№1. Исследовать функцию на условный экстремум.

$U = 3 - 8x + 6y$, если $x^2 + y^2 = 36$

$\varphi(x, y) = 0$

$L = f(x, y) + \lambda \cdot \varphi(x; y)$

$x^2 + y^2 - 36 = 0$

$L = 3 - 8x + 6y + \lambda(x^2 + y^2 - 36)$

$L'_x = (3 - 8x + 6y + \lambda(x^2 + y^2 - 36))'_x = -8 + 2\lambda x$

$L'_y = 6 + 2\lambda y$

$\begin{cases} L'_x = 0 \\ L'_y = 0 \\ \varphi(x; y) = 0 \end{cases} \Rightarrow \begin{cases} -8 + 2\lambda x = 0 \\ 6 + 2\lambda y = 0 \\ x^2 + y^2 = 36 \end{cases} \Rightarrow \begin{cases} x = \frac{4}{2\lambda} \\ y = -\frac{3}{\lambda} \\ \frac{16}{\lambda^2} + \frac{9}{\lambda^2} = 36 \end{cases} \quad \begin{cases} x = \frac{4}{2\lambda} \\ y = -\frac{3}{\lambda} \\ \left[\begin{array}{l} \lambda = \frac{5}{6} \\ \lambda = -\frac{5}{6} \end{array}\right. \end{cases}$

$\frac{16}{\lambda^2} + \frac{9}{\lambda^2} = 36$

$\frac{25}{\lambda^2} = 36$

$\lambda^2 = \frac{25}{36}$

$\left[\begin{array}{l} \lambda = \frac{5}{6} \\ \lambda = -\frac{5}{6} \end{array}\right.$

При $\lambda = \frac{5}{6}$

$\begin{cases} x = \frac{4}{\frac{5}{6}} \\ y = -3 \cdot \frac{6}{5} \end{cases} \quad \begin{cases} x = \frac{24}{5} \\ y = -\frac{18}{5} \end{cases} \quad \begin{cases} x = 4{,}8 \\ y = -3{,}6 \end{cases} \quad M_1(4{,}8; -3{,}6)$

При $\lambda = -\frac{5}{6}$ $\begin{cases} x = -\frac{4 \cdot 6}{5} \\ y = \frac{3 \cdot 6}{5} \end{cases} \quad \begin{cases} x = -4{,}8 \\ y = 3{,}6 \end{cases} \quad M_2(-4{,}8; 3{,}6)$

Дифференциал 2-го порядка

$d^2L = L''_{xx}(dx)^2 + 2L''_{xy}\,dx\,dy + L''_{yy}(dy)^2$

$L''_{xx} = (-8 + 2\lambda x)' = 2\lambda \quad L''_{xy} = 0 \quad L''_{yy} = (6 + 2\lambda y)' = 2\lambda \quad L''_{yx} = 0$

$d^2L = 2\lambda(dx)^2 + 2\lambda(dy)^2$

При $\lambda = \frac{5}{6}$ $d^2L = \frac{5}{3}(dx)^2 + \frac{5}{3}(dy)^2 > 0$ Точка $M_1(4{,}8; -3{,}6)$ – точка минимума

При $\lambda = -\frac{5}{6}$ $d^2L = -\frac{5}{3}(dx)^2 - \frac{5}{3}(dy)^2 < 0$ Точка $M_2(-4{,}8; 3{,}6)$ – точка максимума

№3 Найти производную функции $U = x^2 + y^2 + z^2$ по направлению вектора $\vec{c}(-9, 8, -12)$ в точку $M(8; -12; 9)$.

$U'_x = 2x$ $\quad |\vec{c}| = \sqrt{81+64+144}$

$U'_y = 2y$ $\quad |\vec{c}| = \sqrt{289}$

$U'_z = 2z$ $\quad |\vec{c}| = 17$

$U'_x|_M = 16$ $\quad \vec{c}_0 = \left(-\frac{9}{17}, \frac{8}{17}, -\frac{12}{17}\right)$

$U'_y|_M = -24$ $\quad U'_{\vec{c}}|_M = 16 \cdot \left(-\frac{9}{17}\right) + (-24) \cdot \frac{8}{17} + 18 \cdot \left(-\frac{12}{17}\right)$

$U'_z|_M = 18$ $\quad U'_{\vec{c}}|_M = -\frac{144}{17} - \frac{192}{17} - \frac{216}{17} = -32\frac{8}{17}$

№4. Найти производную функции $U = e^{x^2+y^2+z^2}$ по направлению вектора $\vec{d} = (4, -13, -16)$ в точку $L(-16; 4; -13)$

$U'_x = 2x \cdot e^{x^2+y^2+z^2}$ $\quad U'_x|_L = -32 \cdot e^{256+16+169}$ $\quad U'_x|_L = -32 \cdot e^{441}$

$U'_y = 2y \cdot e^{x^2+y^2+z^2}$ $\quad U'_y|_L = 8 \cdot e^{441}$

$U'_z = 2z \cdot e^{x^2+y^2+z^2}$ $\quad U'_y|_L = -26 \cdot e^{441}$

$U''_{xx} = 2 \cdot (2x^2+1) \cdot e^{x^2+y^2+z^2}$ $\quad |\vec{d}| = \sqrt{16+169+256}$

$U''_{xy} = 4xy\, e^{x^2+y^2+z^2}$ $\quad |\vec{d}| = \sqrt{441}$

$U''_{xz} = 4xz\, e^{x^2+y^2+z^2}$ $\quad |\vec{d}| = 21$ $\quad \vec{d}_0 = \left(\frac{4}{21}; -\frac{13}{21}, -\frac{16}{21}\right)$

$U'_{d}|_L = -32 \cdot e^{441} \cdot \frac{4}{21} - 8e^{441} \cdot \frac{13}{21} + 26 \cdot e^{441} \cdot \frac{16}{21}$

$\cancel{U'_d|_L = 8 \cdot e^{441}\left(\frac{-4 \cdot 4}{21} - \frac{13}{21} + \frac{26 \cdot 2}{21}\right)}$ $\quad U'_d|_L = e^{441}\left(-\frac{128}{21} - \frac{104}{21} + \frac{416}{21}\right)$

$\cancel{U'_d|_L = 8e^{441}\left(-\frac{29}{21} + \frac{52}{21}\right)}$ $\quad U'_d|_L = 8e^{441}\frac{16}{21}$