Hidden Testing and Selective Disclosure of Evidence*

Claudia Herresthal†

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Abstract

A decision maker faces an approval choice under uncertainty. An agent can gather information through sequential testing. Players agree on the optimal choice under certainty, but the decision maker has a higher approval standard than the agent. We compare the case where testing is hidden and the agent can choose whether to disclose his findings to the case where testing is observable. The agent can exploit the additional discretion under hidden testing to his advantage if and only if the decision maker is sufficiently inclined to approve. Hidden testing then yields a Pareto improvement over observable testing if the conflict between players is larger than some threshold, but leaves the decision maker worse off and the agent better off if the conflict is smaller than this threshold.

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JEL codes: D83, D82

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†University of Bonn, Adenauerallee 24-42, 53113 Bonn, Germany (email: claudia.herresthal@uni-bonn.de).

1 Introduction

Pharmaceutical companies have recently come under scrutiny for selective reporting of clinical trial outcomes.[1] As a response to demands for greater transparency, several companies have pledged to register trials and report their outcomes in open online databases (Goldacre et al. [2017]). At first sight, it seems that such transparency would improve regulation because pharmaceutical companies can no longer hide trials with unfavorable outcomes. However, companies may also strategically respond by running fewer trials and this could mean that the regulator has to base his approval decision on weaker evidence.

This paper analyzes how transparent information acquisition affects the likelihood of correct approval decisions and welfare. A decision maker (DM, e.g. a regulator) has to take an approval decision based on evidence gathered by an agent (e.g. a company). However, there is a conflict of interest between the players: the DM is more averse to approving an unsafe product than the agent, that is, the DM has a higher approval standard than the agent. The agent can sequentially run costless tests up to some deadline at which point the DM chooses to approve or not. We first consider a setting of *observable testing*, in which the agent's findings are publicly observed. We contrast this with a setting of *hidden testing*, in which the DM neither observes for how long the agent has tested nor what the agent has found unless the agent chooses to disclose his findings.

One might expect that the DM is better off in equilibrium when testing is observable than when testing is hidden. After all, when testing is hidden, the agent can strategically hide evidence to influence the DM's choice. Provided the DM initially favors approval, we show that the DM is indeed worse off under hidden testing if the conflict between players is small. However, if the conflict is large, then hidden testing yields a strict Pareto improvement over observable testing, that is, not only the agent but also the DM is strictly better off under hidden than observable testing.

This Pareto improvement arises for the following reasons. When testing is observable, the DM can react optimally to the evidence gathered, but the agent acquires evidence strategically. In particular, the agent can manipulate the DM's choice by stopping to test following certain findings while continuing to test following others. We characterize the unique equilibrium and show under which conditions the agent stops testing before the deadline even if further tests could be mutually beneficial. In particular, when the conflict is large and the DM initially favors approval, the agent stops testing before the deadline if running further tests has the downside that the DM may come to favor rejection, even though the agent continues to favor approval.

When testing is hidden, the agent can strategically withhold results from the DM. If the DM initially favors approval, it is indeed the case that the agent exploits this discretion to persuade the DM to approve more often than is optimal from the DM's perspective. However, since the agent has more influence over the final approval choice than under observable testing, the agent has more incentives to keep testing. Moreover, the agent reveals to the DM the findings of the additional

[1] For example, GlaxoSmithKline was accused of having withheld data that suggested that its antidepressant Paxil was linked to suicidal behavior in teenagers (Rabin [2017]), and the Cochrane Review concluded that Roche withheld trial data on its influenza drug Tamiflu to make the drug look more effective (Jefferson et al. [2014]). See also Avorn [2006].

tests run under hidden relative to observable testing if and only if these tests convince the agent that rejection is optimal. Therefore, the additional tests trigger the DM to reject instead of approve if and only if rejection is in both players' interest, resulting in a mutual benefit.

By contrast, if the conflict of interest between players is small instead, the DM is worse off and the agent better off under hidden than observable testing. The reason is that the agent becomes equally informed under each regime, but under hidden testing, the agent's selective disclosure can still lead the DM to make suboptimal choices.

Our analysis can be applied to the context of postmarketing studies for medical drugs. These studies investigate a drug's safety or efficacy, or new indications of a drug after regulatory approval.[2] Frequently, these studies are initiated by companies themselves, but they can also be requested by the regulator as a condition of approval. However, companies often do not carry out the requested studies or do not release all their findings (Glasser et al. [2007]), and it has been argued that regulators lack the power to enforce compliance.[3] In light of this, demands for compulsory trial registration and reporting have been rising.

Our work predicts that registries help to improve regulatory decision making when the conflict of interest is small, but cause adverse welfare effects for all parties when the conflict of interest is large. In particular, without a trial registry, the company can run safety studies in private and withdraw the product if it finds that the product does not meet its own safety standards.[4] In case the company does not withdraw the product, the regulator infers that at least the company's own safety standards are being met and, hence, the regulator will not see a reason to withdraw the product. By contrast, with a trial registry, a company with a relatively low safety standard is discouraged from performing such tests. The reason is that these tests could yield weak evidence that the product is unsafe, sufficient for the regulator to take the product off the market but not sufficient to fail the company's safety standards. Therefore, the company does not run tests that could uncover serious safety concerns, harming both itself and the regulator.

Our results can also be applied in the context of academic publishing. As various replication studies produced findings at odds with those in the initial studies, e.g. Open Science Collaboration [2015], calls for greater transparency in research increased, e.g. Nosek et al. [2018]. One suggestion is to make registration of experiments and the use of result databases a condition for publication. However, as our paper points out, when all experiments need to be documented, a researcher with strong incentives to publish might be reluctant to further investigate a hypothesis that was supported by his existing experiments.[5] The reason is that these additional experiments could contradict his earlier findings and shed some doubt on whether the hypothesis is true. This may not be sufficient to deter the researcher from wanting to publish his findings, but the editor may no longer find the

[2] The studies prior to approval usually only collect evidence on certain patient groups and outside routine clinical practice, e.g. see Smith et al. [2015].

[3] "The only thing the agency can do is take the drug off the market, which is a decision that often would not serve the public health very well.", Peter Lurie, deputy director of the Public Citizen's Health Research Group, available at: https://www.citizen.org/news/fda-report-highlights-poor-enforcement-of-post-marketing-follow-up/.

[4] The company may care about safety standards for liability reasons or to reputational concerns.

[5] The researcher may aim to advance science by supporting a hypothesis if and only if it is true, but be biased in favor of supporting a hypothesis if this increases his chances of publishing.

evidence convincing enough and reject the paper. If the researcher could experiment in private, he would discover whether or not additional experiments strongly reject the hypothesis. In this event, he will not seek publication, thereby reducing the likelihood that a false finding is published.

The paper is structured as follows. Section 2 outlines the model. Observable testing is analyzed in Section 3, hidden testing in Section 4. The welfare comparison can be found in Section 5. Extensions and robustness checks are in Section 6. A detailed comparison with the existing literature can be found in Section 7. Section 8 concludes.

2 Model

Two players, a decision maker (she) and an agent (he), are uninformed about the state of the world $\omega \in \{\omega_0, \omega_1\}$ and share a prior belief $q_0 = \Pr(\omega_1) \in [0,1]$. Time is continuous and finite, $t \in [0,T]$.[6] At time T, the decision maker (DM) chooses an action $a \in \{a_0, a_1\}$ which affects the payoff of player i as follows,[7]

$u_i(a,\omega)$	ω_0	ω_1
a_0	$\frac{s_i}{1-s_i}$	0
a_1	0	1

where $i \in \{DM, A\}$ and $0 < s_A \leq s_{DM} < 1$.[8] In particular, at some belief $q = Pr(\omega_1)$, player i has a higher expected payoff from a_1 than from a_0 if and only if

$$\mathbb{E}_\omega u_i(a_0,\omega) = (1-q)\frac{s_i}{1-s_i} \leq q = \mathbb{E}_\omega u_i(a_1,\omega) \Leftrightarrow s_i \leq q. \tag{1}$$

Hence, if $q < s_A$, both prefer a_0, if $s_A \leq q < s_{DM}$, the DM prefers a_0 while the agent prefers a_1 and if $s_{DM} \leq q$, both prefer a_1. We refer to a_1 as approval, a_0 as rejection and to s_i as player i's approval standard.

When testing is observable, the agent runs publicly observable tests at no cost. When the agent tests, a signal arrives at random according to an exponential distribution with state-dependent arrival rate $\lambda^\omega > 0$. In particular, in state ω, over an interval $[t, t+dt)$, a signal $\theta = \omega$ is observed with probability $\lambda^\omega dt$, that is, the signal perfectly reveals the state. Let h^t denote the history up to time t and H^t the set of all such histories. The (common) posterior belief $q_t = Pr(\omega_1|h^t)$ evolves as follows. If the agent tests and a signal arrives over the interval $[t, t+dt)$, then the belief jumps to 1 if $\theta = \omega_1$ and jumps to 0 if $\theta = \omega_0$. If the agent tests and no signal arrives, the evolution of

[6]The finite horizon represents the agent's resource constraints and can be interpreted as a limiting case of an infinite horizon with a convex cost of testing.

[7]This is without loss. Consider a general payoff function: $u_i(a_0,\omega_0) = \alpha_i$, $u_i(a_0,\omega_1) = \beta_i$, $u_i(a_1,\omega_0) = \gamma_i$ and $u_i(a_1,\omega_1) = \delta_i$, where $\alpha_i, \beta_i, \gamma_i, \delta_i \in \mathbb{R}$ and $\alpha_i > \gamma_i$ and $\delta_i > \beta_i$. Then my results apply for $s_i = \frac{\alpha_i - \gamma_i}{(\alpha_i-\gamma_i)+(\delta_i-\beta_i)}$.

[8]If we instead assumed $0 < s_{DM} \leq s_A < 1$, the results would be a mirror image of the results presented: the labels of states $\omega \in \{\omega_0, \omega_1\}$ and actions $a \in \{a_0, a_1\}$ would have to be switched.

the posterior at time t is described by the following differential equation:[9]

$$\frac{dq_t}{dt} = -\left(\lambda^{\omega_1} - \lambda^{\omega_0}\right) q_t \left(1 - q_t\right). \tag{2}$$

We assume $\lambda^{\omega_0} < \lambda^{\omega_1}$ which implies that the posterior belief drifts down when no signal arrives, that is, no news is bad news. The agent's strategy selects to stop or to continue testing at time t conditional on h^t for each $t \in [0, T]$. The agent stops if he is indifferent between testing and stopping. In addition, once the agent stops, he cannot test again.[10] The DM's strategy selects an action choice $a \in \{a_0, a_1\}$ at time T conditional on h^T. Since all payoff-relevant information contained in history h^t can be summarized by belief q_t for any $t \in [0, T]$, strategies can be thought of as mapping posterior beliefs into action choices.

When testing is hidden, the agent privately chooses when to stop and privately observes the arrival of signals.[11] Further, the agent decides whether to disclose all his findings to the DM or to remain silent. In particular, at time T, the agent's disclosure strategy selects message m conditional on h^T, where the set of feasible messages is given by $M\left(h^T\right) = \left\{\emptyset, h^T\right\}$. Then the DM's strategy selects an action choice a conditional on message m.

In either regime, the solution concept is a perfect Bayesian equilibrium. We impose the additional requirement that, for any off-path history, players update their beliefs about the state according to Bayes' rule using the testing technology.[12]

We restrict attention to situations in which the two players disagree on the approval choice at some feasible posterior belief.[13] In particular, we assume the agent favors approval at the prior, i.e. $s_A \leq q_0$, and the DM favors rejection if testing until the deadline has not produced a signal, i.e. $\underline{q} < s_{DM}$, where

$$\underline{q} \equiv \frac{q_0 \exp\left(-\lambda^{\omega_1} T\right)}{q_0 \exp\left(-\lambda^{\omega_1} T\right) + \left(1 - q_0\right) \exp\left(-\lambda^{\omega_0} T\right)}. \tag{3}$$

3 Observable Testing

When testing is observable, the players share a common posterior belief at any point in time. The agent can influence the DM's approval choice via controlling the flow of information: he can strategically choose to stop testing at certain posterior beliefs but continue testing at others. The agent may therefore stop testing if the DM is willing to approve. However, stopping leaves the agent worse off in the event that further evidence would have led both the DM and the agent to prefer rejection. In this section, we will characterize the unique equilibrium and show under which condition the agent stops testing for strategic reasons.

