Equilicool : User manual

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Abstract

This paper present the object named equilicool. You will find a description of the object, how to build it, examples of activities using it and the theory behind th balancing of the object. Right now, only the chapter theory is done.

Contents

Chapter 1

Introduction

Chapter 2

Building the tool

2.1 The hanger

The hanger was build with a laser cutter. The blueprints can be found at TODO : url.

Then the plank we cut was TODO : details.

2.2 The hook

2.3 Assemble them

Chapter 3

Theory

3.1 Description of the hanger

Let's consider a hanger. We will now gives names to important points and caracteristics of the hanger:

- The hanger has a mass m, and its gravity center is named G.
- The hanger is and by a hook at the point O.
- Each hole have a graduation. The one under the hook is 0 and then their are graduated like in figure 3.1.
- Holes are equally spaced. The distance between two hole is d. and the space between two consecutive hole is d.
- The point of the hole 0 is named H.
- We use the frame of reference $(O, \overrightarrow{u_x}, \overrightarrow{u_y})$ (see figure 3.1).

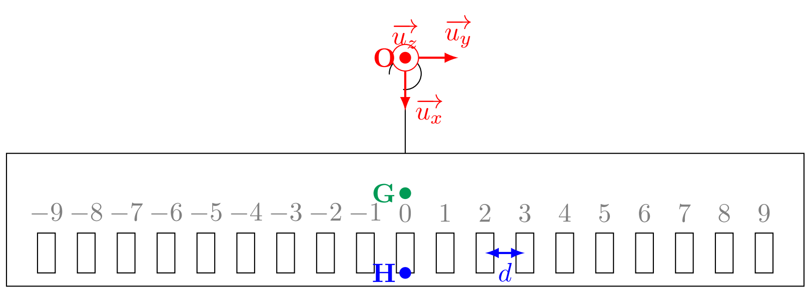

Figure 3.1: A hanger.

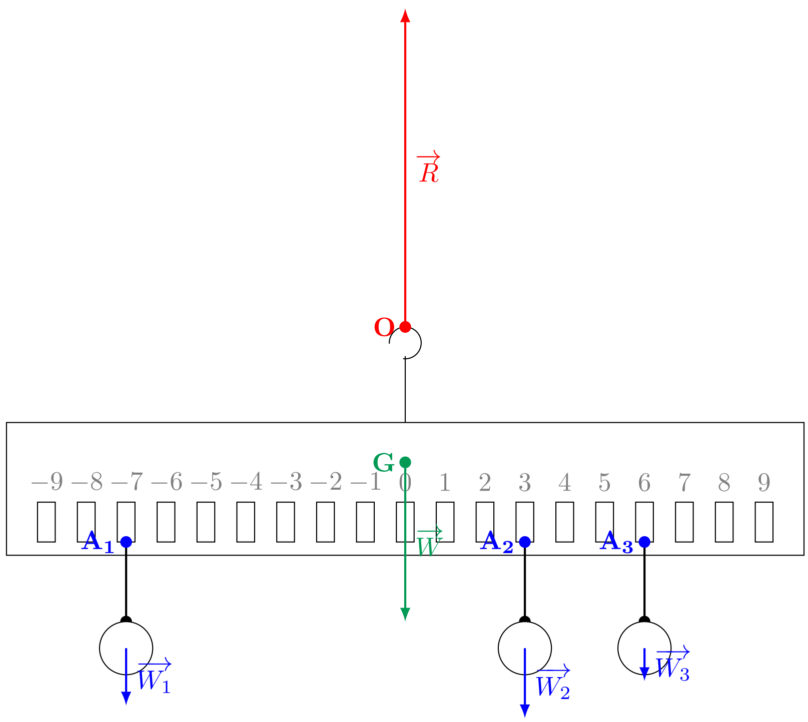

Figure 3.2: A balanced hanger (force vector are at the scale) with three mass.

3.2 Description of forces

Lets now, imagine n objects attached to the hanger. The hanger is consider immobile (it does not fall and does not move like a pendulum). The kth object have a mass m_k and is attached to the position p_k. We name A_k the point where the object is attached. We will detail all the forces applied to the hanger (see figure 3.2):

- Its weight $\overrightarrow{W}$, applied in its gravity center. This force is directed downwards.
- The reaction of the hook $\overrightarrow{R}$, applied in the point O and directed upwards.
- The weight of the kth object $\overrightarrow{W_k}$ applied its forces on the hook in the point $\overrightarrow{A_k}$. This force is directed downwards.

