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A General, Analytic Method for Generating Robust Strategies and Narrative Scenarios

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Robustness is a key criterion for evaluating alternative decisions under conditions of deep uncertainty. However, no systematic, general approach exists for finding robust strategies using the broad range of models and data often available to decision makers. This study demonstrates robust decision making (RDM), an analytic method that helps design robust strategies through an iterative process that first suggests candidate robust strategies, identifies clusters of future states of the world to which they are vulnerable, and then evaluates the trade-offs in hedging against these vulnerabilities. This approach can help decision makers design robust strategies while also systematically generating clusters of key futures interpretable as narrative scenarios. Our study demonstrates the approach by identifying robust, adaptive, near-term pollution-control strategies to help ensure economic growth and environmental quality throughout the 21st century.

Key words: decision making under uncertainty; robust decision making; deep uncertainty; adaptive planning; scenario planning
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1. Introduction

Robustness is an important criterion for good decisions under uncertainty (Rosenhead et al. 1972, Metz et al. 2001). When risks are well defined, quantitative analysis should clearly aim to identify optimal strategies. A variety of powerful analytic methods based on Bayesian decision analysis support the discovery of such choices. When uncertainties are deep,[1] however, robustness may be preferable to optimality as a criterion for evaluating decisions. To date, no general methods have been available for finding robust strategies using the information contained in most computer simulations. This paper aims to describe such a method.

A robust strategy performs relatively well—compared to alternatives—across a wide range of plausible futures. Interest in robustness has been spurred by increased computational capabilities that facilitate the more-extensive calculations often required to implement robustness as a decision criterion, by increasing recognition that many decision problems commonly faced by public- and private-sector decision makers are in fact characterized by deep uncertainty,[2] and by the recognition that decision makers faced with deep uncertainty often use a robustness criterion to assess their options.

Most traditional decision-analytic methods for risk and decision analysis (Morgan and Henrion 1990) are designed to identify optimal strategies contingent on a characterization of uncertainty that obeys the axioms of probability theory. These approaches begin with a single best-estimate description of the relevant system, consisting of a system model that generates outcomes of interest given a choice of strategy and a single set of probability distributions over the model's input parameters to characterize the uncertainties. The analysis recommends the strategy with the optimal expected utility contingent on this model and distributions. This approach generates the best possible strategy when the uncertainties are well characterized by single probability distributions. In their many manifestations, decision-analytic approaches based on

[1] Deep uncertainty is the condition in which analysts do not know or the parties to a decision cannot agree upon (1) the appropriate models to describe interactions among a system's variables, (2) the probability distributions to represent uncertainty about key parameters in the models, or (3) how to value the desirability of alternative outcomes. Similar concepts have been proposed by Knight (1921), who contrasted risk and uncertainty, using the latter to denote unknown factors poorly described by quantifiable probabilities; Ellsberg (2001), whose paradox addresses conditions of ambiguity in which reasonable choices need not follow the axioms of standard probabilistic decision theory; and an increasing literature on ambiguous and imprecise probabilities (de Cooman et al. 2001).

[2] The peaceful end to the Cold War, the terrorist attacks of 9/11, and the rise and fall of the "New Economy" have all contributed to the impression that the future may be less predictable than generally assumed.

optimization have proven extraordinarily useful for a wide range of decision problems.

For situations in which uncertainty exists about the system model or the distributions over its inputs, practitioners of traditional decision-analytic methods often employ sensitivity analyses (Saltelli et al. 2000) to test the optimum strategy's dependence on the specification of model and distributions. This approach is entirely satisfactory when the choice of optimum strategy is relatively insensitive to these key assumptions. When it is not, however, traditional decision analysis may pose both conceptual and practical problems. Optimum strategies lose their prescriptive value as they become increasingly sensitive to the characterization of the uncertainty. In particular, traditional approaches offer no systematic way to choose among several optimum strategies, each contingent on the choice of different prior probability distributions, none of which can be falsified by the available evidence.

In addition, when the optimum strategy is highly sensitive to assumptions, traditional decision analysis may be difficult to employ successfully. It can encourage analysts and decision makers to downplay uncertainty to make predictions more tractable (Sarewitz et al. 2000). It can make it more difficult for parties with different expectations and values to agree on actions because the method requires them first to agree on predictions (Lempert et al. 2004). And it can lead to strategies vulnerable to surprises that might have been countered had available information been used differently (Lempert et al. 2002).

We have been developing novel robust decision-making (RDM) methods to address the challenge of decision making under conditions of deep uncertainty (Bankes 1993, Lempert et al. 2003—henceforth LPB). RDM is an example of the emerging school of computational, multiscenario simulation approaches (Morgan et al. 1999, van Asselt 2000, Metz et al. 2001, Nakicenovic et al. 2000) that aim to incorporate ideas from scenario-based planning (Schwartz 1996) into a quantitative framework. In recent years, formal methods for finding robust strategies have appeared in areas ranging from operations research (Kouvelis and Yu 1997), Bayesian analysis (Berger 1985), control theory (Zhou et al. 1996), and engineering (Ben Haim 2001). RDM focuses on finding such strategies with minimal restrictions on the type of data and simulation model that can be employed in the analysis. The method appears most useful for situations in which the optimum strategy may be highly sensitive to assumptions about model structure and prior probabilities and where the menu of alternative options is sufficiently rich that decision makers can design strategies—potentially nonoptimal for many specific models and priors—but with nonetheless satisfactory performance over a wide range of assumptions. In brief, RDM inverts traditional sensitivity analysis, seeking strategies whose good performance is insensitive to the most significant uncertainties.

RDM characterizes uncertainty with multiple, plausible views of the future, beginning with one or more system models relating strategies to outcomes and a set of many plausible probability distributions over the uncertain input parameters to these models. Each simulation run is interpreted as an instance of traditional Bayesian decision analysis, but RDM runs this analytic machinery numerous times to (1) suggest which strategies are most robust—that is, whose satisfactory performance compared to the other strategies is relatively insensitive to all or most of the uncertainties; (2) identify vulnerabilities of these candidate robust strategies—that is, combinations of model formulations and input parameters in which the strategy performs relatively poorly; (3) suggest new or modified strategies to better hedge against these vulnerabilities; and (4) characterize the trade-offs involved in the choice among such hedging options.

RDM offers a prescriptive, systematic, and quantitative approach for designing and choosing among strategies showing satisfactory performance over numerous assumptions about models and prior probability distributions. When applied in practice, it also may help counter some of the problems faced by traditional decision-analytic methods under conditions of deep uncertainty. RDM is designed to reduce problems of overconfidence by challenging analysts and decision makers to explore a wide range of plausible futures and is designed to facilitate agreement by providing an analytic framework in which parties can agree on near-term actions robust across many expectations and values.

As one important attribute, RDM can help decision makers design robust strategies whose components may not be obvious at the onset. For instance, robust strategies are often adaptive. That is, they evolve over time in response to new information (Lempert et al. 1996). A robust, adaptive strategy might set signposts, the observation of which would suggest the future is evolving along one of several critical paths. The strategy might also specify actions that might be taken in response to observing one or more signposts (Dewar 2001). Given the difficulty of considering the multiplicity of potential paths into the future, even experienced decision makers may have little initial idea about the most robust combinations of signposts and responses. Despite a frequent desire to employ adaptive management approaches in practice, few quantitative methods exist for comparing the performance of different types of adaptive strategies because representing them in a simulation model often requires introducing both structural uncertainty and complicated feedback loops. RDM can assess robust strategies independently of any particular mathematical formalism for the underlying simulation, and so offers an approach to address this lack.

Using the challenge of sustainable development as an example, this study uses RDM to suggest robust near-term pollution-control strategies to help ensure economic growth and environmental quality throughout the 21st century. This focus on long-term environmental sustainability aims to introduce uncertainty that is unambiguously deep and a decision challenge famously sensitive to one's expectations about the future. This particular example is offered here as a methodological demonstration. The computer simulation employed is far too simple to support any policy-relevant conclusions. This RDM application also uses several simplifying assumptions not intrinsic to the method. Nonetheless, the general approach is applicable to computer simulations of arbitrary functional form and complexity, and so provides a foundation to address this and many other decision-making challenges in the presence of deep uncertainty.

