Pearson Edexcel International Advanced Level

Tuesday 16 January 2024

Afternoon (Time: 1 hour 30 minutes)

Paper reference **WMA12/01**

Mathematics
International Advanced Subsidiary/Advanced Level
Pure Mathematics P2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

1.
$$f(x) = ax^3 + 3x^2 - 8x + 2 \qquad \text{where } a \text{ is a constant}$$

Given that when f(x) is divided by ($x - 2$) the remainder is 3, find the value of a.

(3)

Question 1 continued

(Total for Question 1 is 3 marks)

2. Find the coefficient of the term in x^7 of the binomial expansion of

$$\left(\frac{3}{8} + 4x\right)^{12}$$

giving your answer in simplest form.

(3)

Question 2 continued

(Total for Question 2 is 3 marks)

3. The circle C

- has centre $A(3, 5)$
- passes through the point $B(8, -7)$

(a) Find an equation for C.

(3)

The points M and N lie on C such that MN is a chord of C.

Given that MN

- lies above the x-axis
- is parallel to the x-axis
- has length $4\sqrt{22}$

(b) find an equation for the line passing through points M and N.

(3)

(Total for Question 3 is 6 marks)

4. (a) Sketch the curve with equation

$$y = a^{-x} + 4$$

where a is a constant and $a > 1$

On your sketch show

* the coordinates of the point of intersection of the curve with the y-axis

* the equation of the asymptote to the curve.

(3)

x	-4	-1.5	1	3.5	6	8.5
y	13	6.280	4.577	4.146	4.037	4.009

The table above shows corresponding values of x and y for $y = 3^{-\frac{1}{2}x} + 4$

The values of y are given to four significant figures, as appropriate.

Using the trapezium rule with all the values of y in the table,

(b) find an approximate value for

$$\int_{-4}^{8.5} \left(3^{-\frac{1}{2}x} + 4 \right) dx$$

giving your answer to two significant figures.

(3)

(c) **Using the answer to part (b)**, find an approximate value for

(i) $\displaystyle\int_{-4}^{8.5} \left(3^{-\frac{1}{2}x} \right) dx$

(ii) $\displaystyle\int_{-4}^{8.5} \left(3^{-\frac{1}{2}x} + 4 \right) dx + \int_{-8.5}^{4} \left(3^{\frac{1}{2}x} + 4 \right) dx$

(3)

Question 4 continued

(Total for Question 4 is 9 marks)

P7 4 3 1 1 A 0 1 1 3 2

5. (i) Find the value of

$$\sum_{r=1}^{\infty} 6 \times (0.25)^r$$

(ii) A sequence $u_1, u_2, u_3,...$ is defined by

$$u_1 = 3$$

$$u_{n+1} = \frac{u_n - 3}{u_n - 2} \qquad n \in \mathbb{N}$$

(a) Show that this sequence is periodic.

(2)

(b) State the order of this sequence.

(1)

(c) Hence find

$$\sum_{n=1}^{70} u_n$$

(2)

P 7 4 3 1 1 A 0 1 3 3 2

(Total for Question 5 is 8 marks)

6. (a) Given that

$$2\log_4(x+3) + \log_4 x = \log_4(4x+2) + \frac{1}{2}$$

show that

$$x^3 + 6x^2 + x - 4 = 0$$

(4)

(b) Given also that -1 is a root of the equation

$$x^3 + 6x^2 + x - 4 = 0$$

  (i) use algebra to find the other two roots of the equation.

(3)

  (ii) Hence solve

$$2\log_4(x+3) + \log_4 x = \log_4(4x+2) + \frac{1}{2}$$

(1)

P 7 4 3 1 1 A 0 1 7 3 2

Question 6 continued

(Total for Question 6 is 8 marks)

7. Wheat is grown on a farm.

- In year 1, the farm produced 300 tonnes of wheat.

- In year 12, the farm is predicted to produce 4000 tonnes of wheat.

Model A assumes that the amount of wheat produced on the farm will increase by the same amount each year.

(a) Using model A, find the amount of wheat produced on the farm in year 4.
Give your answer to the nearest 10 tonnes.

(3)

Model B assumes that the amount of wheat produced on the farm will increase by the same percentage each year.

