

# The Cubic Polynomial Regression Outperforms Gompertz Equation in Model Fitting of Bacterial Populations

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## Abstract

Population growth rates are often used as a response variable to observe and predict changes to population density and abundance. Mathematical models are applied in food microbiology to determine the growth of bacterial populations on the basis of food safety. This study aims to compare how well phenomenological and mechanistic theory models fit population growth data of bacterial species. By sampling starting parameters from a normal distribution, the polynomial cubic, quadratic and Gompertz equations were successfully fitted to 255 of the 284 unique ID datasets. The Akaike Information Criterion (AIC) was calculated for model comparison. Across all the datasets, the phenomenological cubic equation had the highest total of best fitted growth curves, while the Gompertz model ranked second. This study shows that the mechanistic model has underperformed for unevenly distributed bacterial growth data with small sample sizes and species that demonstrated a death phase. We conclude that model fitting is dependent on the quality of the experimental data. We suggest that model fitting in microbiology should also consider phenomenological models in an area that is rather heavily dependent on mechanistic theories.

## 1 Introduction

1 Population growth rates are used to observe trends in population density and abundance,  
2 from whether these variables are changing, to the rate they occur [Sibly and Hone, 2002].  
3 In addition, they are often applied to predict future projections on population sizes [Sibly  
4 and Hone, 2002]. Population growth rates are important to the application of scientific topics  
5 ranging from population ecology, conservation biology, food microbiology, evolutionary biology  
6 and more [Sibly and Hone, 2002, Sibly et al., 2002]. For example, population growth rate can  
7 be used as a response variable to analyse the success of the invasion of maladaptive traits into  
8 a resident population [Sibly et al., 2002]. Especially in response to current changing conditions  
9 that may alter population dynamics e.g climate change, we need to be able to identify drivers  
10 to the process of population growth, thus helping to prioritize and target management actions  
11 [Eacker et al., 2017]. In microbiology, modelling bacteria populations have formed the basis of  
12 food safety. Being able to effectively give realistic predictions of microbial growth during food  
13 storage over a range of environmental conditions have allowed the minimizing of ineffective,  
14 expensive and slow laboratory challenge testing of foods [Soboleva et al., 2000, Baranyi and  
15 Roberts, 1995, Perni et al., 2005].

16 Measuring bacteria growth is a measure of all cellular processes and captures how the  
 17 cells adapt and survive in their environmental niches [Tonner et al., 2020]. As exemplified in  
 18 figure 1, bacterial growth in batch culture often assumes a sigmoid growth function with three  
 19 growth phases captured by three parameters: lag phase is where no growth occurs in which  
 20 bacteria undergo physiological and regulatory processes that are needed for optimal growth,  
 21 exponential phase is where there is accelerating rapid growth in a certain period of time (lag  
 22 time, also denoted as  $\lambda$ ), thus, reaching a maximal value ( $\mu m$ ) [Rolfe et al., 2012]. The  
 23 final phase is the stationary phase where the carrying capacity ( $A$ ) is reached, growth rates  
 24 decreases and reaches 0 where nutrients have been exhausted [Tonner et al., 2017, Zwietering  
 25 et al., 1990].

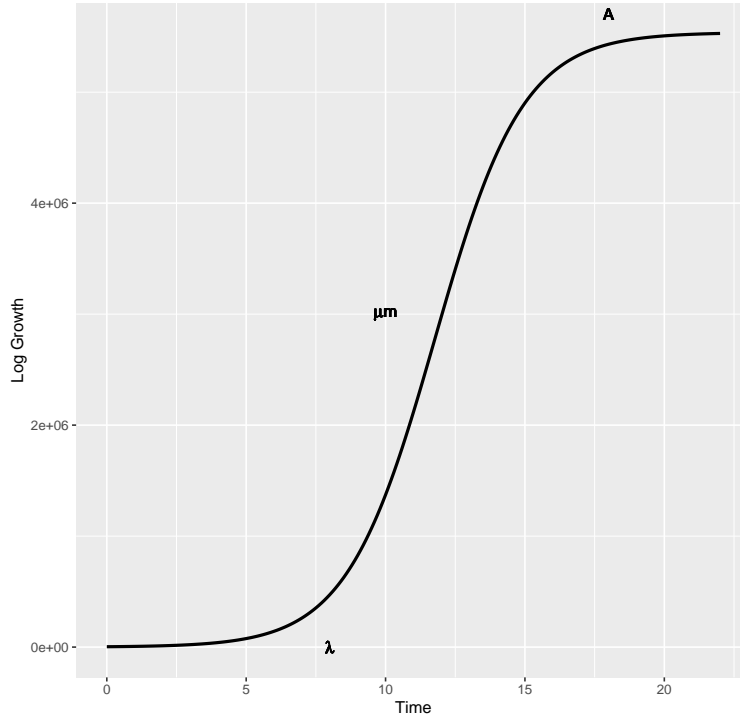


Figure 1: This figure represents a sigmoid growth curve using example logistic data. The three growth phases are captured here by the three parameters: the lag phase is where no growth occurs and is captured by the lag time ( $\lambda$ ).  $\mu m$  is the maximum population growth rate which can be captured by fitting a linear regression line through ( $\lambda$ ) and finding the slope. The last phase is the stationary phase ( $A$ ) where growth rate reaches 0.

26 Previous work has proposed a variety of mathematical models in the form of algebraic

27 curves to describe microbial growth, expressed by functions and equations [Soboleva et al.,  
 28 2000, Peleg and Corradini, 2011]. In this study, I have split such models into two distinct  
 29 forms: phenomenological models and mechanistic theory models. Phenomenological models  
 30 use a mathematical function that is fitted to the data to describe the underlying biological  
 31 process [White and Marshall, 2019]. In contrast, mechanistic models include hypothesis-based  
 32 parameters that have a biological interpretation and explicitly tracks the process of a biological  
 33 system [Geritz and Kisdi, 2012]. This study aims to compare how well these two different  
 34 types of mathematical models fit to population growth data across bacterial species. A large  
 35 part of the literature has been involved in analysing the power between different mechanistic  
 36 theories in modelling bacterial growth, while phenomenological models are often absent in such  
 37 context [Gibson et al., 1988, Adair et al., 1989, Labuza and Fu, 1993, Mackey and Kerridge,  
 38 1988]. Although phenomenological models have substantial predictive power, it can also risk  
 39 not being validated for any biological systems due to the generality of their parameters,  
 40 especially if a prediction is outside the known parameters [Heitzer et al., 1991, Schiraldi and  
 41 Foschino, , Geritz and Kisdi, 2012]. In mathematical models, the fewer parameters there  
 42 are, the greater the explanatory power. The more meaningful and intuitive the parameters  
 43 of a model are, the more phases of microbial growth are covered, providing highly accurate  
 44 models [Esser et al., 2015]. For such reasons, it can be hypothesized that mechanistic models  
 45 can be a better fit within bacterial populations that demonstrate a predictive growth curve.  
 46 The objectives of this study were to (1) fit a quadratic and cubic model to the species datasets  
 47 (2) fit a mechanistic Gompertz model (3) compare the models using the calculated Akaike  
 48 Information Criteria (AIC).

## 49 **2 Materials & Methods**

### 50 **Theory**

51 Two phenomenological models were used in this study: cubic and quadratic equation. The  
 52 cubic equation were fitted using a third degree polynomial regression model and the quadratic  
 53 equation uses a second order polynomial form (parabolic curve).

54 The cubic equation is:

$$y = at^3 + bt^2 + ct + d \quad (1)$$

55 The quadratic equation is:

$$y = bt^2 + ct + d \quad (2)$$

where  $y$  = population abundance at  $t$ ,  $t$  = time and  $a$   $b$   $c$   $d$  are regression coefficients. The number of free parameters are 4 and 3 respectively.

The Gompertz model is one of the most frequently used non-linear sigmoid model fitted to growth curves [Gompertz, 1825]. Originally used to describe human mortality rates, it has now been re-parametrized to be applicable within modelling bacterial growth, becoming one of the most common mechanistic models to be used by microbiologists [Tjørve and Tjørve, 2017, Gibson et al., 1988].

Provided by the literature, the modified Gompertz equation is represented by the general equation:

$$\log(N_t) = N_0 + (N_{max} - N_0)e^{-e^{\frac{r_{max} \exp(1)}{(N_{max} - N_0) \log(10)} + 1} (t_{lag} - t)} \quad (3)$$

where  $N_t$  is the population size at time  $t$ ,  $N_0$  is the initial population size during the lag phase,  $N_{max}$  is the largest population size when the carrying capacity is reached,  $r_{max}$  is the rate of exponential growth,  $t_{lag}$  is the  $x$  intercept (time) to the tangent of the slope of the exponential growth.

## Data

The dataset contains measurements of change in biomass or number of cells of microbes over time (hours). Data was collected through lab experiments across the world, cited from ten research papers [Bae et al., 2014, Bernhardt et al., 2018, Roth, 1962, Galarz et al., 2016, Gill and DeLacy, 1991, Silva et al., 2018, Sivonen, 1990, Stannard et al., 1985, Zwietering et al., 1994, Phillips and Griffiths, 1987]. Single population growth rate curves are identified by unique temperature-species-medium-citation-replicate combination IDs. A total of 284 unique IDs was isolated, identified as ID 0 to 284. Using Python 3.9, I eliminated any negative time and population data and  $NaN$  values due to not making any meaningful biological sense.

## Model Fitting

All model fitting was achieved through utilizing Python 3.9. The modified Gompertz equation (3) was fitted to 284 unique ID using the non-linear least squares regression. I used the Levenberg–Marquardt algorithm of minimization, which is the default algorithm for Python *lmfit* function. The Levenberg-Marquardt algorithm is an effective hybrid technique in solving non-linear equations. It uses both Gauss-Newton and steepest descent approaches to converge to a minimized, optimal solution [Wilson and Mantooth, 2013]. For each unique ID, the four starting values ( $N_0$ ,  $N_{max}$ ,  $r_{max}$ ,  $t_{lag}$ ) were estimated by these methods:  $N_0$  was calculated by isolating the last value (the starting minimum value) within the sample

size of the population measure.  $N_{max}$  was calculated by finding the maximum value within the population measure.  $r_{max}$  and  $t_{lag}$  was calculated by sampling values within a normal distribution ( $N = 100$ ,  $\mu = r_{max}$  or  $t_{lag}$ ,  $\sigma = 0.1$ ), then fitted to find the best minimized parameters (refer to table 3 in the supplementary data for all minimized parameters from the Gompertz equation). Out of 284 ID, I was unable to find suitable minimized starting values for 10 unique datasets (ID 4,14,16,20,88,280,281,282,283,284), therefore was omitted from further data processing. This resulted in 274 unique datasets remaining. For datasets with a sample size of less than the numbers of free parameters in equation (3) ( $df = 0$ ), an output of *NaN* was given due to inadequate sample size for data analysis. Similarly, the cubic (1) and quadratic (2) equations were fitted to the remaining dataset using the default least squares regression (Levenberg–Marquardt algorithm). All datasets were successfully fitted given the estimated starting values of 1. However, unique datasets with a sample size of less than 4 and 3 ( $df = 0$ ) respectively were given an output of *NaN*.

#### Model Selection and Comparison

Model selection was established using a selection criterion: AIC, or rather  $AIC_c$  [Burnham and Anderson, 2004]. AIC is a numerical value by which to rank competition models in terms of information loss. It is used in comparison with other AIC values, where the lowest represents the best approximating model [Symonds and Moussalli, 2011]. In my study, instead of AIC,  $AIC_c$  was used. When  $\frac{n}{K} < 40$  where  $n$  = sample size,  $K$  = total number of parameters in the most complex model,  $AIC_c$  is recommended.  $AIC_c$  is the small sample (second-order bias correction) version of AIC. By using AIC on data with small sample sizes, it increases the change of overfitting [Burnham and Anderson, 2004].

Recent versions of statistical software packages provide AIC values when generating model fitting. AIC values were extracted from the cubic, quadratic, Gompertz model during model fitting. To calculate  $AIC_c$ :

$$AIC = -2(\ln(\text{likelihood})) + 2K\left(\frac{n}{n - K - 1}\right) \quad (4)$$

$$AIC_c = AIC\left(\frac{n}{n - K - 1}\right) \quad (5)$$

where  $n$  is the sample size,  $K$  is the number of free parameters in the model.

AIC scores are often shown as  $\Delta AIC$ . The individual AIC and  $AIC_c$  values are not interpretable on its own as they have scaling issues and are affected by different sample sizes.

115 Therefore,  $AIC_c$  was rescaled by this equation:

$$\Delta_i = AIC_c i - AIC_{c_{min}} \quad (6)$$

116 where  $AIC_{c_{min}}$  is the smallest  $AIC_c$  value between candidate models.  $\Delta_i$  is the information  
 117 loss experienced if we were using the fitted model  $i$ . This transformation will result in the  
 118 best fitted model to have  $\Delta_i = 0$ . The larger the  $\Delta_i$ , the less plausible the fitted model is an  
 119 accurate candidate model for the given dataset [Burnham and Anderson, 2004]. In addition,  
 120 Akaike Weight ( $w_i$ ) was calculated as the probability that the model chosen is the best model  
 121 given the data and the set of candidate models [Wagenmakers and Farrell, 2004].

122 The equation is given below:

$$w_i = \frac{\exp(-\Delta_i/2)}{\sum_{r=1}^R \exp(-\Delta_r/2)} \quad (7)$$

123 where  $\sum_{r=1}^R \exp(-\Delta_r/2)$  is the sum of the value across all models.