2. General Means for Identifying and Assessing Robust Strategies

This paper proposes a general, rule-based approach to design robust strategies from the information in computer-simulation models and to identify vulnerabilities, opportunities, and trade-offs among these strategies systematically. Figure 1 illuminates the basic concept with an exceedingly simple example. The inset graph shows the performance of three strategies—1, 2, and 3—across two states of the world A and B. In State of the World A, Strategy 1 performs best (least cost), Strategy 3 performs worst, and Strategy 2 has intermediate performance relatively closer to Strategy 1. In State of the World B, Strategies 1 and 2 have costs higher than Strategy 3, with Strategy 1 worst by a wide margin.[3]

How should decision makers compare these strategies? If they have reliable information on the likelihood of both states of the world, they should choose the strategy that minimizes expected cost. If decision makers have no information about the likelihood of the state of the worlds, they might choose Strategy 3 to minimize the maximum regret (Savage 1950). In contrast, if decision makers regard World B as plausible but less likely than A, Strategy 2 appears the best choice because it both performs relatively well across both.

The main plot of Figure 1 quantifies this comparison by plotting the expected cost for each strategy as a function of the odds of State of the World B. Strategy 3 has the lowest expected cost when the odds of World B are higher than about 1:1; Strategy 2 for odds higher than 1:100 but lower than 1:1; and Strategy 1 only if the odds of World B are lower than about 1:100. Thus, decision makers willing to satisfice (Simon 1959), that is, accept a strategy whose expected cost is within some range of the optimum, and who regard the odds of State of the World A as more likely than B, should choose Strategy 2 as their most robust choice.

Most real problems lack the convenient simplicity of this example. In particular, there may be many more than two potentially relevant states of the world and no initially obvious strategy with relatively good performance over them all. Thus, this paper proposes a series of steps for reducing problems of arbitrary complexity (limited primarily by available computational resources) to a readily understandable form closer to that shown in Figure 1.

The *regret* of alternative strategies (Savage 1950) provides a conceptually and computationally convenient means to help identify robust strategies and their vulnerabilities. Defined as the difference between the performance of a strategy in some future state of the world, given some value function, and that of what would be the best-performing strategy in that same future state, the regret of strategy s, $s \in \vec{S}$, in future state f, $f \in \vec{F}$, using value m is given as

$$\text{Regret}_m(s, f) = \underset{s'}{\text{Max}}[\text{Performance}_m(s', f)] - \text{Performance}_m(s, f), \tag{1}$$

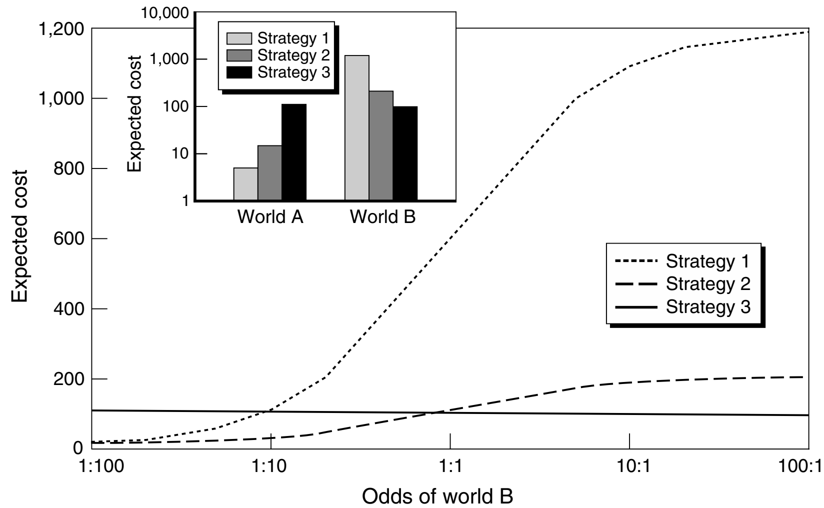
where strategy s' indexes through all strategies to determine the one with optimal performance in future state f. A robust strategy can be defined as one with relatively small regret compared to the alternatives across a wide range of plausible futures.

A regret-based definition of robustness is often preferable to one based directly on the absolute performance of each strategy because regret focuses attention on those states of the world most relevant to the choice among alternative strategies—those in which alternative policies have significantly different outcomes. The regret in Equation (1) preserves the expected value ranking of strategies contingent on any single probabilistic weighting over the future states of the world so that choosing a strategy based on expected regret is the same as

[3] The values shown in Figure 1 are typical of those that might occur in a situation with potentially high-consequence and low-probability events, although the approach here is more general.

[4] This definition is different than that used in narrative scenario-based planning (Schwartz 1996), in which scenarios are members of $\vec{F}$, representing a state of the world independent of any policy choices.

Figure 1 Cost (Inset) and Expected Cost (Main Graph) of Three Strategies in Two Futures: Strategy 2 Has the Lowest Expected Cost If the Odds of World B Are Less Than About 1:1



choosing based on expected performance. But the regret measure also helps identify states of the world in which one strategy performs much better than another. Such information can help decision makers choose among strategies and improve a strategy's performance when they are unsure about the weighting over future states. This study assumes preferences are given by simple functions (e.g., linear or step) that depend solely on regret. Although convenient, this assumption is certainly not true in general. For instance, prospect theory suggests decision makers might assess regret differently in states of the world with relatively favorable and unfavorable performance (e.g., a firm might be less sensitive to regret in states of the world with high profit than those with low profits). However, the computational steps described here can be easily modified to compare strategies using a wide range of different functions for describing preferences.

An RDM analysis uses a futures ensemble given by the cross-product $\vec{E} = \vec{S} \times \vec{F}$ of a set of strategies $\vec{S}$ and a set of plausible future states of the world $\vec{F}$. Each strategy and state of the world is described by a computer-simulation model and the inputs to that model. The ensemble can include both parametric (e.g., input parameters) and structural (e.g., model) uncertainty. In RDM, as with any decision analysis informed by computer simulation, one chooses system models and distributions over their inputs using the best available information. However, RDM's futures ensemble facilitates the application of numeric methods to compare the performance of strategies over multiple system models and multiple probability distributions. A probability distribution is usefully represented by weightings over a discrete set of points, so one can usefully consider the set of "plausible" futures as the union of the sets of all points with nonzero weighting in any of the probability distributions one wants to consider in the RDM analysis.

In general, this set's members will be too numerous to fully generate, so the ensemble exists as a virtual object on the computer in which implications for decision makers' choices can be explored with various sampling strategies (Bankes 1993). RDM consists of the following steps for exploiting the information in the futures ensemble:

Identify Initial Candidate Robust Strategies. The analysis begins by identifying some strategy $s_{\text{candidate}} \in \vec{S}$ as the initial candidate robust strategy. In some applications, the decision makers will propose an initial strategy to consider. In other applications, the analysis proposes an initial candidate by ranking the performance of strategies in $\vec{S}$ contingent on one or more assumed probability weightings across $\vec{F}$. Although this initial step can be identical to that in a traditional expected utility analysis, RDM does not depend on a consensus on such distributions as a necessary input to the calculations. As described below, this initial ranking is only the first step in a process that results in strategies robust over wide ranges of different probability weightings.

Identify Vulnerabilities. The ideal robust strategy would have no vulnerabilities. Such is generally not the case, so the analysis next uses any of a variety of statistical, sensitivity analysis, or other methods to identify and characterize one or more clusters of future states $\vec{F}^{i}_{\text{vuln}}(s_{\text{candidate}}) \in \vec{F}$ where strategy $s_{\text{candidate}}$ performs poorly, defined as those f where the strategy's regret exceeds some satisificing (Simon 1959) level. The example below will use a particularly promising data-mining algorithm to generate low-dimensional $\vec{F}^i_{\text{vuln}}(s_{\text{candidate}})$ clusters in a 41-dimensional space of input parameters that define $\vec{F}$ so that analysts and decision makers can readily understand the potential weaknesses of the strategy $s_{\text{candidate}}$. Note that these $\vec{F}^i_{\text{vuln}}(s_{\text{candidate}})$ are identified without reference to any probability weightings used to suggest $s_{\text{candidate}}$ as a promising strategy.

Suggest Hedges Against Vulnerabilities. The analysis then suggests a relatively small set of alternative strategies that address the vulnerabilities of strategy $s_{\text{candidate}}$. The analysis ranks the strategies $\vec{S}$ according to their performance over each cluster $\vec{F}^i_{\text{vuln}}(s_{\text{candidate}})$ and the set of futures where the strategy $F_{r,t}$ performs relatively well, $\vec{F}_{\sim\text{vuln}}(s_{\text{candidate}}) = \vec{F} - \sum_i \vec{F}^i_{\text{vuln}}(s_{\text{candidate}})$. These rankings define a set of performance trade-offs whose frontier represents a compact set of choices representing a range of different hedges against the vulnerabilities of $s_{\text{candidate}}$.

