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Extended Bridge to CS
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Homework 1
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Question 1

A Convert the following numbers to their decimal representation. Show your work.

(a) $(10011011)_2$

$$\frac{1}{2^7}\frac{0}{2^6}\frac{0}{2^5}\frac{1}{2^4}\frac{1}{2^3}\frac{0}{2^2}\frac{1}{2^1}\frac{1}{2^0} = \frac{1}{128}\frac{0}{64}\frac{0}{32}\frac{1}{16}\frac{1}{8}\frac{0}{4}\frac{1}{2}\frac{1}{1} = 128+16+8+2+1 = (\mathbf{155})_{\mathbf{10}}$$

(b) $(456)_7$

$$\frac{4}{7^2}\frac{5}{7^1}\frac{6}{7^0} = \frac{4}{49}\frac{5}{7}\frac{6}{1} = (4)(49)+(5)(7)+(6)(1) = 196+35+6 = (\mathbf{237})_{\mathbf{10}}$$

(c) $(38A)_{16}$

$$\begin{aligned}\frac{3}{16^2}\frac{8}{16^1}\frac{A(=10)}{16^0} = \frac{3}{256}\frac{8}{16}\frac{10}{1} &= (3)(256)+(8)(16)+(10)(1)\\ &= 768+128+10 = (\mathbf{906})_{\mathbf{10}}\end{aligned}$$

(d) $(2214)_5$

$$\begin{aligned}\frac{2}{5^3}\frac{2}{5^2}\frac{1}{5^1}\frac{4}{5^0} = \frac{2}{125}\frac{2}{25}\frac{1}{5}\frac{4}{1} &= (2)(125)+(2)(25)+(1)(5)+(4)(1)\\ &= 250+50+5+4 = (\mathbf{309})_{\mathbf{10}}\end{aligned}$$

B Convert the following numbers to their binary representation:

(a) $(69)_{10} = \frac{0}{2^7}\frac{1}{2^6}\frac{0}{2^5}\frac{0}{2^4}\frac{0}{2^3}\frac{1}{2^2}\frac{0}{2^1}\frac{1}{2^0} = \frac{0}{128}\frac{1}{64}\frac{0}{32}\frac{0}{16}\frac{0}{8}\frac{1}{4}\frac{0}{2}\frac{1}{1} = (\mathbf{1000101})_{\mathbf{2}}$

(b) $(485)_{10} = \frac{0}{2^9}\frac{1}{2^8}\frac{1}{2^7}\frac{1}{2^6}\frac{1}{2^5}\frac{0}{2^4}\frac{0}{2^3}\frac{1}{2^2}\frac{0}{2^1}\frac{1}{2^0} = \frac{0}{512}\frac{1}{256}\frac{1}{128}\frac{1}{64}\frac{1}{32}\frac{0}{16}\frac{0}{8}\frac{1}{4}\frac{0}{2}\frac{1}{1} = (\mathbf{111100101})_{\mathbf{2}}$

(c) $(6D1A)_{16} = 6 \rightarrow 0110; D \rightarrow 1101; 1 \rightarrow 0001; A \rightarrow 1010 = (\mathbf{0110110100011010})_{\mathbf{2}}$

C Convert the following numbers to their hexadecimal representation:

(a) $(1101011)_2 = (0)110 \rightarrow 6; 1011 \rightarrow B = (\mathbf{6B})_{\mathbf{16}}$

(b) $(895)_{10} = \frac{0}{16^3}\frac{3}{16^2}\frac{7}{16^1}\frac{F}{16^0} = \frac{0}{4096}\frac{3}{256}\frac{7}{16}\frac{F}{1} = (\mathbf{37F})_{\mathbf{16}}$

Question 2

Solve the following, do all calculations in the given base. Show your work.

A
$$\begin{array}{r} (7566)_8 \\ +\ (4515)_8 \\ \hline (\mathbf{14303})_{\mathbf{8}} \end{array}$$

B
$$\begin{array}{r} (10110011)_2 \\ +\ (1101)_2 \\ \hline (\mathbf{11000000})_{\mathbf{2}} \end{array}$$

C
$$\begin{array}{r} (7A66)_{16} \\ +\ (45C5)_{16} \\ \hline (\mathbf{C02B})_{\mathbf{16}} \end{array}$$

D
$$\begin{array}{r} (3022)_5 \\ -\ (2433)_5 \\ \hline (\mathbf{-34})_{\mathbf{5}} \end{array}$$

Question 3

A Convert the following numbers to their 8-bits two's complement representation. Show your work.