124 Using R studios, I then compiled a summary table of all the calculated AIC values for each  
 125 of the unique 274 datasets. However, during the process of  $AIC_c$  calculations, I eliminated  
 126 two datasets (ID 200,212) for having a sample size less than the number of free parameters in  
 127 the quadratic equation ( $N < 3$ ), which is too small for any meaningful analysis. In addition for  
 128 10 data sets,  $-Inf$  and  $Inf$  was outputted. From a closer look, it was due to  $n$  being divided  
 129 by a total value of 0 or less. Along with any datasets with only one successfully calculated  
 130 AIC model ( $N < 4$ ), a total of 18 unique data sets was eliminated from my final table which  
 131 demonstrates the best fitted model for each ID (Supplementary table 4).

### 132 **Computing Languages and Tools**

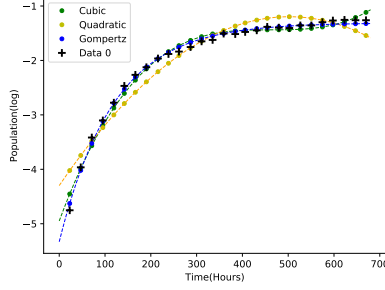
133 Data Wrangling, model fitting, graphing and AIC calculations was performed in Python  
 134 3.9. AIC tables was created in R 4.1.2 using R Studio. Sample graph (figure 1) was cre-  
 135 ated in R 4.1.2. Specific packages used within each computing languages can be found in  
 136 README.md.

## 137 **3 Results**

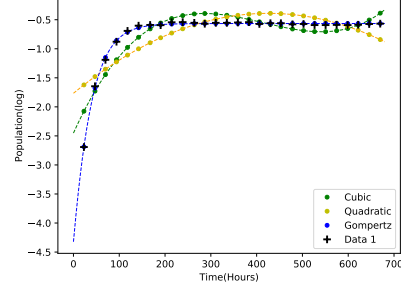
### 138 *(1,2)Fitting phenomenological and mechanistic mathematical models*

139 Out of the 284 unique IDs from the original dataset, I managed to successfully fit 255  
 140 IDs with both phenomenological and mechanistic models. All datasets that were successfully

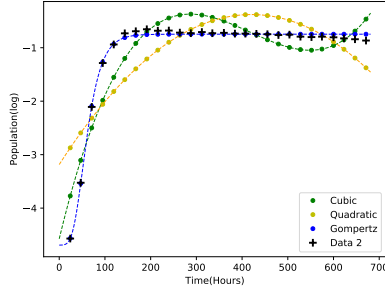
141 fitted had a sample size of  $N \geq 5$ . All curves were plotted with the residuals and overlaid with  
 142 the original dataset points. Some examples of model fitting are shown in Figure 2.



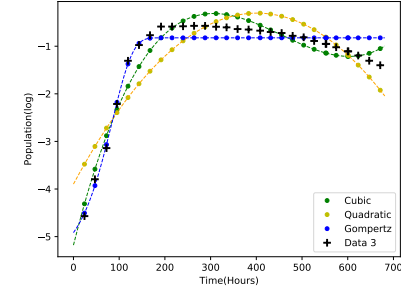
(a) ID 0 with species *Chryseobacterium balustinum* [Bae et al., 2014]



(b) ID 1 with species *Enterobacter.sp.* [Bae et al., 2014]



(c) ID 2 with species *Pantoea agglomerans.1* [Bae et al., 2014]



(d) ID 3 with species *Pantoea agglomerans.2* [Bae et al., 2014]

Figure 2: Figures of model fitting using phenomenological and mechanistic models for ID 0 to 3. Each line represents a specific mathematical model: yellow represents the quadratic equation, green is the cubic equation, blue is the mechanistic Gompertz equation. Datapoints are represented by the black crosses. Points are layed on each theoretical model to compare as residuals with the datapoints.

### 143 (3) Model comparison using Akaike Information Criteria

144 Table 4 in supplementary data demonstrates the best fitted model from each ID identified  
 145 by the lowest  $AIC_c$  score. Table 1 further ranks the three mathematical models by how many  
 146 times a given model fits best across all unique datasets. From my study, the phenomenological  
 147 cubic model fits best for the highest amount of ID bacterial datasets. 65% of the ID is best  
 148 fitted with a Cubic model, 32% fitted with the Gompertz model, while only 3% of the ID is  
 149 best described with the quadratic model (ID 58,70,214,217,239).



Table 1: The total number of each best fitted models across all 255 unique datasets ranked in order from highest to lowest. The cubic mathematical equation was chosen as the best fitted model for the highest number of IDs. This ranking was based on comparisons of  $AIC_c$  scores

Model	Total Number of Each Best Fitted Model Across All ID Based on $AIC_c$ Scores
Cubic	168
Gompertz	82
Quadratic	5
Total	255

Table 2: Total number of pairs between best fitted model and model with  $AIC_c < 2$  across all ID

Best Model	Fitted Models with $AIC_c$ values with $< 2$	Total Number of IDs
Cubic	Quadratic	1
Cubic	Gompertz	3
Quadratic	Cubic	4
Gompertz	Cubic	7
	Total	15

150 Comparing the  $\Delta AIC_c$  of candidate models between each dataset ID, it was observed that  
151 some of the differences between the scores were  $< 2$ . A full list of all mathematical models in  
152 all unique IDs with their  $AIC_c$  values can be found in supplementary table 5. Highlighted  
153 by Burnham and Anderson 2002, when a variable with poor explanatory power is added  
154 to a model, an output of  $\Delta AIC_c < 2$  can occur [Bur, 2002, Arnold, 2010]. Only when the  
155 differences are  $> 2$  is considered significant [Wyss et al., 2012]. In table 2, I can see that 15  
156 IDs demonstrated  $\Delta AIC_c$  values that are not significantly different. Within those 15 unique  
157 IDs, 7 of those indicated that Gompertz model was the best model fitted with the given data,  
158 however, they were insignificantly different in comparison to the cubic model. 4 out of the 5  
159 best fitted quadratic models was not significantly different from the cubic model. While the  
160 best fitted cubic mode of 3 and 1 IDs was not significantly different from the Gompertz and  
161 quadratic model, respectively.

## 162 4 Discussion

163 Microbial growth is an area extensively studied in food microbiology due to causing the most  
 164 common food poisoning [Buchanan et al., 1997, Xiong et al., 1999]. The use of mathemati-  
 165 cal models in fitting and predicting experimental data can help describe the behaviour of  
 166 microorganisms under different environmental conditions (e.g temperature, PH) [Zwietering  
 167 et al., 1990]. In my study, I compared how well two different types of mathematical models:  
 168 phenomenological and mechanistic theory fit across bacterial population growth data. This  
 169 was investigated by fitting the quadratic, cubic and Gompertz equations to each unique ID  
 170 dataset and calculating the  $AIC_c$  values. The lower the  $AIC_c$  value, the best model it is out  
 171 of the candidate set. In contrast to my initial hypothesis, the phenomenological cubic model  
 172 had the highest percentage of being chosen as the best approximate model. The Gompertz  
 173 equation was ranked second.

174 From visual inspection, more than 100 curves were fitted poorly with the Gompertz  
 175 equation as a horizontal line was produced. For example, figure 3 demonstrates four IDs  
 176 (46,68,82,100) with a poor mechanistic curve. This could be mainly due to the quality of data  
 177 provided for model fitting. Although highly used within the literature, the fit of the Gom-  
 178 pertz function can be greatly affected by the quality of the data observations [Labuza and Fu,  
 179 1993]. For a good fit of the function, Labuza T.B, Fu B. and Salazar et al. suggested that  
 180 preferably 15 uniformly spread data points are required through all growth phases [Labuza  
 181 and Fu, 1993, Salazar et al., 2021]. Murphy et al. tested out this hypothesis by undergoing  
 182 random deletion of observations for each growth curve and refitting the Gompertz equation  
 183 after. They found that fitting curves to *Clostridium botulinum* species required evenly spread  
 184 data during the lag, growth and stationary phases. If no counts were made during the growth  
 185 phase, a sigmoid curve could not be fitted, and this was observed in data with approximately  
 186 10 or less. In addition, if there was no observation in the lag or stationary phases, an unre-  
 187 alistic estimate would be produced [Murphy et al., 1996]. All of my unique IDs had a small  
 188 sample size, from  $N = 4$  being the smallest, to  $N = 66$  the largest. Even for IDs (91,90,92,89)  
 189 with the largest sample size ( $N=66,63,57,54$ ), the Gompertz model did not fit properly. I sug-  
 190 gest that this is due to the lack of observations collected during the lag phase, which unlike ID  
 191 90 had, successfully deemed the Gompertz as the best fitted model ( $AIC_c = -216.31, \Delta AIC_c$   
 192  $= 0$ ).

193 More than 20 of the graphs produced during visual inspection demonstrated a pattern of

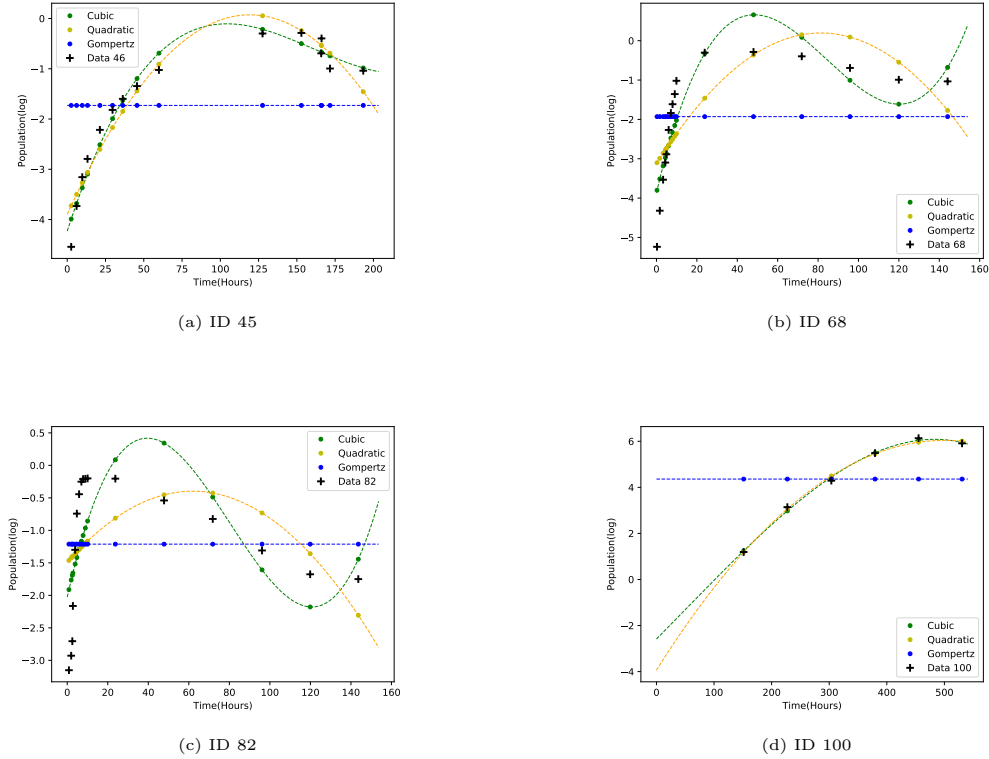


Figure 3: Examples of IDs with poor model fitting using minimized parameters, focusing on the Gompertz equation shown by the blue curve. (a,b,c) All shows a negative trail of decline as  $t$  increases. (d) ID 100 shows a positive curve however due to the lack of sample size ( $N=6$ ), the mechanistic model struggles to fit.

194 decline (refer to (a),(b),(c) in figure 3). This is predicted to be the death phase of bacterial  
 195 species. My modified Gompertz equation did not consider incorporating the death phase in  
 196 explaining bacterial growth curves. As such, the model did not fit the growth well. Addi-  
 197 tionally, the Gompertz model would not always be compatible with microbial growth as the  
 198 model does not extend beyond the stationary phase [Gibson et al., 1988, Xiong et al., 1999].  
 199 As such, other models are found to be preferred and more robust when modelling microbial  
 200 death kinetics [Xiong et al., 1999]. In the context of batch fermentation where bacterial pop-  
 201 ulation demonstrates a death phase, Salazar et al. found that the phenomenological cubic  
 202 model best predicts the overall dynamics of the experimental data in comparison to other  
 203 mechanistic models (Gompertz, Barayani and Vázquez-Murado) based on the results for the

204  $RSS$  (residual sums of squares)  $R^2$  and AIC value [Salazar et al., 2021].

205 It can be suggested that the Gompertz model was ranked second in my study due to the  
206 quality and type of data provided. As such, the phenomenological mathematical models seemed to  
207 be a better fit for predicting bacterial species data. However, what if we consider the power  
208 of fit between phenomenological models? From table 2, I can conclude that the result where  
209 IDs identifying the quadratic model as the best fit is insignificant. Instead, both types of phe-  
210 nomenological model (cubic vs quadratic) was similar in their respective datasets ( $\Delta AIC_c < 2$ ).  
211 In polynomial equations, previous work has shown mixed results regarding model fitting. Mur-  
212 phy et al. found that the cubic model was superior to lower order models (quadratic) in fitting  
213 bacterial population growth [Murphy et al., 1996]. While others have shown that quadratic  
214 models yield more accurate estimates of growth rate and lag time in noisy data sets [Ng and  
215 Schaffner, 1997, Gauch, 1993, McClure et al., 1997]. In some cases, the power of the equations  
216 to reflect the data was approximately the same [Gibson et al., 1988]. This study is unable  
217 to demonstrate a detailed comparison between both types of phenomenological model due to  
218 limited statistical analysis in the  $RSS$  and  $R^2$  value. Nonetheless, this could be investigated  
219 in future works as the main goal of model fitting is to choose the best parsimonious model  
220 that in its simplest state, still correctly predicts experimental results [Gauch, 1993, Gibson  
221 et al., 1988]. In my study, I did not take into consideration other statistical analyses such as  
222 an *ANOVA* of the AIC,  $RSS$ ,  $R^2$  to compare model fits. Consequently, the models predicted  
223 in my study might not be the best fit even though my starting values were successfully mini-  
224 mized. Future work is recommended in repeating this method with larger and evenly spread  
225 samples across all bacterial growth stages while including additional in-depth statistical tests  
226 to achieve more robust conclusions.

