
Question 7

Solve the following questions from the Discrete Math zyBooks:

1. Exercise 3.1.1
 - (a) $27 \in A \Rightarrow \text{True}$
 - (b) $27 \in A \Rightarrow \text{False}$
 - (c) $100 \in B \Rightarrow \text{True}$
 - (d) $E \subseteq C \text{ or } C \subseteq E \Rightarrow \text{False}$
 - (e) $E \subseteq A \Rightarrow \text{True}$
 - (f) $A \subseteq E \Rightarrow \text{False}$
 - (g) $E \in A \Rightarrow \text{False}$
2. Exercise 3.1.2
 - (a) $15 \subset A \Rightarrow \text{False}$
 - (b) $\{15\} \subset A \Rightarrow \text{True}$
 - (c) $\emptyset \subset C \Rightarrow \text{True}$
 - (d) $D \subseteq D \Rightarrow \text{True}$
 - (e) $\emptyset \in B \Rightarrow \text{False}$
3. Exercise 3.1.5
 - (b) $\{3,6,9,12,\dots\} \Rightarrow \mathbf{A = \{x \in \mathbb{Z} : 0 < x \text{ and } x \text{ is an integer multiple of } 3\}; \text{ infinite set}}$
 - (d) $\{0,10,20,30,\dots,1000\} \Rightarrow \mathbf{A = \{x \in \mathbb{N} : x \text{ is a natural multiple of } 10\}; |A| = 101}$
4. Exercise 3.2.1
 - (a) $2 \in X \Rightarrow \text{True}$
 - (b) $\{2\} \subseteq X \Rightarrow \text{True}$
 - (c) $\{2\} \in X \Rightarrow \text{False}$
 - (d) $3 \in X \Rightarrow \text{False}$
 - (e) $\{1,2\} \in X \Rightarrow \text{True}$
 - (f) $\{1,2\} \subseteq X \Rightarrow \text{True}$
 - (g) $\{2,4\} \subseteq X \Rightarrow \text{True}$
 - (h) $\{2,4\} \in X \Rightarrow \text{False}$
 - (i) $\{2,3\} \subseteq X \Rightarrow \text{False}$
 - (j) $\{2,3\} \in X \Rightarrow \text{False}$
 - (k) $|X| = 7 \Rightarrow \text{False}$

Question 8

Solve the following question from the Discrete Math zyBooks:

1. Exercise 3.2.4

$$\begin{aligned} \text{(b) } P(A) &= \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\} \\ \{X \in P(A) : 2 \in X\} &\Rightarrow \{\{\mathbf{2}\}, \{\mathbf{1,2}\}, \{\mathbf{2,3}\}, \{\mathbf{1,2,3}\}\} \end{aligned}$$

Question 9

Solve the following questions from the Discrete Math zyBooks:

1. Exercise 3.3.1

(c) $A \cap C \Rightarrow \{-3, 1, 17\}$

(d) $A \cup (B \cap C) \Rightarrow A \cup \{-5, 1\} \Rightarrow \{-5, -3, 0, 1, 4, 17\}$

(e) $A \cap B \cap C \Rightarrow \{1, 4\} \cap C \Rightarrow \{1\}$

2. Exercise 3.3.3

(a) $\bigcap_{i=2}^5 A_i \Rightarrow A_2 \cap A_3 \cap A_4 \cap A_5 \Rightarrow \{1\}$

$A_2 = \{2^0, 2^1, 2^2\} = \{1, 2, 4\}$

$A_3 = \{3^0, 3^1, 3^2\} = \{1, 3, 9\}$

$A_4 = \{4^0, 4^1, 4^2\} = \{1, 4, 16\}$

$A_5 = \{5^0, 5^1, 5^2\} = \{1, 5, 25\}$

(b) $\bigcup_{i=2}^5 A_i \Rightarrow A_2 \cup A_3 \cup A_4 \cup A_5 \Rightarrow \{1, 2, 3, 4, 5, 9, 16, 25\}$

$A_2 = \{2^0, 2^1, 2^2\} = \{1, 2, 4\}$

$A_3 = \{3^0, 3^1, 3^2\} = \{1, 3, 9\}$

$A_4 = \{4^0, 4^1, 4^2\} = \{1, 4, 16\}$

$A_5 = \{5^0, 5^1, 5^2\} = \{1, 5, 25\}$

(e) $\bigcap_{i=1}^{100} C_i \Rightarrow \{x \in \mathbf{R} : \frac{-1}{100} \leq x \leq \frac{1}{100}\}$

i=1 : $\frac{-1}{1} \leq x \leq \frac{1}{1}$

i=100 : $\frac{-1}{100} \leq x \leq \frac{1}{100}$

(f) $\bigcup_{i=1}^{100} C_i \Rightarrow \{x \in \mathbf{R} : -1 \leq x \leq 1\}$

i=1 : $\frac{-1}{1} \leq x \leq \frac{1}{1}$

i=100 : $\frac{-1}{100} \leq x \leq \frac{1}{100}$

3. Exercise 3.3.4

(b) $P(A \cap B) \Rightarrow \{\emptyset, \{\mathbf{b}\}\}$

$A \cap B \Rightarrow \{b\}$

(d) $P(A) \cup P(B) \Rightarrow \{\emptyset, \{\mathbf{a}\}, \{\mathbf{b}\}, \{\mathbf{c}\}, \{\mathbf{a}, \mathbf{b}\}, \{\mathbf{b}, \mathbf{c}\}\}$