(a) $(124)_{10} = \frac{0}{128}\frac{1}{64}\frac{1}{32}\frac{1}{16}\frac{1}{8}\frac{1}{4}\frac{0}{2}\frac{0}{1} = (\mathbf{01111100})_{\mathbf{2}}$

(b) $(-124)_{10} = \frac{1}{128}\frac{0}{64}\frac{0}{32}\frac{0}{16}\frac{0}{8}\frac{1}{4}\frac{0}{2}\frac{0}{1} = (\mathbf{10000100})_{\mathbf{2}}$

(c) $(109)_{10} = \frac{0}{128}\frac{1}{64}\frac{1}{32}\frac{0}{16}\frac{1}{8}\frac{1}{4}\frac{0}{2}\frac{1}{1} = (\mathbf{01101101})_{\mathbf{2}}$

(d) $(-79)_{10} = \frac{1}{128}\frac{0}{64}\frac{1}{32}\frac{1}{16}\frac{0}{8}\frac{0}{4}\frac{0}{2}\frac{1}{1} = (\mathbf{10110001})_{\mathbf{2}}$

B Convert the following numbers (represented as 8-bit two's complement) to their decimal representation. Show your work.

(a) 8-bit 2's complement: $(00011110)_2 = \frac{0}{128}\frac{0}{64}\frac{0}{32}\frac{1}{16}\frac{1}{8}\frac{1}{4}\frac{1}{2}\frac{0}{1} = 16 + 8 + 4 + 2 = (\mathbf{30})_{\mathbf{10}}$

(b) 8-bit 2's complement: $(11100110)_2 = \frac{1}{128}\frac{1}{64}\frac{1}{32}\frac{0}{16}\frac{0}{8}\frac{1}{4}\frac{1}{2}\frac{0}{1} = -128 + 64 + 32 + 4 + 2 = (\mathbf{-26})_{\mathbf{10}}$

(c) 8-bit 2's complement: $(00101101)_2 = \frac{0}{128}\frac{0}{64}\frac{1}{32}\frac{0}{16}\frac{1}{8}\frac{1}{4}\frac{0}{2}\frac{1}{1} = 32 + 8 + 4 + 1 = (\mathbf{45})_{\mathbf{10}}$

(d) 8-bit 2's complement: $(10011110)_2 = \frac{1}{128}\frac{0}{64}\frac{0}{32}\frac{1}{16}\frac{1}{8}\frac{1}{4}\frac{1}{2}\frac{0}{1} = -128 + 16 + 8 + 4 + 2 = (\mathbf{-98})_{\mathbf{10}}$

Question 4

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.2.4

   (b)

p	q	p∨q	$\neg(p \lor q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

2. Exercise 1.3.4

   (b)

p	q	$p \to q$	$q \to p$	$(p \to q) \to (q \to p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

   (d)

p	q	$\neg q$	$p \leftrightarrow \neg q$	$p \leftrightarrow q$	$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
T	T	F	F	T	T
T	F	T	T	F	T
F	T	F	T	F	T
F	F	T	F	T	T

Question 5

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.2.7

   (b)

   Proposition in words: The applicant must present at least two of the following forms of identification: birth certificate, driver's license, marriage license.

   Logical expression: $(\mathbf{B} \wedge \mathbf{D}) \vee (\mathbf{B} \wedge \mathbf{M}) \vee (\mathbf{D} \wedge \mathbf{M})$

   (c)

   Proposition in words: Applicant must present either a birth certificate or both a driver's license and a marriage license.

   Logical expression: $\mathbf{B} \vee (\mathbf{D} \wedge \mathbf{M})$

2. Exercise 1.3.7

   (b)

   Proposition in words: A person can park in the school parking lot if they are a senior or at least 17 years of age.

   Logical expression: $(\mathbf{s} \vee \mathbf{y}) \rightarrow \mathbf{p}$

   (c)

   Proposition in words: Being 17 years of age is a necessary condition for being able to park in the school parking lot.

   Logical expression: $\mathbf{p} \rightarrow \mathbf{y}$

   (d)

   Proposition in words: A person can park in the school parking lot if and only if the person is a senior and at least 17 years of age.