(b) Using model B, find the amount of wheat produced on the farm in year 2.
Give your answer to the nearest 10 tonnes.

(3)

(c) Calculate, according to the two models, the difference between the total amounts of wheat predicted to be produced on the farm from year 1 to year 12 inclusive.
Give your answer to the nearest 10 tonnes.

(3)

P 7 4 3 1 1 A 0 2 1 3 2

Question 7 continued

(Total for Question 7 is 9 marks)

P 7 4 3 1 1 A 0 2 3 3 2

8. (i) Use a counter example to show that the following statement is **false**

$$\text{``} n^2 + 3n + 1 \text{ is prime for all } n \in \mathbb{N}\text{''}$$

(2)

(ii) Use algebra to prove by exhaustion that for all $n \in \mathbb{N}$

$$\text{``} n^2 - 2 \text{ is } \textbf{not} \text{ a multiple of 4''}$$

(4)

(Total for Question 8 is 6 marks)

P 7 4 3 1 1 A 0 2 5 3 2

9. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 \leqslant x < 360°$, the equation

$$\sin x \tan x = 5$$

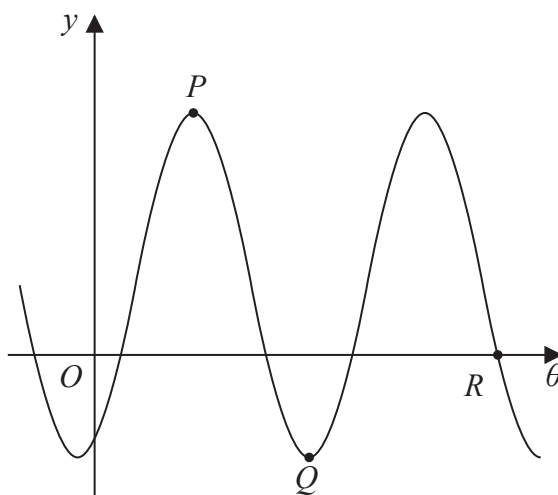giving your answers to one decimal place.

(6)

(ii)

Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$y = A\sin\left(2\theta - \frac{3\pi}{8}\right) + 2$$

where A is a constant and θ is measured in radians.

The points P, Q and R lie on the curve and are shown in Figure 1.

Given that the y coordinate of P is 7

(a) state the value of A,

(1)

(b) find the exact coordinates of Q,

(3)

(c) find the value of θ at R, giving your answer to 3 significant figures.

(4)

P 7 4 3 1 1 A 0 2 7 3 2

Question 9 continued

(Total for Question 9 is 14 marks)

10. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.
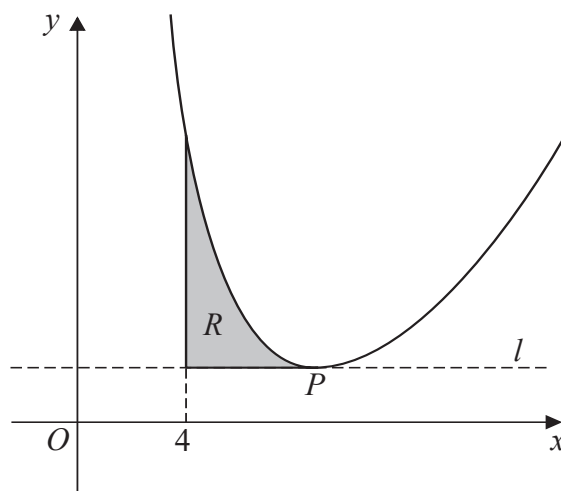

Figure 2

Figure 2 shows a sketch of the curve with equation

$$y = \frac{1}{2}x^2 + \frac{1458}{\sqrt{x^3}} - 74 \qquad\qquad x > 0$$

The point P is the only stationary point on the curve.

(a) Use calculus to show that the x coordinate of P is 9

(4)

The line l passes through the point P and is parallel to the x-axis.

The region R, shown shaded in Figure 2, is bounded by the curve, the line l and the line with equation $x = 4$

(b) Use algebraic integration to find the exact area of R.

(5)

Question 10 continued

(Total for Question 10 is 9 marks)

TOTAL FOR PAPER IS 75 MARKS

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