[9]For details see Liptser and Shiryaev [2013].

[10]These assumptions are chosen to avoid a multiplicity of equilibria which are payoff-equivalent except in a knife-edge case. More details are given in Sections 3 and 4.

[11]The key result remains unchanged if the DM could observe the time at which the agent stops. See also footnote 31 after the results.

[12]This means off-path beliefs satisfy "no-signaling-what-you-don't-know" in the vein of Fudenberg and Tirole [1991].

[13]If the two players were to agree on the optimal action at all feasible posteriors, it is trivial that each player's payoff is the same under hidden and observable testing.

Proposition 1. *Under observable testing, an equilibrium exists and is unique. The equilibrium strategies are as follows. The DM chooses a_1 if and only if $s_{DM} \leq q_T$. The agent stops at time t if a signal has arrived by time t. If no signal has arrived by time t, then*

1. *for $q_0 < s_{DM}$, the agent keeps testing at time t for any $t < T$.*

2. *for $q_0 \geq s_{DM}$, there exists a critical level of the agent's approval standard $\overline{s}_A(s_{DM}, q_0, T)$ such that*

   (a) *if $s_A > \overline{s}_A$, the agent keeps testing at time t for any $t < T$, and*

   (b) *if $s_A \leq \overline{s}_A$, the agent keeps testing at time t for any $t < t^*$ and stops testing at $t = t^*$, where*

$$t^*(s_{DM}, q_0) = \frac{\log\left(\frac{q_0}{1-q_0}\right) - \log\left(\frac{s_{DM}}{1-s_{DM}}\right)}{\lambda^{\omega_1} - \lambda^{\omega_0}}. \tag{4}$$

The critical level $\overline{s}_A \in (0, \underline{q})$ defined for $q_0 \geq s_{DM}$ satisfies

$$\log\left(\frac{\overline{s}_A}{1-\overline{s}_A}\right) = \log\left(\frac{s_{DM}}{1-s_{DM}}\right) - \lambda^{\omega_1}(T - t^*(s_{DM}, q_0)). \tag{5}$$

Proof. We start by showing that the agent's strategy is the unique best reply.[14] The agent always stops testing the first time a signal arrives because any further testing does not influence the DM's belief and final action choice a. Suppose no signal arrived by time t.

Part 1.) We first show that the agent keeps testing at time t if $0 < q_t < s_{DM}$ for any $t < T$. Let $V_t(q, \tau)$ denote the agent's expected continuation value at time t and posterior q when he stops the first time a signal arrives or at time τ. If $\tau = t$, then given the DM's strategy,

$$V_t(q_t, t) = (1 - q_t)\frac{s_A}{1 - s_A}. \tag{6}$$

If instead $\tau = t' > t$, then given the DM's strategy,

$$V_t(q_t, t') = (1 - q_t)\frac{s_A}{1 - s_A}\Pr(q_{t'} < s_{DM}|q_t, \tau = t', \omega_0) + q_t \Pr(q_{t'} \geq s_{DM}|q_t, \tau = t', \omega_1), \tag{7}$$

where $\Pr(q_{t'} < s_{DM}|q_t, \tau = t', \omega)$ is the probability that $q_{t'} < s_{DM}$ at time t' conditional on the belief being q_t at time t, the agent stopping the first time a signal arrives or at time t' and the state being ω. The agent keeps testing at any time t if $0 < q_t < s_{DM}$ since

$$\begin{aligned} V_t(q_t, t') - V_t(q_t, t) > 0 &\Leftrightarrow \frac{\Pr(q_{t'} \geq s_{DM}|q_t, \tau = t', \omega_1)}{\Pr(q_{t'} \geq s_{DM}|q_t, \tau = t', \omega_0)}\frac{q_t}{1 - q_t} > \frac{s_A}{1 - s_A} \\ &\Leftrightarrow \frac{\Pr(\omega_1|q_t, \tau = t', q_{t'} \geq s_{DM})}{\Pr(\omega_0|q_t, \tau = t', q_{t'} \geq s_{DM})} > \frac{s_A}{1 - s_A}, \end{aligned} \tag{8}$$

[14] Recall that we assume that the agent stops if he is indifferent between testing and stopping and that once the agent stops he cannot test again.

where the second step follows by Bayes' rule and the inequality is satisfied for any $t' > t$ since

$$\Pr\left(\omega_1|q_t, \tau = t', q_{t'} \geq s_{DM}\right) > \Pr\left(\omega_1|q_t, \tau = t', q_{t'} = s_{DM}\right) = s_{DM} \geq s_A, \tag{9}$$

and $\frac{x}{1-x} > \frac{s_A}{1-s_A}$ if and only if $x > s_A$ for any $x \in (0,1)$. Then, given $q_0 < s_{DM}$, if no signal has arrived by time t, $0 < q_t < s_{DM}$ for any t. Hence, if no signal has arrived by time t, the agent keeps testing at any $t < T$.

Part 2.) Time $t = t^*$ is the time at which $q_t = s_{DM}$ if no signal has arrived by time t since

$$s_{DM} = \frac{q_0 \exp\left(-\lambda^{\omega_1} t^*\right)}{q_0 \exp\left(-\lambda^{\omega_1} t^*\right) + (1-q_0)\exp\left(-\lambda^{\omega_0} t^*\right)} \Leftrightarrow \exp\left(\left(\lambda^{\omega_1} - \lambda^{\omega_0}\right) t^*\right) = \frac{q_0}{1-q_0}\frac{1-s_{DM}}{s_{DM}} \Leftrightarrow (4)$$

We first show that the agent keeps testing at any time $t < t^*$. Suppose the agent stops testing the first time a signal arrives or at time τ where $\tau \leq t^*$. If no signal has arrived by time τ, then $s_{DM} \leq q_t < 1$. Clearly,

$$V_t\left(q_0, \tau\right) = (1-q_0)\frac{s_A}{1-s_A}\left(1 - \exp\left(-\lambda^{\omega_0}\tau\right)\right) + q_0 \tag{10}$$

increases in τ. Hence, when no signal has arrived, the agent keeps testing at any $t < t^*$. Next, consider $t = t^*$, i.e. $q_{t^*} = s_{DM}$. Suppose the agent stops the first time a signal arrives or at time τ where $\tau > t^*$. Clearly,

$$V_{t^*}\left(s_{DM}, \tau\right) = (1-s_{DM})\frac{s_A}{1-s_A} + s_{DM}\left(1 - \exp\left(-\lambda^{\omega_1}\tau\right)\right), \tag{11}$$

increases in τ. Hence, if $\tau > t^*$, then $\tau = T$ is optimal. Therefore, the agent stops at t^* and receives $V_{t^*}\left(s_{DM}, t^*\right) = s_{DM}$ rather than choosing $\tau = T$ and receiving (11) if and only if

$$V_{t^*}\left(s_{DM}, t^*\right) - V_{t^*}\left(s_{DM}, T\right) \geq 0 \Leftrightarrow \log\left(\frac{s_A}{1-s_A}\right) \leq \log\left(\frac{s_{DM}}{1-s_{DM}}\right) - \lambda^{\omega_1}\left(T - t^*\right), \tag{12}$$

i.e. the agent stops at t^* if and only if $s_A \leq \overline{s}_A$ using the definition of $\overline{s}_A$ in (5). Clearly, $\overline{s}_A > 0$ for $s_{DM} > 0$ and finite T given (5). Further, $\overline{s}_A < \underline{q}$ since $\overline{s}_A$ decreases in s_{DM} given (4) and (5) and, if s_{DM} is at its lower bound, i.e. $s_{DM} = \underline{q}$, then $\overline{s}_A = \underline{q}$.

The DM's strategy is optimal given (1) independent of the agent's strategy for $q_T \neq s_{DM}$. Although the DM is indifferent between a_1 and a_0 at $q_T = s_{DM}$, there can be no equilibrium in which the DM randomizes at $q_T = s_{DM}$. If the DM were to do so, the agent would strictly prefer to stop the first time that the belief is equal to the lowest belief at which the DM chooses a_1 with probability 1 which is not well-defined. □

In equilibrium, given the DM's strategy, the agent anticipates that approval is chosen if and only if the belief at the deadline exceeds the DM's standard. If the DM is initially inclined to reject, i.e. $q_0 < s_{DM}$, the agent has an upside but no downside from testing. The upside is that a signal may arrive and, in this event, the DM takes an action in line with the agent's interest. When no signal arrives, the belief drifts down. In this event, the DM continues to favor rejection and, consequently,

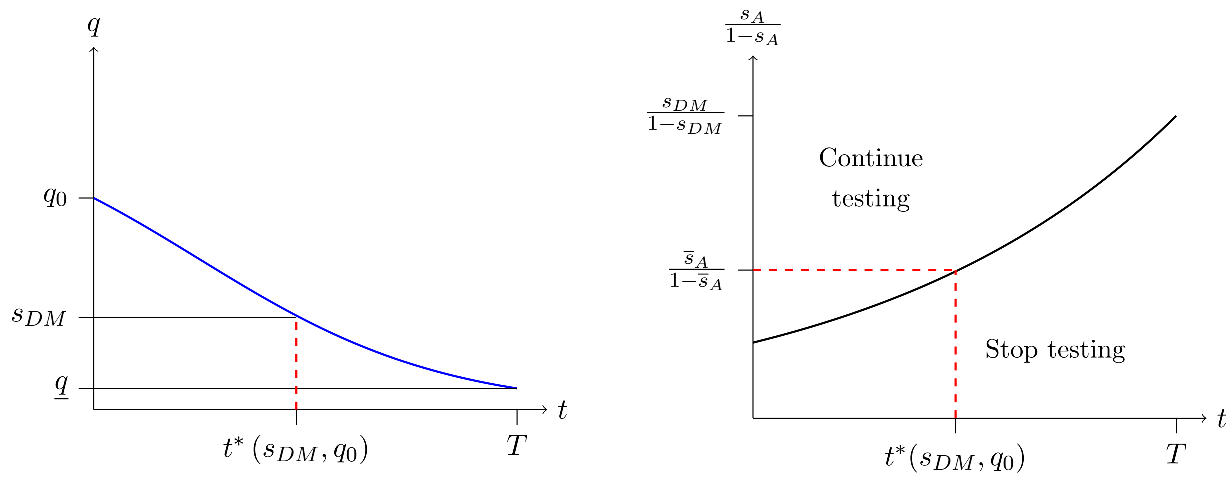

Figure 1: Left panel: The graph shows how the belief drifts down over time when no signal arrives. Time $t^*(s_{DM}, q_0)$ is the time at which the belief reaches the DM's standard. Right panel: The graph divides levels of the agent's standard s_A into those for which the agent stops or continues testing at time t if $q_t = s_{DM}$. If the agent's standard is equal to the critical level $\overline{s}_A$, then the agent is indifferent between testing and stopping if $q_t = s_{DM}$ at time $t^*(s_{DM}, q_0)$.

the agent suffers no downside from testing.

By contrast, the agent has different concerns if the DM is initially inclined to approve, i.e. $q_0 \geq s_{DM}$. The agent has no downside from testing up until time t^*, at which point the belief has drifted down to the DM's standard if no signal has arrived. However, at time t^*, the agent's decision to test is pivotal to the DM's choice even if no signal arrives. If the agent stops, the DM approves. If the agent continues and no signal arrives until the deadline, the DM rejects instead.

Therefore, the agent faces a trade-off at time t^* provided the agent still favors approval when no signal has arrived by the deadline, i.e. provided $s_A < \underline{q}$.[15] By stopping at time t^*, the agent ensures that the DM approves, which is the agent's favored choice at this point. By testing, the agent learns if a signal $\theta = \omega_0$ arrives which causes the DM to reject. In this event, the agent benefits from having tested because, given the signal, the agent favors rejection. However, in the event that no such signal arrives, the agent suffers from having tested because the DM rejects although the agent favors approval.[16] The downside from testing is stronger, the more the agent is inclined towards approval, i.e. the lower the agent's standard. The opportunity to wait for a signal $\theta = \omega_0$ outweighs the downside from a suboptimal choice in case no signal arrives if and only if $\overline{s}_A \leq s_A$.[17] See also Figure 1.