Characterize Deep Uncertainties and Trade-offs Among Strategies. Steps 1 through 3 can reduce a relatively complicated problem to a compact set of choices similar to those shown in Figure 1. As in that example, the choice among the strategies that lie on the trade-off curve in Step 3 depends on the odds ascribed to the future states of the world $\vec{F}^i_{\text{vuln}}(s_{\text{candidate}})$. Decision makers can use this information to choose a new candidate robust strategy and to characterize the deep uncertainties most important to their decision.

Consider Improved Hedging Options and Surprises. The first pass through these steps may yield only strategies whose performance against their vulnerabilities decision makers find unacceptable. However, the RDM analysis also often reveals patterns in the strengths and weaknesses of the strategies that suggest new robust designs $\vec{S}_{\text{improve}} \notin \vec{S}$ (e.g., adding a new policy action to the policy portfolio or new type of milestone to help adjust a strategy over time) that may exploit opportunities and hedge against vulnerabilities in new ways. The analysis then repeats the first three steps with the new set of candidate strategies $\vec{S} + \vec{S}_{\text{improve}}$ to test which—if any—of such new strategies actually offer improved performance. Alternatively, as decision makers and analysts grow confident with promising robust strategies, RDM encourages them to augment the computer simulations to include additional plausible future states of the world $\vec{F}_{\text{new vuln}} \notin \vec{F}$ that might cause the strategy to fail. In so doing, RDM provides a systematic framework for such "red teaming" and allows the characterization of potentially important surprises (Lempert et al. 2002).

RDM does not eliminate the need for analysts to make subjective (in the Bayesian sense) judgments (Lempert et al. 2004). Clearly, the distinction between plausible and implausible simulation models and sets of inputs to those models represents an inference about prior probabilities. But RDM differs from traditional decision-analytic approaches because it addresses the key insight of scenario planning methods—that under conditions we have termed deep uncertainty, multiple, potentially contradictory views of the future may be a better representation of the available scientific information than any single model and a better basis from which to communicate analytic results to an audience with different expectations about the future. Traditional approaches, based on ranking strategies according to a single, best-estimate view of the future (often characterized by known probability distributions over different future states of the world), have difficulty when such rankings are highly sensitive to the underlying assumptions. RDM aims to provide a systematic, analytic approach for identifying strategies insensitive to key uncertainties, in particular, which of multiple, potentially contradictory views of the future will turn out to be correct.

Ultimately, the claim that RDM will help decision makers make better decisions in some important situations than traditional approaches must be empirically tested. The claim derives from the observation that successful decision makers often use heuristics consistent with robustness as the criteria for good decisions and that robustness seems a normative criterion appropriate to conditions in which decision makers believe no single model or prior probability distribution. We make no attempt to prove these claims here. We only observe that such an empirical test requires a flexible method for identifying robust strategies. For instance, we are currently beginning a multiyear series of laboratory and field experiments to test the proposition that decision makers in some circumstances will find decision support tools that identify robust strategies more useful than those based on traditional approaches and to define the conditions under which the proposition is true. Such experiments obviously require the existence of decision support tools based on an approach such as described in this study.[5]

[5] Clearly, one could rigorously examine the claim that under certain conditions, robustness is the normative criterion without the analytic capability to implement robustness criteria. In practice, however, theoretical inquiries are often motivated by the capabilities of new technical artifacts rather than vice versa. The classic example is thermodynamics, which developed in the wake of the development of the steam engine. Thus, it would not be surprising to see theoretical inquiries into normative decision criteria under different conditions follow in the wake of the capability to implement alternative criteria enabled by the advent of new information technology.

3. Demonstrating Robust Milestones for Sustainable Development

These RDM methods will now be employed on a sufficiently complex example to demonstrate the steps outlined above. This paper extends our work in LPB, which used a computer simulation to address the question: What near-term actions should policy makers take today to promote a strong economy and high environmental quality over the 21st century? This sustainability challenge offers an archetypal case of decision making under conditions of deep uncertainty. Despite voluminous data, much remains unpredictable. Decision makers must engage stakeholders who hold a diverse array of values and expectations about the future. This example also helps showcase RDM's ability to assess the best choice of near-term performance milestones for a robust adaptive strategy. As in LPB, this study considers only stylized policies and simple representations of their effect to focus on the decision methodology itself rather than a full, policy-relevant treatment of the sustainability challenge.

3.1. A Simple Scenario Generator

This study uses a simple but highly nonlinear systems dynamics simulation, based on Sanderson's Wonderland model (Groller et al. 1996), to trace the effects of alternatives strategies $\vec{S}$ across many plausible future states $\vec{F}$. This scenario generator[6] tracks the changes in the economy, demographics, and environment in two world regions, representing the developed North and developing South. The economic module tracks output per capita in region $r = N$ or S with the difference equation

$$Y_{r,t+1} = Y_{r,t}\left\{1 + \gamma_r - \mu_r[1 - K_{r,t}(F_{r,t-1}, K_{r',t-1})] - \frac{C_r \tau_{r,t}}{1-\tau_{r,t}} - \Phi(K_{r',t}, \tau_{r',t})\right\}, \qquad (2)$$

where γ_r is the region's exogenous economic growth rate; $K_{r,t}$ is the "carrying capacity," an abstraction representing the state of the environment ranging from 1 (pristine) to 0 (destroyed); μ_r is a factor describing the rate at which decreased carrying capacity slows economic growth; $\tau_{r,t}$ is the level of a pollution tax that aims to preserve environmental quality; C_r scales the cost of these policies; and $\Phi(K_{r',t}, \tau_{r',t})$ is a function representing the impacts on region r of carrying capacity and environmental policy in region r'. The carrying capacity can drop at an exponentially decreasing rate if the annual flow of pollution $F_{r,t}$ exceeds some threshold value. The pollution flow is given by the Kaya equation $F_{r,t} = N_{r,t} Y_{r,t} P_{r,t}$, where $N_{r,t}$ is population and the emissions intensity $P_{r,t}$ (pollution per unit economic output) depends on the policy choices in each region and the exogenous rate of decoupling χ_r—that is, the rate at which pollution per unit of economic output decreases (Azar et al. 2002):

$$P_{r,t+1} = \left(1 - \tau^{eff}_{r,t} - \varphi \tau^{eff}_{r',t}\right)\chi_r P_{r,t}. \qquad (3)$$

The impact of policy at time t is given by $\tau^{eff}_{r,t} = \eta_r \tau_{r,t} + (1-\eta_r)\tau_{r,t-1}$, where the inertia η_r represents any delay in the rate at which policy affects emissions intensity and the coefficient φ represents the effects of policy in one region on the other. Appendix A in LPB describes the rest of the simulation in detail. In brief, the demographic module generates total population from birth and death rates (both a function of income and carrying capacity), and the environmental module describes the response of carrying capacity to changes in the flow of pollution. Overall, the scenario generator has 12 state equations and 41 deeply uncertain input parameters that influence the future evolution of the economy, demographics, environment, the capabilities and values of future decision makers, and the efficacy of various near-term policy actions. (These uncertain inputs include γ_r, C_r, φ, μ_r, χ_r, and η_r.)

The simulation represents near-term policy choices as the first step of a simple two-period decision problem. Today's decision makers choose (first-period) policy interventions in the North and South defined by the time series $\tau_{N,t}$ and $\tau_{S,t}$ for $0 \le t \le T_{\text{2nd-period}}$. The second period begins in $T_{\text{2nd-period}}$, the time (determined endogenously by the simulation) when future decision makers first detect a significant decrease in carrying capacity $K_{r,t}$. In the second decision period, future decision makers are assumed to have perfect information and determine $\tau_{r,t}$ for $t > T_{\text{2nd-period}}$ using utility functions and discount rates of which today's decision makers remain deeply uncertain.

Today's decision makers assess the performance of any near-term strategy s in any particular state of the world f with one of four different weighted averages ($1 \le m \le 4$) of the present value of the $Y_{N,t}$, $Y_{S,t}$, $K_{N,t}$, and $K_{S,t}$ time series. As described in LPB, the weightings represent stakeholders who value the four combinations of (1) economic outcomes only or both economic and environmental outcomes, and (2) outcomes in the North only or global outcomes. Because the present value[7] of these time series will in general be strongly influenced by the system evolution

[6] We use the phrase "scenario generator" rather than "computer-simulation model" to emphasize that the simulations aim not to forecast based on the best-available representation of reality, but rather to create a diverse range of plausible futures suitable for testing the robustness of alternative strategies.