The weight oh an object is proportional to its mass. So :

- $\overrightarrow{P} = mg\overrightarrow{u_x}$ with $g = 9,80665m \cdot s^{-2}$.
- $\overrightarrow{P_k} = m_k g\overrightarrow{u_x}$

The hanger is immobile. So its forces nullify :

$$\begin{aligned}
\overrightarrow{W} + \overrightarrow{R} + \sum_{k=1}^{n} \overrightarrow{W_k} &= \overrightarrow{0} \\
W\overrightarrow{u_x} - R\overrightarrow{u_x} + \sum_{k=1}^{n} W_k\overrightarrow{u_x} &= \overrightarrow{0} \\
R &= W + \sum_{k=1}^{n} W_k \\
R &= g(m + \sum_{k=1}^{n} m_k)
\end{aligned}$$

So, we have the value of the reaction of the hook.

3.3 Condition of a balanced hanger

In this section, we will consider a perfectly balanced hanger. We will search necessary condition of this state.

The only movement the hanger can do is a rotation around the placed i is attached. So a rotation with an axis in O. But the hanger is balanced so it is immobile when it is in a horizontal position. This means that the sum of torques of forces around O is zero.

The formula of the momentum in O of the force $\overrightarrow{F}$ applied in P is : $\overrightarrow{M}_{\overrightarrow{F}/O} = \overrightarrow{OP} \wedge \overrightarrow{F}$. So we will look at the torque of each forces applied on the hanger :

- $$\begin{aligned}
\overrightarrow{M}_{\overrightarrow{R}/O} &= \overrightarrow{OO} \wedge \overrightarrow{R} \\
\overrightarrow{M}_{\overrightarrow{R}/O} &= \overrightarrow{0} \qquad \text{because } \overrightarrow{OO} = \overrightarrow{0}
\end{aligned}$$

- $$\begin{aligned}
\overrightarrow{M}_{\overrightarrow{W}/O} &= \overrightarrow{OG} \wedge \overrightarrow{W} \\
\overrightarrow{M}_{\overrightarrow{R}/O} &= \overrightarrow{0} \qquad \text{because in the case of a balanced hanger, } \overrightarrow{OG} \text{ and } \overrightarrow{W} \text{ are colinear to } \overrightarrow{u_x}
\end{aligned}$$

- $$\begin{aligned}
\overrightarrow{M}_{\overrightarrow{W_k}/O} &= \overrightarrow{OA_k} \wedge \overrightarrow{W_k} \\
\overrightarrow{M}_{\overrightarrow{W_k}/O} &= (\overrightarrow{OH} + \overrightarrow{HA_k}) \wedge \overrightarrow{W_k} \\
\overrightarrow{M}_{\overrightarrow{W_k}/O} &= (\overrightarrow{OH} \wedge \overrightarrow{W_k}) + (\overrightarrow{HA_k} \wedge \overrightarrow{W_k}) \\
\overrightarrow{M}_{\overrightarrow{W_k}/O} &= \overrightarrow{0} + (\overrightarrow{HA_k} \wedge \overrightarrow{W_k}) \qquad \text{because } \overrightarrow{OH} \text{ and } \overrightarrow{W_k} \text{ are colinear to } \overrightarrow{u_x} \\
\overrightarrow{M}_{\overrightarrow{W_k}/O} &= (HA_k\overrightarrow{u_x}) \wedge (W_k\overrightarrow{u_y} \\
\overrightarrow{M}_{\overrightarrow{W_k}/O} &= HA_k.W_k\overrightarrow{u_z} \\
\overrightarrow{M}_{\overrightarrow{W_k}/O} &= p_k.d.m_k.g\overrightarrow{u_z}
\end{aligned}$$

So we can now sum up all the torque and search the case in which this sum nulify :

$$\begin{aligned}
\overrightarrow{M}_{\overrightarrow{R}/O} + \overrightarrow{M}_{\overrightarrow{W}/O} + \sum_{k=1}^{n} \overrightarrow{M}_{\overrightarrow{W_k}/O} &= \overrightarrow{0} \\
\overrightarrow{0} + \overrightarrow{0} + \sum_{k=1}^{n} p_k.d.m_k.g\overrightarrow{u_z} &= \overrightarrow{0} \\
\sum_{k=1}^{n} p_k.d.m_k.g &= 0 \\
\sum_{k=1}^{n} p_k.m_k &= 0
\end{aligned}$$

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So the hanger is balanced if and only if the object check the condition:

$$\sum_{k=1}^{n} p_k.m_k = 0$$

3.4 Summary

The hanger is balanced if and only if, when you sum up the product of masses of the object by the number of their position, yon obtain zero.

Chapter 4

Activities