Applying Drift Diffusion Model to Explain Reaction Time of Risky Choice

Cheol Jun Cho
Undergraduate Student
Computer Science & Engineering

Final project for Seminar in Experimental Psychology: Computational Modeling, 2020 spring

If some contents in slides break, please check below google slide link.

https://docs.google.com/presentation/d/1_DzVJK1cr7zyuuQMeu_s8004d_sJkojv0r0sAUq4qp8/edit#slide=id.g82143f6ab4_0_272

All the material for this project can be found in the project folder. Please check the reproducibility slide for detail.

Please check the material for unreported information.

There are still issues over task 2 but the results are included in this report.

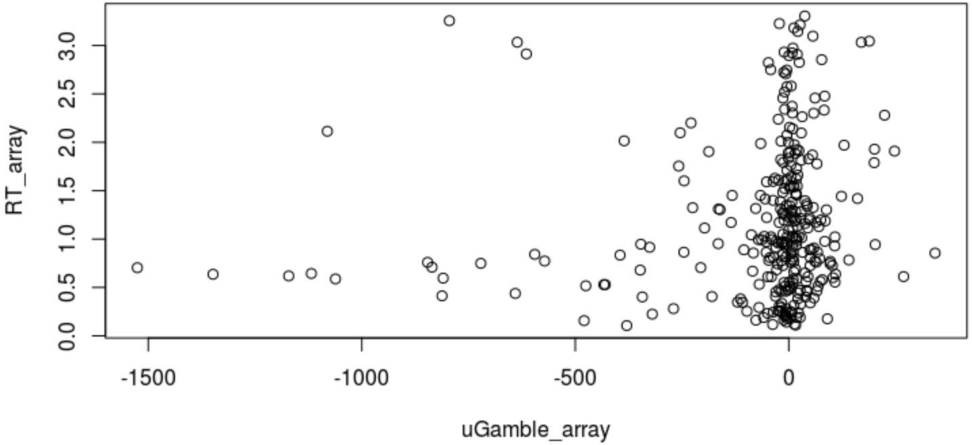
I realized that I also should provide samples in last minute. I am running the code again to save samples. The samples will be uploaded to below google drive link. Please use "fit_*modelname* <- readRDS("*modelname*.rds")" instead of compiling and fitting model.
(Actually, the full results are saved in "summaries" and "plots" folder, so you might not need to run the code again.)

https://drive.google.com/drive/folders/1uYi_V5UaX5EF1lhbWGj2Lc-usnD8BmZH?usp=sharing

Contents

Prospect theory explains choices under risky situation, but tells nothing about information processing

Prospect theory cannot explains how individual reaches to the values.



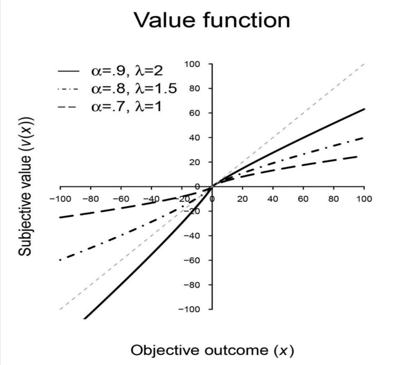
(* utility-RT plot of 5 subjects. utilities are retrieved from prospect model. *)

Reaction time is not "random"

➡ *Explaining reaction time will might unveil the process in risky choice.*

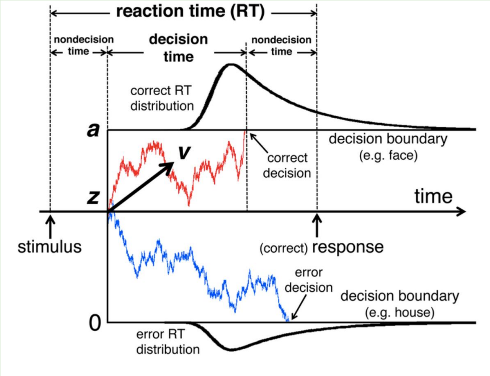
Goal : Combining Drift Diffusion Model with Prospect Theory

prospect theory



+

drift diffusion model



Task 1 : choice between certain and risky option (Rutledge et al., 2014)

Certain Option
(0 or Gain or Loss)

0

VS

Risky Option
(same prob. for Loss & Gain)

219

-142

- Probabilities are fixed to 0.5.
- 3 combination types

	cert	gain	loss
mixed	0	+	-
gain	+	+	0
loss	-	0	-

Task 2 : choice between safe and risky option (Chung et al., 2015)

Safe Option
(only Gain)

258

253

VS

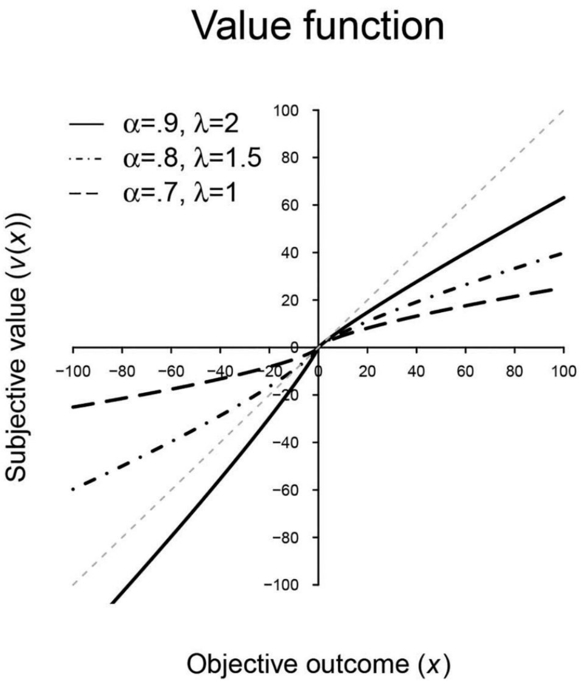
Risky Option
(only Gain)

505

14

- Probabilities are randomly chosen and same for both options.
- Expected value for each option is same.

