7. a) $\tilde{\tau}_1 = \sum_{i=1}^{n} Y_i$

$$\mu = \frac{1}{N} \sum Y_i$$

AASc

- $\text{Viér}(\hat{\tau}_1) = \mathcal{E}[\hat{\tau}_1] - \tau$

$$\cancel{\mathcal{E}[\hat{\tau}_1] = \mathcal{E}\left[\sum_{i=1}^{n} Y_i\right] = \sum_{i=1}^{n} \mathcal{E}[Y_i] \overset{\text{por independencia}}{=} n \cdot \mu}$$

$$\mathcal{E}[\tilde{\tau}_1] = \mathcal{E}\left[\sum_{i=1}^{n} Y_i\right] = \sum_{i=1}^{n} \mathcal{E}[Y_i] = n \cdot \mu$$

$$\text{Vicio}\,(\tilde{\tau}_1) = n \cdot \mu - \sum_{i=1}^{N} Y_i = n \cdot \mu - N \cdot \mu = \mu(n - N)$$

- $\text{Var}(\hat{\tau}_1) = \text{Var}\left(\sum_{i=1}^{n} Y_i\right) = \sum_{i=1}^{n} \text{Var}(Y_i) = n\sigma^2$

AAS$_S$

- $\text{Vies}\,(\hat{\tau}_1) = \mathcal{E}[\hat{\tau}_1] - T$

$$\mathcal{E}[\hat{T}_1] = \mathcal{E}\left[\sum_{i=1}^{n} Y_i\right] = \mathcal{E}\left[\sum_{i=1}^{N} F_i Y_i\right] = \sum_{i=1}^{N} Y_i \mathcal{E}(F_i) = \sum_{i=1}^{N} Y_i \frac{n}{N} = \cancel{N} \mu \cdot \frac{n}{\cancel{N}}$$

$$= \mu \cdot n$$

$$\text{Vicio}(\hat{T}_1) = n \cdot \mu - \sum_{i=1}^{N} Y_i = n \cdot \mu - N \cdot \mu = \mu(n - N)$$

- $\text{Var}(\hat{\tau}_1) = \text{Var}\left(\sum_{i=1}^{n} Y_i\right) = \text{Var}\left(\sum_{i=1}^{N} F_i Y_i\right) = \sum_{i=1}^{N} Y_i^2 \text{Var}(F_i) + \sum_{i \neq j} Y_i Y_j \text{Cov}(F_i, F_j)$

$$= \sum_{i=1}^{N} Y_i^2 \text{Var}(F) - \sum_{i \neq j} Y_i Y_j \frac{\text{Var}(F)}{N-1} = \text{Var}(F)\left(\sum_{i=1}^{N} Y_i^2 - \frac{1}{N-1} \sum_{i \neq j} Y_i Y_j\right) =$$

$$\text{Var}(F)\left[\sum_{i=1}^{N} Y_i^2 - \frac{1}{N-1}\left(-\sum_{i=1}^{N} Y_i^2 + N^2 \mu^2\right)\right] = \text{Var}(F) \frac{N}{N-1} \sum_{i=1}^{N} (Y_i - \mu)^2 =$$

$$\left[\frac{n}{N}\left(1 - \frac{n}{N}\right)\right] \frac{N}{N-1} \sum_{i=1}^{N} (Y_i - \mu)^2 = n\left(1 - \frac{n}{N}\right) \tilde{\sigma}^2$$

em que $\tilde{\sigma}^2 = \frac{1}{N-1} \sum_{i=1}^{N} (Y_i - \mu)^2$

7. b) $\hat{T}_2 = \left(\frac{N}{n}\right)\sum_{i=1}^{n} y_i$

<u>AASc</u>

- $\text{Vício}(\hat{T}_2) = \mathcal{E}[\hat{T}_2] - T$

$$\mathcal{E}[\hat{T}_2] = \mathcal{E}\left[\frac{N}{n}\sum_{i=1}^{n} Y_i\right] = \frac{N}{n}\sum_{i=1}^{n}\mathcal{E}(Y_i) = \frac{N}{\cancel{n}}\cdot \cancel{n}\mu = N.\mu$$

$$\text{Vício}(\hat{T}_2) = N.\mu - \sum_{i=1}^{N} y_i = N.\mu - N.\mu = 0$$

- $\text{Var}(\hat{T}_2) = \text{Var}\left(\frac{N}{n}\sum_{i=1}^{n} Y_i\right) = \frac{N^2}{n^2}\sum_{i=1}^{n}\text{Var}(Y_i) = \frac{N^2}{n^2}\cdot n\cdot\sigma^2 = \frac{N^2}{n}\sigma^2$