   Logical expression: $\mathbf{p} \leftrightarrow (\mathbf{s} \wedge \mathbf{y})$

   (e)

   Proposition in words: Being able to park in the school parking lot implies that the person is either a senior or at least 17 years old.

   Logical expression: $\mathbf{p} \rightarrow (\mathbf{s} \vee \mathbf{y})$

3. Exercise 1.3.9

   (c)

   Proposition in words: The applicant can enroll in the course only if the applicant has parental permission.

   Logical expression: $\mathbf{c} \rightarrow \mathbf{p}$

   (d)

   Proposition in words: Having parental permission is a necessary condition for enrolling in the course.

   Logical expression: $\mathbf{c} \rightarrow \mathbf{p}$

Question 6

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.3.6

   (b) If Joe is eligible for the honors program, then he is maintaining a B average.

   (c) If Rajiv can go on the roller coaster, then he is at least four feet tall.

   (d) If Rajiv is at least four feet tall, then he can go on the roller coaster.

2. Exercise 1.3.10

   (c)

$$(p \vee r) \leftrightarrow (q \wedge r)$$
$$(T \vee unknown) \leftrightarrow (F \wedge unknown)$$
$$T \leftrightarrow F$$
$$\mathbf{F}$$

   (d)

$$(p \wedge r) \leftrightarrow (q \wedge r)$$
$$(T \wedge unknown) \leftrightarrow (F \wedge unknown)$$
$$unknown \leftrightarrow F$$
$$\mathbf{uknown}$$

   (e)

$$p \rightarrow (r \vee q)$$
$$T \rightarrow (unknown \vee F)$$
$$T \rightarrow unknown$$
$$\mathbf{unknown}$$

   (f)

$$(p \wedge q) \rightarrow r$$
$$(T \wedge F) \rightarrow unknown$$
$$F \rightarrow unknown$$
$$\mathbf{unknown}$$

Question 7

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.4.5

(b) $\neg\mathbf{j} \to (\mathbf{l} \vee \neg\mathbf{r}) \equiv (\mathbf{r} \wedge \neg\mathbf{l}) \to \mathbf{j}$

j	l	r	$\neg r$	$l \vee \neg r$	$\neg j$	$\neg j \to (l \vee \neg r)$	$\neg l$	$r \wedge \neg l$	$(r \wedge \neg l) \to j$
T	T	T	F	T	F	T	F	F	T
T	T	F	T	T	F	T	F	F	T
T	F	T	F	F	F	T	T	T	T
T	F	F	T	T	F	T	T	F	T
F	T	T	F	T	T	T	F	F	T
F	T	F	T	T	T	T	F	F	T
F	F	T	F	F	T	F	T	T	F
F	F	F	T	T	T	T	T	F	T

(c) $\mathbf{j} \to \neg\mathbf{l} \not\equiv \neg\mathbf{j} \to \mathbf{l}$

j	l	r	$\neg l$	$j \to \neg l$	$\neg j$	$\neg j \to l$
T	T	T	F	F	F	T
T	T	F	F	F	F	T
T	F	T	T	T	F	T
T	F	F	T	T	F	T
F	T	T	F	T	T	T
F	T	F	F	T	T	T
F	F	T	T	T	T	F
F	F	F	T	T	T	F

(d) $(\mathbf{r} \vee \neg\mathbf{l}) \to \mathbf{j} \not\equiv \mathbf{j} \to (\mathbf{r} \wedge \neg\mathbf{l})$

j	l	r	$\neg l$	$r \vee \neg l$	$(r \vee \neg l) \to j)$	$r \wedge \neg l$	$j \to (r \wedge \neg l)$
T	T	T	F	T	T	F	F
T	T	F	F	F	T	F	F
T	F	T	T	T	T	T	T
T	F	F	T	T	T	F	F
F	T	T	F	T	F	F	T
F	T	F	F	F	T	F	T
F	F	T	T	T	F	T	T
F	F	F	T	T	F	F	T

Question 8

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.5.2

(c)

$$(p \to q) \land (p \to r)$$
$$Conditional\, Identity : (\neg p \lor q) \land (\neg p \lor r)$$
$$Distributive\, Law : \neg p \lor (q \land r)$$
$$Conditional\, Identity : \mathbf{p} \to (\mathbf{q} \land \mathbf{r})$$