Prospect Theory (Kahneman & Tversky, 1971)



λ : *loss aversion*

ρ : *risk preference*

τ : *inverse temperature*

P : *probability*

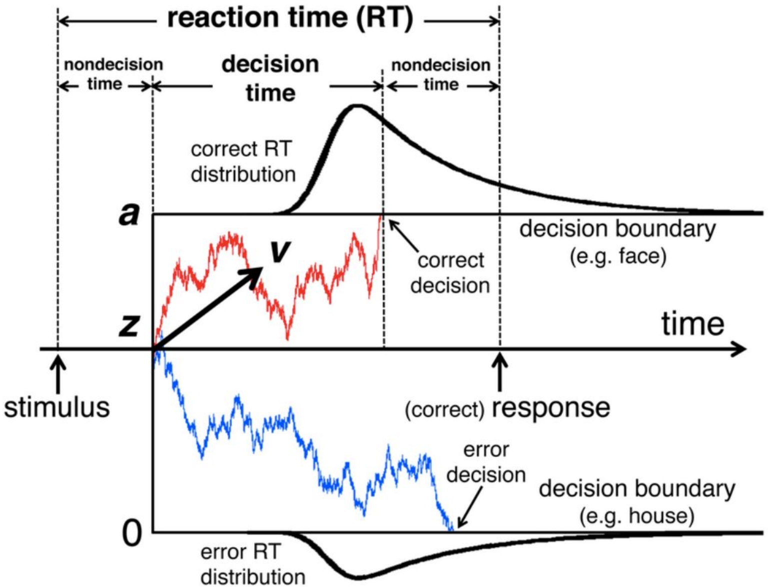
U : *subjective utility*

$$U = \sum_{Gain} P_{Gain} R_{Gain}^{\rho} + \sum_{Loss} \lambda \, P_{Loss} R_{Loss}^{\rho}$$

$$P_{chocie1} = \frac{1}{1 + e^{-\tau(U_1 - U_2)}}$$

$$choice1 \sim Bernoulli(P_{chocie1})$$

Drift Diffusion Model (Ratcliff & Mckoon, 2008)



α : *boundary separation*

β : *bias*

δ : *drift rate*

ndt : *decision time*

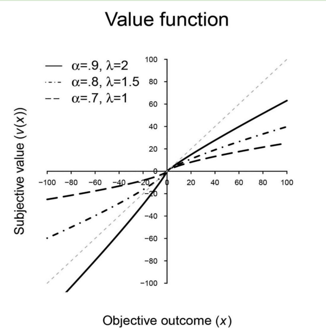
$$RT \sim Wiener(\alpha, \beta, \delta, ndt)$$

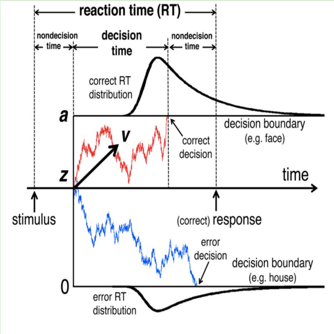
Combined model

The more similar values, the longer reaction time

>> Reflect difference between utilities to drift rate, □.

prospect theory + *drift diffusion model*





H1: *normalized utility difference*

$$U = \sum_{Gain} P_{Gain} R_{Gain}^{\rho} + \sum_{Loss} \lambda\, P_{Loss} R_{Loss}^{\rho}$$

$$P_{chocie1} = \frac{1}{1 + e^{-\tau(U_1 - U_2)}}$$

$$\delta' = \delta\,(2\,P_{choice} - 1)$$

$$RT \sim Wiener(\alpha, \beta, \delta', d)$$

H2: *un-normalized utility difference*

$$U = \sum_{Gain} P_{Gain} R_{Gain}^{\rho} + \sum_{Loss} \lambda\, P_{Loss} R_{Loss}^{\rho}$$

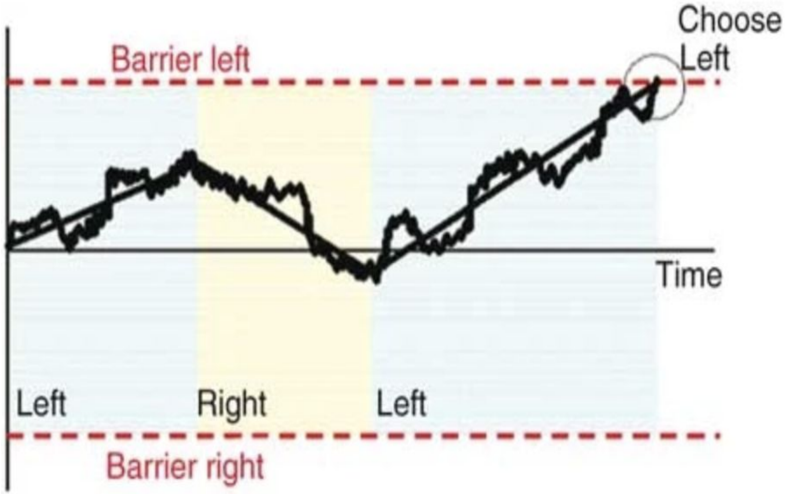
$$P_{chocie1} = \frac{1}{1 + e^{-\tau(U_1 - U_2)}}$$

$$\delta' = \delta\,(U_{chocie} - U_{other})$$

$$RT \sim Wiener(\alpha, \beta, \delta', d)$$

Incorporating eye-gaze bias leads to better performance

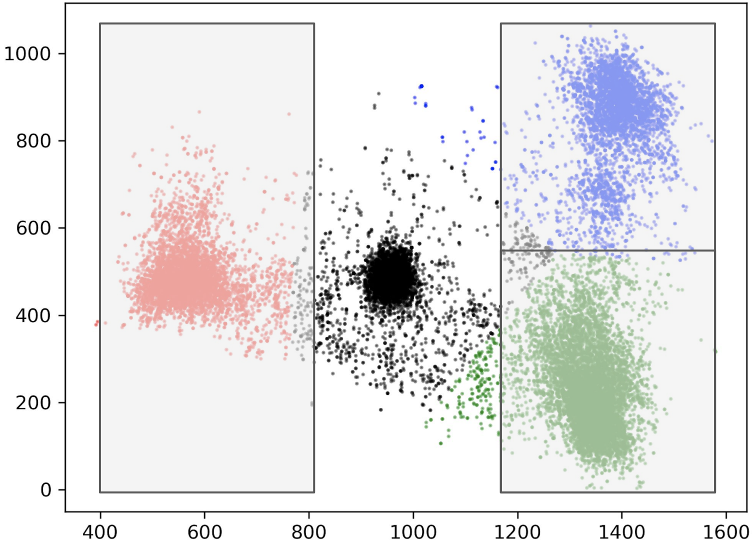
Attentional Drift Diffusion Model
(aDDM, Krajbich et al. 2010)



Attention matters in information processing

>> considering attention by incorporating eye-gaze data

Gaze Coefficient



(* eye-gaze of one subject in task 1. there is a cluster in the middle because of the fixation phase. you can check other plots in "data/behavior_fixation_v2/plot/" *)

Inspired by Gaze-weighted Accumulator Model (GLAM, Thomas et al. 2019)

Gaze Coefficient

$$g_i = \frac{points\ in\ i}{\sum_j points\ in\ j} \quad : \ gaze\ ratio$$

$$g_{i,coeff} = g_i + \theta\,(1 - g_i)\,, \qquad \theta \leq 1$$

If theta is 1, then there is no gaze bias.

If theta is negative, then there is information leakage.