The equilibrium is unique given the assumption that the agent stops if he is indifferent between stopping or testing. The agent is indifferent if a signal has arrived and further testing does not alter

[15] In the extreme case $s_A = 0$, the agent has a downside but no strict upside from testing at time t^*. This extreme case is closely related to Brocas and Carrillo [2007]'s framework when $\underline{\pi} = 0$ and $\alpha < 1 - \beta$.

[16] Note that for the agent to face a downside from testing, the posterior belief needs to drift down in the absence of a signal. If we instead assumed that the belief drifts up, i.e. no news is good news, the agent would always test and the DM could never do strictly better when testing is hidden. For an analysis of a testing technology that allows for the belief to drift up or down over time see Section 6.3.

[17] The equilibrium payoff would be the same if the agent chose at time $t = 0$ until which time $t = k$ he will test for $k \in [0, T]$.

the DM's approval choice. Otherwise, the agent is only indifferent if $s_A = \overline{s}_A$ and no signal has arrived by time t^*. In this knife-edge case, the equilibrium is not unique but we restrict attention to the equilibrium when the agent stops at t^*.

4 Hidden Testing

When testing is hidden, the agent can choose whether or not to disclose his findings. The agent's optimal disclosure depends on how the DM interprets no disclosure and, therefore, multiple equilibrium payoff profiles may exist. Our aim is to understand what happens when the agent exploits the additional discretion under hidden testing to his advantage, that is, we want to characterize agent-preferred equilibria. In the next section, we will then see under which conditions such strategic disclosure by the agent can benefit the DM.

Proposition 2. *Under hidden testing, an equilibrium always exists. There is a critical level of the DM's standard $\overline{s}_{DM}(q_0, T)$ defined by*

$$\log\left(\frac{\overline{s}_{DM}}{1-\overline{s}_{DM}}\right) = \log\left(\frac{q_0}{1-q_0}\right) + \lambda^{\omega_0} T, \tag{13}$$

such that the following are features of an agent-preferred equilibrium,

1. *for $s_A \geq \underline{q}$ or $s_{DM} > \overline{s}_{DM}$,*
   - *the agent's testing strategy is as described in Proposition 1, the agent always discloses, i.e. $m = h^T$ for every h^T,*
   - *the DM chooses a_1 if and only if the agent discloses h^T and the DM prefers a_1 given h^T,*
2. *for $s_A < \underline{q}$ and $s_{DM} \leq \overline{s}_{DM}$,*
   - *if no signal has arrived by time t, the agent keeps testing at time t for any $t < T$, the agent discloses h^T if and only if a signal $\theta = \omega_0$ has arrived by time T,*
   - *the DM chooses a_0 if and only if the agent discloses h^T and the DM prefers a_0 given h^T.*

Proof. Part 1) We first show that an equilibrium with these features exists for all parameter configurations. The DM's strategy is optimal since $m = \emptyset$ does not arise on the equilibrium path and we can assign a belief such that $\Pr(\omega_1 | m = \emptyset) < s_{DM}$. Further, the agent's disclosure strategy is optimal for any h^T for the following reason. If $\Pr\left(\omega_1 | h^T\right) \geq s_{DM}$, then the DM chooses a_1 given $m = h^T$ and the agent prefers a_1 since $s_{DM} \geq s_A$. If $\Pr\left(\omega_1 | h^T\right) < s_{DM}$, both $m = h^T$ and $m = \emptyset$ lead to a_0. Given $m = h^T$ for all h^T, the agent's optimal testing strategy is given by Proposition 1.

Next, we show that this is an agent-preferred equilibrium if $s_A \geq \underline{q}$. Since $\underline{q} > \overline{s}_A$, the agent keeps testing at any t if and only if no signal has arrived by time t for $t < T$ by Proposition 1. Then $m = h^T$ leads to the agent's favored action a for any feasible h^T since $\Pr\left(\omega_1 | h^T\right) \in \left\{0, \underline{q}, 1\right\}$ and $1 > s_{DM} \geq s_A \geq \underline{q} > 0$. Therefore, the agent achieves his first-best payoff and the equilibrium must be agent-preferred.[18]

[18]The agent's first-best payoff is the payoff the agent would achieve if he was also in control of taking action a.

Finally, we show that this is an agent-preferred equilibrium if $s_A < \underline{q}$ and $s_{DM} > \overline{s}_{DM}$. We start by showing that, in any equilibrium, the DM must choose a_0 following $m = \emptyset$. Suppose the DM chose a_1 following $m = \emptyset$ with probability $p \in (0, 1)$. Then, the agent's optimal disclosure would be to choose $m = \emptyset$ if and only if $s_A \leq \Pr\left(\omega_1 | h^T\right) < s_{DM}$. Given $s_{DM} > \overline{s}_{DM} > q_0$, the agent would keep testing if no signal has arrived by time t for any $t < T$ since stopping at time t would yield

$$V_t\left(q_t, t\right) = \left(1 - q_t\right) \frac{s_A}{1 - s_A} \left(1 - p\right) + q_t p. \tag{14}$$

while testing until a signal arrives or $\tau = t' > t$ would yield

$$\begin{aligned} V_t\left(q_t, t'\right) = &\left(1 - q_t\right) \frac{s_A}{1 - s_A} \left(1 - \exp\left(-\lambda^{\omega_0} \tau\right) + \exp\left(-\lambda^{\omega_0} \tau\right)\left(1 - p\right)\right) \\ &+ q_t \left(1 - \exp\left(-\lambda^{\omega_1} \tau\right) + \exp\left(-\lambda^{\omega_1} \tau\right) p\right), \end{aligned} \tag{15}$$

and for any $t' > t$,

$$V_t\left(q_t, t'\right) > V_t\left(q_t, t\right) \Leftrightarrow \left(1 - q_t\right) \frac{s_A}{1 - s_A} p \left(1 - \exp\left(-\lambda^{\omega_0} \tau\right)\right) + q_t \left(1 - p\right)\left(1 - \exp\left(-\lambda^{\omega_1} \tau\right)\right) > 0. \tag{16}$$

But given the agent's strategy, the DM's consistent belief given $m = \emptyset$ would satisfy $\Pr\left(\omega_1 | m = \emptyset\right) = \underline{q}$ and the DM should choose a_0 with probability 1 since $\underline{q} < s_{DM}$, leading to a contradiction.

Next, suppose the DM chose a_1 following $m = \emptyset$ with probability 1. Then, the agent would choose $m = h^T$ if signal $\theta = \omega_0$ arrived and $m = \emptyset$ otherwise.[19] The agent would keep testing at t if and only if no signal has arrived by time t for $t < T$ by (16) with $p = 1$. Given the agent's strategy, the DM's consistent beliefs given $m = \emptyset$ would satisfy

$$\frac{\Pr\left(\omega_1 | m = \emptyset\right)}{\Pr\left(\omega_0 | m = \emptyset\right)} = \frac{q_0}{1 - q_0} \frac{1}{\exp\left(-\lambda^{\omega_0} T\right)} \Leftrightarrow \log\left(\frac{\Pr\left(\omega_1 | m = \emptyset\right)}{\Pr\left(\omega_0 | m = \emptyset\right)}\right) = \log\left(\frac{q_0}{1 - q_0}\right) + \lambda^{\omega_0} T, \tag{17}$$

and, hence, $\Pr\left(\omega_1 | m = \emptyset\right) = \overline{s}_{DM}$ by (13). But then the DM should choose a_0 given these beliefs since $\overline{s}_{DM} < s_{DM}$, leading to a contradiction. Given that the DM's unique equilibrium strategy is to choose a_0 following $m = \emptyset$, the agent's strategy in the equilibrium of Part 1 is his best reply and the equilibrium is agent-preferred.

Part 2) The DM's consistent beliefs given $m = \emptyset$ satisfy (17) and, hence, $\Pr\left(\omega_1 | m = \emptyset\right) = \overline{s}_{DM}$ by (13). Since $\overline{s}_{DM} \geq s_{DM}$, the DM optimally chooses a_1 following $m = \emptyset$. In addition, the agent has no reason to deviate because he achieves his first-best payoff. This implies that the equilibrium is agent-preferred. $\square$

[19] If the agent chose $m = \emptyset$ if and only if no signal arrived, the DM's consistent belief given $m = \emptyset$ would satisfy $\Pr\left(\omega_1 | m = \emptyset\right) = \underline{q}$, leading to a contradiction by the same reasoning as above.

The proposition shows that the agent does not strictly benefit from withholding findings in case the agent prefers rejection when no signal has arrived by the deadline, i.e. $\underline{q} \leq s_A$. Recall that we assume that the DM always prefers rejection in this event, i.e. $\underline{q} < s_{DM}$. Therefore, the agent can wait for a signal until the deadline, disclose his findings and the DM will act in the agent's interest whatever the agent found.

Suppose the agent favors approval when no signal arrived by the deadline, i.e. $\underline{q} > s_A$. The agent would ideally keep testing for as long as no signal has arrived and then recommend approval if and only if he himself favors approval given the evidence. That is, the agent could withhold his findings if he prefers approval and disclose his findings if he prefers rejection. If the agent favors rejection, then the players' interests are aligned and disclosing his findings will lead the DM to reject. If the agent withholds his findings, the DM only infers that the agent favored approval and how the DM optimally reacts to this inference depends on s_{DM}.

If the DM is not sufficiently inclined towards approval, i.e. if $s_{DM} > \overline{s}_{DM}$, then the fact that the findings convinced the agent of approval is insufficient evidence in favor of state ω_1 for the DM to approve. In particular, in the event that the agent does not disclose, the DM's posterior rises to the critical level $\overline{s}_{DM}$ but the DM optimally rejects at this posterior. Therefore, the DM requires hard evidence to choose approval in equilibrium and the agent has no benefit from strategically withholding information he has gathered. The agent effectively faces the same trade-offs as under observable testing.

By contrast, if the DM is sufficiently inclined towards approval, i.e. $s_{DM} \leq \overline{s}_{DM}$, then the DM is willing to approve based on the inference that the agent favors approval. In particular, although the DM does not know if the evidence led the agent to form a posterior that lies above or below the DM's standard, the DM finds it optimal to choose approval conditional on no disclosure. Hence, the agent's recommendation is indeed followed by the DM and the agent achieves his first-best payoff.[20]

The following lemma describes equilibrium payoffs across all equilibria under hidden testing.

Proposition 3. *Suppose testing is hidden.*

1. *If $s_A \geq \underline{q}$ or $s_{DM} > \overline{s}_{DM}$, each player's payoff in the equilibrium in Part 1 of Proposition 2 is their unique equilibrium payoff.*

2. *If $s_A < \underline{q}$ and $s_{DM} \leq \overline{s}_{DM}$, in any given equilibrium, each player's payoff is either equal to their payoff in the equilibrium in Part 1 or Part 2 of Proposition 2.*

Proof. Part 1) The agent can always achieve at least the same payoff as in the equilibrium of Part 1 of Proposition 2 by following the testing strategy in Proposition 1 and reporting $m = h^T$ for any h^T.[21] Since this equilibrium is agent-preferred by Part 1 of Proposition 2, this must be the agent's

[20] If players' payoffs were discounted by the delay in decision making, the agent would reveal signal $\theta = \omega_1$ as soon as it arrives to achieve approval sooner. In equilibrium, following $m = \emptyset$, the DM would then infer that no signal arrived by the deadline and reject, leaving the agent to face the same trade-offs as under observable testing. However, an equilibrium in which $m = \emptyset$ is followed by approval would continue to exist when evidence accrues incrementally. See also footnote 34.

[21] Recall that we assume that the DM's beliefs following $m = h^T$ must be consistent with the testing technology.

unique equilibrium payoff. If the agent's equilibrium payoff is unique, then the DM's equilibrium payoff must also be unique.

Part 2) First, if the DM chooses a_0 following $m = \emptyset$ in equilibrium, the payoff profile must be equal to the one the equilibrium in Part 1 of Proposition 2. This is because the agent's optimal disclosure must result in the DM choosing a_1 if and only if $s_{DM} \leq \Pr\left(\omega_1 | h^T\right)$, because then also $s_A \leq \Pr\left(\omega_1 | h^T\right)$ and, otherwise, any message results in a_0. Therefore, by Proposition 1, the agent's optimal testing strategy is unique and as described in Proposition 1.

Second, if the DM chooses a_1 following $m = \emptyset$ in equilibrium, the payoff profile must be equal to the one in the equilibrium in Part 2 of Proposition 2. This is because the agent's optimal disclosure must result in the DM choosing the agent's preferred action for any h^T because, if $\Pr\left(\omega_1 | h^T\right) < s_A$, the agent reports $m = h^T$ and the DM chooses a_0 and, otherwise, the agent reports $m = \emptyset$ and the DM chooses a_1. Therefore, by Proposition 2, the agent's testing strategy is unique and as described in Part 2 of Proposition 2.