227 The application of mathematical models to population growth data is a widely effective  
228 method to investigate how populations respond to biological interactions. My investigation  
229 highlights the importance of phenomenological equations in food microbiology that often  
230 focuses rather on mechanistic theory models. This provides a reminder for future studies to  
231 consider both types of models in the study of bacterial growth data. When undergoing the  
232 process of model fitting, one should first consider the quality of the experimental data and  
233 the type of species under investigation. Furthermore, a true validation of the model should  
234 come from the ability to correctly predict experimental results and can be confirmed through  
235 independent testing of its parameters [Peleg and Corradini, 2011].

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## 5 Supplementary Data

Table 3: Minimized parameters of the Gompertz equation after sampling a normal distribution for all ID and their AIC values

ID	$N_0$	$N_{max}$	$r_{max}$	$t_{lag}$	AIC
0	-9349.442	-1.309	66.501	-1042.001	-154.622
1	-11.859	-0.565	0.278	-65.634	-210.677
2	-4.692	-0.747	0.15	29.309	-157.373
3	-0.824	-5.562	0.09	129.568	-82.049
5	-4191.507	-0.871	22.573	-1287.587	-168.645
6	-1.412	-18.571	0.092	97.142	-126.798
7	-832.789	-1.788	6.179	-704.082	-123.491
8	-2.11	-107373317.354	38936.046	-4385.96	-147.611
9	-4.211	-2.225	0.017	-18.835	-108.963
10	-5.466	-1.55	0.059	-13.155	-151.785
11	-1.326	-100901.294	64.952	-1745.064	-149.149
12	-0.972	-6.591	0.038	140.605	-177.885
13	-4.899	-4.364	0.279	597.086	-48.289
15	-4.69	-0.886	0.098	52.15	-156.926
17	-3.064	-6.827	0.009	223.544	-142.943
18	-6.243	-1.809	0.032	-60.178	-115.305
19	-4.946	-4.408	0.211	595.456	-48.258
21	-3514.935	-1.009	14.423	-1664.8	-156.061
22	-4.094	-1.143	0.074	-21.236	-111.125
23	-3.359	-0.191	0.822	1.39	-132.213
24	-493.282	6344.123	0.006	-29656.636	0.778
25	-1.384	-1.621	-0.003	243.116	3.566
26	-896.956	8191.764	0.01	-117276.188	-5.703
27	-1.093	-1.093	-0.051	318.351	2.586
28	-2.057	-2.057	-0.043	323.263	11.16
29	-7168.995	-1.363	308.138	-170.33	-101.357
30	-1.651	-1.845	-0.004	381.252	-47.617
31	-2.344	-2.254	1.275	292.702	5.075
32	-664.496	7617.267	0.008	-60582.168	-4.021
33	-1.584	-2.165922005997174e+16	51578958951009.05	-919.843	-37.117
34	-0.719	-1.003	-0.014	550.158	-5.85
35	-813.25	9997.267	0.01	-43376.924	-0.231
36	-1.052	-1.052	-0.02	322.967	15.423
37	-1.524	-1.524	-0.027	322.408	25.529

38	-0.665	-37415.751	252.687	-149.901	-49.753
39	-37.179	492.541	0.001	103.472	-32.987
40	-3.52	-1.511	0.025	-20.775	-18.703
41	-3.022	-0.286	0.17	-8.734	-49.592
42	-3207.061	-1.572	46.88	-437.045	-62.618
43	-1.042	-16.043	0.155	38.685	-40.852
44	-2.263	30695.275	79.679	735201.506	10.875
45	-0.87	-1.334	-0.082	299.739	12.284
46	-1.729	16.729	-0.053	-980.083	15.051
47	25680.304	-1.901	-2100.69	-2796041.394	18.565
48	-2.691	-15863366.403	-141216.352	24533231.854	15.608
49	33.314	-1.752	-1.682	-3701.744	13.241
50	-4.649	-1.692	0.346	-2.921	-28.264
51	-3122.277	-1.221	257.455	-80.567	-54.053
52	-2.448	-1.255	0.079	334.928	-3.165
53	-2.526	570198.252	375.753	30166863.242	4.245
54	-2.331	-1.255	0.088	334.989	-1.136
55	5529.539	-2.172	-332.132	-748200.732	13.212
56	-83901.248	-0.692	-3519.521	183735.801	4.71
57	-3.306	-0.434	0.203	-5.537	-4.166
58	-1.915	-4.318	-0.363	-166.841	37.979
59	-4.236	-0.673	1.132	3.0	-47.131
60	-1.462	-1.255	0.039	334.727	13.112
61	-2.773	2.183	-0.003	-3745.555	-4.329
62	-2.755	-1.255	0.06	334.851	12.334
63	-0.826	-0.826	0.158	334.973	4.674
64	304.903	-3.203	-14.693	-44982.258	21.841
65	-4.552	-0.845	0.387	-4.276	-25.793
66	-2.36	-1.255	0.035	334.908	10.566
67	-1.397	-0.706	-0.034	709.134	6.445
68	-1.93	-1.255	0.107	334.854	19.964
69	927812.261	-1.664	-73833.071	-111779997.404	11.936
70	-3.235	-1.255	0.207	335.003	36.198
71	204.958	-1.595	-10.526	-23635.416	15.579
72	-2.781	15809.633	-118.795	-893381.4	9.56
73	-2.121	-1.255	0.074	335.039	-2.303
74	4.452	-1.951	-0.451	-418.512	7.594
75	-2.222	-1.255	0.05	334.885	6.395
76	-2.327	-1.255	0.074	334.846	-0.085
77	-2.507	-1.255	0.056	334.816	-4.504

78	25.093	-0.668	-1.397	-3260.889	13.123
79	-1.298	-1.255	0.205	334.917	17.644
80	-2.184	27140.916	-186.133	-1316213.465	4.081
81	88.396	-1.823	-4.75	-10087.375	19.715
82	-1.212	-1.212	0.067	334.799	7.751
83	-2.701	16.799	-0.049	-1064.414	-3.934
84	-2.977	-1.255	0.05	334.725	7.465
85	-4.753	-1.255	-0.353	334.805	12.29
86	-2.375	-1.255	0.136	334.976	14.45
87	-1.627	1271043.31	6031.48	15061524.955	11.718
89	8.931	46.449	-0.073	-5288.072	33.906
90	9.752	7.04	0.085	197.715	-199.145
91	8.861	8.861	-0.111	179.366	-46.193
92	7.483	8.284	-0.464	-109.975	20.84
93	-0.123	8.159	0.046	148.328	-30.14
94	5.511	-0.506	0.055	386.433	-12.501
95	4.127	2334.808	4.981	58706.367	37.746
96	4.11	-5.158	-0.232	974.055	29.853
97	710.579	4.289	-31.357	-90750.36	21.433
98	3.824	4.127	-0.167	984.498	21.363
99	-0.425	7.956	0.038	87.536	-31.315
100	4.36	291.447	-0.108	-15527.349	14.707
101	7.193	-1.939	0.093	245.504	-24.775
102	4.321	1.34	-0.115	334.871	31.266
103	907.623	4.221	-41.512	-113274.339	30.054
104	4.119	-0.407	-0.223	1524.973	24.117
105	0.089	3.06	0.066	191.984	-33.178
106	3.998	4.209	-8.565	777.481	16.76
107	0.018	6.423	0.082	131.216	-45.267
108	6.231	-15.141	0.149	124.696	-25.73
109	20.686	4.035	-1.823	-2055.041	28.709
110	4.121	0.622	-0.231	214.429	24.638
111	-0.018	6.08	0.085	114.015	-27.293
112	6.984	-2.486	0.076	293.903	-8.4
113	0.195	6.464	0.113	93.282	-23.447
114	4.048	3.036	-3.861	304.706	35.242
115	4.845	0.35	-0.403	220.64	33.293
116	5.003	0.733	-0.189	213.99	26.12
117	0.166	5.748	0.086	138.839	-42.861
118	4.904	3.419	-10.701	-1646.898	17.485

119	5.369	-0.636	0.093	229.569	-45.797
120	3.396	22.967	-0.01	-6504.836	30.19
121	3.828	-0.541	230692309.709	-1191486071114.843	27.323
122	6.842	-3.821	0.383	51.751	-36.516
123	0.211	8.047	0.062	150.061	-20.749
124	5.892	-7605.59	-484.761	33758.417	12.408
125	2.944	2.944	0.067	-183.749	33.755
126	2.096	5.161	-0.324	408.726	28.719
127	5.094	284079.416	1939.155	5048699.068	33.525
128	4.141	-18.404	-0.31	216.565	26.915
129	6.336	-1.102	0.056	390.35	-31.513
130	-5358.892	57084.113	0.061	-88152.835	5.168
131	4.115	4.013	-0.28	-115.883	34.029
132	2.576	3.163	-0.972	231.761	27.736
133	3.291	-3.369	-0.043	568.971	26.984
134	3.386	-0.322	-2.243	214.491	19.966
135	5.696	-1.008	0.054	358.22	-25.003
136	1.006	6.319	0.073	164.875	-19.134
137	6.734	-0.538	0.079	262.336	-33.795
138	3.648	22.795	-3.814	-791.069	26.695
139	-0.255	2.896	-2.236	225.866	20.703
140	-0.668	6.36	0.503	14.8	-12.625
141	-0.006	7.728	0.075	198.094	-25.463
142	0.232	8.214	0.083	183.197	-18.649
143	3.865	6.837	-0.296	-43.47	36.129
144	-1.384	3.985	-32.515	221.826	29.985
145	3.561	26.645	-0.144	-1759.934	24.254
146	-1.103	2.989	-0.619	1388.814	21.791
147	0.164	6.226	0.059	162.472	-32.596
148	7.281	-0.324	0.128	212.831	-21.451
149	0.268	6.191	0.081	155.126	-20.362
150	48.937	3.649	-0.675	-1541.825	26.244
151	3.8	-2.79	-0.037	857.303	24.529
152	3.563	24.161	-0.058	-3928.374	19.085
153	2.675	3.955	-0.062	-43.621	23.858
154	0.626	7.072	0.079	151.551	-18.254
155	0.241	8.283	0.104	92.258	-44.676
156	3.533	3.533	-0.962	188.171	27.397
157	18493.588	3.075	-460.426	-1387035.453	20.112
158	0.278	6.523	0.623	17.749	-21.076

159	3.195	4.025	-0.122	-109.077	23.493
160	0.113	9.395	0.099	163.323	-0.126
161	4.969	17.178	-0.088	-315.742	35.166
162	-0.126	7.938	0.253	42.545	-16.621
163	0.429	8.563	0.366	23.671	-26.366
164	3.413	-0.731	-37.85	221.568	27.502
165	7.329	-0.538	-0.41	903.785	17.597
166	3.335	7.026	1.646	1.415	-20.641
167	4.687	3.767	1.023	6.198	-16.294
168	7.767	-72.591	-0.135	2985.95	13.727
169	6.266	-13.831	-0.983	213.489	13.154
170	-2.659	4.678	-0.724	214.482	5.063
171	7.523	-1.016	-6.294	214.404	17.099
172	5.627	-0.541	-0.279	518.213	9.091
173	9.799	3.892	3.377	4.546	-20.548
174	10.692	1.53	2.705	6.381	-17.045
175	5.917	-0.514	-0.39	1067.614	8.419
176	9.378	4.066	3.287	5.6	-18.068
177	4.502	-12.769	-0.535	213.617	6.113
178	4.815	2.0	-0.059	214.177	4.291
179	6.94	-0.539	-0.264	616.384	12.126
180	6.879	-10.887	-23.628	214.358	15.204
181	8.62	3.752	6.202	2.399	-41.651
182	6.45	-0.541	-0.34	633.115	14.076
183	5.942	1.299	-0.079	214.043	15.559
184	5.773	46.176	-0.145	-5944.509	13.256
185	11.272	-6518774.952	311774.183	-28.564	-9.701
186	-2.666	8.467	-0.953	214.325	12.307
187	7.71	-14.778	-1.47	213.605	13.277
188	6.661	6383715.637	-38390.503	-499854175.969	9.724
189	7.498	-0.541	-33690173.017	408526626373.911	14.535
190	6.763	-0.012	-0.135	692.24	13.9
191	9.307	1.906	11.006	0.966	-9.48
192	7.638	-5.084	-0.18	213.55	16.618
193	8.942	0.685	12.728	0.93	-8.661
194	6.558	-11.366	-3.43	213.843	13.025
195	7.41	412.648	-1.962	-45643.507	18.208
196	7.274	0.66	-0.183	1011.485	15.796
197	2.99	2.488	1259.339	2512106.027	16.32
198	3.783	3.239	-54.831	-177012.127	19.874