Joint model

incorporate eye-gaze in 3 different levels of H1

H3: *choice-level attention*

$$U = \sum_{Gain} P_{Gain} R_{Gain}^{\rho} + \sum_{Loss} \lambda\, P_{Loss} R_{Loss}^{\rho}$$

$$P_{chocie1} = \frac{1}{1 + e^{-\tau(U_1 - U_2)}}$$

$$\delta' = \underline{g_{choice,coeff}}\,\delta\,(2\,P_{choice} - 1)$$

$$RT \sim Wiener(\alpha, \beta, \delta', d)$$

H4: *utility-level attention*

$$U = \sum_{Gain} P_{Gain} R_{Gain}^{\rho} + \sum_{Loss} \lambda\, P_{Loss} R_{Loss}^{\rho}$$

$$P_{chocie1} = \frac{1}{1 + e^{-\tau\underline{(g_{1,coeff} U_1 - g_{2,coeff} U_2)}}}$$

$$\delta' = \delta\,(2\,P_{choice} - 1)$$

$$RT \sim Wiener(\alpha, \beta, \delta', d)$$

H3: *value-level attention*

$$U = \sum_{Gain} \underline{g_{Gain,coeff}} P_{Gain} R_{Gain}^{\rho} + \sum_{Loss} \underline{g_{Loss,coeff}} \lambda\, P_{Loss} R_{Loss}^{\rho}$$

$$P_{chocie1} = \frac{1}{1 + e^{-\tau(U_1 - U_2)}}$$

$$\delta' = \delta\,(2\,P_{choice} - 1)$$

$$RT \sim Wiener(\alpha, \beta, \delta', d)$$

Total

5 Models

H1: ***normalized utility difference***

$$U = \sum_{Gain} P_{Gain} R_{Gain}^{\rho} + \sum_{Loss} \lambda\, P_{Loss} R_{Loss}^{\rho}$$

$$P_{chocie1} = \frac{1}{1 + e^{-\tau(U_1 - U_2)}}$$

$$\delta' = \delta\ (2\, P_{choice} - 1)$$

$$RT \sim Wiener(\alpha, \beta, \delta', d)$$

H2: ***un-normalized utility difference***

$$U = \sum_{Gain} P_{Gain} R_{Gain}^{\rho} + \sum_{Loss} \lambda\, P_{Loss} R_{Loss}^{\rho}$$

$$P_{chocie1} = \frac{1}{1 + e^{-\tau(U_1 - U_2)}}$$

$$\delta' = \delta\ (U_{chocie} - U_{other})$$

$$RT \sim Wiener(\alpha, \beta, \delta', d)$$

H3: ***choice-level attention***

$$U = \sum_{Gain} P_{Gain} R_{Gain}^{\rho} + \sum_{Loss} \lambda\, P_{Loss} R_{Loss}^{\rho}$$

$$P_{chocie1} = \frac{1}{1 + e^{-\tau(U_1 - U_2)}}$$

$$\delta' = g_{choice,coeff}\, \delta\ (2\, P_{choice} - 1)$$

$$RT \sim Wiener(\alpha, \beta, \delta', d)$$

H4: ***utility-level attention***

$$U = \sum_{Gain} P_{Gain} R_{Gain}^{\rho} + \sum_{Loss} \lambda\, P_{Loss} R_{Loss}^{\rho}$$

$$P_{chocie1} = \frac{1}{1 + e^{-\tau(g_{1,coeff} U_1 - g_{2,coeff} U_2)}}$$

$$\delta' = \delta\ (2\, P_{choice} - 1)$$

$$RT \sim Wiener(\alpha, \beta, \delta', d)$$

H3: ***value-level attention***

$$U = \sum_{Gain} g_{Gain,coeff} P_{Gain} R_{Gain}^{\rho} + \sum_{Loss} g_{Loss,coeff} \lambda\, P_{Loss} R_{Loss}^{\rho}$$

$$P_{chocie1} = \frac{1}{1 + e^{-\tau(U_1 - U_2)}}$$

$$\delta' = \delta\ (2\, P_{choice} - 1)$$

$$RT \sim Wiener(\alpha, \beta, \delta', d)$$

Hierarchical Bayesian model

Group level parameters are defined for all parameters (*mu* & *sigma*).

Rstan is used.

All models use following setting.

iter = **4000**, *warm up* = **1000**, *chain* = **4,** *thinning* = **1**, *init* = **random**,
adapt delta = **0.95**, *max tree depth* = **10**, *step size* = **1**

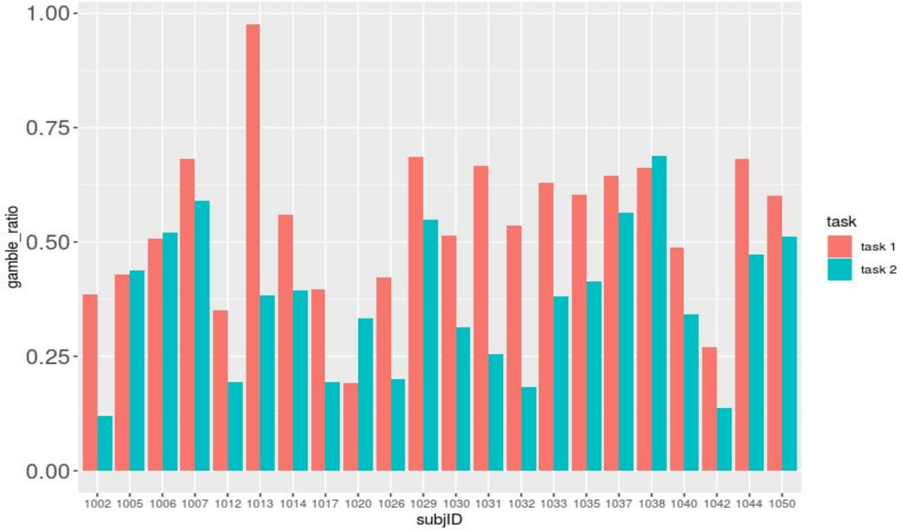
Mattirck is also applied.