Finally, no other equilibrium payoff would arise if the DM chose a_1 following $m = \emptyset$ with some probability $p \in (0, 1)$. Suppose the DM chose a_1 following $m = \emptyset$ with some probability $p \in (0, 1)$. Then the agent chooses $m = \emptyset$ if and only if no signal arrived by time T and $s_A \leq \Pr\left(\omega_1 | h^T\right) < s_{DM}$. If $q_0 \geq s_{DM}$, the agent keeps testing at time t if and only if no signal has arrived by time t for $t < t^*$ by Proposition 1. If $q_0 \geq s_{DM}$ and the agent optimally stops at time t^*, the equilibrium payoff is as in Part 1 of Proposition 2. If $q_0 \geq s_{DM}$ and the agent does not stop at time t^* or if $q_0 < s_{DM}$, the DM's consistent beliefs given $m = \emptyset$ would lead her to choose a_0 with probability 1 by the same reasoning as in the proof of Part 1 of Proposition 2. With this we have exhausted all possibilities for the DM's action following $m = \emptyset$. □

The agent can always guarantee himself at least the same payoff as under observable testing by following the same testing strategy as under observable testing and then disclosing his findings. If the DM is not sufficiently inclined to approve, i.e. $s_{DM} > \overline{s}_{DM}$, or if the players both favor rejection when no signal has arrived by the deadline, i.e. $s_A \geq \underline{q}$, then this is also the highest payoff the agent can achieve as shown by Proposition 2, implying that this is the unique equilibrium payoff.

Otherwise, i.e. if $s_A < \underline{q}$ and $s_{DM} \leq \overline{s}_{DM}$, there are two possible profiles of equilibrium payoffs. If the DM rejects following no disclosure in equilibrium, the agent achieves the same payoff as under observable testing. If the DM accepts following no disclosure in equilibrium, the agent achieves his first-best payoff as in Part 2 of Proposition 2. These are the only two possible equilibrium payoffs because there cannot be an equilibrium in which the agent keeps testing if the belief is weakly below the DM's standard and the DM randomizes following no disclosure. This is because, if the DM were to accept with a probability strictly between 0 and 1 following no disclosure, the agent would withhold information only if no signal arrived by the deadline since $s_A < \underline{q}$ but, in this event, the DM would strictly prefer rejection since we assume $\underline{q} < s_{DM}$.[22]

[22]Proposition 3 would still apply if the agent chose at time $t = 0$ until which time $t = k$ he would test for $k \in [0, T]$.

5 Welfare Comparison

This section will compare players' payoffs across hidden and observable testing and show under which conditions both players are strictly better off under hidden testing. As the previous section has shown, under hidden testing, the agent is never strictly worse off than under observable testing. However, it is unclear how the DM's payoff compares across regimes. Under observable testing, the DM can react optimally to the information generated by the agent's testing, but the agent may not exhaust all testing opportunities even if no signal has arrived. Under hidden testing, the agent exhausts all such testing opportunities but may strategically withhold findings which can hinder the DM from reacting optimally to the information gathered. Therefore, the DM faces a trade-off between hidden and observable testing.

Proposition 4. *In an agent-preferred equilibrium under hidden testing,*

1. *if $s_A \geq \underline{q}$ or $s_{DM} > \overline{s}_{DM}$, each player has the same payoff as under observable testing,*
2. *if $\overline{s}_A < s_A < \underline{q}$ or $q_0 < s_{DM} \leq \overline{s}_{DM}$, the agent has a strictly higher payoff and the DM a strictly lower payoff than under observable testing,*
3. *if $s_A \leq \overline{s}_A$ and $s_{DM} \leq q_0$, each player has a strictly higher payoff than under observable testing.*[23]

Proof. Part 1) This is implied by Part 1 of Proposition 2.

Part 2 and 3), i.e. $s_A < \underline{q}$ and $s_{DM} \leq \overline{s}_{DM}$. The agent must be strictly better off in the agent-preferred equilibrium under hidden testing than under observable testing. This is because, by Proposition 3, the agent's payoff under hidden testing is either equal to the payoff under observable testing or equal to the payoff in the equilibrium in Part 2 of Proposition 2. The equilibrium in Part 2 of Proposition 2 is agent-preferred by Proposition 2. Hence, the agent is strictly better off in the agent-preferred equilibrium under hidden testing than under observable testing.

By Propositions 1 and 2, the DM's payoff under hidden testing is the same as under observable testing if a signal arrives in the interval $[0, t^*]$. Suppose no signal has arrived by t^*.

Since $s_{DM} \leq \overline{s}_{DM}$, under hidden testing, the DM chooses a_0 if and only if $\theta = \omega_0$ arrives over time horizon $(t^*, T]$. Therefore, the DM's payoff under hidden testing is

$$s_{DM} + (1 - s_{DM})\left(1 - \exp\left(-\lambda^{\omega_0}\left(T - t^*\right)\right)\right) \frac{s_{DM}}{1 - s_{DM}}. \tag{18}$$

If $\overline{s}_A < s_A$ or $q_0 < s_{DM}$, then, under observable testing, the agent keeps testing at time t if and only if no signal has arrived by time t for $t^* \leq t < T$, and the DM chooses a_1 if and only if $\theta = \omega_1$ arrives over time horizon $(t^*, T]$. Therefore, the DM's payoff under observable testing is

$$s_{DM}\left(1 - \exp\left(-\lambda^{\omega_1}\left(T - t^*\right)\right)\right) + (1 - s_{DM}) \frac{s_{DM}}{1 - s_{DM}}. \tag{19}$$

[23] Note that $\overline{s}_A < \underline{q}$ and $q_0 < \overline{s}_{DM}$ so that three parts of Proposition 4 exhaust all possibilities.

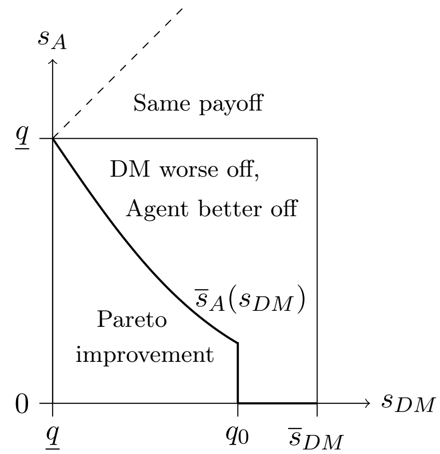

Figure 2: Payoff comparison between the agent-preferred equilibrium under hidden testing and the equilibrium under observable testing for combinations of s_{DM} and s_A with $s_A \leq s_{DM}$ and $\underline{q} < s_{DM}$.

Hence, the DM's payoff is strictly higher under observable than hidden testing since

$$(18) < (19) \Leftrightarrow \lambda^{\omega_0} < \lambda^{\omega_1}. \tag{20}$$

Otherwise, i.e. if $s_A \leq \overline{s}_A$ and $s_{DM} \leq q_0$, under observable testing, the agent stops at $t = t^*$ when no signal has arrived. Then $q_{t^*} = s_{DM}$ and the DM chooses a_1. Therefore, the DM's payoff is strictly higher under hidden than observable testing since

$$s_{DM} + (1 - s_{DM})\left(1 - \exp\left(-\lambda^{\omega_0}\left(T - t^*\right)\right)\right)\frac{s_{DM}}{1 - s_{DM}} > s_{DM}. \tag{21}$$

□

The interesting payoff comparison is for the region in which the agent can strictly benefit from withholding information under hidden testing, that is, when the DM is sufficiently inclined to accept, i.e. $s_{DM} \leq \overline{s}_{DM}$, and players disagree on the optimal action when no signal has arrived by the deadline, i.e. $s_A < \underline{q}$.[24] See also Figure 2.

Then if the conflict between the two players is sufficiently small, i.e. $\overline{s}_A < s_A$, or the DM is initially inclined to reject, i.e. $q_0 < s_{DM}$, the agent keeps testing when no signal has arrived under either regime.[25] Therefore, under observable testing, the DM has access to all obtainable information and can react optimally. Hidden testing is then worse for the DM since the agent's strategic manipulation leads her to approve in situations in which, knowing the full evidence, she would have rejected.

Corollary 1. *The agent-preferred equilibrium under hidden testing yields a strict Pareto improvement over observable testing if and and only if $s_A \leq \overline{s}_A$ and $s_{DM} \leq q_0$.*

[24]Recall that we assume throughout that $\underline{q} < s_{DM}$.

[25]Recall that, for the knife-edge case $s_A = \overline{s}_A$, we assume that the agent stops at time t^* under observable testing if he is indifferent between testing and stopping. If we instead assumed that the agent keeps testing at time t^* if indifferent, the case $s_A = \overline{s}_A$ would be included in Part 2 instead of Part 3 of Proposition 4.

However, if the conflict between players is instead large, i.e. $s_A \leq \bar{s}_A$, and the DM favors approval at the prior, i.e. $s_{DM} \leq q_0$, both can be strictly better off under hidden testing. Under observable testing, the agent stops testing before the deadline even when no signal has arrived. Indeed, in this situation, neither the DM nor the agent learns about whether a signal $\theta = \omega_0$ would have arrived in the time remaining until the deadline. This is a mutual loss since both players would have preferred rejection in this event. By contrast, in an agent-preferred equilibrium under hidden testing, the agent will learn about whether a signal $\theta = \omega_0$ arrives in the time remaining and he will disclose it truthfully. If no such signal arrives, the DM approves, just as she would have done under observable testing. Therefore, both players benefit from hidden testing.[26]

Proposition 4 has focused on the agent-preferred equilibrium under hidden testing. When considering the full range of equilibrium payoffs under hidden testing, the payoff comparisons of Proposition 4 hold in the weak sense.[27] This implies that hidden testing offers a Pareto improvement in a weak sense if and only if $s_A \leq \bar{s}_A$ and $s_{DM} \leq q_0$, that is, no player is strictly worse off in any equilibrium under hidden testing than in the unique equilibrium under observable testing and there exists at least one equilibrium under hidden testing in which each player is strictly better off than under observable testing.[28]

6 Robustness and Extensions

The previous section has shown that both players can be better off under hidden than observable testing. This section will show what would happen if the DM could choose some intermediate regime in which the agent's findings become public with some probability. Moreover, this section will show that a strict Pareto improvement also exists if the agent can only disclose signals (but not the absence of signals) or if the testing technology produces incremental evidence in favor of either state. Proofs for this section can be found in the supplementary appendix.

6.1 Intermediate regime

The DM never observes the agent's findings directly when testing is hidden, while the DM is guaranteed to observe the agent's findings when testing is observable. Could the DM benefit from a regime with an intermediate degree of transparency? Suppose the agent acquires evidence in private but, at time T, before the agent decides whether to disclose his findings, the DM observes the history h^T with probability $p \in [0, 1]$, independent of the agent's findings. If the DM does not observe the history, the agent chooses whether or not to disclose it.[29] For this exercise, we assume

[26] If the agent had state-independent preferences, i.e. $s_A \leq 0$, then the equilibrium payoff profile under hidden testing would be unique and equal to the payoff profile under observable testing. The reason is that, if $s_{DM} \leq q_0$, the agent would not test at all under either regime as the DM chooses a_1 without any tests. Moreover, if $s_{DM} > q_0$, there is no equilibrium under hidden testing in which the DM chooses a_1 following $m = \emptyset$ because, if the DM were to choose a_1 following $m = \emptyset$, the agent would not test and send $m = \emptyset$. Hence, the agent faces the same trade-offs regarding testing under either regime.

[27] i.e. replace "strictly" with "weakly".

[28] This follows from Part 2 of Proposition 3.

[29] Hence, $p = 0$ corresponds to hidden testing and $p = 1$ to observable testing.

that the agent keeps testing if he is indifferent between testing and stopping.