199	NaN	NaN	NaN	NaN	NaN
200	NaN	NaN	NaN	NaN	NaN
201	NaN	NaN	NaN	NaN	NaN
202	NaN	NaN	NaN	NaN	NaN
203	18.264	4.307	-0.106	214.487	15.354
204	3.6	0.01	0.177	214.353	20.167
205	-20.423	3.25	-29.523	13378.983	19.066
206	NaN	NaN	NaN	NaN	NaN
207	NaN	NaN	NaN	NaN	NaN
208	5.382	-0.655	0.346	71.113	15.338
209	4.247	4.247	-0.07	71.133	12.573
210	-89.595	3.985	8293.57	-5468531.163	20.362
211	NaN	NaN	NaN	NaN	NaN
212	NaN	NaN	NaN	NaN	NaN
213	3.426	7.703	0.018	1902.728	17.824
214	0.033	8.06	0.025	280.436	-11.625
215	4.403	4.317	-0.979	-594.868	32.162
216	0.831	6.193	0.096	24.494	-33.605
217	4.398	219.96	1.92	573.966	15.361
218	5.399	0.992	0.584	54.69	-32.969
219	0.998	9.996	-0.034	-1338.901	-170.082
220	0.992	4.986	0.048	126.163	-70.437
221	0.1	5.251	0.086	-21.175	-56.405
222	5.839	6.06	-0.077	-671.074	25.138
223	1.001	6.681	0.431	45.43	-31.914
224	1.014	1.014	-4.583	532.454	-160.231
225	3.788	-46.003	-0.047	3176.171	32.122
226	0.897	6.468	0.101	43.074	-22.3
227	1.053	6.6	0.351	13.569	-31.876
228	1.021	1.026	-0.13	98.938	-97.923
229	1.019	1.019	-0.002	-475.41	-148.592
230	5.429	0.885	0.045	543.013	-51.2
231	0.99	6.398	0.151	33.028	-42.293
232	5.522	0.263	0.232	59.234	-26.714
233	-0.117	6.481	0.361	-3.569	-29.053
234	1.01	3.839	0.104	80.313	-66.916
235	4.078	0.996	0.145	378.139	-56.378
236	0.973	5.218	0.14	90.897	-45.719
237	6.228	-13.701	0.177	70.048	-19.768
238	4.162	9.895	-0.296	-136.245	15.971



239	4.189	7.43	-0.067	-130.428	22.601
240	1.075	0.062	0.0	76.141	-209.238
241	0.588	4.678	0.04	-9.954	-68.078
242	0.707	4.832	0.181	2.357	-27.795
243	4.238	5.933	-0.097	-49.941	12.926
244	3.494	5.413	0.579	102.279	9.948
245	1.005	1.035	0.0	78.945	-191.966
246	5.572	0.91	0.064	352.817	-50.871
247	0.992	5.861	0.17	24.778	-25.558
248	5.766	-1215095956133.912	1085353830.651	-2148.105	-27.922
249	-0.226	6.034	0.269	-7.197	-53.57
250	-2351.688	28780.735	0.037	-36843.495	-10.625
251	12.973	142.987	-0.325	-9207.776	66.01
252	4.058	22.745	0.57	8.186	-14.166
253	15.145	2567.852	29.445	9615.969	63.637
254	15.735	60.062	-0.583	-1282.942	56.205
255	6.741	-16.032	0.173	-648.894	26.441
256	19.874	-20.234	0.195	194.661	-66.502
257	20.736	-17.128	0.509	76.835	-11.344
258	2.334	21.358	0.854	-2.203	-27.814
259	3.869	21.74	1.593	2.839	-20.861
260	15.26	225.444	0.599	1763.852	55.105
261	4.205	10.843	0.098	76.843	-15.62
262	14.329	42.4	0.092	11322.691	72.774
263	3.424	20.494	0.521	2.894	-8.098
264	4.118	21.221	0.966	2.137	-35.073
265	21.242	-34.804	1.527	26.182	-7.493
266	15.973	143.982	0.452	6585.188	43.24
267	0.061	1.355	0.01	11.587	-21.432
268	-929.895	6979.469	0.009	-170901.986	-17.464
269	0.075	1.814	0.014	27.608	-25.34
270	0.782	215.06	-442.389	-174149.146	-5.197
271	0.99	0.623	10.852	-166.057	-10.364
272	1.06	-3011.845	-39.463	10437.716	-6.742
273	0.522	1.789	0.009	11.554	-33.537
274	0.587	1.615	0.013	25.19	-30.933
275	-7281.137	13.75	345.25	-120.552	-4.729
276	8.378	1234.242	7.687	6560.772	28.772
277	NaN	NaN	NaN	NaN	NaN
278	-0.264	14.561	0.394	23.561	-1.304

279	NaN	NaN	NaN	NaN	NaN
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Table 4: Best fitted mathematical models for each ID dataset and their sample size, AIC,  $AIC_c$ ,  $\Delta AIC_c$ ,  $w_i$  values

ID	Model	Sample Size	AIC	$AIC_c$	$\Delta AIC_c$	$w_i$
0	Gompertz	28	-154.62	-188.24	0	1
1	Gompertz	28	-210.68	-256.48	0	1
2	Gompertz	28	-157.37	-191.58	0	1
3	Gompertz	30	-82.05	-98.46	0	0.69
5	Gompertz	28	-168.65	-205.31	0	1
6	Gompertz	28	-126.8	-154.36	0	1
7	Gompertz	28	-123.49	-150.34	0	0.97
8	Gompertz	28	-147.61	-179.7	0	0.93
9	Cubic	28	-127.72	-155.48	0	1
10	Gompertz	28	-151.78	-184.78	0	1
11	Gompertz	28	-149.15	-181.57	0	1
12	Gompertz	28	-177.89	-216.56	0	1
13	Cubic	28	-69.79	-84.96	0	0.69
15	Gompertz	28	-156.93	-191.04	0	1
17	Cubic	28	-143.46	-174.65	0	0.57
18	Cubic	28	-115.77	-140.94	0	0.57
19	Cubic	28	-67.1	-81.69	0	0.8
21	Gompertz	28	-156.06	-189.99	0	0.99
22	Gompertz	21	-111.13	-145.85	0	1
23	Gompertz	28	-132.21	-160.96	0	1
24	Cubic	26	-32.28	-39.97	0	1
25	Cubic	26	-20.59	-25.49	0	1
26	Cubic	22	-63.24	-81.84	0	1
27	Cubic	23	-72.55	-92.7	0	1
28	Cubic	23	-38.83	-49.61	0	1
29	Gompertz	23	-101.36	-129.51	0	1
30	Cubic	21	-70.15	-92.08	0	1
31	Cubic	25	-19.28	-24.11	0	1
32	Cubic	23	-31.63	-40.42	0	1
33	Gompertz	23	-37.12	-47.43	0	1
34	Cubic	32	-54.64	-64.75	0	1
35	Cubic	25	-38.63	-48.28	0	1
36	Cubic	23	-16.33	-20.86	0	1
37	Cubic	23	-9.44	-12.06	0	1
38	Gompertz	23	-49.75	-63.57	0	1
39	Cubic	21	-71.29	-93.56	0	1
40	Cubic	22	-45.13	-58.4	0	1

41	Gompertz	32	-49.59	-58.78	0	1
42	Gompertz	21	-62.62	-82.19	0	1
43	Cubic	23	-83.91	-107.22	0	1
44	Cubic	12	-18.9	-32.4	0	1
45	Cubic	17	-55.29	-78.33	0	1
46	Cubic	15	-34.31	-51.47	0	1
47	Cubic	15	-20.77	-31.16	0	0.97
48	Cubic	14	-21.73	-33.81	0	1
49	Cubic	15	-20.8	-31.2	0	1
50	Gompertz	13	-28.26	-45.93	0	0.95
51	Gompertz	12	-54.05	-92.66	0	1
52	Cubic	13	-27.45	-44.61	0	1
53	Cubic	14	-18.96	-29.49	0	0.99
54	Cubic	14	-31.31	-48.71	0	1
55	Cubic	16	-38.57	-56.1	0	1
56	Cubic	16	-6.86	-9.98	0	0.89
57	Cubic	16	-10.55	-15.35	0	0.99
58	Quadratic	17	-14.58	-19.06	0	0.61
59	Gompertz	16	-47.13	-68.55	0	1
60	Cubic	15	-11.83	-17.74	0	0.99
61	Cubic	14	-27.71	-43.11	0	1
62	Cubic	13	-23.69	-38.5	0	1
63	Cubic	16	-8.68	-12.63	0	0.93
64	Cubic	15	-9.44	-14.16	0	0.98
65	Gompertz	16	-25.79	-37.52	0	0.97
66	Cubic	14	-12.83	-19.96	0	0.92
67	Cubic	16	-5.23	-7.61	0	0.88
68	Cubic	16	-4.49	-6.54	0	1
69	Cubic	14	-5.04	-7.84	0	0.99
70	Quadratic	15	26.16	35.67	0	0.56
71	Cubic	17	-12.27	-17.39	0	1
72	Cubic	17	-16.54	-23.43	0	0.99
73	Cubic	15	-24.23	-36.34	0	1
74	Cubic	16	-15.68	-22.81	0	0.99
75	Cubic	16	-16.41	-23.87	0	0.99
76	Cubic	15	-29.19	-43.79	0	1
77	Cubic	16	-33.33	-48.48	0	1
78	Cubic	18	4.65	6.43	0	0.74
79	Cubic	18	5.31	7.35	0	0.95
80	Cubic	16	-20.49	-29.81	0	1

81	Cubic	17	5.86	8.3	0	0.97
82	Cubic	17	-2.06	-2.92	0	0.95
83	Cubic	15	-21.06	-31.6	0	0.99
84	Cubic	15	-16.28	-24.42	0	1
85	Cubic	17	-3.44	-4.88	0	0.74
86	Cubic	17	-9.53	-13.5	0	0.98
87	Cubic	17	-13.01	-18.44	0	1
89	Cubic	54	-66.66	-73.46	0	0.92
90	Gompertz	63	-199.14	-216.31	0	1
91	Cubic	66	-129.48	-140.09	0	1
92	Cubic	57	-21.7	-23.79	0	0.89
93	Cubic	9	-34.48	-77.57	0	0.99
94	Gompertz	8	-12.5	-33.34	0	0.99
95	Cubic	14	-22.93	-35.67	0	1
96	Cubic	11	-18.3	-33.56	0	1
97	Cubic	9	-11.65	-26.22	0	0.96
98	Cubic	9	-24.14	-54.31	0	1
99	Gompertz	10	-31.32	-62.63	0	0.6
100	Cubic	6	-19.34	-116.03	0	1
101	Cubic	13	-26.37	-42.84	0	0.78
102	Cubic	13	-39.4	-64.03	0	1
103	Cubic	12	-43.41	-74.42	0	1
104	Cubic	9	-19.28	-43.38	0	1
105	Gompertz	10	-33.18	-66.36	0	1
106	Cubic	7	-12.62	-44.17	0	0.99
107	Gompertz	13	-45.27	-73.56	0	1
108	Gompertz	12	-25.73	-44.11	0	0.62
109	Cubic	11	-32.84	-60.2	0	0.97
110	Cubic	9	-27.07	-60.91	0	1
111	Gompertz	11	-27.29	-50.04	0	1
112	Gompertz	7	-8.4	-29.4	0	0.99
113	Cubic	10	-32	-64.01	0	1
114	Cubic	13	-20.89	-33.94	0	1
115	Cubic	12	-25.85	-44.32	0	1
116	Cubic	9	-25.25	-56.81	0	1
117	Gompertz	11	-42.86	-78.58	0	1
118	Cubic	7	-5.56	-19.45	0	0.94
119	Gompertz	11	-45.8	-83.96	0	1
120	Cubic	13	-19.63	-31.89	0	1
121	Cubic	12	-41.67	-71.44	0	1

122	Cubic	8	-38.6	-102.93	0	0.94
123	Gompertz	10	-20.75	-41.5	0	1
124	Cubic	6	-16.41	-98.48	0	1
125	Cubic	13	-21.12	-34.32	0	1
126	Cubic	11	-18.87	-34.59	0	0.85
127	Cubic	12	-21.7	-37.2	0	0.74
128	Cubic	9	-12.29	-27.65	0	1
129	Gompertz	10	-31.51	-63.03	0	0.95
130	Cubic	6	-14.56	-87.36	0	1
131	Cubic	13	-23.35	-37.95	0	1
132	Cubic	12	-14.51	-24.88	0	0.99
133	Cubic	10	-23.96	-47.92	0	1
134	Cubic	8	-20.31	-54.16	0	1
135	Gompertz	10	-25	-50.01	0	1
136	Gompertz	6	-19.13	-114.81	0	1
137	Cubic	14	-36.94	-57.47	0	0.92
138	Cubic	12	-27.82	-47.7	0	1
139	Cubic	9	-21.74	-48.91	0	1
140	Cubic	6	-14.02	-84.11	0	0.98
141	Gompertz	9	-25.46	-57.29	0	1
142	Gompertz	7	-18.65	-65.27	0	0.99
143	Cubic	13	-23.82	-38.71	0	1
144	Cubic	12	-14.57	-24.98	0	0.99
145	Cubic	9	-23.3	-52.43	0	1
146	Cubic	9	-25.66	-57.72	0	1
147	Gompertz	10	-32.6	-65.19	0	0.97
148	Cubic	14	-23.49	-36.54	0	0.83
149	Gompertz	7	-20.36	-71.27	0	1
150	Cubic	10	-15.41	-30.81	0	1
151	Cubic	9	-16.31	-36.69	0	1
152	Cubic	8	-22.27	-59.39	0	1
153	Cubic	10	-32.05	-64.09	0	1
154	Gompertz	7	-18.25	-63.89	0	1
155	Gompertz	13	-44.68	-72.6	0	1
156	Cubic	12	-22.12	-37.92	0	0.99
157	Cubic	8	-14.65	-39.06	0	1
158	Gompertz	8	-21.08	-56.2	0	0.97
159	Cubic	9	-22.14	-49.82	0	1
160	Cubic	6	-9.9	-59.4	0	1
161	Cubic	11	-18.16	-33.29	0	1