[7] For simplicity, this example uses simple discounting to compare costs and benefits over time. In general, RDM could

in times past $T_{\text{2nd-period}}$, the efficacy of today's choices will be strongly influenced by the choices of decision makers in the second period.

Given deep uncertainty about the key factors driving the sustainability problem, LPB demonstrated that no fixed choice of $\tau_{N,t}$ and $\tau_{S,t}$ is robust. That is, no predetermined time series gives reasonable performance using all four value functions over all possible combinations of plausible values for the 41 uncertain input parameters. LPB suggests that today's decision makers should instead set 10-year emissions intensity milestones and adjust policies annually to meet these goals. Thus, milestone strategy s is defined by a choice (M_N, M_S) that yields near-term innovation policies given by

$$\tau_{r,t}^{M} = \begin{cases} \dfrac{(\gamma_r - M_r)}{[1-(1-\eta_r)^{10}](1-\chi_r)}\dfrac{t}{10} & \\ \qquad \text{if } \gamma_r - M_r \geq 0 \text{ and } 0 \leq t \leq 10 & \\ 0 \qquad \text{if } \gamma_r - M_r < 0 \text{ and } 0 \leq t \leq 10 & \\ \tau_{r,10}^{M} \quad \text{if } 10 < t \leq T_{\text{2nd-period}} & \end{cases}. \tag{4}$$

This design of an adaptive strategy is inspired both by LPB's assessment of the vulnerabilities of fixed strategies and by a common response practice of decision makers faced with problems of deep uncertainty. The time series described by Equation (4) assumes that decision makers acquire perfect information about the parameters χ_r and η_r immediately after choosing the milestones, and thus are only a crude approximation of the evolution of an actual near-term adaptive strategy. Nonetheless, this representation of policy and the modified Wonderland simulation provide a useful platform for demonstrating RDM.

3.2. Identify Initial Candidate Robust Strategies

This study now uses the modified Wonderland scenario generator to create the futures ensemble $\vec{E} = \vec{S} \times \vec{F}$ and then follows the steps outlined in §2 to identify a robust choice of milestones (M_N, M_S) and characterize its residual vulnerabilities.

The set of strategies $\vec{S}$ is defined by a full-factorial experimental design over all combinations of the 10-year emissions intensity milestones (0%, 1%, 2%, 3%, and none[8]) for both north and south. The resulting 25 "milestone" strategies are labeled by an M followed by the milestone values for the north and south. Small numbers represent more aggressive targets. For instance, M00, the no-increase strategy considered in LPB, sets near-term milestones that demand both north and south pursue policies that give a policy-induced decoupling rate (that is, the sum of the rates due to policy and exogenous trends) equal to the rate of economic growth, while M3X sets no target in the south and requires the north to keep its decoupling rate within only three percentage points of economic growth. For many pollutants, the decoupling rate has historically run a few percentage points behind the economic growth rate (Azar et al. 2002).

The set of future states $\vec{F}$ is generated by sampling a diverse range of Wonderland input parameters consistent with available data that constrain both the simulation's inputs and outputs. The range of plausible values for each uncertain input is drawn from the sustainability literature as in LPB. For instance, we choose the northern economic growth rate, γ_N, to vary between 0% and 4% ($/yr) and the northern decoupling rate, χ_N, to vary between −1% and 5% (pollution/$-yr).[9] Five independent, 1,000-point latin hypercube experimental designs generate an initial sample over the 41-dimensional space of input parameters. To ensure consistency with an output constraint, this 5,000-point sample is then pruned to eliminate any state whose population trajectory (in the absence of any environmental or economic constraints) falls outside plausible bounds.[10] The resulting sample of 2,259 future states aims to represent a diverse, plausible set consistent with available data.

Calculating the regrets using Equation (1) for each strategy in each state yields the futures ensemble $\vec{E} = \vec{S} \times \vec{F}$, where $\vec{E}$ is a database with 56,475 records. Each record has 44 entries: 41 listing the parameter values for one of the 2,259 states, 2 listing the milestones for one of the 25 strategies, and 1 reporting the regret of that strategy in that state.[11] This study focuses on performance calculated with the most stressing of LPB's four measures—"World Green HDI," a variant of the United Nation's Human Development Index (HDI; United Nations 1990). This measure represents stakeholders interested in global economic and environmental outcomes. Not emphasized here but introduced more fully in LPB, RDM provides the

straightforwardly employ a wide range of other formulations, from hyperbolic discounting to a multiattribute (near-term and long-term) framing.

[8] A milestone M_r = none yields $\tau_{r,t} = 0$ for $0 \leq t \leq T_{\text{2nd-period}}$.

[9] Table A1 in LPB gives plausible ranges for all 41 Wonderland input parameters.

[10] As in LPB, this study defines plausible future population scenarios as those with a 2050 population between 700 million and 1.5 billion in the North and between 7 billion and 15 billion in the South.

[11] The manipulation of these futures ensembles was facilitated by Evolving Logic's Computer Assisted Reasoning system (CARs) software. CARs encapsulates many types of modeling formalism and helps users to carry out the steps necessary for conducting RDM.

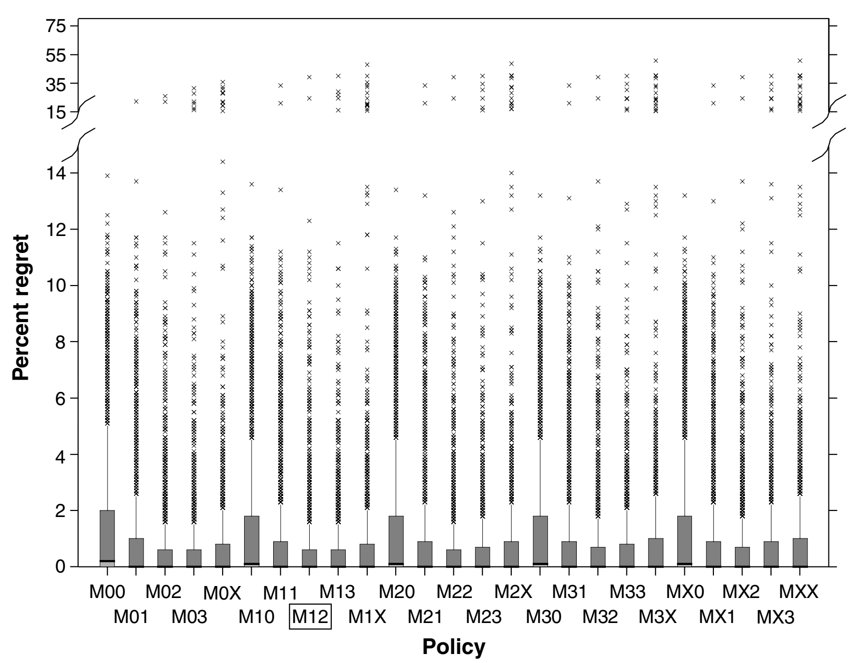
important ability to address robustness across multiple value systems as well as multiple expectations about the future.

The futures ensemble $\vec{E}$ may now be used to suggest an initial candidate robust strategy. One could simply choose the strategy with the optimal expected regret contingent on some weighting across the states of the world. The database representation of the futures ensemble $\vec{E}$ also makes it computationally straightforward to employ other criteria based on the needs of the decision makers and the structure of the problem. Here, we rank strategies according to their upper-quartile regret because in this particular sustainability example, it was useful to avoid an explicit, initial probabilistic weighting of states (such weightings being applied at later stages of the analysis). The upper-quartile regret, as opposed to the median regret, is useful because the latter is trivially zero or near zero for all strategies in this example because it is costless and unnecessary to achieve the milestones for cases in which the exogenous decoupling rate exceeds the economic growth rate. Note that the final results shown in Figure 7 would be similar had the analysis begun with an initial strategy based on the mean expected regret.

Figure 2 shows the distribution of regret across the future states of the world $\vec{F}$. The top of each box represents the upper-quartile regret for each strategy s over the states $\vec{F}$ (that is, the strategy has regret less than this value in three-quarters of the states), the vertical line atop each box represents regrets within 3/2 the interquartile range, and the x marks show the regrets for the remaining worst states for each strategy. Note that only five strategies—M00, M10, M20, M30, and MX0—have a nonzero median regret (indicated by a line across their boxes), and that all strategies have a zero lower quartile. The scalloping pattern of these regrets suggests that the best-performing options have a laxer southern milestone of 2% (M02, M12, M22, M32, and MX2). Note that if well-characterized probability estimates for some input parameters were available, we could also average over the corresponding records in the futures ensemble, thus reducing the dimensionality of $\vec{E}$, recalculate Figure 2 using expected regrets over those distributions, and focus the RDM analysis on the remaining deeply uncertain dimensions of the decision problem.