22 Subjects, for each subject, **65.4** tasks (1) and **105.8** tasks (2) in average

Multimodal data (eye-gaze, pupil size, face expression)

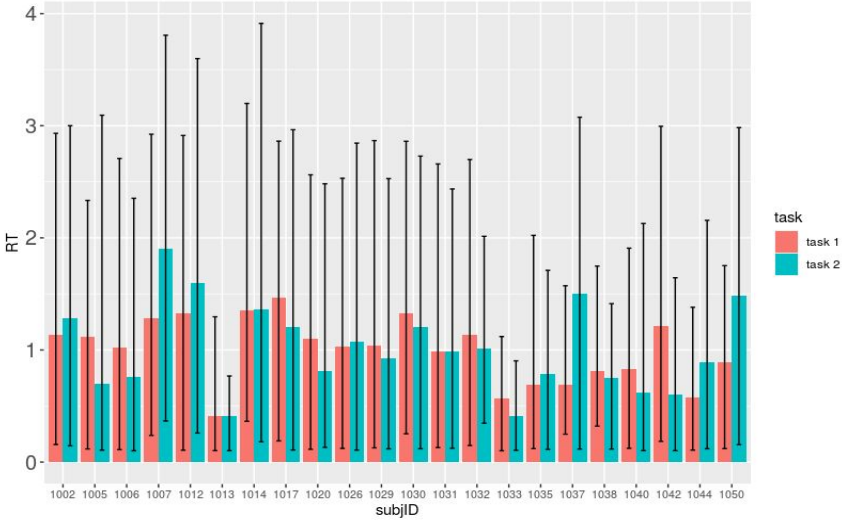
Gamble Ratio

	task 1	*task 2*
MEAN/SD	0.54/0.17	0.37/0.16



Reaction Time

	task 1	*task 2*
MEAN/SD	1.00/0.29	1.01/0.39



Summary of fitting

Only group parameters, “**mu~**”, indicating mean of individual parameters are included.
Values of below table mean “**posterior mean / Rhat**.”
Full information is in “summaries” folder.
Regarding R hat, fitted models of task 2 using eye-gaze are not reliable.

task	model	mu_rho	mu_lambda	mu_tau	mu_alpha	mu_beta	mu_delta	mu_ndt	mu_theta
task 1	prospect	0.89/1.00	1.40/1.00	0.06/1,00					
	H1	0.96/1.03	1.37/1.01	0.15/1.07	1.94/1.00	0.50/1.00	0.88/1.04	0.09/1.00	
	H2	1.29/2,24	0.08/32.56	2.68/1.50	1.97/1,39	0.53/3.26	0.42/911.60	0.09/15.09	
	H3	1.04/1.05	1.12/1.09	0.33/1.11	1.93/1.00	0.48/1.00	0.64/1.09	0.09/1.00	0.73/1.09
	H4	1.07/2.74	1.56/2.90		1.87/1.63	0.50/1.30	0.64/3.61	0.09/1.06	0.68/1.96
	H5	0.97/1.01	1.36/1.00	0.14/1.01	1.93/1.00	0.50/1.00	0.87/1.01	0.09/1.00	0.98/1.00
task 2	prospect	0.27/1.00		9.01/1.00					
	H1	0.30/1.00		7.47/1.00	1.89/1.00	0.50/1.00	1.26/1.00	0.09/1.00	
	H2	0.33/1.08		1.28/1.05	1.85/1.00	0/60/10.53	1/60/37.92	1.60/37.92	
	H3	0.34/1.94		6.38/1.30	1.88/1.02	0.49/1.00	1.13/1.89	0.09/1.00	0.50/2.00
	H4	0.94/9.32			1.80/1.26	0.49/1.62	1.27/5.66	0.09/1.01	-0.25/4.66
	H5	0.57/4.59		9.52/1.23	1.84/1.25	00.50/1.07	0.93/2.82	0.09/1.02	0.01/5.76

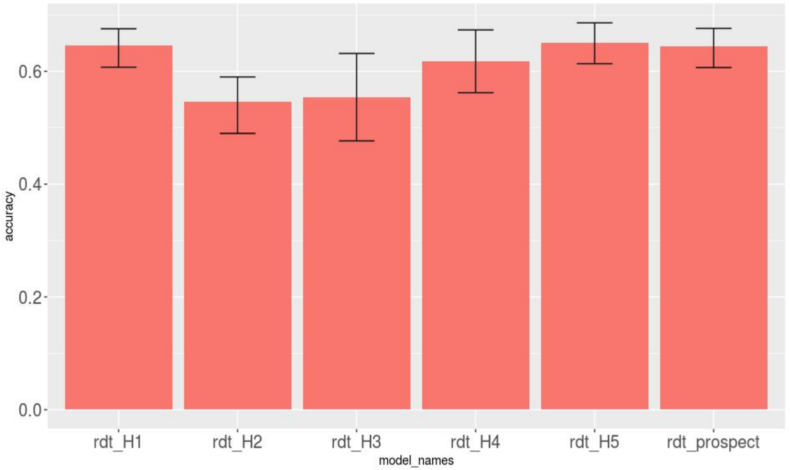
Model Comparison

task	model	param #	LOOIC
task 1	H1	7	4598.90
	H2	6	479928765954.72
	H3	8	4649.58
	H4	8	4756.32
	H5	8	**4570.91**
task 2	H1	6	7583.10
	H2	5	3259360800273.2
	H3	7	7868.18
	H4	7	8194.57
	H5	7	7939.47

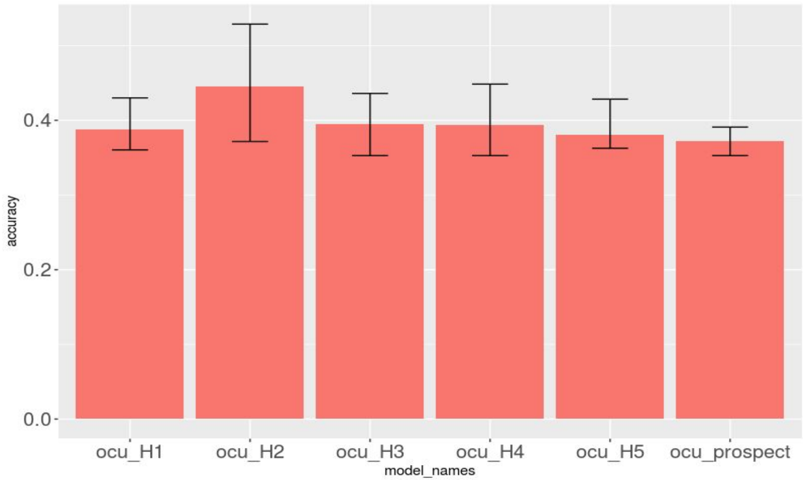
LOOICs of H2 model show abnormal value because H2 fitting is not done properly as shown in the previous slide.

