Lecture6

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1 笔记

1.1 最大信息熵分布

利用优化方法求解最大信息熵分布:

$$
\begin{aligned}
\max_p J(p) = -\int_{-\infty}^{\infty} p(x)\log p(x)dx \\
s.t. \quad \int_{-\infty}^{\infty} p(x)dx = 1 \\
\int_{-\infty}^{\infty} xp(x)dx = \mu \\
\int_{-\infty}^{\infty} (x-\mu)^2 p(x)dx = \delta^2
\end{aligned}
$$

引入拉格朗日乘子 α, β, γ 得到广义的优化目标:

$$
\begin{aligned}
L(p,\alpha,\beta,\gamma) = &-\int_{-\infty}^{\infty} p(x)\log p(x)dx \\
&+\alpha(\int_{-\infty}^{\infty} p(x)dx - 1) \\
&+\beta(\int_{-\infty}^{\infty} xp(x)dx - \mu) \\
&+\gamma(\int_{-\infty}^{\infty} (x-\mu)^2 p(x)dx - \delta^2)
\end{aligned}
$$

$$
\hat{g}(p,\alpha,\beta,\gamma,x) = -p(x)\log p(x) + \alpha p(x) + \beta x p(x) + \gamma(x-\mu)^2 p(x)
$$

利用 E-L 方程

$$
\begin{aligned}
0 &= \frac{\partial \hat{g}}{\partial p} \\
0 &= -\log p(x) - 1 + \alpha + \beta x + \gamma (x-\mu)^2
\end{aligned}
$$

化简可得:

$$
\begin{aligned}
p(x) =& e^{\alpha - 1 + \beta x - \gamma (x-\mu)^2} \\
=& c_1 e^{\gamma (x - c_2)^2}
\end{aligned}
$$

利用约束条件可以求得:

$$
\begin{aligned}
c_1 &= \frac{1}{\sqrt{2\pi}} \\
c_2 &= \mu \\
\gamma &= -\frac{1}{2\omega^2}
\end{aligned}
$$

1.2 横截条件

1.2.1 终端时刻固定、状态自由

状态变量 $x(t) : [t_0, t_f] \to \mathbf{R}^n$ 连续可微。在给定初始时刻 t_0 状态为 $x(t_0) = x_0$，终端时刻固定为 t_f，终端状态自由。函数 g 取值于 $\mathbf{R}$，且二阶可微。则状态变量 x 最小化性能指标

$$
J(x) = \int_{t_0}^{t_f} g(x(t), \dot{x}(t), t) dt
$$

的必要条件为对任意时刻 $t \in [t_0, t_f]$，均满足

$$
0 = \frac{\partial g}{\partial x}(x(t), \dot{x}(t), t) - \frac{d}{dt}[\frac{\partial g}{\partial \dot{x}}(x(t), \dot{x}(t), t)]
$$

以及在终端时刻满足:

$$
0 = \frac{\partial g}{\partial \dot{x}}(x(t_f), \dot{x}(t_f), t_f)
$$

1.2.2 终端时刻自由、状态固定

状态变量 $x(t) : [t_0, t_f] \to \mathbf{R}^n$ 连续可微。在给定初始时刻 t_0 状态为 $x(t_0) = x_0$，终端时刻自由，终端状态固定为 $x(t_f) = x_f$。函数 g 取值于 $\mathbf{R}$，且二阶可微。则状态变量 x 最小化性能指标

$$
J(x) = \int_{t_0}^{t_f} g(x(t), \dot{x}(t), t) dt
$$

的必要条件为对任意时刻 $t \in [t_0, t_f]$，均满足

$$0 = \frac{\partial g}{\partial x}(x(t), \dot{x}(t), t) - \frac{d}{dt}[\frac{\partial g}{\partial \dot{x}}(x(t), \dot{x}(t), t)]$$

以及在终端时刻满足：

$$0 = g(x(t_f), \dot{x}(t_f), t_f) - \frac{\partial g}{\partial \dot{x}}(x(t_f), \dot{x}(t_f), t_f)\dot{x}(t_f)$$

2 作业

2.1 小车倒立摆

假设小车的质量为 M，小球的质量为 m，倒立摆的长度为 l，摆距离平衡状态的倾角为 θ 拉格朗日方程建模：

$$\begin{aligned}
&T_{car} = \frac{1}{2}M\dot{x}^2 \\
&T_{ball} = \frac{1}{2}m(l^2\dot{\theta}^2 + \dot{x}^2 - 2l\cos\theta\dot{\theta}\dot{x}) \\
&V_{car} = 0 \\
&V_{ball} = mgl\cos\theta \\
&L = T - V = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}m(l^2\dot{\theta}^2 + \dot{x}^2 - 2l\cos\theta\dot{\theta}\dot{x}) - mgl\cos\theta
\end{aligned}$$

列写欧拉拉格朗日方程：

$$\begin{aligned}
F &= \frac{\partial L}{\partial x} - \frac{d}{dt}[\frac{\partial L}{\partial \dot{x}}] \\
0 &= \frac{\partial L}{\partial \theta} - \frac{d}{dt}[\frac{\partial L}{\partial \dot{\theta}}]
\end{aligned}$$

化简可得：

$$\begin{aligned}
&F = (M+m)\ddot{x} + ml\sin\theta\dot{\theta}^2 - ml\cos\theta\ddot{\theta} \\
&0 = mgl\sin\theta + ml^2\ddot{\theta} + ml\cos\theta\ddot{x}
\end{aligned}$$

在 $\theta = 0$ 处线性化，得到：

$$\begin{aligned}
&F = (M+m)\ddot{x} - ml\ddot{\theta} \\
&0 = mgl\theta + ml^2\ddot{\theta} - ml\ddot{x} \\
&\ddot{x} = \frac{F - mg\theta}{M} \\
&\ddot{t}heta = \frac{F - (m+M)\theta}{Ml}
\end{aligned}$$

取状态变量 $x_1 = \theta, x_2 = \dot{\theta}, x_3 = x, x_4 = \dot{x}$ 得到状态空间模型

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{(m+M)g}{Ml} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{mg}{M} & 0 & 0 & 0 \end{bmatrix}$$