

insert name of the cool tool name here : User
manual

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Abstract

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Chapter 1

Introduction

Chapter 2

Building the tool

2.1 Blueprint

Chapter 3

Theory

3.1 Description of the hanger

Let's consider a hanger. We will now give names to important points and characteristics of the hanger:

- The hanger has a mass m , and its gravity center is named G .
- The hanger is held by a hook at the point O .
- Each hole has a graduation. The one under the hook is 0 and then they are graduated like in figure 3.1.
- Holes are equally spaced. The distance between two holes is d . and the space between two consecutive holes is d .
- The point of the hole 0 is named H .
- We use the frame of reference $(O, \vec{u}_x, \vec{u}_y)$ (see figure 3.1).

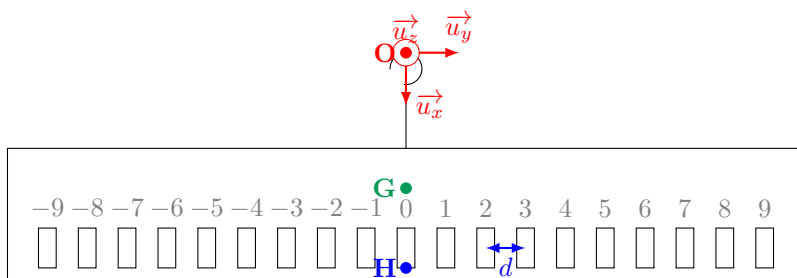


Figure 3.1: A hanger.

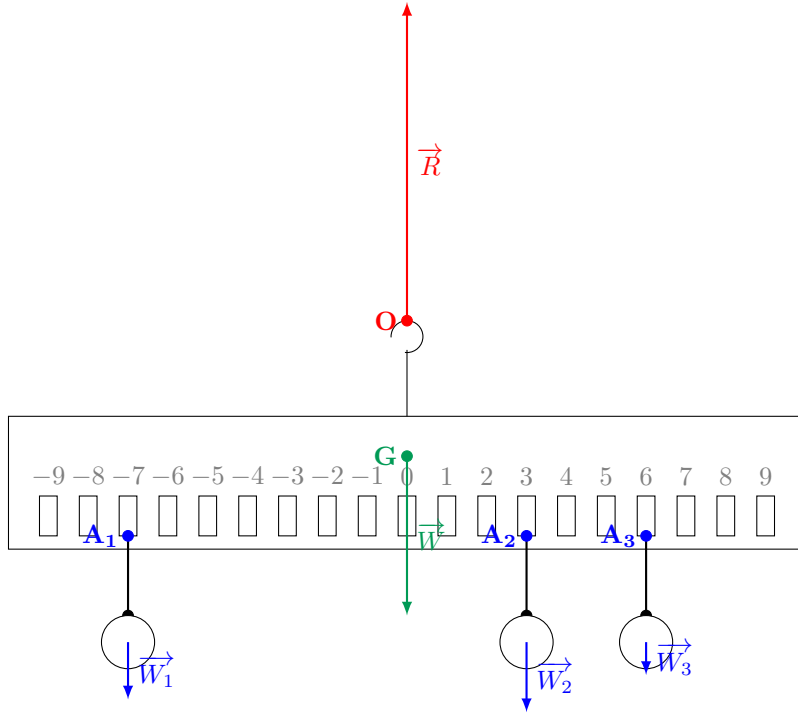


Figure 3.2: A balanced hanger (force vector are at the scale) with three mass.

3.2 Description of forces

Lets now, imagine n objects attached to the hanger. The hanger is consider immobile (it does not fall and does not move like a pendulum). The k th object have a mass m_k and is attached to the position p_k . We name A_k the point where the object is attached. We will detail all the forces applied to the hanger (see figure 3.3):

- Its weight \vec{W} , applied in its gravity center. This force is directed downwards.
- The reaction of the hook \vec{R} , applied in the point O and directed upwards.
- The weight of the k th object \vec{W}_k applied its forces on the hook in the point A_k . This force is directed downwards.