Proposition 5. *If $s_A < \overline{s}_A$ and $s_{DM} \leq q_0$, the probability p^* of observing h^T that maximizes the DM's payoff in the Pareto-best equilibrium is given by*

$$p^* = \frac{1 - \exp\left(-\lambda^{\omega_0}\left(T - t^*\left(s_{DM}, q_0\right)\right)\right)}{\exp\left(-\lambda^{\omega_1}\left(T - t^*\left(s_{DM}, q_0\right)\right)\right)\left(\frac{s_{DM}}{1-s_{DM}}\right)\left(\frac{1-s_A}{s_A}\right) - \exp\left(-\lambda^{\omega_0}\left(T - t^*\left(s_{DM}, q_0\right)\right)\right)} \tag{22}$$

where $t^*\left(s_{DM}, q_0\right)$ is defined by (4) and $p^* \in (0,1)$. Given $p = p^*$, the agent's payoff is equal to his payoff under observable testing.

When a Pareto improvement from hidden testing exists, the agent acquires more information under hidden testing because he can ensure that this additional information influences the DM's approval choice in line with his interests. However, to have an incentive to acquire more information, the agent only needs to have this advantage with a probability just high enough to make him indifferent between acquiring more information or not at time t^*. The larger is the conflict of interest, the lower must be the probability p^* with which the DM observes the evidence in order for the agent to keep testing when no signal has arrived by time t^*.[30]

6.2 Disclosure

The Pareto improvement continues to exist if the agent could only disclose the arrival of a signal but not the absence of arrival.[31] To see this, note that, in the equilibrium described in Part 2 of Proposition 2, disclosure occurs if and only if a signal $\theta = \omega_0$ arrives. The agent effectively recommends an approval choice and the DM is willing to implement the recommended choice. Therefore, the Pareto improvement would also exist if the agent could merely send a cheap talk message or if the approval decision was delegated to the agent.[32]

6.3 Incremental evidence

We have assumed that signals fully reveal the state. Does a Pareto improvement also exist if the testing technology generates only incremental evidence which can be either in favor of approval or rejection?[33] To show this, assume instead that testing generates a Brownian motion X with state-dependent drift, that is, we assume X starts at $X_0 = 0$ and evolves according to

$$dX_t = \mu_\omega dt + dW_t, \tag{23}$$

[30]If $s_A \geq \overline{s}_A$ or $s_{DM} > q_0$, the DM's payoff is maximized by $p^* = 1$ because the agent never stops testing under observable testing when no signal has arrived and so the DM achieves her first-best payoff.

[31]Further, it is not crucial that the agent privately observes the time at which he stops testing. If it were publicly observed, the agent could always test until the deadline and then follow the same disclosure strategy as outlined in Part 2 of Proposition 2.

[32]If the agent were uncertain about s_{DM} and believed $\Pr\left(s_{DM} > \overline{s}_{DM}\right) > 0$, then he would prefer to disclose a signal $\theta = \omega_1$ and achieve approval for certain rather than not disclose and face a positive probability of rejection. The agent would then face the same trade-offs as under observable testing. However, if the DM could send a cheap talk message about whether or not $s_A \leq \overline{s}_A$ and $s_{DM} \leq q_0$, Corollary 1 would still apply.

[33]In the original model, testing generates either conclusive evidence when a signal arrives or incremental evidence in favor of rejection when no signal arrives.

where W is a standard Wiener process independent of ω and $\mu_0 = -1 < 0 < 1 = \mu_1$.

Then, under observable testing, the (common) posterior belief at time t, $q_t = Pr\left(\omega_1 | h^t\right)$, depends only on X_t. It is useful to track posterior beliefs using the posterior log-likelihood ratio Q_t, where Q_t satisfies

$$Q_t = \log\left(\frac{q_t}{1-q_t}\right) = \log\left(\frac{q_0}{1-q_0}\right) + \log\left(\frac{\phi\left(\frac{X_t - t}{\sqrt{t}}\right)}{\phi\left(\frac{X_t + t}{\sqrt{t}}\right)}\right) = Q_0 + 2X_t, \tag{24}$$

and ϕ is the density of a standard normal distribution. When testing is hidden, disclosing h^T is equivalent to disclosing the endpoint of the path X_T as the endpoint contains all payoff-relevant information the agent has. We impose no restrictions on s_A, s_{DM}, q_0 and T other than $s_A \leq s_{DM}$. All remaining assumptions stay unchanged from Section 2.

Proposition 6. *Suppose testing generates a Brownian motion X. There exists a critical level of the agent's standard $\widehat{s}_A\left(s_{DM}, T\right)$ and a critical level of the DM's standard $\widehat{s}_{DM}\left(q_0, T, s_A\right)$ such that the agent-preferred equilibrium under hidden testing yields a strict Pareto improvement over observable testing if $s_A \leq \widehat{s}_A$ and $s_{DM} \leq \widehat{s}_{DM}$, where $\widehat{s}_A \in (0, s_{DM})$ is defined by*

$$\log\left(\frac{\widehat{s}_A}{1-\widehat{s}_A}\right) = \log\left(\frac{s_{DM}}{1-s_{DM}}\right) + \log\left(1 - \frac{\sqrt{T}}{\phi\left(\sqrt{T}\right) + \sqrt{T}\Phi\left(\sqrt{T}\right)}\right), \tag{25}$$

and $\widehat{s}_{DM} \in (q_0, 1)$ is defined by

$$\log\left(\frac{\widehat{s}_{DM}}{1-\widehat{s}_{DM}}\right) = Q_0 + \log\left(\frac{1 - \Phi\left(\frac{\frac{1}{2}\left(\log\left(\frac{s_A}{1-s_A}\right) - Q_0\right) - T}{\sqrt{T}}\right)}{1 - \Phi\left(\frac{\frac{1}{2}\left(\log\left(\frac{s_A}{1-s_A}\right) - Q_0\right) + T}{\sqrt{T}}\right)}\right), \tag{26}$$

where ϕ is the density and Φ the CDF of a standard Normal distribution. Moreover, for any q_0 and T, the region in which $s_A \leq \widehat{s}_A$ and $s_{DM} \leq \widehat{s}_{DM}$ is non-empty.

When testing is hidden, the agent-preferred equilibrium has a similar structure to the one in the original model: the agent discloses if he prefers rejection and does not disclose if he prefers approval. Conditional on the inference that the agent tested until the deadline and prefers approval, the DM is willing to approve.[34] Hence, the agent achieves his first-best payoff by strategically withholding information. Unlike in the original model, the critical level $\widehat{s}_{DM}$ of the DM's standard is a function of the agent's standard. This is because the agent's belief at the deadline can take any value in

[34] Note that, even if players' payoffs were subject to some discounting, there would exist an equilibrium under hidden testing in which $m = \emptyset$ is followed by approval. At any time, the agent would face a trade-off between the action being taken sooner and being based on better information. Hence, if the agent continued until the deadline and reports $m = \emptyset$, the DM would infer that the agent did not find highly convincing evidence in favor of approval or rejection earlier yet favors approval at the deadline. Based on this inference, the DM would approve if s_{DM} is sufficiently low.

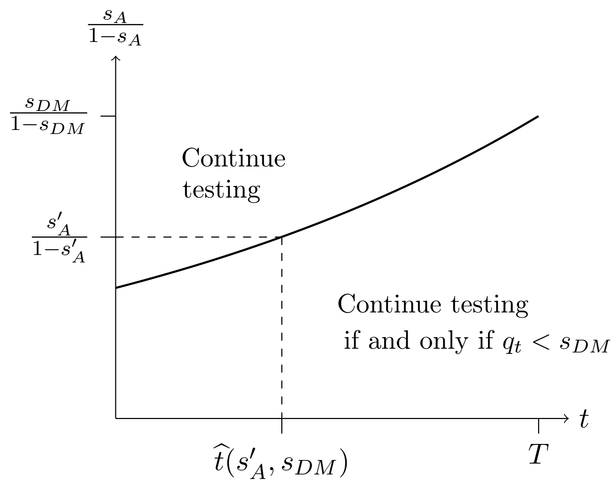

Figure 3: The graph shows the critical time $\widehat{t}$ for any given s_A holding s_{DM} fixed, i.e. for all $t < \widehat{t}$, the agent keeps testing at time t and, for all $t \geq \widehat{t}$, the agent continues testing at time t if and only if $q_t < s_{DM}$.

$(0, 1)$ and, therefore, the fact that the agent prefers approval is weaker evidence in favor of state ω_1 if the agent is more inclined to approve.[35]

When testing is observable, the agent faces a trade-off between testing and stopping when the belief is equal to the DM's standard for the same reasons as in the original model. However, unlike in the original model, this event can arise with positive probability at any point in time. If the event $q_t = s_{DM}$ arises shortly before the deadline, the agent's testing within the remaining time is relatively unlikely to move the belief below the agent's standard but relatively likely to move the belief above the agent's but below the DM's standard. Therefore, the downside of testing outweighs the upside. By contrast, earlier on in the game, when there is more time remaining until the deadline, the belief is more likely to drift towards the correct state and, therefore, it is less likely that testing leads players to disagree on the optimal approval choice. We show that there exists some critical time $\widehat{t}$ such that the agent prefers to keep testing in the event $q_t \geq s_{DM}$ if and only if $t < \widehat{t}$. This critical time $\widehat{t}$ decreases as the conflict between players increases, that is, as the agent's standard drops. See also Figure 3.

Since the agent's strategy under observable testing is much more complex than in the original model, it is harder to compute distributions over final beliefs and, hence, equilibrium payoffs. However, to show that a Pareto improvement exists, it is sufficient to analyze a situation in which the agent's standard is low enough such that he would stop in the event $q_t = s_{DM}$ under observable testing irrespective of when it occurs. If $q_0 \geq s_{DM}$, this implies that the belief never drops below the DM's standard and the DM approves with probability 1. Then hidden testing yields a Pareto improvement for the same reason as in the original model: both players benefit from hidden testing because the agent tests more under hidden testing and, if the additional tests lead to rejection being chosen, then both players prefer rejection.

[35] Since a history for which the players disagree always arises with positive probability, such an agent-preferred equilibrium exists for any s_A.

Unlike in the original model, there are parameter combinations for which a strict Pareto improvement exists even if the DM is initially inclined to reject, i.e. $q_0 < s_{DM}$. The reason is that, when testing is observable, the agent stops testing if the belief rises to the DM's standard. Recall that this could never happen in the original model as the belief could jump up but not drift up. In the event that the agent stops at $q_t = s_{DM}$ for $t < T$, hidden testing yields a mutual advantage as described above. However, with incremental evidence, hidden testing now also yields a disadvantage for the DM in the event that the belief never reaches the DM's standard over the time horizon $[0, T]$ and lies above the agent's standard at time T. In this event, the DM prefers rejection but only rejects if testing is observable. As the proof shows, if the DM is willing to implement the agent's recommended choice under hidden testing, i.e. if $s_{DM} \leq \widehat{s}_{DM}$, then the benefit of hidden testing must outweigh the cost for the DM.

7 Related literature

Our work builds on the extensive literature on strategic information acquisition in sender-receiver games. Existing work has shown that, when incentives are misaligned, the DM values commitment to take ex-post suboptimal actions to improve information acquisition e.g. Szalay [2005], and, for this reason, limiting transparency can be beneficial, e.g. Prat [2005]. In our paper, hidden testing has features of a limited liability insurance issued by the DM to the agent, but the fact that the DM benefits from hidden testing is in contrast with findings on limited liability insurance by Mackowiak and Wiederholt [2012].

In a cheap talk context, Argenziano et al. [2016] study how different transparency regimes and allocations of decision rights affect costly information acquisition. Deimen and Szalay [2019] compare delegation and communication when the agent can acquire different types of information and the degree of conflict arises endogenously.

Our focus is the verifiable disclosure of information. Unlike in optimal persuasion, e.g. Kamenica and Gentzkow [2011], the agent in our setting faces a given testing technology and only has the flexibility to continue or to stop acquiring information. Brocas and Carrillo [2007] also study how an agent exerts influence via controlling the flow of information, but they assume the agent has state-independent preferences and find that payoffs are equal across hidden and observable testing. Other related work also assumes the agent's preferences are state-independent. Henry and Ottaviani [2019] and McClellan [2020] compare payoffs under different forms of commitment when an agent controls a flow of public information, testing is costly and players' payoffs are discounted. Che et al. [2021] study persuasion when an agent chooses the directionality of public Poisson signals to influence a DM over time.[36] Assuming testing is hidden and costly, Felgenhauer and Schulte [2014] study persuasion when the agent can selectively disclose individual signals.

That incentives to acquire costly information can be higher under voluntary rather than mandatory disclosure is shown by e.g. Matthews and Postlewaite [1985], Farrell [1985] and Shavell [1994].