162	Gompertz	11	-16.62	-30.47	0	0.97
163	Gompertz	9	-26.37	-59.32	0	1
164	Cubic	10	-40.03	-80.05	0	1
165	Cubic	7	-6.74	-23.59	0	1
166	Gompertz	6	-20.64	-123.85	0	0.95
167	Cubic	7	-45.64	-159.73	0	1
168	Cubic	6	-24.27	-145.63	0	1
169	Cubic	6	-9.4	-56.42	0	1
170	Cubic	6	-30.51	-183.07	0	1
171	Cubic	9	-13.77	-30.98	0	1
172	Cubic	9	-32.82	-73.84	0	1
173	Cubic	6	-21.31	-127.86	0	0.91
174	Gompertz	6	-17.04	-102.27	0	0.62
175	Cubic	6	-25.4	-152.4	0	1
176	Gompertz	7	-18.07	-63.24	0	1
177	Cubic	7	-31.13	-108.96	0	1
178	Cubic	7	-27.3	-95.54	0	1
182	Cubic	7	-19.37	-67.78	0	1
183	Cubic	7	-23.05	-80.68	0	1
184	Cubic	7	-18.04	-63.13	0	1
188	Cubic	6	-20.27	-121.6	0	1
189	Cubic	6	-21.14	-126.85	0	1
190	Cubic	6	-8.97	-53.83	0	1
192	Cubic	6	-7.65	-45.9	0	1
193	Cubic	6	-18.85	-113.13	0	1
194	Cubic	7	-38.08	-133.28	0	1
195	Cubic	7	-18.6	-65.09	0	1
196	Cubic	7	-20.57	-71.98	0	1
197	Cubic	6	-20.11	-120.68	0	1
198	Cubic	7	-9.72	-34.03	0	1
204	Cubic	7	-7.29	-25.53	0	1
205	Cubic	5	-3.74	-13.09	0	1
208	Gompertz	6	15.34	-61.35	0	1
209	Cubic	5	-18.47	-110.81	0	1
213	Gompertz	6	17.82	-71.29	0	1
214	Quadratic	6	-2.47	7.4	0	0.53
215	Cubic	19	-50.68	-304.11	0	1
216	Gompertz	13	-33.6	-201.63	0	1
217	Quadratic	7	-9.22	-11.67	0	0.72
218	Gompertz	10	-32.97	-53.57	0	1

219	Cubic	17	-181.43	-634.99	0	1
220	Gompertz	13	-70.44	-140.87	0	1
221	Gompertz	9	-56.4	-79.91	0	0.91
222	Cubic	14	-22.21	-36.09	0	1
223	Gompertz	11	-31.91	-71.81	0	1
224	Cubic	16	-164.97	-256.62	0	0.98
225	Cubic	17	-41.14	-75.43	0	1
226	Gompertz	12	-22.3	-32.44	0	0.99
227	Gompertz	9	-31.88	-45.16	0	1
228	Cubic	10	-99.72	-170.94	0	0.82
229	Cubic	16	-168.27	-378.61	0	1
230	Gompertz	17	-51.2	-102.4	0	1
231	Gompertz	11	-42.29	-61.52	0	1
232	Gompertz	8	-26.71	-37.85	0	1
233	Gompertz	5	-29.05	-53.26	0	1
234	Gompertz	8	-66.92	-178.44	0	1
236	Gompertz	10	-45.72	-121.92	0	1
237	Gompertz	8	-19.77	-30.75	0	0.6
238	Cubic	9	-8.89	-17.79	0	0.86
239	Quadratic	9	4.42	8.85	0	0.98
240	Cubic	19	-210.37	-473.34	0	0.78
241	Gompertz	17	-68.08	-153.18	0	1
242	Gompertz	8	-27.8	-37.72	0	0.99
243	Cubic	8	-10.16	-14.39	0	0.96
244	Cubic	7	-5.7	-15.21	0	1
245	Cubic	18	-193.59	-516.24	0	0.9
246	Gompertz	14	-50.87	-178.05	0	1
247	Gompertz	7	-25.56	-35.39	0	1
248	Gompertz	8	-27.92	-43.43	0	1
249	Gompertz	8	-53.57	-187.5	0	1
250	Cubic	10	-22.59	-60.25	0	1
251	Cubic	16	-17.27	-46.04	0	1
252	Cubic	18	-20.6	-41.21	0	1
253	Cubic	16	-18.04	-26.23	0	1
254	Cubic	13	-17.82	-24.68	0	1
255	Cubic	9	-30.64	-44.56	0	0.86
256	Gompertz	20	-66.5	-108.07	0	1
257	Gompertz	14	-11.34	-25.52	0	0.65
258	Gompertz	17	-27.81	-37.09	0	0.84
259	Gompertz	15	-20.86	-32.45	0	0.97



260	Cubic	13	-18.17	-25.74	0	0.98
261	Gompertz	9	-15.62	-23.43	0	0.98
262	Cubic	19	-26.36	-42.84	0	0.91
263	Gompertz	12	-8.1	-18.22	0	0.95
264	Gompertz	13	-35.07	-47.6	0	1
265	Gompertz	12	-7.49	-12.85	0	0.61
266	Cubic	10	-19.28	-31.33	0	0.99
267	Cubic	7	-21.63	-37.08	0	0.47
268	Cubic	7	-25.95	-51.9	0	0.98
269	Cubic	6	-34.97	-122.39	0	1
270	Cubic	6	-32.57	-114	0	1
271	Cubic	7	-24.09	-144.55	0	1
272	Cubic	7	-19.76	-118.56	0	1
273	Gompertz	7	-33.54	-117.38	0	0.9
274	Gompertz	7	-30.93	-108.26	0	0.98
275	Cubic	7	-5.14	-17.98	0	0.67
276	Cubic	7	1.45	5.07	0	0.81
278	Cubic	5	-12.92	-45.23	0	1

Table 5: All mathematical models for each ID dataset and their sample size, AIC,  $AIC_c$ ,  $\Delta AIC_c$ ,  $w_i$  values

ID	Model	Sample Size	AIC	$AIC_c$	$\Delta AIC_c$	$w_i$
0	Cubic	28	-122.25	-148.83	39.41	0
0	Quadratic	28	-77.88	-90.86	97.38	0
0	Gompertz	28	-154.62	-188.24	0	1
1	Cubic	28	-88.28	-107.48	149	0
1	Quadratic	28	-65.63	-76.57	179.91	0
1	Gompertz	28	-210.68	-256.48	0	1
2	Cubic	28	-53.13	-64.68	126.9	0
2	Quadratic	28	-28.26	-32.97	158.61	0
2	Gompertz	28	-157.37	-191.58	0	1
3	Cubic	30	-80.74	-96.89	1.57	0.31
3	Quadratic	30	-42.56	-49.11	49.35	0
3	Gompertz	30	-82.05	-98.46	0	0.69
5	Cubic	28	-148.32	-180.57	24.74	0
5	Quadratic	28	-110.24	-128.61	76.69	0
5	Gompertz	28	-168.65	-205.31	0	1
6	Cubic	28	-109.09	-132.8	21.56	0
6	Quadratic	28	-56.31	-65.7	88.66	0
6	Gompertz	28	-126.8	-154.36	0	1
7	Cubic	28	-117.72	-143.32	7.02	0.03
7	Quadratic	28	-82.28	-95.99	54.35	0
7	Gompertz	28	-123.49	-150.34	0	0.97
8	Cubic	28	-143.24	-174.37	5.33	0.07
8	Quadratic	28	-122.21	-142.57	37.13	0
8	Gompertz	28	-147.61	-179.7	0	0.93
9	Cubic	28	-127.72	-155.48	0	1
9	Quadratic	28	-100.12	-116.81	38.67	0
9	Gompertz	28	-108.96	-132.65	22.83	0
10	Cubic	28	-125.44	-152.71	32.07	0
10	Quadratic	28	-65.75	-76.71	108.07	0
10	Gompertz	28	-151.78	-184.78	0	1
11	Cubic	28	-136.95	-166.73	14.84	0
11	Quadratic	28	-95.47	-111.39	70.19	0
11	Gompertz	28	-149.15	-181.57	0	1
12	Cubic	28	-127.21	-154.86	61.7	0
12	Quadratic	28	-83.72	-97.68	118.88	0
12	Gompertz	28	-177.89	-216.56	0	1
13	Cubic	28	-69.79	-84.96	0	0.69

13	Quadratic	28	-71.43	-83.34	1.62	0.31
13	Gompertz	28	-48.29	-58.79	26.17	0
15	Cubic	28	-79.9	-97.28	93.76	0
15	Quadratic	28	-37.66	-43.94	147.1	0
15	Gompertz	28	-156.93	-191.04	0	1
17	Cubic	28	-143.46	-174.65	0	0.57
17	Quadratic	28	-143.52	-167.44	7.21	0.02
17	Gompertz	28	-142.94	-174.02	0.63	0.42
18	Cubic	28	-115.77	-140.94	0	0.57
18	Quadratic	28	-105.36	-122.92	18.01	0
18	Gompertz	28	-115.31	-140.37	0.57	0.43
19	Cubic	28	-67.1	-81.69	0	0.8
19	Quadratic	28	-67.6	-78.87	2.82	0.2
19	Gompertz	28	-48.26	-58.75	22.94	0
21	Cubic	28	-147.82	-179.95	10.03	0.01
21	Quadratic	28	-127.29	-148.5	41.49	0
21	Gompertz	28	-156.06	-189.99	0	0.99
22	Cubic	21	-88.4	-116.02	29.83	0
22	Quadratic	21	-55.48	-68.53	77.32	0
22	Gompertz	21	-111.13	-145.85	0	1
23	Cubic	28	-36.14	-43.99	116.96	0
23	Quadratic	28	-27.7	-32.32	128.64	0
23	Gompertz	28	-132.21	-160.96	0	1
24	Cubic	26	-32.28	-39.97	0	1
24	Quadratic	26	-13.91	-16.44	23.53	0
24	Gompertz	26	0.78	0.96	40.93	0
25	Cubic	26	-20.59	-25.49	0	1
25	Quadratic	26	-6.11	-7.22	18.27	0
25	Gompertz	26	3.57	4.42	29.91	0
26	Cubic	22	-63.24	-81.84	0	1
26	Quadratic	22	-39.26	-47.99	33.85	0
26	Gompertz	22	-5.7	-7.38	74.45	0
27	Cubic	23	-72.55	-92.7	0	1
27	Quadratic	23	-42.71	-51.71	40.99	0
27	Gompertz	23	2.59	3.3	96	0
28	Cubic	23	-38.83	-49.61	0	1
28	Quadratic	23	-16.17	-19.57	30.04	0
28	Gompertz	23	11.16	14.26	63.87	0
29	Cubic	23	-40.03	-51.15	78.36	0
29	Quadratic	23	-25.24	-30.55	98.96	0

29	Gompertz	23	-101.36	-129.51	0	1
30	Cubic	21	-70.15	-92.08	0	1
30	Quadratic	21	-61.71	-76.23	15.84	0
30	Gompertz	21	-47.62	-62.5	29.58	0
31	Cubic	25	-19.28	-24.11	0	1
31	Quadratic	25	-10.69	-12.73	11.38	0
31	Gompertz	25	5.07	6.34	30.45	0
32	Cubic	23	-31.63	-40.42	0	1
32	Quadratic	23	-21.66	-26.22	14.2	0
32	Gompertz	23	-4.02	-5.14	35.28	0
33	Cubic	23	-24.02	-30.7	16.73	0
33	Quadratic	23	-12.31	-14.9	32.53	0
33	Gompertz	23	-37.12	-47.43	0	1
34	Cubic	32	-54.64	-64.75	0	1
34	Quadratic	32	-43.71	-49.95	14.8	0
34	Gompertz	32	-5.85	-6.93	57.82	0
35	Cubic	25	-38.63	-48.28	0	1
35	Quadratic	25	-21.6	-25.72	22.56	0
35	Gompertz	25	-0.23	-0.29	47.99	0
36	Cubic	23	-16.33	-20.86	0	1
36	Quadratic	23	-1.3	-1.58	19.29	0
36	Gompertz	23	15.42	19.71	40.57	0
37	Cubic	23	-9.44	-12.06	0	1
37	Quadratic	23	6	7.26	19.32	0
37	Gompertz	23	25.53	32.62	44.68	0
38	Cubic	23	-28.66	-36.63	26.95	0
38	Quadratic	23	-12.85	-15.56	48.02	0
38	Gompertz	23	-49.75	-63.57	0	1
39	Cubic	21	-71.29	-93.56	0	1
39	Quadratic	21	-50.94	-62.92	30.64	0
39	Gompertz	21	-32.99	-43.3	50.27	0
40	Cubic	22	-45.13	-58.4	0	1
40	Quadratic	22	-24.19	-29.57	28.83	0
40	Gompertz	22	-18.7	-24.2	34.19	0
41	Cubic	32	-31.24	-37.03	21.75	0
41	Quadratic	32	-23.92	-27.34	31.44	0
41	Gompertz	32	-49.59	-58.78	0	1
42	Cubic	21	-47.43	-62.26	19.93	0
42	Quadratic	21	-32.03	-39.57	42.62	0
42	Gompertz	21	-62.62	-82.19	0	1

43	Cubic	23	-83.91	-107.22	0	1
43	Quadratic	23	-41.79	-50.59	56.63	0
43	Gompertz	23	-40.85	-52.2	55.02	0
44	Cubic	12	-18.9	-32.4	0	1
44	Quadratic	12	-13.64	-20.46	11.94	0
44	Gompertz	12	10.88	18.64	51.04	0
45	Cubic	17	-55.29	-78.33	0	1
45	Quadratic	17	-36.9	-48.26	30.07	0
45	Gompertz	17	12.28	17.4	95.73	0
46	Cubic	15	-34.31	-51.47	0	1
46	Quadratic	15	-27.51	-37.51	13.96	0
46	Gompertz	15	15.05	22.58	74.04	0
47	Cubic	15	-20.77	-31.16	0	0.97
47	Quadratic	15	-17.61	-24.02	7.14	0.03
47	Gompertz	15	18.56	27.85	59.01	0
48	Cubic	14	-21.73	-33.81	0	1
48	Quadratic	14	-12.09	-16.92	16.88	0
48	Gompertz	14	15.61	24.28	58.08	0
49	Cubic	15	-20.8	-31.2	0	1
49	Quadratic	15	-13.69	-18.67	12.53	0
49	Gompertz	15	13.24	19.86	51.06	0
50	Cubic	13	-24.6	-39.98	5.95	0.05
50	Quadratic	13	-16.02	-23.14	22.79	0
50	Gompertz	13	-28.26	-45.93	0	0.95
51	Cubic	12	-23.84	-40.87	51.79	0
51	Quadratic	12	-15.47	-23.21	69.45	0
51	Gompertz	12	-54.05	-92.66	0	1
52	Cubic	13	-27.45	-44.61	0	1
52	Quadratic	13	-19.79	-28.58	16.03	0
52	Gompertz	13	-3.16	-5.14	39.47	0
53	Cubic	14	-18.96	-29.49	0	0.99
53	Quadratic	14	-14.29	-20.01	9.48	0.01
53	Gompertz	14	4.25	6.6	36.09	0
54	Cubic	14	-31.31	-48.71	0	1
54	Quadratic	14	-25.65	-35.92	12.79	0
54	Gompertz	14	-1.14	-1.77	46.94	0
55	Cubic	16	-38.57	-56.1	0	1
55	Quadratic	16	-24.38	-32.51	23.59	0
55	Gompertz	16	13.21	19.22	75.32	0
56	Cubic	16	-6.86	-9.98	0	0.89

