to increase its reduction rate if and when it learns that the MOC is vulnerable. However, in those futures with very low λ^* the MOC is highly unlikely to collapse whether or not it is vulnerable because, as noted earlier, the critical threshold rises with declining climate sensitivity. In such circumstances, SF unnecessarily increases its rate of emissions reductions and thus suffers high regret compared to BAU. We label this cluster of cases the *Overreaction Scenario*.

Fig. 7 shows the fraction of high-regret cases for the SF path for each of the points in our experimental design. The two scenarios characterize these high-regret cases reasonably well, although miss some high-regret cases with a vulnerable MOC and climate sensitivity between values of about 4 and 6. Overall, the catastrophic climate sensitivity and overreaction scenarios have coverage of 86% and density of 72%.

These two scenarios now support a choice task—evaluating the tradeoffs among the SF, BAU, EUM, and LDC paths. Fig. 8 shows the expected regret for the four options as a function of the probability ascribed to the overreaction and catastrophic climate sensitivity scenarios, labeled here as p_{OR} and p_{CCS}, respectively. The upper-left-hand panel shows that SF has low regret when p_{CCS} lies in below about 60% and is greater than about one-half the probability ascribed to the overreaction scenario, that is $p_{CCS} > 0.45 p_{OR}$. EUM has low regret when p_{CCS} is less than about 40%. LDC has low regret when the probability ascribed to this scenario is greater than about 40%. Thus relative to EUM, SF increases the range of probability for which it can successfully

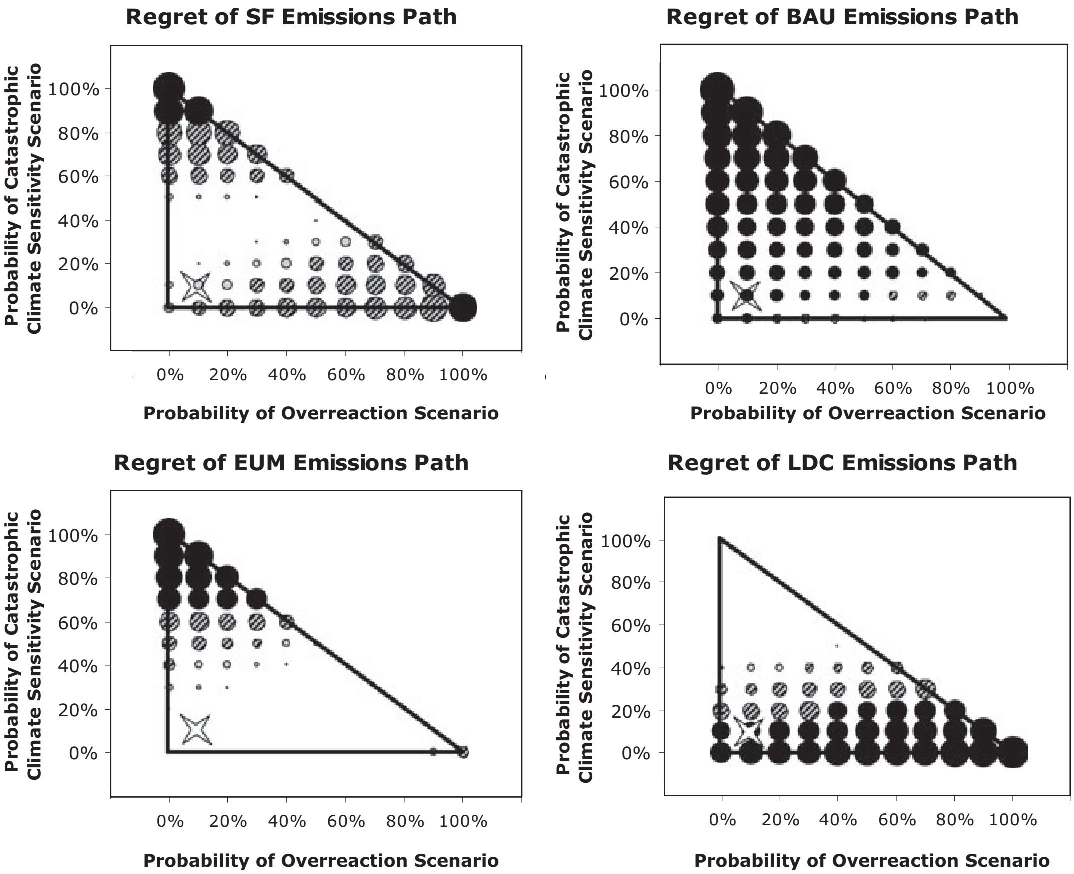

Fig. 8. Expected regret of SF, BUA, EUM, and LDC paths as a function of the probability ascribed to the "Overreaction" and "Catastrophic Climate Change Sensitivity" scenarios. The star at (9%, 9%) indicates best-estimate probabilities as reported in MILK.

address the catastrophic scenario, but at the cost of performing more poorly when the probability of the overreaction case is high.

The parameter probability density functions used by MLK suggest that the joint probability of these two scenarios is approximately 9%, as shown by the star in Fig. 8. The figure thus suggests that if decisionmakers are confident in the MLK probability estimate, they should choose EUM. If they worry p_{CCS} may be significantly higher than the MLK estimate, then SF may be a reasonable choice, but only if decisionmakers believe p_{OR} is sufficiently low. Decisionmakers should only choose LDC if they are confident MLK has significantly underestimated p_{CCS}.