<u>AASs</u>

- $\text{Vício}(\hat{T}_2) = \mathcal{E}[\hat{T}_2] - T$

$$\mathcal{E}[\hat{T}_2] = \mathcal{E}\left(\frac{N}{n}\sum_{i=1}^{n} Y_i\right) = \frac{N}{n}\mathcal{E}\left(\sum_{i=1}^{n} Y_i\right) = \frac{N}{n}\mathcal{E}\left(\sum_{i=1}^{N} F_i Y_i\right) = \frac{N}{n}\sum_{i=1}^{N} Y_i\,\mathcal{E}(F_i) =$$

$$\frac{\cancel{N}}{\cancel{n}}\cdot N\mu\cdot\frac{\cancel{n}}{\cancel{N}} = N\mu$$

$$\text{Vício}(\hat{T}_2) = N\mu - \sum_{i=1}^{N} y_i = N\mu - N\mu = 0$$

- $\text{Var}(\hat{T}_2) = \text{Var}\left(\frac{N}{n}\sum_{i=1}^{n} Y_i\right) = \frac{N^2}{n^2}\text{Var}\left(\sum_{i=1}^{n} Y_i\right) = \frac{N^2}{n^2}\text{Var}\left(\sum_{i=1}^{N} F_i Y_i\right) = \frac{N^2}{n^2}\Big[\sum_{i=1}^{N} Y_i^2\,\text{Var}(F_i)$

$$+ \sum_{i\neq j} Y_i Y_j \text{Cov}(F_i, F_j)\Big] = \frac{N^2}{n^2}\left[\sum_{i=1}^{N} Y_i^2\,\text{Var}(F) - \sum_{i\neq j} Y_i Y_j \frac{\text{Var}(F)}{N-1}\right] =$$

$$\frac{N^2}{n^2}\text{Var}(F)\left[\sum_{i=1}^{N} Y_i^2 - \frac{1}{N-1}\sum_{i\neq j} Y_i Y_j\right] = \frac{N^2}{n^2}\text{Var}(F)\left[\sum_{i=1}^{N} Y_i^2 - \frac{1}{N-1}\left(-\sum_{i=1}^{N} Y_i^2 + N^2\mu^2\right)\right]$$

$$= \frac{N^2}{n^2}\text{Var}(F)\frac{N}{N-1}\sum_{i=1}^{N}(Y_i-\mu)^2 = \frac{N^2}{n^2}\left[\frac{n}{N}\left(1-\frac{n}{N}\right)\right]\frac{N}{N-1}\sum_{i=1}^{N}(Y_i-\mu)^2 =$$

$$= \frac{N^2}{n}\left(1-\frac{n}{N}\right)\tilde{\sigma}^2$$

em que $\tilde{\sigma}^2 = \frac{1}{N-1}\sum_{i=1}^{N}$

7.c) Para escolher qual o melhor estimador em relação ao parametro, usararei o Erro Quadrado Medio, pois, ele é usado para avaliar a precisão dos estimadores comparando o erro entre os valores estimados e os valores reais.

AAS$_c$

$$EQM(\hat{T}_1) = n\sigma^2 + [\mu(n-N)]^2 .$$

$$EQM(\hat{T}_2) = \frac{N^2}{n}\sigma^2 + 0^2 = \frac{N^2}{n}\sigma^2 .$$

AAS$_s$

$$EQM(\hat{T}_1) = n\left(1 - \frac{n}{N}\right)\tilde{\sigma}^2 + [\mu(n-N)]^2 .$$

$$EQM(\hat{T}_2) = \frac{N^2}{n}\left(1 - \frac{n}{N}\right)\tilde{\sigma}^2 + 0 = \frac{N^2}{n}\left(1 - \frac{n}{N}\right)\tilde{\sigma}^2 .$$

No caso da Amostragem Aleatória Simples com Reposição (AAS$_c$), $\hat{T}_2$ é preferível, pois, em cenários gerais não apresenta dependência do viés e depende de menos parametros para ser estimado.

No caso da Amostragem Aleatória Simples sem Reposição (AAS$_s$), $\hat{T}_2$ também é preferível pelos mesmos motivos.