56	Quadratic	16	-4.41	-5.87	4.1	0.11
56	Gompertz	16	4.71	6.85	16.83	0
57	Cubic	16	-10.55	-15.35	0	0.99
57	Quadratic	16	-3.08	-4.11	11.24	0
57	Gompertz	16	-4.17	-6.06	9.29	0.01
58	Cubic	17	-12.85	-18.2	0.86	0.39
58	Quadratic	17	-14.58	-19.06	0	0.61
58	Gompertz	17	37.98	53.8	72.87	0
59	Cubic	16	-12.57	-18.29	50.27	0
59	Quadratic	16	-1.98	-2.63	65.92	0
59	Gompertz	16	-47.13	-68.55	0	1
60	Cubic	15	-11.83	-17.74	0	0.99
60	Quadratic	15	-5.32	-7.25	10.49	0.01
60	Gompertz	15	13.11	19.67	37.41	0
61	Cubic	14	-27.71	-43.11	0	1
61	Quadratic	14	-22.54	-31.56	11.55	0
61	Gompertz	14	-4.33	-6.73	36.37	0
62	Cubic	13	-23.69	-38.5	0	1
62	Quadratic	13	-14.01	-20.24	18.26	0
62	Gompertz	13	12.33	20.04	58.55	0
63	Cubic	16	-8.68	-12.63	0	0.93
63	Quadratic	16	-5.57	-7.42	5.21	0.07
63	Gompertz	16	4.67	6.8	19.43	0
64	Cubic	15	-9.44	-14.16	0	0.98
64	Quadratic	15	-4.52	-6.16	8	0.02
64	Gompertz	15	21.84	32.76	46.92	0
65	Cubic	16	-20.8	-30.25	7.27	0.03
65	Quadratic	16	-13.66	-18.22	19.3	0
65	Gompertz	16	-25.79	-37.52	0	0.97
66	Cubic	14	-12.83	-19.96	0	0.92
66	Quadratic	14	-10.7	-14.98	4.98	0.08
66	Gompertz	14	10.57	16.44	36.39	0
67	Cubic	16	-5.23	-7.61	0	0.88
67	Quadratic	16	-2.76	-3.68	3.93	0.12
67	Gompertz	16	6.45	9.38	16.99	0
68	Cubic	16	-4.49	-6.54	0	1
68	Quadratic	16	4.26	5.68	12.22	0
68	Gompertz	16	19.96	29.04	35.58	0
69	Cubic	14	-5.04	-7.84	0	0.99
69	Quadratic	14	1.85	2.59	10.43	0.01

69	Gompertz	14	11.94	18.57	26.41	0
70	Cubic	15	24.12	36.19	0.52	0.44
70	Quadratic	15	26.16	35.67	0	0.56
70	Gompertz	15	36.2	54.3	18.63	0
71	Cubic	17	-12.27	-17.39	0	1
71	Quadratic	17	-4.51	-5.9	11.49	0
71	Gompertz	17	15.58	22.07	39.46	0
72	Cubic	17	-16.54	-23.43	0	0.99
72	Quadratic	17	-9.92	-12.97	10.46	0.01
72	Gompertz	17	9.56	13.54	36.98	0
73	Cubic	15	-24.23	-36.34	0	1
73	Quadratic	15	-16.79	-22.9	13.44	0
73	Gompertz	15	-2.3	-3.46	32.89	0
74	Cubic	16	-15.68	-22.81	0	0.99
74	Quadratic	16	-10.65	-14.2	8.61	0.01
74	Gompertz	16	7.59	11.05	33.86	0
75	Cubic	16	-16.41	-23.87	0	0.99
75	Quadratic	16	-10.05	-13.41	10.46	0.01
75	Gompertz	16	6.4	9.3	33.17	0
76	Cubic	15	-29.19	-43.79	0	1
76	Quadratic	15	-19.93	-27.18	16.61	0
76	Gompertz	15	-0.09	-0.13	43.66	0
77	Cubic	16	-33.33	-48.48	0	1
77	Quadratic	16	-22.08	-29.44	19.04	0
77	Gompertz	16	-4.5	-6.55	41.93	0
78	Cubic	18	4.65	6.43	0	0.74
78	Quadratic	18	6.63	8.53	2.1	0.26
78	Gompertz	18	13.12	18.17	11.74	0
79	Cubic	18	5.31	7.35	0	0.95
79	Quadratic	18	10.42	13.39	6.05	0.05
79	Gompertz	18	17.64	24.43	17.08	0
80	Cubic	16	-20.49	-29.81	0	1
80	Quadratic	16	-13.61	-18.14	11.66	0
80	Gompertz	16	4.08	5.94	35.74	0
81	Cubic	17	5.86	8.3	0	0.97
81	Quadratic	17	11.9	15.56	7.25	0.03
81	Gompertz	17	19.72	27.93	19.63	0
82	Cubic	17	-2.06	-2.92	0	0.95
82	Quadratic	17	2.36	3.09	6.01	0.05
82	Gompertz	17	7.75	10.98	13.9	0

83	Cubic	15	-21.06	-31.6	0	0.99
83	Quadratic	15	-15.52	-21.17	10.43	0.01
83	Gompertz	15	-3.93	-5.9	25.7	0
84	Cubic	15	-16.28	-24.42	0	1
84	Quadratic	15	-9.33	-12.73	11.7	0
84	Gompertz	15	7.47	11.2	35.62	0
85	Cubic	17	-3.44	-4.88	0	0.74
85	Quadratic	17	-2.09	-2.74	2.14	0.26
85	Gompertz	17	12.29	17.41	22.29	0
86	Cubic	17	-9.53	-13.5	0	0.98
86	Quadratic	17	-4.31	-5.63	7.87	0.02
86	Gompertz	17	14.45	20.47	33.98	0
87	Cubic	17	-13.01	-18.44	0	1
87	Quadratic	17	-5.61	-7.34	11.1	0
87	Gompertz	17	11.72	16.6	35.04	0
89	Cubic	54	-66.66	-73.46	0	0.92
89	Quadratic	54	-63.59	-68.68	4.78	0.08
89	Gompertz	54	33.91	37.37	110.83	0
90	Cubic	63	-121.6	-132.08	84.23	0
90	Quadratic	63	-85.9	-91.72	124.59	0
90	Gompertz	63	-199.14	-216.31	0	1
91	Cubic	66	-129.48	-140.09	0	1
91	Quadratic	66	-121.25	-129.07	11.02	0
91	Gompertz	66	-46.19	-49.98	90.11	0
92	Cubic	57	-21.7	-23.79	0	0.89
92	Quadratic	57	-18.31	-19.69	4.09	0.11
92	Gompertz	57	20.84	22.84	46.63	0
93	Cubic	9	-34.48	-77.57	0	0.99
93	Quadratic	9	-18.05	-32.5	45.07	0
93	Gompertz	9	-30.14	-67.81	9.75	0.01
94	Cubic	8	-9.2	-24.54	8.8	0.01
94	Quadratic	8	-5.12	-10.24	23.09	0
94	Gompertz	8	-12.5	-33.34	0	0.99
95	Cubic	14	-22.93	-35.67	0	1
95	Quadratic	14	-6.3	-8.81	26.86	0
95	Gompertz	14	37.75	58.72	94.39	0
96	Cubic	11	-18.3	-33.56	0	1
96	Quadratic	11	-4.93	-7.75	25.8	0
96	Gompertz	11	29.85	54.73	88.29	0
97	Cubic	9	-11.65	-26.22	0	0.96



97	Quadratic	9	-10.91	-19.63	6.58	0.04
97	Gompertz	9	21.43	48.22	74.44	0
98	Cubic	9	-24.14	-54.31	0	1
98	Quadratic	9	-13.99	-25.18	29.13	0
98	Gompertz	9	21.36	48.07	102.38	0
99	Cubic	10	-30.9	-61.79	0.84	0.4
99	Quadratic	10	-22.34	-37.24	25.39	0
99	Gompertz	10	-31.32	-62.63	0	0.6
100	Cubic	6	-19.34	-116.03	0	1
100	Quadratic	6	-19.3	-57.9	58.12	0
100	Gompertz	6	14.71	88.24	204.27	0
101	Cubic	13	-26.37	-42.84	0	0.78
101	Quadratic	13	-11.98	-17.3	25.54	0
101	Gompertz	13	-24.78	-40.26	2.58	0.22
102	Cubic	13	-39.4	-64.03	0	1
102	Quadratic	13	-29.16	-42.12	21.91	0
102	Gompertz	13	31.27	50.81	114.84	0
103	Cubic	12	-43.41	-74.42	0	1
103	Quadratic	12	-17.58	-26.36	48.05	0
103	Gompertz	12	30.05	51.52	125.94	0
104	Cubic	9	-19.28	-43.38	0	1
104	Quadratic	9	-16.93	-30.47	12.91	0
104	Gompertz	9	24.12	54.26	97.65	0
105	Cubic	10	-10.78	-21.56	44.79	0
105	Quadratic	10	-9.02	-15.04	51.31	0
105	Gompertz	10	-33.18	-66.36	0	1
106	Cubic	7	-12.62	-44.17	0	0.99
106	Quadratic	7	-14.4	-33.59	10.57	0.01
106	Gompertz	7	16.76	58.66	102.82	0
107	Cubic	13	-37.36	-60.71	12.85	0
107	Quadratic	13	-11.45	-16.54	57.02	0
107	Gompertz	13	-45.27	-73.56	0	1
108	Cubic	12	-24.9	-42.69	1.42	0.3
108	Quadratic	12	-26.67	-40.01	4.1	0.08
108	Gompertz	12	-25.73	-44.11	0	0.62
109	Cubic	11	-32.84	-60.2	0	0.97
109	Quadratic	11	-33.77	-53.06	7.14	0.03
109	Gompertz	11	28.71	52.63	112.84	0
110	Cubic	9	-27.07	-60.91	0	1
110	Quadratic	9	-25.25	-45.46	15.45	0

110	Gompertz	9	24.64	55.44	116.35	0
111	Cubic	11	-8.53	-15.64	34.4	0
111	Quadratic	11	-10.08	-15.84	34.2	0
111	Gompertz	11	-27.29	-50.04	0	1
112	Cubic	7	-5.52	-19.33	10.07	0.01
112	Quadratic	7	-7.31	-17.07	12.33	0
112	Gompertz	7	-8.4	-29.4	0	0.99
113	Cubic	10	-32	-64.01	0	1
113	Quadratic	10	-27.63	-46.05	17.95	0
113	Gompertz	10	-23.45	-46.89	17.11	0
114	Cubic	13	-20.89	-33.94	0	1
114	Quadratic	13	-13.73	-19.84	14.1	0
114	Gompertz	13	35.24	57.27	91.21	0
115	Cubic	12	-25.85	-44.32	0	1
115	Quadratic	12	-21.57	-32.36	11.96	0
115	Gompertz	12	33.29	57.07	101.39	0
116	Cubic	9	-25.25	-56.81	0	1
116	Quadratic	9	-18.64	-33.55	23.26	0
116	Gompertz	9	26.12	58.77	115.58	0
117	Cubic	11	-4.5	-8.25	70.32	0
117	Quadratic	11	-4.15	-6.53	72.05	0
117	Gompertz	11	-42.86	-78.58	0	1
118	Cubic	7	-5.56	-19.45	0	0.94
118	Quadratic	7	-6.04	-14.1	5.35	0.06
118	Gompertz	7	17.48	61.2	80.65	0
119	Cubic	11	-37.8	-69.3	14.66	0
119	Quadratic	11	-18.52	-29.1	54.86	0
119	Gompertz	11	-45.8	-83.96	0	1
120	Cubic	13	-19.63	-31.89	0	1
120	Quadratic	13	-12.56	-18.15	13.75	0
120	Gompertz	13	30.19	49.06	80.95	0
121	Cubic	12	-41.67	-71.44	0	1
121	Quadratic	12	-38.07	-57.11	14.33	0
121	Gompertz	12	27.32	46.84	118.28	0
122	Cubic	8	-38.6	-102.93	0	0.94
122	Quadratic	8	-26.51	-53.01	49.92	0
122	Gompertz	8	-36.52	-97.38	5.56	0.06
123	Cubic	10	-14.97	-29.93	11.57	0
123	Quadratic	10	-3.8	-6.33	35.17	0
123	Gompertz	10	-20.75	-41.5	0	1