$P(A) \Rightarrow \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

$P(B) \Rightarrow \{\emptyset, \{b\}, \{c\}, \{b, c\}\}$

Question 10

Solve the following questions from the Discrete Math zyBooks:

1. Exercise 3.5.1

(b) $B \times A \times C \Rightarrow (\text{foam, tall, non-fat})$

(c) $B \times C \Rightarrow \{(\text{foam, non-fat}), (\text{foam, whole}), (\text{no-foam, non-fat}), (\text{no-foam, whole})\}$

2. Exercise 3.5.3

(b) $\mathbb{Z}^2 \subseteq \mathbb{R}^2 \Rightarrow \text{True}$

(c) $\mathbb{Z}^2 \cap \mathbb{Z}^3 = \emptyset \Rightarrow \text{False}$

(e) If $A \subseteq B$, then $A \times C \subseteq B \times C \Rightarrow \text{True}$

3. Exercise 3.5.6

(d) $\{0\}^2 = \{0\} \times \{0\} \Rightarrow (0, 0) \mid \{1\}^2 = \{1\} \times \{1\} \Rightarrow (1, 1)$

$\{0\} \cup \{(0, 0)\} \Rightarrow \{0, (0, 0)\} \mid \{1\} \cup \{(1, 1)\} \Rightarrow \{1, (1, 1)\}$

$\{xy : \text{where } x \in \{0\} \cup \{0\}^2 \text{ and } y \in \{1\} \cup \{1\}^2\} \Rightarrow \{(0, 1), (0, (1, 1)), ((0, 0), 1), ((0, 0), (1, 1))\}$

(e) $\{a\}^2 \Rightarrow \{a\} \times \{a\} \Rightarrow (a, a)$

$\{a\} \cup \{a\}^2 \Rightarrow \{a, (a, a)\}$

$\{xy : x \in \{aa, ab\} \text{ and } y \in \{a\} \cup \{a\}^2\} \Rightarrow \{(aa, a), (aa, (a, a)), (ab, a), (ab, (a, a))\}$

4. Exercise 3.5.7

(c) $A \times B \Rightarrow \{ab, ac\}$

$A \times C \Rightarrow \{aa, ab, ad\}$

$(A \times B) \cup (A \times C) \Rightarrow \{aa, ab, ac, ad\}$

(f) $A \times B \Rightarrow \{ab, ac\}$

$P(A \times B) \Rightarrow \{\emptyset, \{\mathbf{ab}\}, \{\mathbf{ac}\}, \{\mathbf{ab}, \mathbf{ac}\}\}$

(g) $P(A) \Rightarrow \{\emptyset, \{a\}\}$

$P(B) \Rightarrow \{\emptyset, \{b\}, \{c\}, \{b, c\}\}$

$P(A) \times P(B) \Rightarrow \{(\emptyset, \emptyset), (\emptyset, \{\mathbf{b}\}), (\emptyset, \{\mathbf{c}\}), (\emptyset, \{\mathbf{b}, \mathbf{c}\}), (\{\mathbf{a}\}, \emptyset), (\{\mathbf{a}\}, \{\mathbf{b}\}), (\{\mathbf{a}\}, \{\mathbf{c}\}), (\{\mathbf{a}\}, \{\mathbf{b}, \mathbf{c}\})\}$

Question 11

Solve the following questions from the Discrete Math zyBooks:

1. Exercise 3.6.2

(b)

$$(B \cup A) \cap (\overline{B} \cup A)$$

Distributive Law: $(B \cap \overline{B}) \cup A$

Complement Law: $\emptyset \cup A$

Commutative Law: $A \cup \emptyset$

Identity Law: **A**

(c)

$$\overline{A \cap \overline{B}}$$

De Morgan's Law: $\overline{A} \cup \overline{\overline{B}}$

Double Complement Law: **A** \cup **B**

2. Exercise 3.6.3

(b) $A = \{1, 2\}$ and $B = \{1\}$

$$B \cap A \Rightarrow \{1\}$$

$$\{1, 2\} - \{1\} \Rightarrow \{2\} \neq A$$

(d) $B = \{1, 2\}$ and $A = \{1\}$

$$(B - A) \cup A$$

$$B - A \Rightarrow \{2\}$$

$$\{2\} \cup \{1\} \Rightarrow \{1, 2\} \neq A$$

3. Exercise 3.6.4

(b)

$$A \cap (B - A)$$

Subtraction Law: $A \cap (B \cap \overline{A})$

Commutative Law: $A \cap (\overline{A} \cap B)$

Associative Law: $(A \cap \overline{A}) \cap B$

Complement Law: $\emptyset \cap B$

Commutative Law: $B \cap \emptyset$

Domination Law: \emptyset

(c)

$$A \cup (B - A)$$

$$\text{Subtraction Law: } A \cup (B \cap \overline{A})$$

$$\text{Distributive Law: } (A \cup B) \cap (A \cup \overline{A})$$

$$\text{Complement Law: } (A \cup B) \cap U$$

$$\text{Identity Law: } \mathbf{A} \cup \mathbf{B}$$