Accuracy on observed choice data with estimated parameters

Task 1



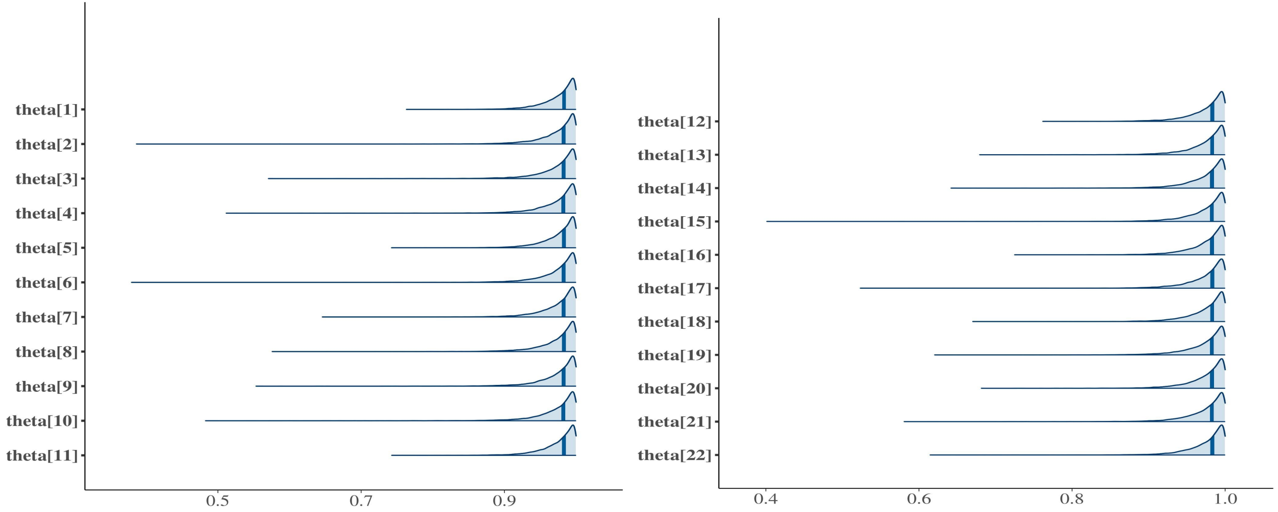
Task 2



The accuracy of prospect model of task 2 is under 0.5, which means it is not properly done.
>> All works on task 2 should be re-checked and overhauled.

Posterior of individual theta in H5 of task 1

Thetas are very close to 1, which means zero gaze-bias.



Discussion

*As the results of task 2 are unreliable in overall, following discussions **only consider the results of task 1**.*

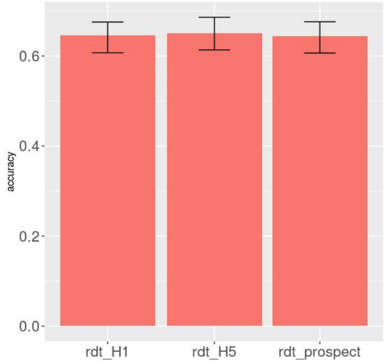
Estimated parameters (*rho, lambda*) of H1, H5 are close to those of prospect model.

Accuracy of H1, H5 is on a par with prospect model.

Adopting difference of utility to drift rate is valid approach.

task	model	mu_rho	mu_lambda	mu_tau
	prospect	0.89	1.40	0.06
	H1	0.96	1.37	0.15
task 1	H5	0.97	1.36	0.14

(* posterior mean of group parameters involved in prospect theory *)



(* model-accuracy of task 1 *)

Fitting H2 is not properly done.

Normalizing difference of utility might stabilize model fitting and lead to better performance.

task	model	mu_rho	mu_lambda	mu_tau	mu_alpha	mu_beta	mu_delta	mu_ndt
task 1	H2	1.29/2,24	0.08/32.56	2.68/1.50	1.97/1,39	0.53/3.26	0.42/911.60	0.09/15.09

(* posterior mean/R hat of group parameters of H2*)

task	model	param #	LOOIC
task 1	H2	6	479928765954.72

(* LOOIC of H2*)

H5 model is best among H3 ~ H5.

Attention is involved in evaluating each value, rather than integrated utility or choice.

task	model	param #	LOOIC
task 1	H3	8	4649.58
	H4	8	4756.32
	H5	8	**4570.91**

(* LOOIC of H3,H4,H5*)

No big difference between LOOIC of H1 and H5.

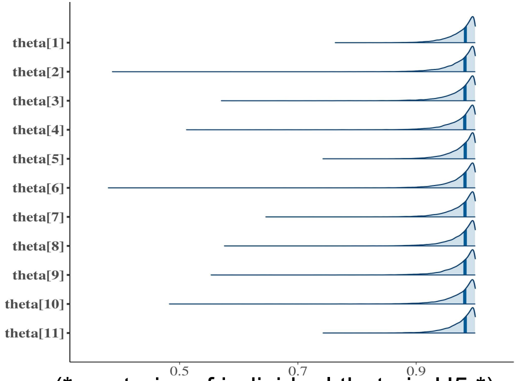
Estimation of theta approximates to 1.

Applying attention to prospect+drift diffusion model has no significant improvement.

The possible reason is that inner process of risky choice is not upon accumulating evidence, which means that **putting attention to one value doesn't always make that value higher.**

task	model	param #	LOOIC
task 1	H1	7	4598.90
	H5	8	**4570.91**

(* LOOIC of H1,H5*)



(* posterior of individual theta in H5 *)

Future work

- Explore non-accumulating model to incorporate attention in risky choice task.
- Find way to consider double-edged, negative and positive, effect of eye-gaze on evaluating subjective utility.
- Recheck and fix the overall pipeline of task 2, from data processing to model fitting.
- Try to feed both datasets from task 1,2 to a single model. It might help stabilizing fitting task 2 models with task 1 data, as there is no loss term in task 2.

Reference

Kahneman, D., & Tversky, A. (1979). Prospect theory: An analysis of decision under risk. Econometrica, 47, 263–291. http://dx.doi.org/10 .2307/1914185 Kahneman,

Ratcliff, R., and Mckoon, G. (2008). The diffusion decision model: theory and data for two-choice decision tasks. Neural Comput. 20, 873–922. doi: 10.1162/neco.2008.12-06-420

Rutledge, R. B., Skandali, N., Dayan, P., & Dolan, R. J. (2014). A computational and neural model of momentary subjective well-being. Proceedings of the National Academy of Sciences, 111(33), 12252-12257.

Chung, D., Christopoulos, G. I., King-Casas, B., Ball, S. B., & Chiu, P. H. (2015). Social signals of safety and risk confer utility and have asymmetric effects on observers' choices. Nature neuroscience, 18(6), 912.

Krajbich, I., Armel, C., Rangel, A. (2010). Visual fixations and the computation and comparison of value in simple choice. Nature neuroscience, 13, 1292–1298.

Thomas, A. W., Molter, F., Krajbich, I., Heekeren, H. R., & Mohr, P. N. C. (2019). Gaze bias differences capture individual choice behaviour. Nature Human Behaviour, 3(6), 625–635. https://doi.org/10.1038/s41562-019-0584-8

REPRODUCIBILITY

The task1 is named as “rdt” and the task 2 is named as “ocu.”

All R implementations are in “R” folder, for example “R/*taskname_modelname*.R.”

All stan codes for models are in “model” folder, for example “model/*taskname_modelname*.stan.” The stan codes are implemented based on ddm and prospect model in “hBayesDM” package.

The basic common processing of data and result is in “R/basic_*taskname*.R.”