The strategy M12 emerges as the best initial candidate using the upper quartile of regret. With its modest milestones, M12 has a regret less than 0.6% in three-quarters of the futures in the ensemble. The M03 strategy performs slightly better measured by the mean, 0.64% versus 0.68%. Interestingly, M12's maximum regret of 39% is among the highest of the strategies. In contrast, MX0, which minimizes the maximum regret at 13.2%, produces among the highest upper-quartile regrets of 1.8%.

Figure 2 **Distribution of Regret (%) for 25 Milestone Strategies Over Sample of 2,259 Plausible Futures**



Notes. The top of each box shows each strategy's upper-quartile regret, the "whisker" above each box represents observations within 3/2 the interquartile range, and x marks represent all other observations. The line across some boxes indicates a nonzero median regret. Strategy M12 has the lowest upper-quartile regret.

3.3. Identify Vulnerabilities

The analysis has now suggested a candidate robust strategy. In this example, this selection was based on a count of states in which strategies have high regret. In other examples, candidate strategies could be selected based on expected regret contingent on some prior probabilities, or just be a strategy of particular interest to the decision maker. Whatever the starting point, RDM now begins a process of finding a strategy more robust than the initial candidate, beginning with an exploration of its potential weaknesses. Despite its relatively superior performance over the full set of plausible states, M12 may not be the most robust strategy for two reasons. First, the strategy may have serious vulnerabilities more effectively hedged by alternative milestone strategies. Second, M12's overall performance is by no means acceptable, even if it exceeds that of the alternatives yet considered. The strategy has an upper-quartile regret of 0.6%, well above the 0.1% threshold acceptable regret in LPB. This section and §3.4 will address the first concern; §3.5 will address the second.

The study employs Friedman and Fisher's (1999) "patient" rule induction method (PRIM) to identify and characterize the clusters of future states in which the strategy M12 has high regret. PRIM is a data-mining algorithm designed to generate a set of low-dimensional "boxes" in high-dimensional data containing regions where the value of a particular function is large (or small) compared to its value outside these boxes. Many methods exist that might usefully generate such clusters. PRIM seems particularly useful for RDM because it aims to optimize both the classification accuracy of the boxes; that is, the percentage of large function values they contain, and the interpretability of the boxes; that is, the simplicity of the rules needed to define them. For instance, Friedman and Fisher use PRIM to identify from marketing data key characteristics (e.g., age, gender, marital status) of likely consumers of a new product.[12]

This study implemented PRIM using publicly available software[13] that inputs a data set and outputs rules defining several alternative n-dimensional boxes and two measures of the ability of each box to approximate the desired clusters: the mean value of the data within the box (ideally large relative to the mean value of data outside the box) and the size of the box (ideally large enough to represent a meaningful region of the output space). These measures are generally inversely correlated because a larger box may include a lower density of high-value data. In this application, it is convenient to apply PRIM to a binary transformation of the outputs in the futures ensemble. Any regret exceeding a threshold of 2% World Green HDI units is given a value of 1; all others are set to 0. (This threshold for unacceptable regret derives from LPB.) With such a transformation, consistent with the concept of robustness as a satisficing criterion, the mean within any PRIM box is equivalent to its density of high-regret states. The percentage of all high-regret states captured in the box equals the product of the box mean and size divided by the ensemble mean. An ideal cluster should contain both a high density of high-regret states (e.g., most states in the box are vulnerabilities of the strategy) and span most of such states in the ensemble (e.g., most vulnerable states are in the box). The binary transformation is a convenient simplifying assumption that allows use of the existing PRIM computer code without modification to generate these two useful measures of merit for the clusters in our application.

When applied to the 2,259 records in the futures ensemble $\vec{E}$ with regrets for strategy M12 (with these regrets transformed into 1s and 0s), the PRIM method suggests rules for several dozen boxes. After inspecting the alternatives—which in a real policy exercise would be done with participation of key decision makers and stakeholders—we chose the rules shown in Figure 3 as a compelling characterization of a set of states to which M12 is highly vulnerable. In these states, the South's economic growth (γ_S) is high, and exogenous decoupling rates (χ_S) are low. The cost of policy intervention to speed decoupling (C_S) is high, and the environment can sustain a high level of pollution. M12 has a regret of 2% or greater in 256 (11%) of the states in $\vec{E}$, and the chosen cluster contains 50% of these high-regret states. Forty-eight percent of the states within the cluster have high regret.[14] Many stakeholders in the sustainability debate—such as industrialists and the economic ministers of developing countries—often strongly voice their expectations that adoption of emission milestones such as those in M12 would lead to states characterized by costly and unnecessary environmental regulations. With a bit of license, we can call this PRIM-generated cluster the "Southern Julian Simon" (SJS) states.[15]

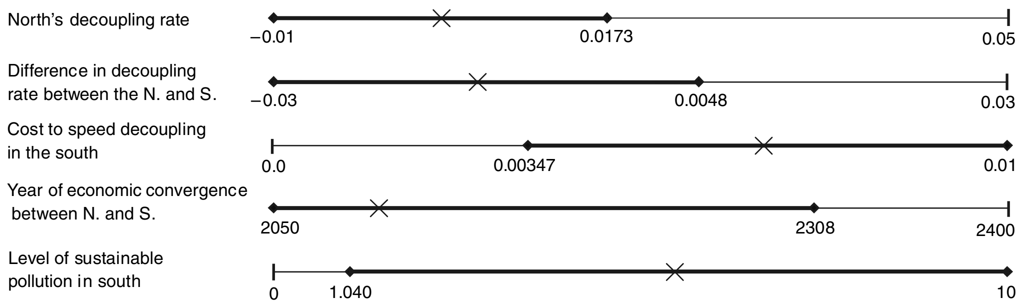
[12] Note that while many sensitivity analysis methods, such as those used in LPB, require particular experimental designs, PRIM can employ the nonhypercubic designs used in this study.

[13] Available at http://stat.stanford.edu/~jhf/SuperGEM.html.

[14] In principle, one could apply PRIM repeatedly to yield multiple boxes and capture additional high-regret future states. But this analysis considers only one cluster because subsequent iterations of PRIM on the remaining futures revealed no new boxes substantively different from the first chosen.

[15] PRIM does not guarantee the ability to find such meaningful clusters. Although clearly important, RDM can be conducted without them as was done in LPB. PRIM, an algorithm developed for other purposes, was used here for RDM without significant modification. Further research may provide additional or more-effective algorithms for generating RDM clusters for a wide variety of decision problems.

Figure 3 Ranges of Parameter Values (Heavy Lines) Corresponding to the PRIM-Generated Rules That Define the "Southern Julian Simon" Cluster of States



Notes. Light lines indicate full range of each input parameter. Values for the representative SJS future discussed in the text indicated by x marks.

To better understand why M12 performs poorly in this cluster, one can return to the Wonderland simulation and examine in detail the time evolution of key system outputs in a representative future. For instance, M03 is the optimal strategy for the five input parameter values indicated by x marks in Figure 3.[16] In this future, global output per capita under M03 increases from about $5,000 to over $23,000 by 2100 unconstrained by any environmental damage. In contrast, per capita economic output with M12 stagnates at about $6,000. M12 fails in this cluster because its attempts to limit emissions are unnecessary and exceedingly costly.

3.4. Characterizing Trade-offs

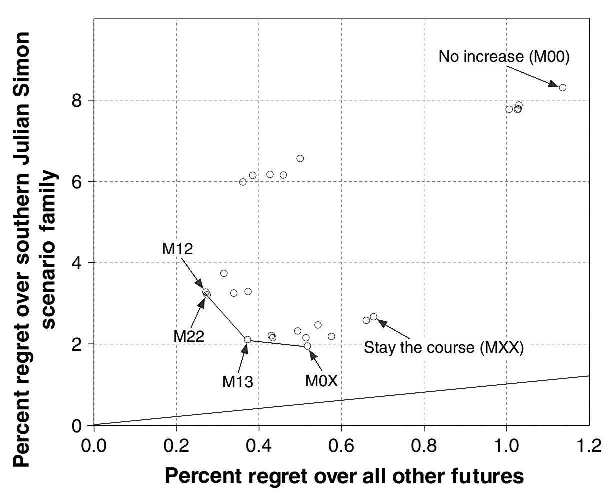
The SJS states represent a potentially important vulnerability of the M12 strategy. RDM now identifies alternative strategies that perform nearly as well as M12 across the full set of plausible states, but better where it is most vulnerable.