[36] Nikandrova and Pancs [2018], Che and Mierendorff [2019] and Mayskaya [2019] also use a Poisson testing technology, but focus on optimal stopping by a single player.

However, in our paper, the reason the agent acquires more information is not to avoid the skeptical inference that he is hiding unfavorable outcomes.

Polinsky and Shavell [2012] also show that social welfare can be higher under voluntary rather than mandatory disclosure in a model with transfers. They study a firm that can acquire costly information about product harm to increase the gains from trade with consumers. The firm benefits from additional information under both mandatory and voluntary disclosure but is willing to pay a higher cost for information acquisition if disclosure is voluntary. By contrast, in our paper, the agent acquires less information under observable than under hidden testing even though information is costless. In addition, our paper shows that that social welfare can increase in the sense of a strict Pareto improvement in a sender-receiver game with no transfers.

Previous work on sender-receiver games with no transfers produced different welfare results, in particular, that hidden testing always benefits the DM but harms the agent. In particular, Henry [2009] studies an agent whose ideal action choice differs from the DM's ideal action by a constant amount independent of the state. The agent commits to a quantity of costly research ex ante. If his choice is hidden, the agent conducts more research to have a larger pool of signals to select from but interestingly, this leaves the DM better informed since he can infer all signals due to unraveling in the vein of Milgrom [1981] and Grossman [1981].

Moreover, Felgenhauer and Loerke [2017] study an agent who desires approval independent of the state. The agent tests sequentially and can decide how informative each test will be. Surprisingly, they find that the agent runs only a single test in any Pareto-undominated equilibrium, whether testing is observable or hidden. However, if testing is hidden the agent runs a more informative test, because this makes it credible that he will not run further tests even if the outcome is unfavorable.[37]

In these papers, when disclosure is voluntary, the agent would benefit from the power to commit to disclosing all his findings as this would allow him to restore the outcome under observable testing. In our paper, the agent has the option to credibly disclose all his findings, yet a situation can arise in which the DM strictly benefits from hidden testing due to players having partially aligned interests.

In the context of medical trials, Dahm et al. [2009] find that a compulsory trial registry combined with a voluntary results database implements full transparency, but deters investment in research relative to a voluntary results database alone.[38] Further, Dahm et al. [2018] show that stricter monitoring of firms' reporting discourages information acquisition. They find that imperfect enforcement of mandatory disclosure may be optimal, e.g. if, at some intermediate level of monitoring, firms run trials but only some disclose negative findings, then, with stricter monitoring, some of these firms may stop running trials at all. We find that intermediate transparency is optimal for the DM even if tailored to a single agent. If the agent does not become sufficiently informed when transparency is high, the DM should grant the agent just enough discretion to make the agent indifferent between

[37] Libgober [2020] shows that a receiver may be better off if the sender cannot verifiably disclose certain dimensions of an experiment because it may lead the sender to choose a different experiment that is more desirable along observed dimensions.

[38] Yoder [2020] studies incentives for research when tests are pre-registered and rewards can be conditioned on either the distribution of test outcomes or the outcome realization. Gall and Maniadis [2019] analyze a tournament between researchers with and without certain possibilities of inflating outcomes at a cost and find that preventing researchers from selective reporting also discourages more severe questionable research practices such as fabrication.

supplying more information or not.

Despite the agent facing different incentives for information acquisition and disclosure, our results share with Che and Kartik [2009] that the DM may be better informed when the agent is more biased.[39] Studying delegation to exogenously informed agents, Li and Suen [2004] find that more biased agents are delegated to more often. Moreover, Di Tillio et al. [2017] show that various welfare effects including a Pareto improvement can arise from manipulation of an RCT when the agent is privately informed ex ante.[40] Di Tillio et al. [2021] study more generally for which evidence distributions a selected sample is more informative than a random sample. Their work shares with this paper the feature that the agent's selective disclosure is informative for the DM but, in our paper, this is due to partially aligned interests rather than distributional properties.

8 Conclusion

This paper studies a DM facing an approval choice under uncertainty. An agent can acquire information through sequential testing, but the agent's ideal approval standard lies below the DM's. How is welfare affected when testing is hidden and the agent chooses whether or not to disclose his findings compared to when testing is observable? Our key contribution is to identify conditions under which hidden testing yields a strict Pareto improvement over observable testing. Under these conditions, when testing is observable, the agent does not exhaust all tests to become informed even though he bears no direct costs for conducting these tests. The reason for stopping is that additional information could lead the DM to change her action choice against the agent's interest. When testing is hidden, the agent can exploit the fact that their interests are partially aligned and strategically withhold evidence such that the DM acts in the agent's interest. However, due to this additional discretion under hidden testing, the agent has a reason to become better informed and, therefore, not only the DM but also the agent benefit from hidden testing.

While the current paper analyzes the interaction between an DM and an agent, there are other interesting consequences of transparency when several decision makers use the evidence as a basis for their choices or several agents supplying evidence. In particular, interesting externalities between agents arise, e.g. if an agent with a higher standard than the DM is added to the game. Then the agent with the higher standard would continue to supply evidence if the belief has dropped to the DM's standard, encouraging the agent with the lower standard to keep testing as well. These and related questions can be explored in future research.

[39] See also Gerardi and Yariv [2008] and Dur and Swank [2005].

[40] Janssen [2018] studies a perfectly informed agent who tests sequentially to seek approval from the DM and finds that the DM can never strictly benefit from hidden testing.

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A Supplementary Appendix

A.1 Proof of Proposition 5

Among all equilibria under hidden testing ($p = 0$), the agent-preferred equilibrium in Part 2 of Proposition 2 achieves the Pareto-best payoff profile by Propositions 3 and 4. Fixing the strategy profile to be as in Part 2 of Proposition 2, the DM's payoff strictly increases in p:

$$\begin{aligned}
&\frac{\partial}{\partial p}\left[(1-q_0)\frac{s_{DM}}{1-s_{DM}}\left[(1-\exp\left(-\lambda^{\omega_0}(T)\right))+p\exp\left(-\lambda^{\omega_0}(T)\right)\right]\right.\\
&\qquad\left.+q_0\left[(1-\exp\left(-\lambda^{\omega_1}(T)\right))+(1-p)\exp\left(-\lambda^{\omega_1}(T)\right)\right]\right]>0\\
&\Leftrightarrow (1-q_0)\frac{s_{DM}}{1-s_{DM}}\exp\left(-\lambda^{\omega_0}(T)\right)-q_0\exp\left(-\lambda^{\omega_1}(T)\right)>0\\
&\Leftrightarrow \frac{s_{DM}}{1-s_{DM}}>\frac{q_0\exp\left(-\lambda^{\omega_1}(T)\right)}{(1-q_0)\exp\left(-\lambda^{\omega_0}(T)\right)}\Leftrightarrow s_{DM}>\underline{q}.
\end{aligned}$$

We next show that the strategy profile in Part 2 of Proposition 2 is an equilibrium if and only if $p \leq p^*$. Fixing the agent's testing strategy and the DM's strategy, the agent's disclosure strategy is optimal independent of p, since the agent's disclosure matters only in the continuation game that follows when the DM does not observe h^T and exogenous disclosure is independent of the agent's findings. Further, given the agent's strategy, the DM's strategy is optimal independent of p.[41] However, the agent's testing strategy is not necessarily optimal if $p > 0$. In particular, if $p > 0$, the agent faces a trade-off between testing and stopping if and only if $t = t^*$ by the same argument as in the proof of Proposition 1. To find the highest level of p such that the agent test at $t = t^*$, suppose no signal has arrived by time t^*, i.e. $q_{t^*} = s_{DM}$. If the agent stops at time t^*, the DM chooses a_1. If not, the agent could disclose h^T and the DM would choose a_1. Hence, the agent's continuation value from stopping at t^* is $V_{t^*}(s_{DM}, t^*) = s_{DM}$. If the agent keeps testing at time t^*, the agent will keep testing at all $t > t^*$ if no signal arrived by time t, since he does so even if $p = 1$ by the proof of Proposition 1. Hence, the agent's continuation value from testing at t^* is

$$\begin{aligned}
V_{t^*}(s_{DM},T) = (1-p)&\left[(1-s_{DM})\left(1-\exp\left(-\lambda^{\omega_0}(T-t^*)\right)\right)\frac{s_A}{1-s_A}+s_{DM}\right]\\
&+p\left[(1-s_{DM})\frac{s_A}{1-s_A}+s_{DM}\left(1-\exp\left(-\lambda^{\omega_1}(T-t^*)\right)\right)\right].
\end{aligned}$$

Therefore, the agent is indifferent between testing or not at time t^* if and only if

$$\begin{aligned}
&V_{t^*}(s_{DM},t^*) = V_{t^*}(s_{DM},T)\\
&\quad\Leftrightarrow p^* = \frac{1-\exp\left(-\lambda^{\omega_0}(T-t^*(s_{DM},q_0))\right)}{\exp\left(-\lambda^{\omega_1}(T-t^*(s_{DM},q_0))\right)\left(\frac{s_{DM}}{1-s_{DM}}\right)\left(\frac{1-s_A}{s_A}\right)-\exp\left(-\lambda^{\omega_0}(T-t^*(s_{DM},q_0))\right)}.
\end{aligned}$$

If $p > p^*$, the agent stops testing at time t^* when no signal has arrived. Proposition 4 shows that the DM's payoff is strictly higher if the agent tests at time t^* and $p = 0$ than if the agent stops

[41] I.e. the DM reacts in the same way to disclosure of h^T by the agent or by chance.

at time t^* even when $p = 1$. Therefore, the DM's payoff is maximized at $p = p^*$.

If $s_A = \overline{s}_A$, then $p^* = 1$, p^* decreases as s_A decreases and $\lim_{s_A \to 0} p^* = 0$ for any s_{DM}, T, q_0. Hence, $p^* \in (0, 1)$.

A.2 Proof of Proposition 6

We derive Lemma A.1 on observable testing in subsection A.2.1 and Lemma A.2 on hidden testing in subsection A.2.2. Based on these, we prove Proposition 6 in subsection A.2.3.

A.2.1 Observable Testing

Lemma A.1. *Under observable testing, an equilibrium exists and is unique. The equilibrium strategies are as follows. The DM chooses a_1 if and only if $s_{DM} \leq q_T$. There exists some critical time $\widehat{t}(s_A, s_{DM})$ such that*

1. *for any $t < \widehat{t}$, the agent keeps testing at time t,*

2. *for $t \geq \widehat{t}$, the agent keeps testing at time t if and only if $q_t < s_{DM}$.*

The critical time $\widehat{t}$ is defined by $\widehat{t} = \max\{0, T - R\}$ where $R \in [0, \infty)$ satisfies

$$\log\left(1 - \frac{\sqrt{R}}{\phi\left(\sqrt{R}\right) + \sqrt{R}\Phi\left(\sqrt{R}\right)}\right) = \log\left(\frac{s_A}{1 - s_A}\right) - \log\left(\frac{s_{DM}}{1 - s_{DM}}\right), \tag{27}$$

and ϕ is the density and Φ the CDF of a standard Normal distribution. The LHS of (27) strictly decreases in R.

Proof. The DM's strategy is optimal by the same arguments as in the proof of Proposition 1. We show that the agent's strategy is a best reply. First, the agent tests at time t if $q_t < s_{DM}$ by the same argument as in the proof of Proposition 1. Next, consider time t with $q_t > s_{DM}$ and suppose the agent tests until the first time τ that $q = s_{DM}$. Therefore, $q_T \geq s_{DM}$ and $V_t(q_t, \tau) = V_t(q_t, t)$, i.e. the agent cannot be strictly worse off by testing until time τ.

