MAT1856/APM466: Mathematical Finance **Winter 2019**

Assignment #1: Yield Curves

Professor: Luis Seco, TA: Jonathan Mostovoy

Note: Please bring any questions about this assignment to your TA's, Jonathan's, weekly office hour.

1.1 Introduction

You are to follow the Canadian Government bond prices for consecutive days in a two-week period (10 days, starting Jan 14), for the purposes of calculating a yield curve (ytm curve), spot curve, and forward curve for each day. Please obtain this data from CanadianFixedIncome.ca.

This assignment is split into 2 parts, the first asks questions about some fundamentals of fixed income and mathematical finance. The second will be an empirical exercise in generating yield curves, and in particular the 1, 2, 3, 4 and 5 year rates, and analyzing these rates.

1.2 Expectations

1. You may use Excel or any programming language of your choice to answer the "empirical questions".
2. Please have your final report typset using LaTeX, for which overleaf.com is particularly useful.
3. Each of the "fundamental questions" must be answered in clear and coherent sentences - no math.
4. An Appendix including all your code, and a bibliography must be included.
5. You may, and are encouraged, to discuss how to do these questions with your peers. However, your write-up must be done individually, and the sharing of your write-up before the deadline is prohibited.

Additional Notes: Marks will be awarded for each question as either full-, half-, or zero-marks according to if the question was answered with a few small mistakes, substantial mistakes but fundamental idea still correct, or fundamental idea wrong / no answer respectively. -10 marks (each) if expectations 2, 3, or 4 not adhered to.

2 Questions

2.1 Fundamental Questions - 25 points

1. *(4 points)* Why shouldn't you use bonds with an initial term (maturity − issue date) of 30 years for calculating points on the yield curve less than 5 years out? Should the on-the-run Canadian 5 year bond be used to calculate the 5 year point of the yield curve instead of a 10 year bond even if a 10 year bond has a maturity closer to exactly 5 years away? *(Please provide at least two reasons for each.)*

   (Hint: Coupons, and financial flows.)

2. *(6 points)* In the second part of this assignment, you are to derive the spot curve up to 5 years out via bootstrapping for each day in your data-set. Please list a subset of bonds for which you will feed into a bootstrapping algorithm and explain your choice for each bond chosen. You may use a day count which rounds days to the nearest month (I.e., Jan 10th becomes Jan 1, Mar 20 becomes Apr 1, and the length of time between Mar 20 and Jan 10 would be 3 months).

   (Note: To easily refer to a bond, please use the following convention: "CAN 2.5 Jun 24" refers to the Canadian Government bond with a maturity in June 24 and a coupon of 2.5).

3. *(15 points total)* The final part of this assignment asks you to treat the 1,2,3,4,5 year points of the yield curve as time series, and compute the eigenvalues and eigenvectors of the associated 5x5 covariance matrix of log-returns.

   (a) *(10 points)* Explain what the distribution of eigenvalues tells you about the characteristics of the time series associated with the 1,2,3,4,5 year points on the yield curve.

   (b) *(5 points)* Give two examples of financial classes for which you would expect to have the covariance matrices of the log-returns of assets within said classes produce completely different distributions of eigenvalues, and explain why this would be so. Examples of financial classes could be "US Stocks", "Canadian Government Bonds", "Euro Hedge Funds", "FX Swaps", etc.

4. *(Bonus: 10 points, math allowed)* Describe the motivation and mathematics behind a mathematical model other than bootstrapping that at least two central banks (state which banks) use to derive their long-term spot curves. Please also state two advantages and one disadvantage of the described model.

   (Complete answers only - no half marks.)

2.2 Empirical Questions - 75 points

5. *(40 points total)*

   (a) *(10 points)* Provide a well-labeled plot with a yield curve (ytm curve) corresponding to each day of data superimposed on-top of each other. You may use any interpolation technique you deem appropriate provided you include a reasonable explanation for the technique used.

   (b) *(15 points)* Write a pseudo-code (a simple explanation of an algorithm) for how you would derive the spot curve with terms ranging from 1-5 years from your chosen bonds in part 2. (Please also recall the day convention simplifications provided in part 2 as well.) Then a well-labeled plot with a spot curve corresponding to each day of data superimposed on-top of each other.

   (c) *(15 points)* Write a pseudo-code for how you would derive the 1-year forward curve with terms ranging from 2-5 years from your chosen bonds in part 2 (I.e., a curve with the first point being the 1yr-1yr forward rate and the last point being the 1yr-4yr rate). Then provide a well-labeled plot with a forward curve corresponding to each day of data superimposed on-top of each other.

6. *(20 points)* Calculate two covariance matrices for the time series of daily log-returns of yield, and forward rates (no spot rates). In other words, first calculate the covariance matrix of the random variables X_i, for $i = 1, \ldots, 5$, where each random variable X_i has a time series $X_{i,j}$ given by:

$$X_{i,j} = \log(r_{i,j+1}/r_{i,j}), \quad j = 1, \ldots, 9$$

   then do the same for the following forward rates - the 1yr-1yr, 1yr-2yr, 1yr-3yr, 1yr-4yr.

7. *(15 points)* Calculate the eigenvalues and eigenvectors of both covariance matrices.