Figure 4 compares the upper-quartile regret values for each of the 25 milestone strategies over two sets of future states: those in the SJS, $\vec{F}_{SJS}$, and those in the set of all other future states, $\vec{F}_{\sim SJS} = \vec{F} - \vec{F}_{SJS}$. Not surprisingly, M12 has the lowest regret in the latter. In the SJS future states, M12 still performs better than many alternatives, although worse than 12 other milestone strategies. In particular, M0X has lowest regret over $\vec{F}_{SJS}$.

If decision makers could confidently agree on estimates of the probability of the $\vec{F}_{SJS}$ future states, they could derive an optimal policy from the data in Figure 4. But under conditions of deep uncertainty, decision makers lack the information to support and advocate such a determination. Instead, Figure 4 suggests that decision makers should consider the four alternatives that trace out the low-regret frontier—M12, M22, M13, and M0X. The choice among these four is not obvious because each alternative involves different risks and a different allocation of burdens between the north and south. Those who believe the SJS future states relatively more likely would advocate M0X with no milestones in the south and strict milestones in the north. Those who believe these states less likely would advocate M12 or M22 with more equal northern and southern milestones. The M13 strategy, with lax southern and more stringent northern milestones, hedges more across both types of states.

In general, RDM does not determine the best strategy. Rather, it uses the information in computer simulations to reduce complicated, multidimensional, deeply uncertain problems to a small number of key trade-offs for decision makers to ponder. Such results can be particularly useful when the optimal strategies are sensitive to the assumptions, but the available information is nonetheless sufficient to support the design and evaluation of robust strategies relatively insensitive to the assumptions. Here, the analysis suggests that whatever the expectations of different deci-

Figure 4 Upper-Quartile Regret (%) for 25 Milestone Strategies Over the SJS and the Non-SJS States



Note. Strategies on the frontier of the trade-off curve are labeled.

[16] The other 36 input parameters take their nominal values as listed in Table A1 in LPB.

sion makers, they should choose only among the M12, M22, M13, and M0X strategies. For instance, many participants in the sustainability debate advocate a "Stay the Course" strategy (MXX). Figure 4 suggests they should consider M0X instead, as it performs as well in the SJS future states but is better hedged across the full ensemble. In addition, such results can have important value of information implications. Of all the uncertainties decision makers might seek to reduce, those most valuable to the choice among milestone strategies relate to the likelihood of the SJS states.

In an actual policy-relevant exercise, robust decision making would next probe for the vulnerabilities of the best options that emerge from Figure 4. These explorations would in general emphasize robustness as a multiattribute criterion, requiring reasonable performance over many states and values. For instance, PRIM might identify new clusters of states in which strategies such as M13 and M0X have high regret using the World Green HDI measure. In addition, PRIM might identify states where "Stay the Course" and M0X perform quite differently using regret based on LPB's North Green HDI measure.

Rather than pursue such explorations with the milestone strategies, however, this study turns instead to identifying strategies of a type more robust than any yet considered.

3.5. Consider Improved Hedging Options

None of the milestone strategies shown in Figure 4 performs satisfactorily. The best third-quartile performance over the SJS states is M0X's 1.94%, and the best performance over M12's satisficing states is M12's 0.27%. Both are significantly larger than the values LPB describes as acceptable. When faced with a set of unpalatable alternatives, decision makers naturally seek additional options. Often such options seek to improve strategies' adaptivity to achieve greater robustness. RDM can help structure and focus this process of designing improved adaptive strategies more robust than the current alternatives. While applied here to sustainable development, this approach is completely general and can provide analytic support for adaptive decision making in a wide range of institutions and problem areas.

The inability of any milestone strategy of the type in Equation (4) to perform well across both the states where aggressive goals are unneeded and costly and those states where aggressive near-term action is inexpensive and vital suggests consideration of strategies that pursue aggressive decoupling milestones subject to a cost constraint. In climate-change policy, some have proposed "safety valve" strategies that set emissions-reduction targets contingent on a cost constraint implemented, for instance, via tradable emissions permits that governments agree to sell in unlimited numbers at some maximum price, thus providing an upper bound to the cost of any pollution-control policy (McKibbin and Wilcoxen 2002). Thus today's decision makers might set cost thresholds in the north and south, T_N and T_S, in addition to the 10-year emissions intensity milestones, so that near-term pollution taxes are given by

$$\tau_{r,t}^{SV} = \begin{cases} \tau_{r,t}^{M} & \text{if } C_r(\tau_{r,t})^2 \le T_r \text{ and } 0 \le t \le 10 \\ \sqrt{\dfrac{T_r}{C_r}} & \text{if } C_r(\tau_{r,t})^2 > T_r \text{ and } 0 \le t \le 10\,, \\ \tau_{r,10}^{SV} & \text{if } 10 < t \le T_{\text{2nd-period}} \end{cases} \tag{5}$$

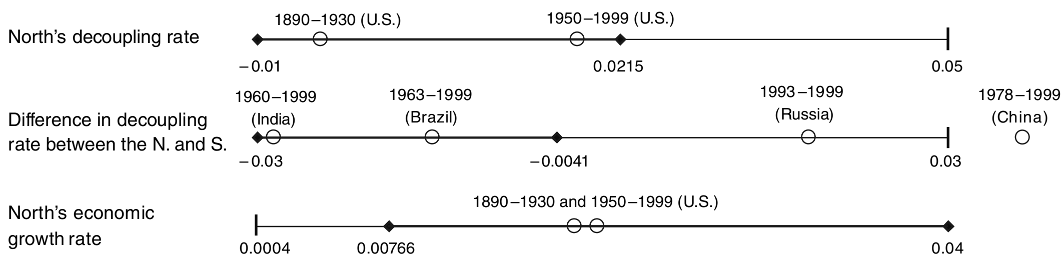
where the $\tau_{r,t}^{M}$ are given by Equation (4). These adaptive safety-valve strategies monitor annual costs as they pursue policies to achieve the milestones. If the costs ever exceed the predetermined thresholds, the policies are relaxed to meet the cost target, irrespective of whether the milestones can be achieved.

RDM seeks to find the most robust choice of milestones and thresholds (M_N, M_S, T_N, T_S). The new set of strategies $\vec{S} + \vec{S}_{\text{improved}}$ is now given by the previous 25 milestone strategies plus a full-factorial experimental design of 400 different combinations of near-term milestones (0%, 1%, 2%, 3%, and none) and cost thresholds (0.2%, 0.5%, 1%, and 1.5%) for both the north and south. The analysis generates a 960,075-member sample of the futures ensemble $\vec{E} = \vec{S} \times \vec{F}$ by calculating the regret of each of the 425 strategies over the previous experimental design of 2,259 future states $\vec{F}$.

The strategy SV00.005.002[17] emerges as the best initial candidate using the upper-quartile regret based on the World Green HDI measure. The SV00.005.002 strategy has more aggressive near-term emissions intensity targets than the best-performing milestone strategies—no emissions intensity increase in the north or south—but also strict cost constraints on policies aiming to achieve these milestones. Each year, these policies can spend no more than 0.5% and 0.2% of GDP in the north and south, respectively. The path to the milestones is not followed during any year the costs to do so are too high. This combination of aggressive targets and cost constraints gives SV00.005.002 an upper-quartile regret over the full futures ensemble of 0.27%, more than 55% lower than that of the best milestone strategies considered previously.

[17] Safety-valve strategy names consist of three parts: the north and south milestones, the north cost threshold, and the south cost threshold.

Figure 5 **Ranges of Parameter Values (Heavy Lines) Corresponding to the PRIM-Generated Rules to Define the LGD States**



Note. Open circles denote historical values from LPB Figure 5.1 for the United States, China, India, Brazil, and Russia over various periods.

