The Leontief Production Function and The Harrod-Domar Controversy

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Leontief production function

$$Y = F(K, L) = \min(AK, BL)$$

where $A > 0, B > 0$ are constants. With fixed proportions,

- if the available capital stock and labor force happen to be such taht $AK = BL$, then all workers and machines are fully employed;
- if K and L are such that $AK > BL$ then only the quantity of capital $(B/A) \cdot L$ is used.
- If $AK < BL$, then only the amount of labor $(A/B) \cdot K$ is used, and the remainder is unemployed.

The assumption of no substitution between capital and labor led Harrod and Domar to predict that capitalist economies would have undesirable outcomes.

Controversy The output per capita from the production[1] above:

$$y = \min(Ak, B).$$

- For $k < B/A$, capital is fully employed, and $y = Ak$.
- For $k > B/A$, the quantity of capital used is constant, and Y is the constant multiple B of labor L.

The growth rate

$$\dot{k}/k = s \cdot [\min(Ak, B)]/k - (n + \delta).$$

- For $k \leq B/A$, $s \cdot [\min(Ak, B)]/k$ is a horizontal line.
- For $k > B/A$, this term is a downward-sloping curve that approaches zero as k goes to infinity.

Assume first that the saving rate is low enough so that $sA < n + \delta$. The saving curve $s \cdot f(k)/k$ never crosses the $n + \delta$ line, so there is no positive steady-state value k^*. The economy shrinks in per capita terms.

Assume the saving rate is high enough so that $sA > n + \delta$, the saving curve eventually crosses the $n + \delta$ line at the point $k^* > B/A$.

[1] Is it feasible?

Since $k^* > B/A$, the steady state features idle machines but no unemployed workers. k is constant in the steady state, the quantity K grows along with L at the rate n. The quantity of idle machines also grows at the rate n.

The only way to reach a steady state is for the parameters of the model to satisfy the condition $sA = n + \delta$. Since the four parameters are all exogenous, there is no reason for the quality to hold. Hence the conclusion from Harrod and Domar was that an economy would, in all probability, reach one of two undesirable outcomes.