Finally, consider time t with $q_t = s_{DM}$. We start by showing that, for a given s_A, there exists a critical level of the time remaining $R(s_{DM}, s_A) \in [0, \infty)$ such that the agent stops if and only if $T - t \leq R(s_{DM}, s_A)$. First, note that the continuation value of stopping at time t, $V_t(s_{DM}, t)$, is independent of the time remaining. Second, the continuation value from using the optimal stopping time, denoted by $V_t(s_{DM}, \tau^*)$, must satisfy $V_t(s_{DM}, \tau^*) \geq V_t(s_{DM}, t)$ by optimality. Third, $V_t(s_{DM}, \tau^*)$ cannot strictly decrease as the time remaining increases because the agent's set of possible stopping times increases.[42] The above implies that, if there exists t such that $V_t(s_{DM}, \tau^*) = V_t(s_{DM}, t)$, then for any $t' > t$, $V_{t'}(s_{DM}, \tau^*) = V_{t'}(s_{DM}, t)$. To show that there

[42] To see why, suppose $q_t = s_{DM}$ and suppose the deadline T increases to $T' > T$. If the deadline is at time T and the agent optimally stops at some time $\tau \leq T$, then he can achieve the same payoff by stopping at time τ when the deadline is $T' > T$. However, if the deadline is at T', the agent also has the option to continue testing at T and, therefore, his payoff must be weakly higher.

exists t such that $V_t(s_{DM}, \tau^*) = V_t(s_{DM}, t)$, compare the continuation value of stopping at t with the continuation value of testing until the deadline T:

$$V_t(s_{DM}, t) \geq V_t(s_{DM}, T) \Leftrightarrow \frac{s_{DM}}{1-s_{DM}} \frac{\Pr(q_T < s_{DM} \mid q_t = s_{DM}, \tau = T, \omega_1)}{\Pr(q_T < s_{DM} \mid q_t = s_{DM}, \tau = T, \omega_0)} \geq \frac{s_A}{1-s_A}$$
$$\Leftrightarrow \frac{\Phi\left(-\sqrt{T-t}\right)}{\Phi\left(\sqrt{T-t}\right)} \geq \frac{s_A}{1-s_A} \frac{1-s_{DM}}{s_{DM}}. \tag{28}$$

As $t \to T$, the LHS of (28) tends to 1 and the inequality is satisfied for any $s_A \leq s_{DM}$, i.e. stopping yields a strictly higher continuation value sufficiently close to the deadline T.

The next step is to show that the critical level of the time remaining R satisfies (27). First, let $R' \in (0, T)$ be an amount of time remaining such that $V_t(s_{DM}, T - R') \geq V_t(s_{DM}, T)$, implying that the agent stops at $q = s_{DM}$ if $T - t \leq R'$. Second, fix some $\epsilon \in (0, R')$. Consider t with $T - t \geq R'$ and $q_t = s_{DM}$. Consider the following stopping rule τ_ϵ: the agent tests at any time in $[t, t+\epsilon)$ and then the agent stops at the first time that $q \geq s_{DM}$. Take T to be sufficiently large. Then by the previous argument, as $T - t$ increases above R', there exists a critical level of the time remaining R_ϵ at which the agent is indifferent between stopping at $T - R_\epsilon$ and following stopping rule τ_ϵ,[43] i.e.

$$V_{T-R_\epsilon}(s_{DM}, \tau_\epsilon) = V_{T-R_\epsilon}(s_{DM}, T - R_\epsilon)$$
$$\Leftrightarrow \frac{\Pr\left(\max_{k \in [T-R_\epsilon+\epsilon, T]} q_k < s_{DM} \mid q_{T-R_\epsilon} = s_{DM}, \omega_1\right)}{\Pr\left(\max_{k \in [T-R_\epsilon+\epsilon, T]} q_k < s_{DM} \mid q_{T-R_\epsilon} = s_{DM}, \omega_0\right)} = \frac{s_A}{1-s_A} \frac{1-s_{DM}}{s_{DM}}, \tag{29}$$

where

$$\Pr\left(\max_{k \in [T-R_\epsilon+\epsilon, T]} q_k < s_{DM} \mid q_{T-R_\epsilon} = s_{DM}, \omega\right) =$$
$$\int_{-\infty}^{S_{DM}} \Pr\left(\Sigma_\epsilon = \frac{1}{2}(x - S_{DM}) \mid \Sigma_0 = 0, \omega\right) \Pr\left(\max_{k \in [0, R_\epsilon - \epsilon]} \Sigma_k < \frac{1}{2}(S_{DM} - x) \mid \Sigma_0 = 0, \omega\right) dx, \tag{30}$$

and $\Sigma_t \equiv \frac{1}{2} Q_t = \mu_\omega dt + dW_t$ and $S_{DM} \equiv \log\left(\frac{s_{DM}}{1-s_{DM}}\right)$.

The next step is to show that, in the limit as $\epsilon \to 0$, (29) is equivalent to (27). Define

$$m_\epsilon(x; \omega) \equiv Pr\left(\max_{k \in \left[0, \overline{R}_\epsilon - \epsilon\right]} \Sigma_k < \frac{1}{2}(S_{DM} - x) \mid \Sigma_0 = 0, \omega\right). \tag{31}$$

[43] The final action choice is affected by following stopping time τ_ϵ instead of stopping at time $T - R_\epsilon$ if the belief drops below the DM's standard within $[T - R_\epsilon, T - R_\epsilon + \epsilon)$ and never exceeds the DM's standard within $[T - R_\epsilon + \epsilon, T]$.

The Taylor expansion of $m_\epsilon(x;\omega)$ about $x = S_{DM}$ is given by

$$m'_\epsilon(S_{DM};\omega)(x - S_{DM}) + r_{1,\epsilon}(x;\omega). \tag{32}$$

Substituting (32) into (30):

$$\begin{aligned}
&\int_{-\infty}^{S_{DM}} \frac{1}{\sqrt{2\pi\epsilon}} \exp\left(-\frac{1}{2}\left(\frac{\frac{1}{2}(x - S_{DM}) - \mu_\omega \epsilon}{\sqrt{\epsilon}}\right)^2\right)\left[m'_\epsilon(S_{DM};\omega)(x - S_{DM}) + r_{1,\epsilon}(x;\omega)\right] dx \\
&= m'_\epsilon(S_{DM};\omega)\int_{-\infty}^{S_{DM}} \frac{1}{\sqrt{2\pi\epsilon}} \exp\left(-\frac{1}{2}\left(\frac{\frac{1}{2}(x - S_{DM}) - \mu_\omega \epsilon}{\sqrt{\epsilon}}\right)^2\right)(x - S_{DM})\, dx \\
&+ \int_{-\infty}^{S_{DM}} \frac{1}{\sqrt{2\pi\epsilon}} \exp\left(-\frac{1}{2}\left(\frac{\frac{1}{2}(x - S_{DM}) - \mu_\omega \epsilon}{\sqrt{\epsilon}}\right)^2\right) r_{1,\epsilon}(x;\omega)\, dx.
\end{aligned} \tag{33}$$

Therefore, taking the limit of the LHS of (29) as $\epsilon \to 0$ yields:

$$\lim_{\epsilon\to 0} \frac{\Pr\left(\max\limits_{k\in\left[T-\overline{R}_\epsilon+\epsilon,T\right]} q_k < s_{DM} \mid q_{T-\overline{R}_\epsilon} = s_{DM}, \omega_1\right)}{\Pr\left(\max\limits_{k\in\left[T-\overline{R}_\epsilon+\epsilon,T\right]} q_k < s_{DM} \mid q_{T-\overline{R}_\epsilon} = s_{DM}, \omega_0\right)} = \lim_{\epsilon\to 0} \frac{m'_\epsilon(S_{DM};\omega_1) + n_\epsilon(S_{DM};\omega_1)}{m'_\epsilon(S_{DM};\omega_0) + n_\epsilon(S_{DM};\omega_0)}, \tag{34}$$

where

$$n_\epsilon(S_{DM};\omega) = \frac{\int_{-\infty}^{S_{DM}} \frac{1}{\sqrt{2\pi\epsilon}} \exp\left(-\frac{1}{2}\left(\frac{\frac{1}{2}(x - S_{DM}) - \mu_\omega \epsilon}{\sqrt{\epsilon}}\right)^2\right) r_{1,\epsilon}(x;\omega)\, dx}{\int_{-\infty}^{S_{DM}} \frac{1}{\sqrt{2\pi\epsilon}} \exp\left(-\frac{1}{2}\left(\frac{\frac{1}{2}(x - S_{DM}) - \mu_\omega \epsilon}{\sqrt{\epsilon}}\right)^2\right)(x - S_{DM})\, dx}. \tag{35}$$

To calculate $m'_\epsilon(S_{DM};\omega)$, we use the following standard result repeated here for convenience: the CDF of $\max_{t\in[0,T]} \tilde{W}_t$ of a drifted Brownian motion $\tilde{W}_t = \gamma t + W_t$ over $t \in [0,T]$ with $\tilde{W}_0 = 0$ is given by:

$$\Pr\left(\max_{t\in[0,T]} \tilde{W}_t < a\right) = \Phi\left(\frac{a - \gamma T}{\sqrt{T}}\right) - \exp(2\gamma a)\,\Phi\left(\frac{-a - \gamma T}{\sqrt{T}}\right) \tag{36}$$

for $a \geq 0$. Hence,

$$\begin{aligned}
m_\epsilon(x;\omega) =& \Phi\left(\frac{\frac{1}{2}(S_{DM} - x) - \mu_\omega(R_\epsilon - \epsilon)}{\sqrt{R_\epsilon - \epsilon}}\right) \\
&- \exp(\mu_\omega(S_{DM} - x))\,\Phi\left(\frac{-\frac{1}{2}(S_{DM} - x) - \mu_\omega(R_\epsilon - \epsilon)}{\sqrt{R_\epsilon - \epsilon}}\right),
\end{aligned} \tag{37}$$

$$m'_\epsilon(S_{DM};\omega) = -\frac{1}{\sqrt{R_\epsilon - \epsilon}}\phi\left(-\mu_\omega\sqrt{R_\epsilon - \epsilon}\right) + \mu_\omega\Phi\left(-\mu_\omega\sqrt{R_\epsilon - \epsilon}\right). \tag{38}$$

Suppose $\lim_{\epsilon\to 0} n_\epsilon(S_{DM};\omega) = 0$. Then (34) and (38) yield

$$\lim_{\epsilon\to 0} \frac{\Pr\left(\max_{k\in[T-R_\epsilon+\epsilon,T]} q_k < s_{DM} \mid q_{T-R_\epsilon} = s_{DM}, \omega_1\right)}{\Pr\left(\max_{k\in[T-R_\epsilon+\epsilon,T]} q_k < s_{DM} \mid q_{T-R_\epsilon} = s_{DM}, \omega_0\right)} = 1 - \frac{\sqrt{R}}{\phi\left(\sqrt{R}\right) + \sqrt{R}\Phi\left(\sqrt{R}\right)}. \tag{39}$$

Therefore, in the limit as $\epsilon \to 0$, (29) is equivalent to (27). Hence, the critical time t must satisfy $\widehat{t} = \max\{0, T-R\}$ where R is defined by (27). See also Figure 3.

The LHS of (27) strictly decreases in R:

$$\frac{\partial}{\partial R}\left[\log\left(1 - \frac{\sqrt{R}}{\phi\left(\sqrt{R}\right) + \sqrt{R}\Phi\left(\sqrt{R}\right)}\right)\right] = \frac{\partial}{\partial R}\left[1 - \frac{1}{g(R)}\right] = \frac{1}{1-\frac{1}{g(R)}}\left[\frac{g'(R)}{g(R)^2}\right] < 0, \tag{40}$$

where

$$g(R) = \frac{1}{\sqrt{R}}\phi\left(\sqrt{R}\right) + \Phi\left(\sqrt{R}\right), \tag{41}$$

$$g'(R) = -\frac{\exp\left(-\frac{1}{2}R\right)}{2\sqrt{2\pi}R^{\frac{3}{2}}} < 0, \tag{42}$$

and $g(R) \geq 1$ for all $R \geq 0$ as $\lim_{R\to\infty} g(R) = 1$.