PRIM next identifies and characterizes vulnerabilities of the SV00.005.002 strategy. The rules shown in Figure 5 provide the most compelling cluster. These states have low exogenous decoupling rates in the north and south as well as moderate-to-high global economic growth rates. As discussed below, for these states, the emissions milestone and cost constraint of SV00.005.002 is appropriate for the north but are too stringent and hinder economic growth in the south. This PRIM-generated cluster can be labeled the "Low Global Decoupling" (LGD) future states, $\vec{F}_{LGD}$. Of the 2,259 states in the futures ensemble $\vec{E}$, $SV00.005.002$ has large regret, 2% or greater, in 192 (8.5%). $\vec{F}_{LGD}$ contains 58% of these states and spans only 18% of $\vec{E}$. The cluster is, however, less dense with high-regret states than $\vec{F}_{SJS}$ was for M12. Only about 28% of $\vec{F}_{LGD}$ are high-regret states for SV00.005.002.

Figure 6 next compares the regret of each of the 400 safety-valve and 25 milestone strategies over two sets of states—those in the LGD future states $\vec{F}_{LGD}$ and in the non-LGD future states. SV02.005.010 has the lowest third-quartile regret in the latter, while SV01.015.015 performs best in the former. These two strategies are joined by SV02.010.015, SV02.005.015, and SV02.005.005 in tracing the frontier of a trade-off curve of best hedging options.[18] All but the lowest-right strategy along the trade-off curve, call for strict emission milestones in the north (0% over GDP growth) and modest emission milestones in the south (2% over GDP growth). Moving down along the curve towards lower regret in LGD states, southern cost constraints relax from 0.5% of GDP to 1.5%, while northern cost constraints remain fixed at 0.5%. This suggests that in LGD states, strict southern cost constraints are triggered too early (e.g., before sufficient emission reduction occurs). Moving leftwards up the curve towards lower regret in non-LGD states, southern milestones are relaxed to 2% and northern cost constraints are tightened to 0.5%. In non-LGD states, performance suffers if northern cost constraints are too loose (e.g., the north spends too much reducing emissions) or if the south is too aggressive in its reductions.

These five safety-valve strategies, the result of reframing the policy options as near-term milestones combined with cost limits, yield considerable performance improvements over the original milestone-only strategies also shown in Figure 6.

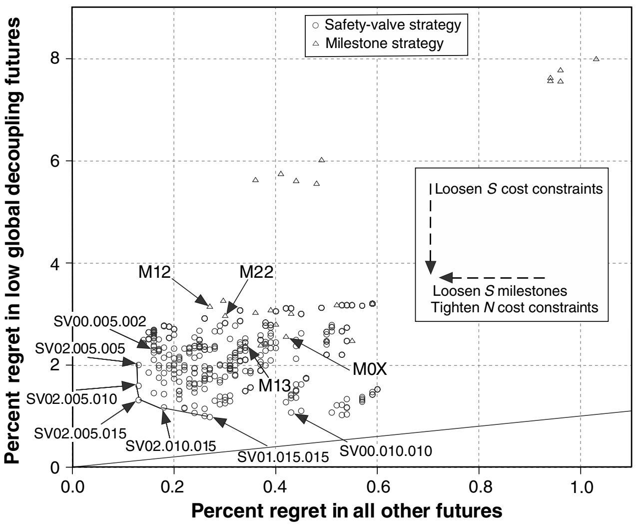
RDM can now reduce this complicated, multidimensional, deeply uncertain decision challenge to a simple form reminiscent of our simple example summarized in Figure 1. Figure 7 plots the expected regret of four of the five strategies along the trade-off curve in Figure 6—SV02.005.010, SV02.005.015, SV02.010.015, and SV01.015.015—as a function of the odds of an LGD state relative to the other plausible states.[19] The LGD states constitute about 18% of the entire sample of states. If all states were equally likely (labeled 1:1), SV02.005.015 would be the best choice. The SV02.005.010 strategy is a slightly better choice if the odds of an LGD state are close to 1:100, whereas SV01.015.015 is best when the odds are about 3:1 or more of an LGD rather than a future state of any other type.

Consistent with Watson and Buede's (1987) concept of policy region analysis, Figure 7 also marks the "robust regions" of odds in which each strategy has an expected regret within 20% of the best possible for those odds. The strategy SV02.005.010, with its aggressive northern milestone and modest southern milestone but tight cost constraints, is only robust if decision makers are confident the odds of an LGD state are less than about 3:1. SV01.015.015, with its aggressive north and south milestones and willingness to accept higher costs to achieve them, is robust if decision makers are certain that the odds of an LGD state are greater than about 1:1. Strategies SV02.005.015 and SV02.010.015, with their laxer southern milestones but willingness to accept high costs to

[18] Using the third-quartile measure, SV00.005.002—the best-performing strategy over the entire set of futures—need not lie on the trade-off curve.

[19] The relative odds of an LGD future are given by $\phi = [p/(1-p)] \times [(N - N_{LGD})/N_{LGD}]$ where N is the total number of states in $\vec{F}$, N_{LGD} is the number of states in $\vec{F}_{LGD}$, and p is the probability weighting assigned to the LGD states.

Figure 6 Upper-Quartile Regret (%) for 400 Safety-Valve and 25 Milestone Strategies Over the LGD ($\vec{F}_{LGD}$) and the Non-LGD States ($\vec{F} - \vec{F}_{LGD}$)

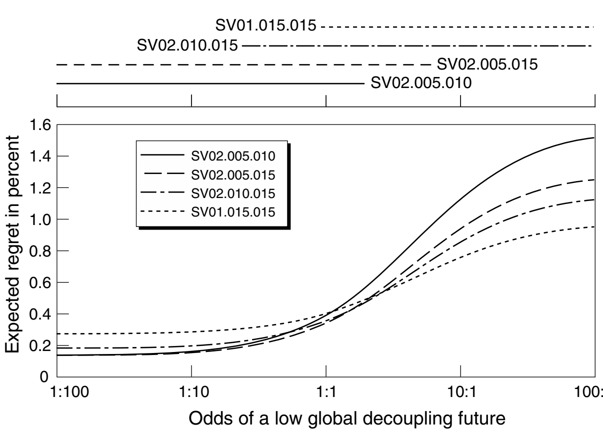


Notes. Strategies on the frontier of the trade-off curve are labeled and marked with line. The four milestone strategies on the frontier of Figure 4 are also indicated. Their regret has increased, as the least-regret strategy is now a safety-valve strategy.

achieve it (at the apex of the trade-off curve shown in Figure 6), are robust over the widest range of odds, collectively spanning the entire range.

RDM also helps decision makers focus their attention on the most valuable information needed to inform their choices. Of all the data relevant to the Wonderland simulation, those affecting the likelihood of the LGD future states are most crucial to the choice among robust strategies. Figure 5 lays historic data on economic growth and decoupling rates over the range of parameter value ranges that define $\vec{F}_{LGD}$. Actual economic growth in the United States was consistent with the assumptions underlying $\vec{F}_{LGD}$ across both the 19th and 20th centuries. The LGD future states assume northern decoupling rates characteristic of the United States in the 19th century, but lower than those in the 20th. Twentieth-century decoupling rates of countries in the south fall both within and outside the range assumed by $\vec{F}_{LGD}$. To the extent that decision makers take this historical data to suggest that the LGD states are possible, but not likely, they should consider SV02.010.015 or SV02.005.015 as the most robust strategy.

Figure 7 Expected Regret of Best Safety-Valve Strategies from Figure 6 as a Function of the Odds of an LGD Future State



Note. Bars at the top of the figure show regions where each strategy is within 20% of best strategy for those odds.

4. Summarizing Deeply Uncertain Information for Decision Makers

Decision makers in the public and private sectors face many decision challenges characterized by deep uncertainty. Computer simulations provide an important means to assemble and trace the implications of information that can inform such decisions. But analysts have lacked general methods for using such simulations to support one of the key decision criteria most appropriate under conditions of deep uncertainty—robustness.

We have presented a general, systematic, quantitative method for developing robust strategies using the information contained in computer simulations and data. This rule-based approach works in principle with any type of formalism that maps, deterministically or stochastically, input assumptions onto outputs representing potential consequences.[20] RDM envisions an iterative process initialized by generating a set of alternative strategies and a diverse set of plausible states. An initial screening suggests a small number of strategies that perform well, that have low regret, over the entire set of states. Statistical tools such as the PRIM then help divide the set of states into clusters in which these candidate strategies perform particularly poorly (have high regret) and clusters in which they perform reasonably well. These readily interpretable clusters can be used to support visualizations that compare the performance of alternative strategies and define key trade-offs among the best-available hedges against these vulnerabilities. These results, often aided by additional computer exploration, can also help users design new types of strategies to hedge against a broader range of contingencies. This shifts the available policy trade-off curves significantly towards regions of even greater robustness.