124	Cubic	6	-16.41	-98.48	0	1
124	Quadratic	6	-13.16	-39.49	58.99	0
124	Gompertz	6	12.41	74.45	172.93	0
125	Cubic	13	-21.12	-34.32	0	1
125	Quadratic	13	-2.93	-4.23	30.09	0
125	Gompertz	13	33.75	54.85	89.17	0
126	Cubic	11	-18.87	-34.59	0	0.85
126	Quadratic	11	-19.83	-31.17	3.42	0.15
126	Gompertz	11	28.72	52.65	87.24	0
127	Cubic	12	-21.7	-37.2	0	0.74
127	Quadratic	12	-23.38	-35.08	2.13	0.26
127	Gompertz	12	33.52	57.47	94.67	0
128	Cubic	9	-12.29	-27.65	0	1
128	Quadratic	9	-7.54	-13.57	14.08	0
128	Gompertz	9	26.91	60.56	88.2	0
129	Cubic	10	-28.58	-57.16	5.86	0.05
129	Quadratic	10	-12.26	-20.44	42.59	0
129	Gompertz	10	-31.51	-63.03	0	0.95
130	Cubic	6	-14.56	-87.36	0	1
130	Quadratic	6	-12.46	-37.39	49.97	0
130	Gompertz	6	5.17	31.01	118.37	0
131	Cubic	13	-23.35	-37.95	0	1
131	Quadratic	13	-1.02	-1.48	36.47	0
131	Gompertz	13	34.03	55.3	93.25	0
132	Cubic	12	-14.51	-24.88	0	0.99
132	Quadratic	12	-10.05	-15.07	9.81	0.01
132	Gompertz	12	27.74	47.55	72.42	0
133	Cubic	10	-23.96	-47.92	0	1
133	Quadratic	10	-6.75	-11.25	36.67	0
133	Gompertz	10	26.98	53.97	101.89	0
134	Cubic	8	-20.31	-54.16	0	1
134	Quadratic	8	-13.33	-26.67	27.5	0
134	Gompertz	8	19.97	53.24	107.41	0
135	Cubic	10	-16.81	-33.63	16.38	0
135	Quadratic	10	-12.02	-20.03	29.98	0
135	Gompertz	10	-25	-50.01	0	1
136	Cubic	6	-17.27	-103.61	11.19	0
136	Quadratic	6	-17.13	-51.4	63.41	0
136	Gompertz	6	-19.13	-114.81	0	1
137	Cubic	14	-36.94	-57.47	0	0.92

137	Quadratic	14	-17.67	-24.74	32.73	0
137	Gompertz	14	-33.8	-52.57	4.9	0.08
138	Cubic	12	-27.82	-47.7	0	1
138	Quadratic	12	-17.3	-25.95	21.75	0
138	Gompertz	12	26.7	45.76	93.46	0
139	Cubic	9	-21.74	-48.91	0	1
139	Quadratic	9	-17.72	-31.9	17.01	0
139	Gompertz	9	20.7	46.58	95.49	0
140	Cubic	6	-14.02	-84.11	0	0.98
140	Quadratic	6	-14.89	-44.68	39.42	0
140	Gompertz	6	-12.63	-75.75	8.35	0.02
141	Cubic	9	-12.69	-28.55	28.74	0
141	Quadratic	9	0.9	1.63	58.92	0
141	Gompertz	9	-25.46	-57.29	0	1
142	Cubic	7	-15.84	-55.43	9.84	0.01
142	Quadratic	7	0.29	0.68	65.95	0
142	Gompertz	7	-18.65	-65.27	0	0.99
143	Cubic	13	-23.82	-38.71	0	1
143	Quadratic	13	1.51	2.18	40.88	0
143	Gompertz	13	36.13	58.71	97.42	0
144	Cubic	12	-14.57	-24.98	0	0.99
144	Quadratic	12	-10.74	-16.1	8.88	0.01
144	Gompertz	12	29.99	51.4	76.38	0
145	Cubic	9	-23.3	-52.43	0	1
145	Quadratic	9	-15.47	-27.84	24.59	0
145	Gompertz	9	24.25	54.57	107	0
146	Cubic	9	-25.66	-57.72	0	1
146	Quadratic	9	-14.75	-26.54	31.18	0
146	Gompertz	9	21.79	49.03	106.75	0
147	Cubic	10	-28.95	-57.9	7.29	0.03
147	Quadratic	10	-7.56	-12.6	52.59	0
147	Gompertz	10	-32.6	-65.19	0	0.97
148	Cubic	14	-23.49	-36.54	0	0.83
148	Quadratic	14	-0.8	-1.12	35.42	0
148	Gompertz	14	-21.45	-33.37	3.17	0.17
149	Cubic	7	-15.4	-53.92	17.35	0
149	Quadratic	7	-2.39	-5.59	65.68	0
149	Gompertz	7	-20.36	-71.27	0	1
150	Cubic	10	-15.41	-30.81	0	1
150	Quadratic	10	-5.85	-9.75	21.07	0

150	Gompertz	10	26.24	52.49	83.3	0
151	Cubic	9	-16.31	-36.69	0	1
151	Quadratic	9	-14.06	-25.3	11.39	0
151	Gompertz	9	24.53	55.19	91.88	0
152	Cubic	8	-22.27	-59.39	0	1
152	Quadratic	8	-16.74	-33.48	25.91	0
152	Gompertz	8	19.09	50.89	110.28	0
153	Cubic	10	-32.05	-64.09	0	1
153	Quadratic	10	-7.96	-13.26	50.83	0
153	Gompertz	10	23.86	47.72	111.81	0
154	Cubic	7	-14.33	-50.15	13.74	0
154	Quadratic	7	-2.94	-6.86	57.03	0
154	Gompertz	7	-18.25	-63.89	0	1
155	Cubic	13	-36.7	-59.64	12.96	0
155	Quadratic	13	-11.23	-16.23	56.37	0
155	Gompertz	13	-44.68	-72.6	0	1
156	Cubic	12	-22.12	-37.92	0	0.99
156	Quadratic	12	-18.44	-27.66	10.25	0.01
156	Gompertz	12	27.4	46.97	84.88	0
157	Cubic	8	-14.65	-39.06	0	1
157	Quadratic	8	-13.03	-26.07	12.99	0
157	Gompertz	8	20.11	53.63	92.69	0
158	Cubic	8	-18.5	-49.32	6.88	0.03
158	Quadratic	8	-5.92	-11.84	44.36	0
158	Gompertz	8	-21.08	-56.2	0	0.97
159	Cubic	9	-22.14	-49.82	0	1
159	Quadratic	9	-2.67	-4.81	45.01	0
159	Gompertz	9	23.49	52.86	102.68	0
160	Cubic	6	-9.9	-59.4	0	1
160	Quadratic	6	-0.38	-1.13	58.27	0
160	Gompertz	6	-0.13	-0.76	58.64	0
161	Cubic	11	-18.16	-33.29	0	1
161	Quadratic	11	3.65	5.74	39.04	0
161	Gompertz	11	35.17	64.47	97.76	0
162	Cubic	11	-12.67	-23.22	7.25	0.03
162	Quadratic	11	-1.77	-2.78	27.69	0
162	Gompertz	11	-16.62	-30.47	0	0.97
163	Cubic	9	-17.08	-38.44	20.89	0
163	Quadratic	9	-6.1	-10.97	48.35	0
163	Gompertz	9	-26.37	-59.32	0	1

164	Cubic	10	-40.03	-80.05	0	1
164	Quadratic	10	-7.21	-12.02	68.03	0
164	Gompertz	10	27.5	55	135.06	0
165	Cubic	7	-6.74	-23.59	0	1
165	Quadratic	7	-3.9	-9.1	14.49	0
165	Gompertz	7	17.6	61.59	85.18	0
166	Cubic	6	-19.68	-118.08	5.77	0.05
166	Quadratic	6	-19.69	-59.08	64.77	0
166	Gompertz	6	-20.64	-123.85	0	0.95
167	Cubic	7	-45.64	-159.73	0	1
167	Quadratic	7	-27.48	-64.11	95.62	0
167	Gompertz	7	-16.29	-57.03	102.7	0
168	Cubic	6	-24.27	-145.63	0	1
168	Quadratic	6	-24.66	-73.99	71.64	0
168	Gompertz	6	13.73	82.36	227.99	0
169	Cubic	6	-9.4	-56.42	0	1
169	Quadratic	6	-8.61	-25.82	30.6	0
169	Gompertz	6	13.15	78.93	135.35	0
170	Cubic	6	-30.51	-183.07	0	1
170	Quadratic	6	-25.96	-77.88	105.19	0
170	Gompertz	6	5.06	30.38	213.45	0
171	Cubic	9	-13.77	-30.98	0	1
171	Quadratic	9	-7.28	-13.11	17.87	0
171	Gompertz	9	17.1	38.47	69.45	0
172	Cubic	9	-32.82	-73.84	0	1
172	Quadratic	9	-33.63	-60.54	13.3	0
172	Gompertz	9	9.09	20.46	94.29	0
173	Cubic	6	-21.31	-127.86	0	0.91
173	Quadratic	6	-14.91	-44.72	83.14	0
173	Gompertz	6	-20.55	-123.29	4.57	0.09
174	Cubic	6	-16.88	-101.27	1	0.38
174	Quadratic	6	-18.71	-56.12	46.15	0
174	Gompertz	6	-17.04	-102.27	0	0.62
175	Cubic	6	-25.4	-152.4	0	1
175	Quadratic	6	-27.32	-81.95	70.45	0
175	Gompertz	6	8.42	50.52	202.92	0
176	Cubic	7	-9.75	-34.11	29.13	0
176	Quadratic	7	-6.86	-16.02	47.22	0
176	Gompertz	7	-18.07	-63.24	0	1
177	Cubic	7	-31.13	-108.96	0	1

177	Quadratic	7	-29.59	-69.04	39.92	0
177	Gompertz	7	6.11	21.39	130.36	0
178	Cubic	7	-27.3	-95.54	0	1
178	Quadratic	7	-18.94	-44.18	51.35	0
178	Gompertz	7	4.29	15.02	110.55	0
179	Cubic	5	-26.43	-Inf	NA	NA
179	Quadratic	5	-17.38	-86.91	Inf	NA
179	Gompertz	5	12.13	Inf	Inf	NA
180	Cubic	5	-15.75	-Inf	NA	NA
180	Quadratic	5	-9.42	-47.12	Inf	NA
180	Gompertz	5	15.2	Inf	Inf	NA
181	Cubic	5	-22.28	-Inf	NA	NA
181	Quadratic	5	-3.26	-16.31	Inf	NA
181	Gompertz	5	-41.65	-Inf	NA	NA
182	Cubic	7	-19.37	-67.78	0	1
182	Quadratic	7	-15.55	-36.28	31.5	0
182	Gompertz	7	14.08	49.27	117.05	0
183	Cubic	7	-23.05	-80.68	0	1
183	Quadratic	7	-21.25	-49.58	31.1	0
183	Gompertz	7	15.56	54.46	135.14	0
184	Cubic	7	-18.04	-63.13	0	1
184	Quadratic	7	-14.23	-33.21	29.92	0
184	Gompertz	7	13.26	46.39	109.53	0
185	Cubic	5	-9.9	-Inf	NA	NA
185	Quadratic	5	-11.42	-57.1	Inf	NA
185	Gompertz	5	-9.7	-Inf	NA	NA
186	Cubic	5	-24.52	-Inf	NA	NA
186	Quadratic	5	-26.51	-132.53	Inf	NA
186	Gompertz	5	12.31	Inf	Inf	NA
187	Cubic	5	-20.5	-Inf	NA	NA
187	Quadratic	5	-15.96	-79.78	Inf	NA
187	Gompertz	5	13.28	Inf	Inf	NA
188	Cubic	6	-20.27	-121.6	0	1
188	Quadratic	6	-22.07	-66.21	55.38	0
188	Gompertz	6	9.72	58.34	179.94	0
189	Cubic	6	-21.14	-126.85	0	1
189	Quadratic	6	-20.13	-60.38	66.47	0
189	Gompertz	6	14.54	87.21	214.06	0
190	Cubic	6	-8.97	-53.83	0	1
190	Quadratic	6	-7.76	-23.27	30.56	0

190	Gompertz	6	13.9	83.4	137.23	0
191	Cubic	5	-60.41	-Inf	NA	NA
191	Quadratic	5	-19.03	-95.15	Inf	NA
191	Gompertz	5	-9.48	-Inf	NA	NA
192	Cubic	6	-7.65	-45.9	0	1
192	Quadratic	6	-5.91	-17.74	28.16	0
192	Gompertz	6	16.62	99.71	145.61	0
193	Cubic	6	-18.85	-113.13	0	1
193	Quadratic	6	-12.11	-36.32	76.81	0
193	Gompertz	6	-8.66	-51.97	61.16	0
194	Cubic	7	-38.08	-133.28	0	1
194	Quadratic	7	-32.96	-76.9	56.38	0
194	Gompertz	7	13.03	45.59	178.87	0
195	Cubic	7	-18.6	-65.09	0	1
195	Quadratic	7	-19.92	-46.47	18.62	0
195	Gompertz	7	18.21	63.73	128.81	0
196	Cubic	7	-20.57	-71.98	0	1
196	Quadratic	7	-20.82	-48.57	23.41	0
196	Gompertz	7	15.8	55.29	127.27	0
197	Cubic	6	-20.11	-120.68	0	1
197	Quadratic	6	-12.15	-36.44	84.24	0
197	Gompertz	6	16.32	97.92	218.6	0
198	Cubic	7	-9.72	-34.03	0	1
198	Quadratic	7	-0.94	-2.2	31.83	0
198	Gompertz	7	19.87	69.56	103.59	0
199	Cubic	4	NA	NA	NA	NA
199	Quadratic	4	-14.52	-Inf	NA	NA
199	Gompertz	4	NA	NA	NA	NA
201	Cubic	4	NA	NA	NA	NA
201	Quadratic	4	-19.82	59.47	NA	NA
201	Gompertz	4	NA	NA	NA	NA
202	Cubic	4	NA	NA	NA	NA
202	Quadratic	4	-16.02	-Inf	NA	NA
202	Gompertz	4	NA	NA	NA	NA
203	Cubic	7	-18.42	73.68	Inf	NA
203	Quadratic	7	-17.61	-Inf	NA	NA
203	Gompertz	7	15.35	-61.41	Inf	NA
204	Cubic	7	-7.29	-25.53	0	1
204	Quadratic	7	-4.56	-10.65	14.89	0
204	Gompertz	7	20.17	70.58	96.12	0