The accuracy reports are obtained by “*taskname*_accuracy.R.”

REPRODUCIBILITY

All result summaries including posterior mean, R hat and looic, can be found in “summaries” folder.

All plots are in “plots” folder. It includes trace plots and posterior distribution for all parameters of each model.

Data are in “data/behavior_fixation_v2/*taskname*” for each task. The csv file for each subject including behavior and eye-gaze fixation data is in this folder. Also, you can find plots for eye-gaze in “data/behavior_fixation_v2/plot” folder. Eye-gaze is processed by clustering using K-means in “scikit-learn package.”

APPENDIX - POSTERIOR DISTRIBUTION - mu_rho

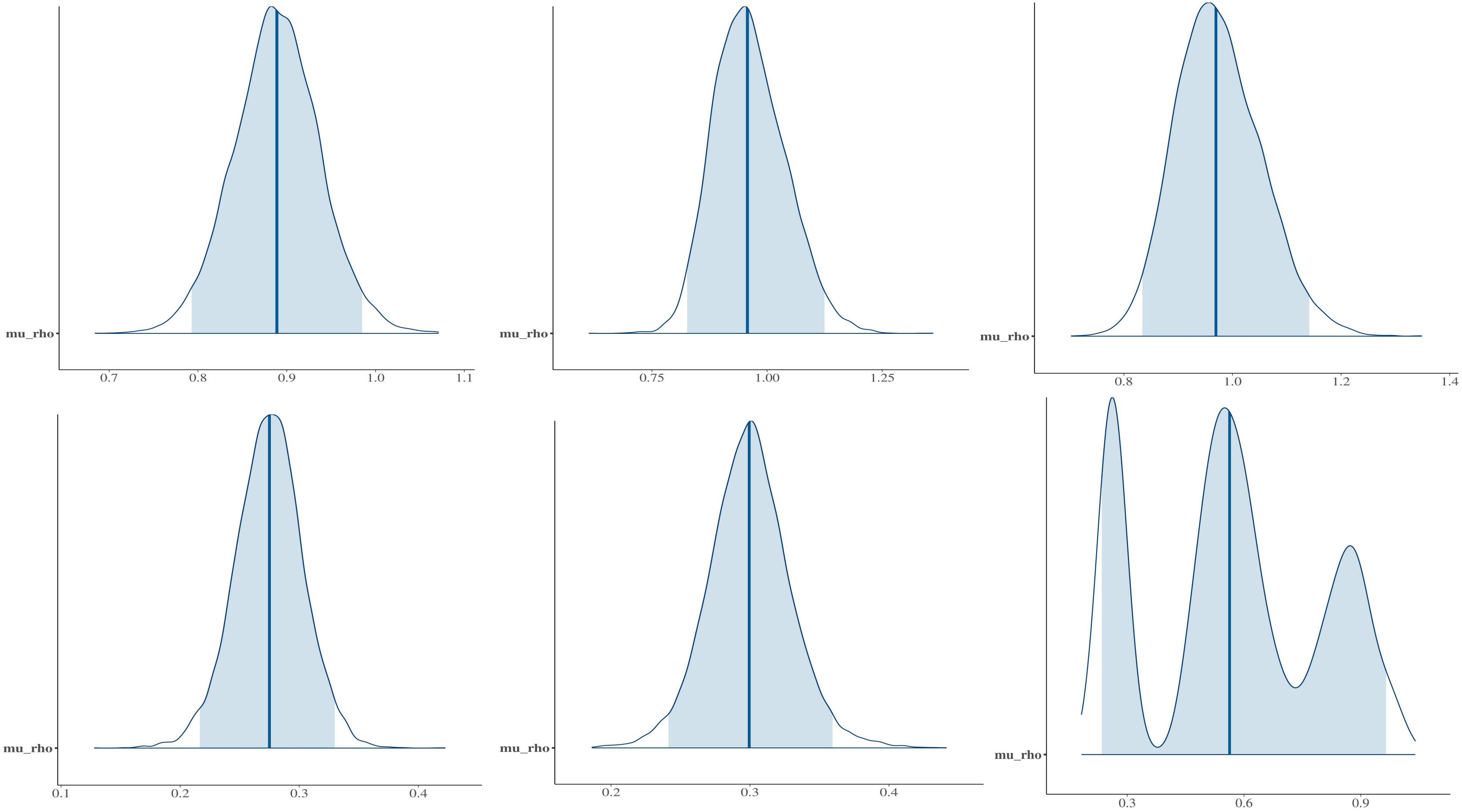
prospect

H1

H5

Task 1

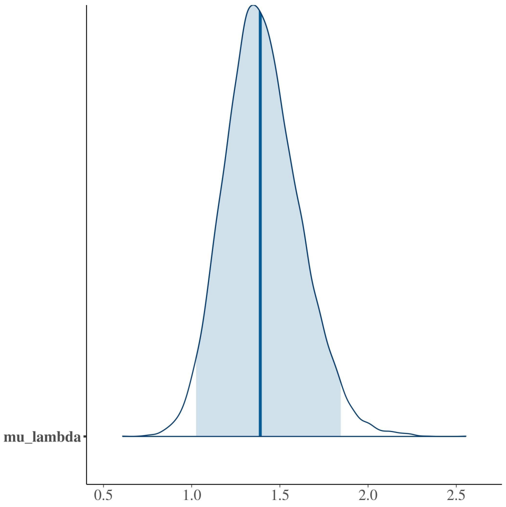
Task 2



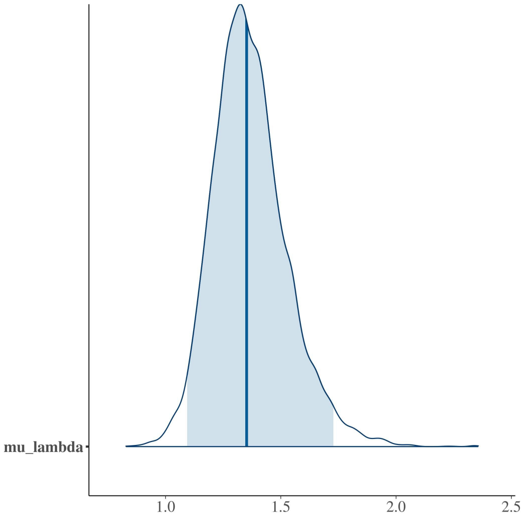
APPENDIX - POSTERIOR DISTRIBUTION - mu_lambda