The weight oh an object is proportional to its mass. So :

- $\vec{P} = mg\vec{u}_x$ with $g = 9,80665m \cdot s^{-2}$.
- $\vec{P}_k = m_k g\vec{u}_x$

The hanger is immobile. So its forces nullify :

$$\begin{aligned}\vec{W} + \vec{R} + \sum_{k=1}^n \vec{W}_k &= \vec{0} \\ W\vec{u}_x - R\vec{u}_x + \sum_{k=1}^n W_k\vec{u}_x &= \vec{0} \\ R &= W + \sum_{k=1}^n W_k \\ R &= g(m + \sum_{k=1}^n m_k)\end{aligned}$$

So, we have the value of the reaction of the hook.

3.3 Condition of a balanced hanger

In this section, we will consider a perfectly balanced hanger. We will search necessary condition of this state.

The only movement the hanger can do is a rotation around the placed it is attached. So a rotation with an axis in O . But the hanger is balanced so it is immobile when it is in a horizontal position. This means that the sum of torques of forces around O is zero.

The formula of the momentum in O of the force \vec{F} applied in P is : $\vec{M}_{\vec{F}/O} = \vec{OP} \wedge \vec{F}$. So we will look at the torque of each forces applied on the hanger :

- $\begin{aligned}\vec{M}_{\vec{R}/O} &= \vec{OO} \wedge \vec{R} \\ \vec{M}_{\vec{R}/O} &= \vec{0} \quad \text{because } \vec{OO} = \vec{0}\end{aligned}$
- $\begin{aligned}\vec{M}_{\vec{W}/O} &= \vec{OG} \wedge \vec{W} \\ \vec{M}_{\vec{W}/O} &= \vec{0} \quad \text{because in the case of a balanced hanger, } \vec{OG} \text{ and } \vec{W} \text{ are colinear to } \vec{u}_x\end{aligned}$
- $\begin{aligned}\vec{M}_{\vec{W}_k/O} &= \vec{OA_k} \wedge \vec{W}_k \\ \vec{M}_{\vec{W}_k/O} &= (\vec{OH} + \vec{HA_k}) \wedge \vec{W}_k \\ \vec{M}_{\vec{W}_k/O} &= (\vec{OH} \wedge \vec{W}_k) + (\vec{HA_k} \wedge \vec{W}_k) \\ \vec{M}_{\vec{W}_k/O} &= \vec{0} + (\vec{HA_k} \wedge \vec{W}_k) \quad \text{because } \vec{OH} \text{ and } \vec{W}_k \text{ are colinear to } \vec{u}_x \\ \vec{M}_{\vec{W}_k/O} &= (HA_k\vec{u}_x) \wedge (W_k\vec{u}_y) \\ \vec{M}_{\vec{W}_k/O} &= HA_k.W_k\vec{u}_z \\ \vec{M}_{\vec{W}_k/O} &= p_k.d.m_k.g\vec{u}_z\end{aligned}$

So we can now sum up all the torque and search the case in which this sum

$$\begin{aligned}
\vec{M}_{\vec{R}/O} + \vec{M}_{\vec{W}/O} + \sum_{k=1}^n \vec{M}_{\vec{W}_k/O} &= \vec{0} \\
\vec{0} + \vec{0} + \sum_{k=1}^n p_k \cdot d \cdot m_k \cdot g \vec{u}_z &= \vec{0} \\
\sum_{k=1}^n p_k \cdot d \cdot m_k \cdot g &= 0 \\
\sum_{k=1}^n p_k \cdot m_k &= 0
\end{aligned}$$

nulify :

So the hanger is balanced if and only if the object check the condition:

$$\sum_{k=1}^n p_k \cdot m_k = 0$$

3.4 In the general case : the unbalanced hanger

In this section, the hanger is inclined by an angle θ .

Chapter 4

Activities