205	Cubic	5	-3.74	-13.09	0	1
205	Quadratic	5	1.35	3.15	16.24	0
205	Gompertz	5	19.07	66.73	79.82	0
206	Cubic	4	NA	NA	NA	NA
206	Quadratic	4	1.08	5.39	NA	NA
206	Gompertz	4	NA	NA	NA	NA
207	Cubic	4	NA	NA	NA	NA
207	Quadratic	4	-13.88	-Inf	NA	NA
207	Gompertz	4	NA	NA	NA	NA
208	Cubic	6	-4.28	17.11	78.46	0
208	Quadratic	6	0.09	Inf	Inf	0
208	Gompertz	6	15.34	-61.35	0	1
209	Cubic	5	-18.47	-110.81	0	1
209	Quadratic	5	-20.12	-60.36	50.45	0
209	Gompertz	5	12.57	75.44	186.25	0
210	Cubic	8	-32.01	-Inf	NA	NA
210	Quadratic	8	-15.97	-79.83	Inf	NA
210	Gompertz	8	20.36	Inf	Inf	NA
211	Cubic	4	NA	NA	NA	NA
211	Quadratic	4	-5.89	-11.77	NA	NA
211	Gompertz	4	NA	NA	NA	NA
213	Cubic	6	-13.94	55.74	127.03	0
213	Quadratic	6	1.41	Inf	Inf	0
213	Gompertz	6	17.82	-71.29	0	1
214	Cubic	6	-5.1	7.66	0.25	0.47
214	Quadratic	6	-2.47	7.4	0	0.53
214	Gompertz	6	-11.63	17.44	10.04	0
215	Cubic	19	-50.68	-304.11	0	1
215	Quadratic	19	-51.1	-153.3	150.81	0
215	Gompertz	19	32.16	192.97	497.08	0
216	Cubic	13	-23.31	-139.86	61.77	0
216	Quadratic	13	-19.13	-57.38	144.24	0
216	Gompertz	13	-33.6	-201.63	0	1
217	Cubic	7	-7.24	-9.83	1.84	0.28
217	Quadratic	7	-9.22	-11.67	0	0.72
217	Gompertz	7	15.36	20.85	32.52	0
218	Cubic	10	-11.62	-18.88	34.69	0
218	Quadratic	10	-8.14	-11.75	41.82	0
218	Gompertz	10	-32.97	-53.57	0	1
219	Cubic	17	-181.43	-634.99	0	1

219	Quadratic	17	-180.26	-420.62	214.38	0
219	Gompertz	17	-170.08	-595.29	39.71	0
220	Cubic	13	-39.91	-79.82	61.05	0
220	Quadratic	13	-36.13	-60.22	80.66	0
220	Gompertz	13	-70.44	-140.87	0	1
221	Cubic	9	-53.18	-75.34	4.57	0.09
221	Quadratic	9	-23.27	-30.44	49.47	0
221	Gompertz	9	-56.4	-79.91	0	0.91
222	Cubic	14	-22.21	-36.09	0	1
222	Quadratic	14	0.55	0.8	36.89	0
222	Gompertz	14	25.14	40.85	76.94	0
223	Cubic	11	-6.35	-14.29	57.52	0
223	Quadratic	11	-0.32	-0.57	71.23	0
223	Gompertz	11	-31.91	-71.81	0	1
224	Cubic	16	-164.97	-256.62	0	0.98
224	Quadratic	16	-166.22	-232.71	23.92	0
224	Gompertz	16	-160.23	-249.25	7.38	0.02
225	Cubic	17	-41.14	-75.43	0	1
225	Quadratic	17	-30.57	-48.03	27.4	0
225	Gompertz	17	32.12	58.89	134.32	0
226	Cubic	12	-14.33	-20.85	11.59	0
226	Quadratic	12	-16.32	-21.76	10.67	0
226	Gompertz	12	-22.3	-32.44	0	0.99
227	Cubic	9	-12.28	-17.4	27.75	0
227	Quadratic	9	-13.21	-17.28	27.88	0
227	Gompertz	9	-31.88	-45.16	0	1
228	Cubic	10	-99.72	-170.94	0	0.82
228	Quadratic	10	-101.18	-151.77	19.18	0
228	Gompertz	10	-97.92	-167.87	3.08	0.18
229	Cubic	16	-168.27	-378.61	0	1
229	Quadratic	16	-169.66	-305.39	73.22	0
229	Gompertz	16	-148.59	-334.33	44.28	0
230	Cubic	17	-44.55	-89.1	13.3	0
230	Quadratic	17	-41.16	-68.6	33.8	0
230	Gompertz	17	-51.2	-102.4	0	1
231	Cubic	11	-25.02	-36.39	25.12	0
231	Quadratic	11	-21.06	-28.08	33.43	0
231	Gompertz	11	-42.29	-61.52	0	1
232	Cubic	8	-12.06	-17.08	20.76	0
232	Quadratic	8	-6.58	-8.6	29.24	0

232	Gompertz	8	-26.71	-37.85	0	1
233	Cubic	5	-22.55	-41.34	11.93	0
233	Quadratic	5	-11.09	-17.43	35.83	0
233	Gompertz	5	-29.05	-53.26	0	1
234	Cubic	8	-31.18	-83.14	95.3	0
234	Quadratic	8	-15.32	-30.65	147.8	0
234	Gompertz	8	-66.92	-178.44	0	1
235	Cubic	14	-25.16	-Inf	NA	NA
235	Quadratic	14	-22.08	-110.42	Inf	NA
235	Gompertz	14	-56.38	-Inf	NA	NA
236	Cubic	10	-4.95	-13.2	108.71	0
236	Quadratic	10	-2.52	-5.05	116.87	0
236	Gompertz	10	-45.72	-121.92	0	1
237	Cubic	8	-19.24	-29.93	0.82	0.4
237	Quadratic	8	-14.55	-20.37	10.38	0
237	Gompertz	8	-19.77	-30.75	0	0.6
238	Cubic	9	-8.89	-17.79	0	0.86
238	Quadratic	9	-8.5	-14.17	3.62	0.14
238	Gompertz	9	15.97	31.94	49.73	0
239	Cubic	9	6.39	17.04	8.19	0.02
239	Quadratic	9	4.42	8.85	0	0.98
239	Gompertz	9	22.6	60.27	51.42	0
240	Cubic	19	-210.37	-473.34	0	0.78
240	Quadratic	19	-212.25	-382.05	91.29	0
240	Gompertz	19	-209.24	-470.79	2.55	0.22
241	Cubic	17	-60.07	-135.15	18.02	0
241	Quadratic	17	-54.96	-98.92	54.25	0
241	Gompertz	17	-68.08	-153.18	0	1
242	Cubic	8	-20.87	-28.32	9.4	0.01
242	Quadratic	8	-8.42	-10.67	27.05	0
242	Gompertz	8	-27.8	-37.72	0	0.99
243	Cubic	8	-10.16	-14.39	0	0.96
243	Quadratic	8	-6.25	-8.17	6.22	0.04
243	Gompertz	8	12.93	18.31	32.7	0
244	Cubic	7	-5.7	-15.21	0	1
244	Quadratic	7	1.55	3.1	18.3	0
244	Gompertz	7	9.95	26.53	41.73	0
245	Cubic	18	-193.59	-516.24	0	0.9
245	Quadratic	18	-195.03	-390.07	126.17	0
245	Gompertz	18	-191.97	-511.91	4.33	0.1

246	Cubic	14	-28.94	-101.29	76.76	0
246	Quadratic	14	-11.45	-26.73	151.32	0
246	Gompertz	14	-50.87	-178.05	0	1
247	Cubic	7	-6.55	-9.06	26.32	0
247	Quadratic	7	-7.48	-9.62	25.77	0
247	Gompertz	7	-25.56	-35.39	0	1
248	Cubic	8	-18.58	-28.9	14.54	0
248	Quadratic	8	-3.44	-4.82	38.61	0
248	Gompertz	8	-27.92	-43.43	0	1
249	Cubic	8	-20.93	-73.25	114.25	0
249	Quadratic	8	-4.41	-10.29	177.21	0
249	Gompertz	8	-53.57	-187.5	0	1
250	Cubic	10	-22.59	-60.25	0	1
250	Quadratic	10	-23.21	-46.43	13.82	0
250	Gompertz	10	-10.62	-28.33	31.92	0
251	Cubic	16	-17.27	-46.04	0	1
251	Quadratic	16	-0.72	-1.44	44.61	0
251	Gompertz	16	66.01	176.03	222.07	0
252	Cubic	18	-20.6	-41.21	0	1
252	Quadratic	18	-5.52	-9.19	32.01	0
252	Gompertz	18	-14.17	-28.33	12.87	0
253	Cubic	16	-18.04	-26.23	0	1
253	Quadratic	16	-10.32	-13.75	12.48	0
253	Gompertz	16	63.64	92.56	118.8	0
254	Cubic	13	-17.82	-24.68	0	1
254	Quadratic	13	-8.98	-11.55	13.13	0
254	Gompertz	13	56.2	77.82	102.5	0
255	Cubic	9	-30.64	-44.56	0	0.86
255	Quadratic	9	-30.65	-40.86	3.7	0.14
255	Gompertz	9	26.44	38.46	83.02	0
256	Cubic	20	-57.64	-93.66	14.41	0
256	Quadratic	20	-55.67	-80.41	27.65	0
256	Gompertz	20	-66.5	-108.07	0	1
257	Cubic	14	-10.61	-23.88	1.64	0.28
257	Quadratic	14	-11.67	-21.01	4.51	0.07
257	Gompertz	14	-11.34	-25.52	0	0.65
258	Cubic	17	-24.73	-32.97	4.11	0.11
258	Quadratic	17	-25.17	-31.47	5.62	0.05
258	Gompertz	17	-27.81	-37.09	0	0.84
259	Cubic	15	-15.59	-24.25	8.2	0.02

259	Quadratic	15	-17.14	-23.99	8.46	0.01
259	Gompertz	15	-20.86	-32.45	0	0.97
260	Cubic	13	-18.17	-25.74	0	0.98
260	Quadratic	13	-13.71	-17.93	7.81	0.02
260	Gompertz	13	55.11	78.07	103.8	0
261	Cubic	9	-10.22	-15.33	8.1	0.02
261	Quadratic	9	-4.08	-5.57	17.86	0
261	Gompertz	9	-15.62	-23.43	0	0.98
262	Cubic	19	-26.36	-42.84	0	0.91
262	Quadratic	19	-26.49	-38.26	4.58	0.09
262	Gompertz	19	72.77	118.26	161.1	0
263	Cubic	12	-4.4	-9.9	8.32	0.01
263	Quadratic	12	-6.32	-11.38	6.84	0.03
263	Gompertz	12	-8.1	-18.22	0	0.95
264	Cubic	13	-22.57	-30.62	16.97	0
264	Quadratic	13	-14.08	-17.83	29.77	0
264	Gompertz	13	-35.07	-47.6	0	1
265	Cubic	12	-6.38	-10.94	1.9	0.24
265	Quadratic	12	-6.7	-10.04	2.8	0.15
265	Gompertz	12	-7.49	-12.85	0	0.61
266	Cubic	10	-19.28	-31.33	0	0.99
266	Quadratic	10	-15.36	-22.19	9.15	0.01
266	Gompertz	10	43.24	70.27	101.6	0
267	Cubic	7	-21.63	-37.08	0	0.47
267	Quadratic	7	-23.06	-34.58	2.49	0.13
267	Gompertz	7	-21.43	-36.74	0.34	0.4
268	Cubic	7	-25.95	-51.9	0	0.98
268	Quadratic	7	-26.13	-43.55	8.35	0.02
268	Gompertz	7	-17.46	-34.93	16.97	0
269	Cubic	6	-34.97	-122.39	0	1
269	Quadratic	6	-23.18	-54.08	68.31	0
269	Gompertz	6	-25.34	-88.69	33.69	0
270	Cubic	6	-32.57	-114	0	1
270	Quadratic	6	-33.56	-78.31	35.69	0
270	Gompertz	6	-5.2	-18.19	95.81	0
271	Cubic	7	-24.09	-144.55	0	1
271	Quadratic	7	-25.85	-77.54	67	0
271	Gompertz	7	-10.36	-62.18	82.36	0
272	Cubic	7	-19.76	-118.56	0	1
272	Quadratic	7	-20.89	-62.67	55.9	0

272	Gompertz	7	-6.74	-40.45	78.11	0
273	Cubic	7	-32.29	-113.02	4.36	0.1
273	Quadratic	7	-33.1	-77.23	40.15	0
273	Gompertz	7	-33.54	-117.38	0	0.9
274	Cubic	7	-28.57	-99.99	8.28	0.02
274	Quadratic	7	-29.04	-67.77	40.5	0
274	Gompertz	7	-30.93	-108.26	0	0.98
275	Cubic	7	-5.14	-17.98	0	0.67
275	Quadratic	7	0.64	1.5	19.49	0
275	Gompertz	7	-4.73	-16.55	1.43	0.33
276	Cubic	7	1.45	5.07	0	0.81
276	Quadratic	7	3.42	7.97	2.9	0.19
276	Gompertz	7	28.77	100.7	95.63	0
277	Cubic	4	NA	NA	NA	NA
277	Quadratic	4	4.3	10.03	NA	NA
277	Gompertz	4	NA	NA	NA	NA
278	Cubic	5	-12.92	-45.23	0	1
278	Quadratic	5	-0.7	-1.64	43.59	0
278	Gompertz	5	-1.3	-4.57	40.66	0
279	Cubic	4	NA	NA	NA	NA
279	Quadratic	4	-15.37	-Inf	NA	NA
279	Gompertz	4	NA	NA	NA	NA