Task 1

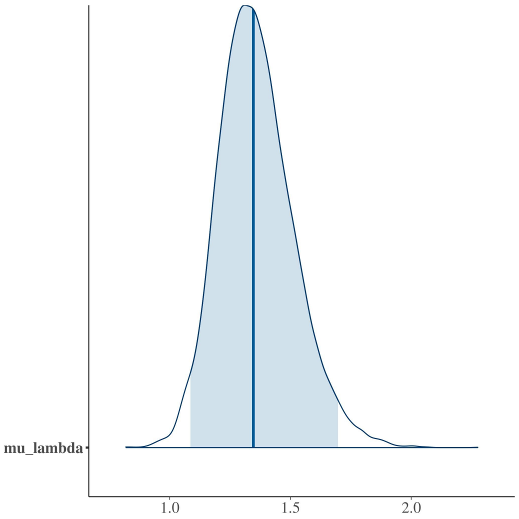
prospect



H1



H5

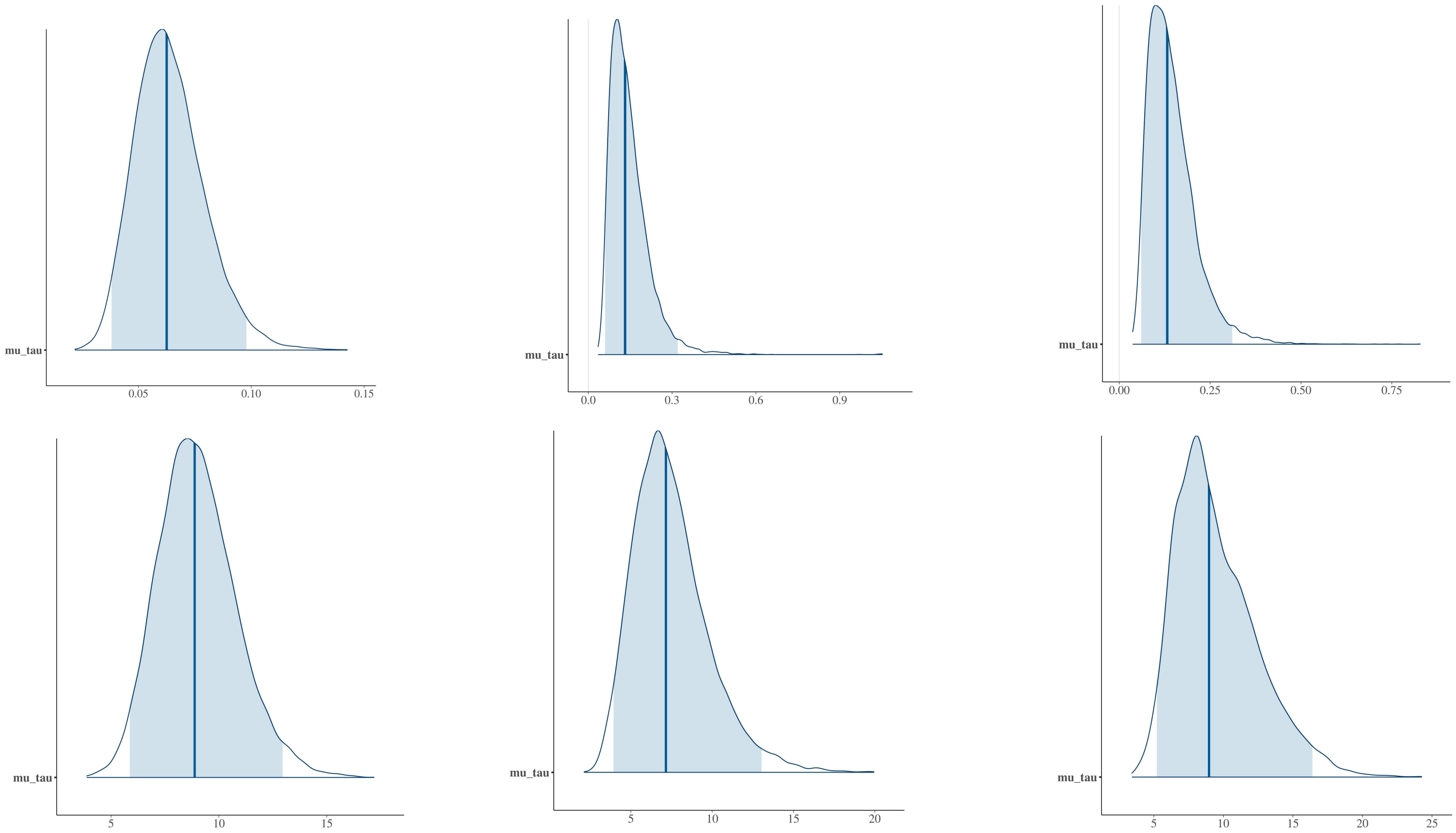


APPENDIX - POSTERIOR DISTRIBUTION - mu_tau

prospect H1 H5

Task 1

Task 2



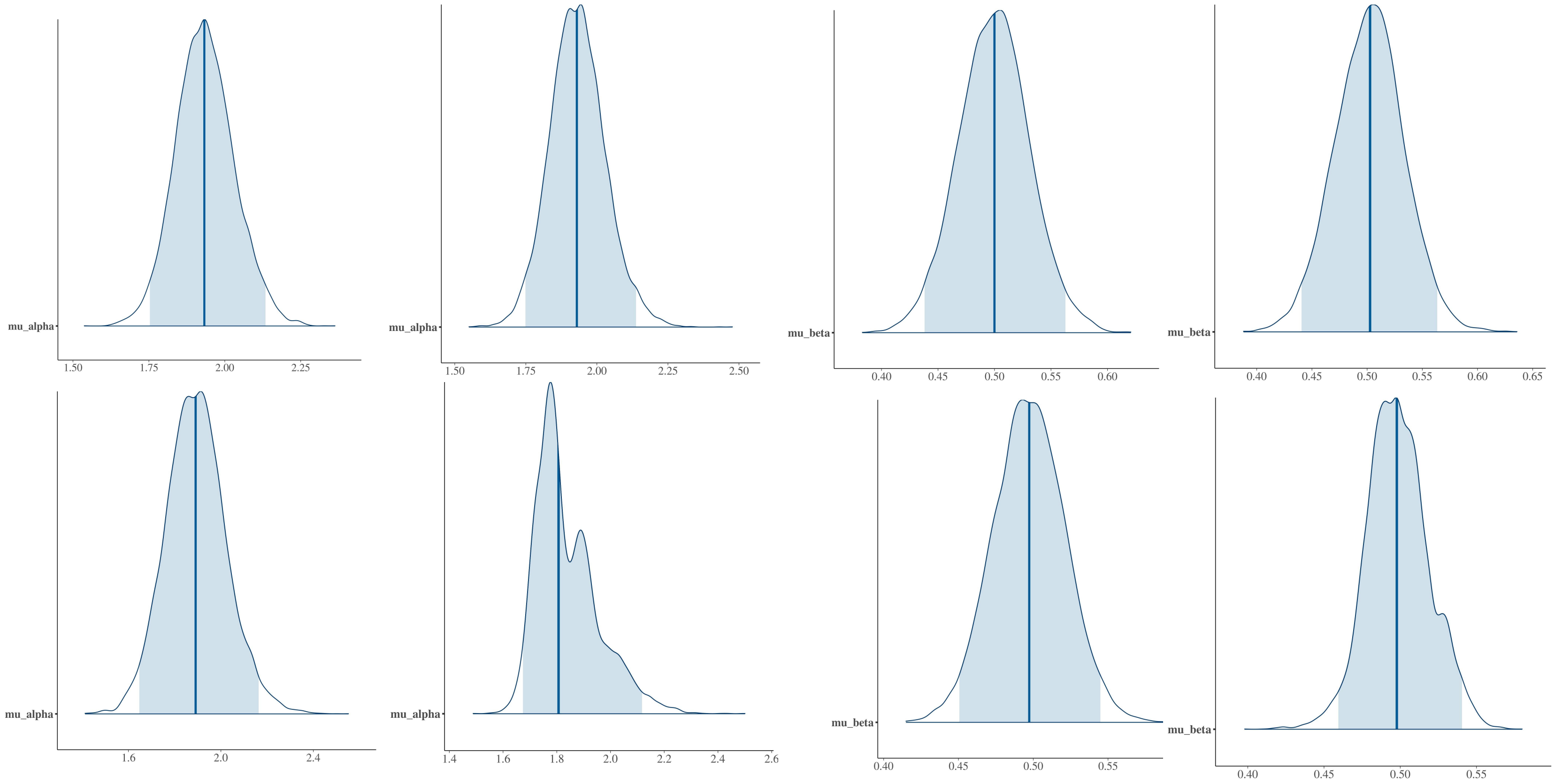
APPENDIX - POSTERIOR DISTRIBUTION - mu_alpha, mu_beta