The rest of the proof shows that $\lim_{\epsilon\to 0} n_\epsilon(S_{DM};\omega) = 0$. The remainder term in (32):

$$r_{1,\epsilon}(x;\omega) = m''_\epsilon(S_{DM};\omega)(x - S_{DM}) + r_{2,\epsilon}(x;\omega), \tag{43}$$

Rewrite (35) as

$$\begin{aligned} &\frac{m''_\epsilon(S_{DM};\omega)\int_{-\infty}^{S_{DM}} \frac{1}{\sqrt{2\pi\epsilon}}\exp\left(-\frac{1}{2}\left(\frac{\frac{1}{2}(x-S_{DM})-\mu_\omega\epsilon}{\sqrt{\epsilon}}\right)^2\right)(x-S_{DM})^2 dx}{\int_{-\infty}^{S_{DM}} \frac{1}{\sqrt{2\pi\epsilon}}\exp\left(-\frac{1}{2}\left(\frac{\frac{1}{2}(x-S_{DM})-\mu_\omega\epsilon}{\sqrt{\epsilon}}\right)^2\right)(x-S_{DM}) dx} \\ &+ \frac{\int_{-\infty}^{S_{DM}} \frac{1}{\sqrt{2\pi\epsilon}}\exp\left(-\frac{1}{2}\left(\frac{\frac{1}{2}(x-S_{DM})-\mu_\omega\epsilon}{\sqrt{\epsilon}}\right)^2\right) r_{2,\epsilon}(x;\omega) dx}{\int_{-\infty}^{S_{DM}} \frac{1}{\sqrt{2\pi\epsilon}}\exp\left(-\frac{1}{2}\left(\frac{\frac{1}{2}(x-S_{DM})-\mu_\omega\epsilon}{\sqrt{\epsilon}}\right)^2\right)(x-S_{DM}) dx}. \end{aligned} \tag{44}$$

The first term of (44) can be written as

$$m''_\epsilon(S_{DM};\omega)\frac{\int_{-\infty}^{0} \frac{1}{\sqrt{2\pi\epsilon}}\exp\left\{-\frac{1}{2}\left(\frac{y-\mu_\omega\epsilon}{\sqrt{\epsilon}}\right)^2\right\} y^2 dy}{\int_{-\infty}^{0} \frac{1}{\sqrt{2\pi\epsilon}}\exp\left\{-\frac{1}{2}\left(\frac{y-\mu_\omega\epsilon}{\sqrt{\epsilon}}\right)^2\right\} y dy} = m''_\epsilon(S_{DM};\omega)\frac{E\left(Y^2|Y<0\right)}{E\left(Y|Y<0\right)}, \tag{45}$$

where $Y \sim N\left(\mu_\omega \epsilon, \sqrt{\epsilon}\right)$. Then for $Z \sim N(0,1)$,

$$E\left(Y|Y<0\right)=E\left(\sqrt{\epsilon}Z+\mu_\omega\epsilon|Z<-\mu_\omega\sqrt{\epsilon}\right)=\sqrt{\epsilon}\left[\sqrt{\epsilon}\mu_\omega-\frac{\phi\left(-\mu_\omega\sqrt{\epsilon}\right)}{\Phi\left(-\mu_\omega\sqrt{\epsilon}\right)}\right] \tag{46}$$

$$E\left(Y^2|Y<0\right)=E\left(\left(\sqrt{\epsilon}Z+\mu_\omega\epsilon\right)^2|Z<-\mu_\omega\sqrt{\epsilon}\right)=\epsilon+\epsilon^2-\mu_\omega\epsilon\sqrt{\epsilon}\frac{\phi\left(-\mu_\omega\sqrt{\epsilon}\right)}{\Phi\left(-\mu_\omega\sqrt{\epsilon}\right)} \tag{47}$$

Substituting (46) and (47) into (45) and taking limits yields

$$\lim_{\epsilon\to 0}\frac{E\left(Y^2|Y<0\right)}{E\left(Y|Y<0\right)}m''_\epsilon\left(S_{DM};\omega\right)=m''_\epsilon\left(S_{DM};\omega\right)\lim_{\epsilon\to 0}\frac{\sqrt{\epsilon}+\sqrt{\epsilon}\epsilon-\mu_\omega\epsilon\frac{\phi\left(-\mu_\omega\sqrt{\epsilon}\right)}{\Phi\left(-\mu_\omega\sqrt{\epsilon}\right)}}{\left[\sqrt{\epsilon}\mu_\omega-\frac{\phi\left(-\mu_\omega\sqrt{\epsilon}\right)}{\Phi\left(-\mu_\omega\sqrt{\epsilon}\right)}\right]}=\frac{0}{-\sqrt{\frac{2}{\pi}}}=0.$$

The same reasoning can be applied reiteratively to the second term of (44). $\square$

A.2.2 Hidden testing

Lemma A.2. *Suppose testing is hidden. There exists a critical level of the DM's standard $\widehat{s}_{DM}\left(q_0,T,s_A\right)$, where $\widehat{s}_{DM}\in\left(q_0,1\right)$ is defined by*

$$\log\left(\frac{\widehat{s}_{DM}}{1-\widehat{s}_{DM}}\right)=Q_0+\log\left(\frac{1-\Phi\left(\frac{\frac{1}{2}\left(\log\left(\frac{s_A}{1-s_A}\right)-Q_0\right)-T}{\sqrt{T}}\right)}{1-\Phi\left(\frac{\frac{1}{2}\left(\log\left(\frac{s_A}{1-s_A}\right)-Q_0\right)+T}{\sqrt{T}}\right)}\right), \tag{48}$$

such that if $s_{DM}\leq\widehat{s}_{DM}$, an equilibrium exists, and the following are features of an agent-preferred equilibrium,

- *the agent tests until $t=T$, and discloses h^T if and only if he prefers a_0 given h^T,*
- *the DM chooses a_0 if and only if the agent discloses h^T and the DM prefers a_0 given h^T.*

Proof. Given that the agent sends $m=\emptyset$ if and only if $q_T\geq s_A$, the DM's consistent beliefs following $m=\emptyset$ satisfy

$$\frac{\Pr\left(\omega_1|m=\emptyset\right)}{\Pr\left(\omega_0|m=\emptyset\right)}=\frac{q_0}{1-q_0}\frac{\Pr\left(q_T\geq s_A|q_0,\tau=T,\omega_1\right)}{\Pr\left(q_T\geq s_A|q_0,\tau=T,\omega_0\right)}=\frac{q_0}{1-q_0}\frac{1-\Phi\left(\frac{\frac{1}{2}\left(\log\left(\frac{s_A}{1-s_A}\right)-Q_0\right)-T}{\sqrt{T}}\right)}{1-\Phi\left(\frac{\frac{1}{2}\left(\log\left(\frac{s_A}{1-s_A}\right)-Q_0\right)+T}{\sqrt{T}}\right)}, \tag{49}$$

hence, $\Pr\left(\omega_1|m=\emptyset\right)=\widehat{s}_{DM}$ by (48). Since $s_{DM}\leq\widehat{s}_{DM}$, the DM optimally chooses a_1 following $m=\emptyset$. In addition, the agent has no reason to deviate from his strategy because he achieves his first-best payoff. This also implies that the equilibrium is agent-preferred. $\square$

A.2.3 Welfare comparison

We now prove Proposition 6. Define $\widehat{s}_A(s_{DM}, T)$ such that (27) is satisfied when $s_A = \widehat{s}_A$ and $R = T$.

Under hidden testing, the agent can guarantee himself at least the same payoff as under observable testing by following the same testing strategy as in Lemma A.1 and reporting $m = h^T$ for any h^T. Then the agent's equilibrium payoff is strictly less than his first-best payoff as the agent stops before time T with positive probability. In the agent-preferred equilibrium in Lemma A.2, the agent obtains his first-best payoff. Hence, the agent must have a strictly higher payoff in the agent-preferred equilibrium under hidden testing than in the equilibrium under observable testing.

We will distinguish between two cases to compare the DM's payoff under hidden and observable testing: i) $s_{DM} \leq q_0$ and $s_A \leq \widehat{s}_A$ and ii) $q_0 < s_{DM} \leq \widehat{s}_{DM}$ and $s_A \leq \widehat{s}_A$.[44]

In both case i) and case ii), by Lemma A.2, in the agent-preferred equilibrium under hidden testing, a_1 is chosen with probability $\Pr(q_T \geq s_A | q_0, \tau = T)$, i.e. with the probability that $q_T \geq s_A$ given the prior is q_0 and the agent tests until time T.

Consider case i), i.e. $s_{DM} \leq q_0$ and $s_A \leq \widehat{s}_A$. By Lemma A.1, $\widehat{t} = 0$ and, hence, under observable testing a_1 is chosen with probability 1. Then the DM has a strictly higher payoff under hidden than observable testing since

$$(1 - q_0) \frac{s_{DM}}{1 - s_{DM}} \Pr(q_T < s_A | q_0, \tau = T, \omega_0) + q_0 \Pr(q_T \geq s_A | q_0, \tau = T, \omega_1) > q_0$$
$$\Leftrightarrow \frac{s_{DM}}{1 - s_{DM}} > \frac{\Pr(\omega_1 | q_T < s_A, q_0, \tau = T)}{\Pr(\omega_0 | q_T < s_A, q_0, \tau = T)}$$

which holds since

$$\frac{s_{DM}}{1 - s_{DM}} \geq \frac{s_A}{1 - s_A} = \frac{\Pr(\omega_1 | q_T = s_A, q_0, \tau = T)}{\Pr(\omega_0 | q_T = s_A, q_0, \tau = T)} > \frac{\Pr(\omega_1 | q_T < s_A, q_0, \tau = T)}{\Pr(\omega_0 | q_T < s_A, q_0, \tau = T)}.$$

The set of s_{DM} for which $s_{DM} \leq q_0$ is non-empty. In addition, the set of s_A for which $s_A \leq \widehat{s}_A$ is non-empty since $\widehat{s}_A(s_{DM}, T) > 0$ for any s_{DM} and finite T by Lemma A.1. Hence, for any q_0 and T the region for which $s_{DM} \leq q_0$ and $s_A \leq \widehat{s}_A$ is non-empty.

Next, consider case ii), i.e. $s_A \leq \widehat{s}_A$ and $q_0 < s_{DM} \leq \widehat{s}_{DM}$, or equivalently,

$$Q_0 < \log\left(\frac{s_{DM}}{1 - s_{DM}}\right) \leq Q_0 + \log\left(\frac{1 - \Phi\left(\frac{\frac{1}{2}\left(\log\left(\frac{s_A}{1-s_A}\right) - Q_0\right) - T}{\sqrt{T}}\right)}{1 - \Phi\left(\frac{\frac{1}{2}\left(\log\left(\frac{s_A}{1-s_A}\right) - Q_0\right) + T}{\sqrt{T}}\right)}\right). \tag{50}$$

By Lemma A.1, a_1 is chosen with probability $\Pr\left(\max_{t \in [0,T]} q_t \geq s_{DM} | q_0, \tau = T\right)$. Then the DM has

[44] Note that $q_0 < \widehat{s}_{DM}$ by Lemma A.2.

a strictly higher payoff under hidden than observable testing if and only if

$$\Pr\left(q_T < s_A | q_0, \tau = T, \omega_0\right) + \frac{q_0}{1-q_0}\frac{1-s_{DM}}{s_{DM}} \Pr\left(q_T \geq s_A | q_0, \tau = T, \omega_1\right) >$$
$$\Pr\left(\max_{t\in[0,T]} q_t < s_{DM} | q_0, \tau = T, \omega_0\right) + \frac{q_0}{1-q_0}\frac{1-s_{DM}}{s_{DM}} \Pr\left(\max_{t\in[0,T]} q_t \geq s_{DM} | q_0, \tau = T, \omega_1\right). \tag{51}$$

The LHS of (51) can be written as

$$\Phi\left(\frac{\frac{1}{2}\left(S_A - Q_0\right) + T}{\sqrt{T}}\right) - \exp\left(Q_0 - S_{DM}\right)\Phi\left(\frac{\frac{1}{2}\left(S_A - Q_0\right) - T}{\sqrt{T}}\right), \tag{52}$$

where $S_i \equiv \log\left(\frac{s_i}{1-s_i}\right)$ for $i \in \{A, DM\}$, and the RHS can be written as

$$\begin{aligned}
&\Phi\left(\frac{\frac{1}{2}\left(S_{DM} - Q_0\right) + T}{\sqrt{T}}\right) - \exp\left(Q_0 - S_{DM}\right)\Phi\left(\frac{-\frac{1}{2}\left(S_{DM} - Q_0\right) + T}{\sqrt{T}}\right) \\
&- \exp\left(Q_0 - S_{DM}\right)\left[\Phi\left(\frac{\frac{1}{2}\left(S_{DM} - Q_0\right) - T}{\sqrt{T}}\right) - \exp\left(S_{DM} - Q_0\right)\Phi\left(\frac{-\frac{1}{2}\left(S_{DM} - Q_0\right) - T}{\sqrt{T}}\right)\right] \\
=&1 - \exp\left(Q_0 - S_{DM}\right),
\end{aligned} \tag{53}$$

using (36). Substituting (52) and (53) into (51), we can see that (51) is implied by the second inequality of (50).