In addition to informing the choice among the four emission reduction paths, the scenarios also support a decision structuring task—augmenting the set of options considered. In particular, decisionmakers and analysts might identify modifications to SF's learning algorithm that would eliminate its poor performance in the overreaction scenario without degrading its performance in the catastrophic climate sensitivity scenario or introducing any significant new vulnerabilities. If decisionmakers believed they had identified such a path, the RDM analysis would be rerun to characterize any vulnerabilities of this new version of SF and to evaluate its tradeoffs with the alternative options.

5. COMPARISON AND CONCLUSIONS

This study compares two approaches—info-gap and RDM—that can help decisionmakers evaluate potential robust strategies. The study uses each approach to evaluate alternative GHG emissions reduction paths given the potential for nonlinear threshold response in the climate system, deep uncertainty about any such abrupt change and other key parameters, and the ability to learn about any thresholds over time.

In their broad characteristics, info-gap and RDM share many similarities. Both represent uncertainty with sets of multiple plausible representations of the future, rather than a unique probability density function over future states of the world. Both incorporate the concept of robust satisficing—that, under some circumstances, decisionmakers may prefer strategies that perform acceptably well over a wide range of conditions to strategies that maximize performance under expected conditions. Both use quantified system models to relate decisionmakers' choices to potential consequences. Both info-gap and RDM provide decision support in the form of trade-off curves comparing alternative strategies rather than provide any definitive, unique ordering of options.

For the GHG emission reduction decision considered here, info-gap and RDM make broadly similar recommendations that nonetheless differ in their particulars. Both approaches suggest BAU is a poor choice. Both suggest that LDC might be favored by decisionmakers primarily concerned with futures that deviate significantly from current best estimates. Info-gap shows similar performance for SF and EUM and suggests the former offers more robustness for a small cost in optimal performance. RDM offers a less positive view of SF, suggesting the path offers more robustness than EUM against catastrophic climate change, but increases the risk of overreacting in some cases where climate change proves small.

The two methods reach these insights by following different analytic paths: treating losses and gains in different ways, taking different approaches to imprecise information, and arranging their analyses in different orders.

Info-gap explicitly considers both the potential gains if conditions turn out better than expected alongside losses if they turn out worse. RDM does not explicitly differentiate between losses and gains. However, RDM's scenario discovery process can identify cases representing each situation—for instance, in the overreaction scenario, SF fails to produce gains as large as BAU or EUM—and enable decisionmakers to trade one against the other.

The info-gap decision analysis asks decisionmakers to set minimum and aspirational performance levels and to favor the strategies that meet these levels, respectively, over the widest and narrowest range of uncertainty. The approach does not prescribe any rules for balancing between the most robust and most opportune strategies. RDM considers imprecise probabilities and suggests probability thresholds—the likelihood ascribed to a scenario that might cause a decisionmaker to choose an alternative strategy.

Although both approaches stress the need for constructive iteration through the analysis, the two sequence their steps in opposite orders with resulting implications for their style and content.

Info-gap first builds an uncertainty model and then uses its system model to evaluate the performance of a set of decision options over the range of uncertainties. The approach requires a series of judgments from analysts and decisionmakers in constructing the uncertainty model and specifying the required minimum and aspirational performance levels. The analysis provides visualizations used to adjudicate the tradeoffs between alternative options' robustness and opportunity. Although many variants of info-gap have been implemented and published, three basic elements are always present: (i) an uncertainty model, (ii) a system model, and (iii) minimum and aspirational levels applied to robustness and opportuneness trade-off curves.

In contrast, RDM begins with the set of decision options and aims to characterize the uncertainties in a way that usefully highlights the tradeoffs among those options. The approach requires a series of judgments, including: the strategies with which to begin the analysis, a regret-based or absolute performance criteria, the benchmark levels that constitute acceptable performance, and the set of scenarios (among those suggested by the scenario discovery algorithms) that best characterize the vulnerabilities of the strategies under consideration. The implementation of RDM considered here aims to characterize the cases where a proposed strategy does not perform acceptably. This information not only helps decisionmakers choose among decision options, it can also help them identify an improved set of options.[13]

This comparison of two approaches for assessing robust strategies suggests at least two strands of further study. First, empirical research on decisionmakers' preferences could measure how the different balance of attributes provided by the info-gap and RDM approaches affect decisionmakers' understanding and policy preferences in alternative decision contexts. As an example of such evaluations, recent work has compared water managers' responses to

[13] These differences between the methods are often softened in practice. Some info-gap studies have identified the sets of conditions that correspond to the points on the robustness curve. Some RDM studies have treated losses and gains differently.

RDM, traditional scenario approaches, and expected utility analyses.[34,41] These evaluations suggest that water managers found that RDM provided more useful information for decision making, but found it less easy to explain than the other two types of analyses. Comparative evaluations of how the info-gap and RDM approaches affect decisionmakers might provide useful insights on these two methods and on robust decision approaches more broadly.

Second, it would be useful to understand in general the conditions where alternative robust decision approaches—such as info-gap, RDM, robust optimization, and others—give similar and differing assessments of options. Given the diversity of definitions of robustness, and the differing judgments called for in implementing alternative robust decision methods, it is perhaps surprising they often reach similar results. A deeper understanding of why and when this is the case could help improve the foundations of robust decision methods.

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