mu_alpha | mu_beta

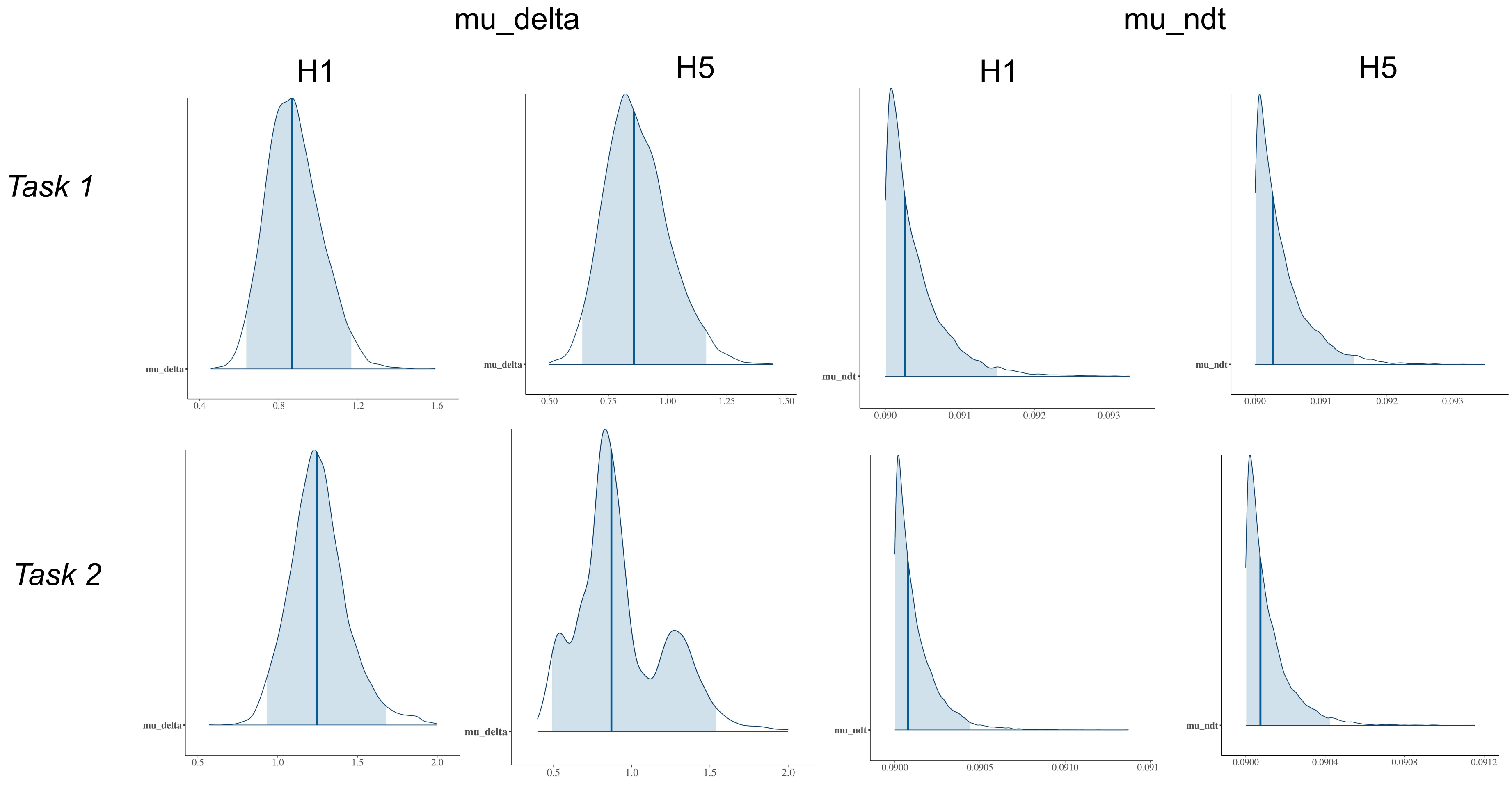
H1 | H5 | H1 | H5

Task 1

Task 2



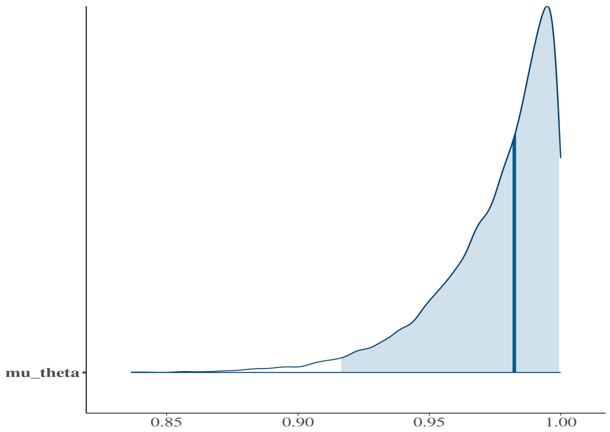
APPENDIX - POSTERIOR DISTRIBUTION - mu_delta, mu_ndt



APPENDIX - POSTERIOR DISTRIBUTION - mu_theta

H5

Task 1



Task 2