MIE562: Preliminary Report

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I. Dataset

Data for crop selling price, amount of production, and variable costs were gathered from the U.S. Department of Agriculture [1]. Data for crop maturity time interval and growing season were gathered from crop management information presented by the Province of Manitoba [2]. The optimal growing time was calculated simply by taking the mean of the maturity time and the growing season was converted to a set of time indices. The dataset will be used as a simulation of real-world farming problems and will be used in our time-indexed mixed integer programming model and constraint programming model.

Crop Type	Selling price (dollars per bushel)	Yield (bushel per acre)	Variable cost (dollars per acre)	Time to maturity LB (days)	Time to maturity UB (days)	Optimal growing time (days)	Growing season (months)	Growing season (time index interval)
Barley	5.15	75.45	179	60	90	75	Early March to late August	60-240
Corn	4.8	181	346	110	120	115	Mid October	280-285
Oats	3.3	65.56	130	80	90	85	Early March to late June	60-180
Rice	14.1	128.03	558	150	210	180	Early March to late July	60-210
Sorghum	4.7	69.49	144	110	140	125	Early March to late August	60-240
Soybean	10.5	51.5	192	90	150	120	Early May to late May	120-150
Wheat	6.5	49.09	135	90	100	95	Early February to late May, early September to late December	30-150, 210-360

Table 1. Data of project-specified variables

As described in our problem definition, the amount of crop yield and the cost of farming are dependent on the length of time that crops grow on the field. Specifically, there is an exponential relationship between yield and time and a linear relationship between the variable cost and time. The purpose of an exponential relationship, in this case, is that we want the yield of crops to be half of its optimal when the growing time of crops is at minimum or maximum. The linear relationship ensures the cost of farming goes up steadily as growth time increases. An example of the changes in crop yield, variable cost, and the resulting gross profit of one type of crop is demonstrated in the table below:

Time (days on the field)	Yield (bushel per acre)	Cost (dollars per acre)	Gross profit (dollars)
80	32.78	122.32	-14.1478
81	40.44	123.85	9.601384
82	47.50	125.38	31.38155
83	54.02	126.91	51.3461
84	60.02	128.44	69.63644
85	65.56	129.97	86.383
86	60.02	131.49	66.57844
87	54.02	133.02	45.2301
88	47.50	134.55	22.20755
89	40.44	136.08	-2.63062
90	32.78	137.61	-29.4378

Table 2. The relationship between days on the field and the yield, cost and profit for oats

II. Time-indexed MIP model

We have accomplished two algorithms based on time-indexed MIP, and CP algorithms is on the way.

First algorithms add an assumption not mentioned in Project Statement. We assumes that growth time is fixed all the time.

Second algorithms is the solution for the exact problem we defined in Project Statement.

Refer to Project Statement part, and mip.py (Algorithm 1) and mip_norelax.py(Algorithm 2).

Main difficulties of this project are: 1. 360 days is a big number. 2. Every year plan means we need to deal with growth periods that cross two year. 3. Growth time of crop is dynamic, greatly enhance the computation cost. 4. Dynamic growth time can not be directly embedded into constraints, we model this by propose a new binary variable z. 5. Choice of grow crop j on field i is not once. The times can be range from 0 to as much as possible, which means claiming variables before model is impossible.

Here is 4 cases of how my codes perform. We rely on data.csv to generate test samples.

Sample 1:
1 Field, a_1 = 10
1 Workforce
All crops available

Sample 2: (Max possible field number, otherwise out of memory)
6 Field, a_i = i, i=1,2,3,4,5,6
1 Workforce
2 Crop (Barley + Corn)

Case 1:

Sample 1 + Algorithm 1

Crop	0	1	2	3	4	5	6
Sowing Time	/	280	/	60	/	/	/

G Gross profit

17700.230

CPU Time

0.430

Case 2:

Sample 1 + Algorithm 2

Crop	0	1	2	3	4	5	6
Sowing Time	/	280	/	64	/	/	/
Growth Time	73	115	80	180	110	144	93

G Gross profit

17700.230

CPU Time

90.174

Case 3:

Sample 2 + Algorithm 1

S_ij Sow date for crop j on land i

Crop\Land	0	1	2	3	4	5

0	69, 151, 229	64, 149	126, 203	88, 166	127, 202	78,156
1	/	280	284	283	282	281

G Gross profit

19467.403

CPU Time

0.698

Case 4:

Sample 2 + Algorithm 2

Schedule:

P_ij Growth of crop j on land i

Crop\Land	0	1	2	3	4	5
0	75	/	71	71	72	/
1	/	115	115	115	115	115

S_ij Sow date for crop j on land i

Crop\Land	0	1	2	3	4	5
0	74,164,239	62	60,131,202	67,138,209	63	/
1	/	281	280	283	284	282

G Gross profit

21695.207

CPU Time

1620.721

References

[1] “Agricultural Baseline Database,” *USDA ERS - Agricultural Baseline Database*. [Online]. Available: https://www.ers.usda.gov/data-products/agricultural-baseline-database/. [Accessed: 04-Nov-2022].

[2] “Agriculture: Province of Manitoba,” *Province of Manitoba - Agriculture*. [Online]. Available: https://www.gov.mb.ca/agriculture/crops/crop-management/index.html. [Accessed: 04-Nov-2022].

Problem Definition: Agricultural Production Planning

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1 Problem Statement

The objective of this project is to achieve an optimum agricultural production plan that maximises gross profit per year under a set of constraints concerning different categories of land, labour, market, and crop characteristics.

Objects in this problem can be simplified to: Field, Crops, Farmer and Money.