RDM may make an important contribution to the growing debate over the best means to characterize deep uncertainty for decision makers (Lempert

[20] Bankes et al. (2000) describes an application of robust decision methods using scenarios generated entirely from alternative fits to data, without any computer-simulation model.

et al. 2004). The traditional decision-analytic approach encourages characterizing uncertainties, however deep, by placing the best-available probability distributions over future outcomes. The practitioners of scenario-based planning (Schwartz 1996) advocate an alternative approach—characterizing deep uncertainty with detailed descriptions of a small set of fundamentally different paths into the future. These narratives can help organizations grapple with the implications of future states they might regard as inconvenient, controversial, or violating conventional wisdom. Placing probabilities on such scenarios can defeat their purpose, causing decision makers to either focus on those deemed most likely or reject the analysis entirely because they disagree with the probability estimates. However, scenario planning offers no systematic means to compare alternative strategies and any particular choice of scenarios out of the many possible may seem arbitrary.

RDM may offer a synthesis between the communicative power of narrative scenarios and the rigor of quantitative decision analysis. RDM produces clusters such as the "Southern Julian Simon" or "Low Global Decoupling" future states easily interpretable as narrative scenarios. Unweighted by probabilities, they may then be considered even by decision makers who find them unlikely or inconvenient. But unlike narrative scenario-based planning, RDM offers systematic, quantitative justification for the choice of these particular scenarios—they represent key vulnerabilities of candidate strategies—and a framework for using them to generate and compare alternative decisions.

RDM is, of course, not useful for all decision problems. The method requires more computation than traditional decision analysis and thus most naturally addresses situations in which optimum strategies display significant sensitivity to assumptions. In addition, not all decision challenges offer robust solutions. In some situations, no amount of effort will suggest strategies that perform reasonably well across all or most plausible states of the world. RDM appears most promising for deeply uncertain decision problems with a sufficiently rich array of decision options and enough information, expressible in one or more computer models, so that taking advantage of known constraints on the future one can discover well-crafted strategies robust across a wide range of uncertain possibilities. In particular, the approach provides a systematic means to design adaptive decision strategies that use near-term milestones to achieve satisfactory performance across a broad span of plausible states. Given the ubiquity of deeply uncertain policy challenges, the differences over goals and assumptions that frequently divide stakeholders, and decision makers' use of adaptive strategies and near-term milestones to address such conditions, the methods described here could prove broadly useful in a wide number of fields.

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References
Azar, C., J. Holmberg, S. Karlsson. 2002. *Decoupling—Past Trends and Prospects for the Future.* Report, Environmental Advisory Council, Ministry of the Environment, Stockholm, Sweden.

Bankes, S. C. 1993. Exploratory modeling for policy analysis. *Oper. Res.* **41**(3) 435–449.

Bankes, S. C. 2002. Tools and techniques for developing policies for complex and uncertain systems. *Proc. National Acad. Sci.* **99**(13) 7263–7266.

Bankes, S. C., S. W. Popper, R. J. Lempert. 2000. *Incorporating Adversarial Reasoning in COAA Planning.* Evolving Logic, Topanga, CA.

Ben Haim, Y. 2001. *Information-Gap Decision Theory: Decisions Under Severe Uncertainty.* Academic Press, London, UK.

Berger, J. O. 1985. *Statistical Decision Theory and Bayesian Analysis.* Springer-Verlag, New York.

Brooks, A., S. Bankes, B. Bennett. 1999. An application of exploratory analysis: The weapon mix problem. *Military Oper. Res.* **4**(1) 67–80.

de Cooman, G., T. L. Fine, T. Seidenfeld. 2001. *Proceedings of the Second International Symposium on Imprecise Probabilities and Their Applications*. Shaker Publishing, Maastricht, The Netherlands.

Dewar, J. A. 2001. *Assumption Based Planning: A Tool for Reducing Avoidable Surprises.* Cambridge University Press, Cambridge, UK.

Ellsberg, D. 2001. *Risk, Ambiguity, and Decision*. Garland Publishing, New York.

Friedman, J. H., N. I. Fisher. 1999. Bump hunting in high-dimensional data. *Statist. Comput.* **9**(2) 123–143.

Groller, E., R. Wegenkittl, A. Milik, A. Prskawetz, G. Feichtinger, W. C. Sanderson. 1996. The geometry of wonderland. *Chaos, Solutions, Fractals* **7**(12) 1989–2006.

Knight, F. H. 1921. *Risk, Uncertainty, and Profit*. Houghton Mifflin, Boston, MA.

Kouvelis, P., G. Yu. 1997. *Robust Discrete Optimization and Its Applications*. Kluwer Academic Publishers, Dordrecht, The Netherlands.

Lempert, R. J., S. W. Popper, S. C. Bankes. 2002. Confronting surprise. *Soc. Sci. Comput. Rev.* **20**(4) 420–440.

Lempert, R. J., S. W. Popper, S. C. Bankes. 2003. *Shaping the next one hundred years: New methods for quantitative, long-term policy analysis*. MR-1626, RAND, Santa Monica, CA.

Lempert, R. J., M. E. Schlesinger, S. C. Bankes. 1996. When we don't know the costs or the benefits: Adaptive strategies for abating climate change. *Climatic Change* **33** 235–274.

Lempert, R. J., N. Nakicenovic, D. Sarewitz, M. Schlesinger. 2004. Characterizing climate-change uncertainties for decisionmakers. *Climatic Change* **65** 1–9.

Lempert, R. J., M. E. Schlesinger, S. C. Bankes, N. G. Andronova. 2000. The impact of variability on near-term climate-change policy choices. *Climatic Change* **45**(1) 129–161.

McKibbin, W. J., P. J. Wilcoxen. 2002. *Climate Change Policy After Kyoto.* Brookings Institution Press, Washington, D.C.

Metz, B., O. Davidson, R. Swart, J. Pan, eds. 2001. Climate change 2001: Mitigation. Contribution of Working Group III to the Third Assessment [TAR] Report of the Intergovernmental Panel on Climate Change (IPCC), Cambridge University Press, Cambridge, UK.

Morgan, M. G., M. Henrion. 1990. *Uncertainty: A Guide to Dealing with Uncertainty in Quantitative Risk and Policy Analysis*. Cambridge University Press, Cambridge, UK.

Morgan, M. G., M. Kandlikar, J. Risebey, H. Dowlatabadi. 1999. Why conventional tools for policy analysis are often inadequate for problems of global change. *Climatic Change* **41** 271–281.

Nakicenovic, N., J. Alcamo, G. Davis, B. de Vries, J. Fenhann, S. Gaffin, K. Gregory, A. Grübler. 2000. Special report on emissions scenarios. Working Group III, Intergovernmental Panel on Climate Change (IPCC), Cambridge University Press, Cambridge, UK.

Robalino, D. A., R. J. Lempert. 2000. Carrots and sticks for new technology: Crafting greenhouse gas reductions policies for a heterogeneous and uncertain world. *Integrated Assessment* **1** 1–19.

Rosenhead, M. J., M. Elton, S. K. Gupta. 1972. Robustness and optimality as criteria for strategic decisions. *Oper. Res. Quart.* **23**(4) 413–430.

Saltelli, A., K. Chan, E. M. Scott. 2000. *Sensitivity Analysis*. John Wiley & Sons, New York.

Sarewitz, D., R. A. Pielke, Jr., R. Byerly, eds. 2000. *Prediction: Science, Decision Making, and the Future of Nature.* Island Press, Washington, D.C.

Savage, L. J. 1950. *The Foundations of Statistics.* Wiley & Sons, New York.

Schwartz, P. 1996. *The Art of the Long View.* Double Day, New York.

Simon, H. 1959. Theories of decision-making in economic and behavioral science. *Amer. Econom. Rev.* **49**(3) 253–283.

United Nations. 1990. *Human Development Report.* Oxford University Press, Oxford, UK.

van Asselt, M. B. A. 2000. *Perspectives on Uncertainty and Risk.* Kluwer Academic Publishers, Dordrecht, The Netherlands.

van der Heijden, K. 1996. *Scenarios: The Art of Strategic Conversation.* Wiley & Sons, Chichester, UK.

Victor, D. 2001. *The Collapse of the Kyoto Protocol and the Struggle to Slow Global Warming.* Princeton University Press, Princeton, NJ.

Watson, S. R., D. M. Buede. 1987. *Decision Synthesis.* Cambridge University Press, Cambridge, UK.

Zhou, K., J. Doyle, K. Glover. 1996. *Robust and Optimum Control Theory.* Prentice-Hall, Englewood Cliffs, NJ.