Specifically, the optimization model is attained by assuming the following:

- Total cultivation area is bounded and each field can grow only one type of crop at a time.
- The harvested fields can be planted again only after a fixed amount of time.
- The farm (set of fields) is operated by a fixed number of farmers, and each farmer can work on at most one job at a time.
- A farmer is adequate for sowing or harvesting for any field.
- There can be only one farmer working on a field at the same day.
- Choices of crops are limited throughout the year.
- Growing time of each crop is within a range, with the distribution of yield over its growing time defined by exponential function.
- Every crop has one or two sowing season.
- The maintenance cost of each crop are fixed throughout the year, regardless of seasons.

- Products can be sold immediately after harvest.
- On the same field, the same crop will have the same growth time.

Referring to the constraints listed above, two models are chosen based on Mixed Integer Linear Programming (MILP) and Constraint Programming (CP). Solvers are respectively Gurobi and CPLEX.

2 General definition

2.1 Subscripts definition

M fields

$$i \in \{1, 2, ..., M\}$$

N crops

$$j \in \{1, 2, ..., N\}$$

L farmers

$$k \in \{1, 2, ..., L\}$$

360 days (in a year)

$$t \in \{1, 2, ..., 360\}$$

2.2 Variable definition

Name	Subscripts	Variable
Field gross profit (in dollars)	i,j	h_{ij}
Field area (in acres)	i	a_i
Crop LB growth time (in days)	j	$p_{j,LB}$
Crop UB growth time (in days)	j	$p_{j,UB}$
Crop optimal growth time (in days)	j	$\bar{p}_j$
Crop gross profit per acre (in dollars)	j	g_j
Crop selling price per bushel (in dollars)	j	r_j
Crop optimal yield per acre (in bushels)	j	$\bar{b}_j$
Crop yield per acre (in bushels)	j	$b_j(p_j)$
Crop yield decay rate	j	d_j
Crop maintenance cost per day per acre (in dollars)	j	m_j
Crop LB sowing date (in days)	j	$l_{j,LB}$
Crop UB sowing date (in days)	j	$l_{j,UB}$

Table 1: Definition Table

2.3 Variable relations

Relation 1 is used for calculating crop j's gross profit per acre.

$$g_j = r_j b_j(p_j) - m_j p_j \tag{1}$$

Relation 2 is used for calculating field i planted with crop j's gross profit.

$$h_{ij} = a_i g_j \tag{2}$$

Relation 3 is used for determining yield of crop j as a function of growing time.

$$b_j(p_j) = \bar{b}_j(2 - e^{-d_j|p_j - \bar{p}_j|/\bar{p}_j}) \tag{3}$$

Relation 4 is used for determining yield decay rate. At LB/UB growing time, the yield will be half of optimal yield at optimal growing time.

$$d_j = \frac{\ln 3/2}{|p_{j,UB} - \bar{p}_j|/\bar{p}_j} \tag{4}$$

Relation 5 is used for determining optimal growing time for crop j.

$$\bar{p}_j = (p_{j,LB} + p_{j,UB})/2 \tag{5}$$

2.4 Constraints in sentences

No.	Name	Definition
1	Field	One field can only sow, grow or harvest one crop at one time.
2	Workforce	The number of farmers who sow or harvest at any time is below L.
3	Growth time	Growth time must be between its LB growth time and UB growth time.
4	Season	Any crop can only be sowed between its LB sowing date and UB sowing date.
5	Switch	For some decision variables, they can only be assigned value 1 or 0.

Table 2: Constraint defintion table

3 MIP

3.1 Time-indexed MIP

Obejective function

$$\text{MAX} \sum_i \sum_j \sum_t x_{ijt} g_{ij} a_i \tag{6}$$

Relation for p_{ij}:

Name	Subscripts	Variable
Field plant status (1- sowing at time t, 0 - all other)	i,j,t	x_{ijt}
Crop growth time on field (1- time is right, 0 - not this time)	i,j,t	z_{ijt}
Crop gowth time (in days)	i,j	p_{ij}

Table 3: Decision variables table

$$\sum_{t \in [p_{j,LB}, p_{j,UB})} z_{ijt} t = p_{ij} \tag{7}$$

$$\sum_{t \in [p_{j,LB}, p_{j,UB})} z_{ijt} = 1 \tag{8}$$

Switch constraint

$$x_{ijt} \in \{0, 1\} \tag{9}$$

$$z_{ijt} \in \{0, 1\} \tag{10}$$

Field constraint

$$\sum_j \sum_{t' \in T_{jt}} x_{i,j,t'} \leq 1, \forall i, t \tag{11}$$

$$T_{jt} \equiv \{t - p_{ij} + 1, ..., t\} \text{ or } \{t - p_{ij} + 361, ..., 360, 1, ..., t\}$$

Workforce constraint

$$\sum_i \sum_j (x_{i,j,t} + x_{i,j,(t-p_{ij}+1)}) \leq L, \forall t \tag{12}$$

Growth time constraint (automatically fit by definition)

$$p_{j,LB} \leq p_{ij} \leq p_{j,UB} \tag{13}$$

Season constraint

$$\sum_t L_{jt} x_{ijt} = 0 \ \forall i, j \tag{14}$$

$L_{jt} = 1$ iff t out of growing season of crop j, otherwise $L_{jt} = 0$.

4 CP

Name	Subscripts	Variable
Field growth optional interval	i,j,k	$[x_{ijk}, x_{ijk} + p_{ijk})$
Crop growth time (in days)	i,j	p_{ij}

Table 4: Decision variables table

Name	Subscripts	Variable
UB of crop growth times on field	i,j	K_{ij}

Table 5: New variables for CP

$$K_{ij} = \left\lfloor \frac{a_i}{p_{j,LB}} \right\rfloor \tag{15}$$

$$k \in \{1, 2, ...K_{ij}\}$$