(f)

$$\neg(p \lor (\neg p \land q)$$
$$De\, Morgan's\, Law : \neg p \land \neg(\neg p \land q)$$
$$De\, Morgan's\, Law : \neg p \land (\neg\neg p \lor \neg q)$$
$$Double\, Negative : \neg p \land (p \lor \neg q)$$
$$Distributive\, Law : (\neg p \land p) \lor (\neg p \land \neg q)$$
$$Complement\, Law : F \lor (\neg p \land \neg q)$$
$$Identity\, Law : \neg \mathbf{p} \land \neg \mathbf{q}$$

(i)

$$(p \land q) \to r$$
$$Conditional\, Identity : \neg(p \land q) \lor r$$
$$De\, Morgan's\, Law : (\neg p \lor \neg q) \lor r$$
$$Associatve\, Law : \neg p \lor (\neg q \lor r)$$
$$Communtative\, Law : \neg p \lor (r \lor \neg q)$$
$$Associative\, Law : (\neg p \lor r) \lor \neg q$$
$$De\, Morgan's\, Law : \neg(p \land \neg r) \lor \neg q$$
$$Conditiona\, Identity : (\mathbf{p} \land \neg \mathbf{r}) \to \neg \mathbf{q}$$

2. Exercise 1.5.3
   (c)

$$\begin{gathered}
\neg r \vee (\neg r \to p) \\
Conditional\,Identity : \neg r \vee (\neg\neg r \vee p) \\
Double\,Negative : \neg r \vee (r \vee p) \\
Associative\,Law : (\neg r \vee r) \vee p \\
Complement\,Law : T \vee p \\
Domination\,Law : T
\end{gathered}$$

   (d)

$$\begin{gathered}
\neg(p \to q) \to \neg q \\
Conditional\,Identity : \neg(\neg p \vee q) \to \neg q \\
De\,Morgan's\,Law : (\neg\neg p \wedge \neg q) \to \neg q \\
Double\,Negative : (p \wedge \neg q) \to \neg q \\
Conditional\,Identity : \neg(p \wedge \neg q) \vee \neg q \\
De\,Morgan's\,Law : (\neg p \vee \neg\neg q) \vee \neg q \\
Double\,Negative : (\neg p \vee q) \vee \neg q \\
Associative\,Law : \neg p \vee (q \vee \neg q) \\
Complement\,Law : \neg p \vee T \\
Domination\,Law : T
\end{gathered}$$

Question 9

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.6.3

   (c)

   English Statement: There is a number that is equal to its square.

   Logical Expression: $\exists \mathbf{x}(\mathbf{x} = \mathbf{x^2})$

   (d)

   English Statement: Every number is less than or equal to its square plus 1.

   Logical Expression: $\forall \mathbf{x}(\mathbf{x} \leq \mathbf{x^2} + \mathbf{1})$

2. Exercise 1.7.4

   (b)

   English Statement: Everyone was well and went to work yesterday.

   Logical Statement: $\forall \mathbf{x}(\neg \mathbf{S}(\mathbf{x}) \wedge \mathbf{W}(\mathbf{x}))$

   (c)

   English Statement: Everyone who was sick did not go to work.

   Logical Statement: $\forall \mathbf{x}(\mathbf{S}(\mathbf{x}) \to \mathbf{W}(\mathbf{x}))$

   (d)

   English Statement: Yesterday someone was sick and went to work.

   Logical Statement: $\exists \mathbf{x}(\mathbf{S}(\mathbf{x}) \wedge \mathbf{W}(\mathbf{x}))$

Question 10

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.7.9
   (c) True
   (d) True
   (e) True
   (f) True
   (g) False
   (h) True
   (i) True
2. Exercise 1.9.2
   (b) True
   (c) True
   (d) False
   (e) False
   (f) True
   (g) False
   (h) True
   (i) True

Question 11

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.10.4
   (c)
   English Statement: There are two numbers whose sum is equal to their product.
   Logical Expression: $\exists \mathbf{x} \exists \mathbf{y}(\mathbf{x}+\mathbf{y}=\mathbf{x}\mathbf{y})$
   (d)
   English Statement: The ratio of every two positive numbers is also positive.
   Logical Expression: $\forall \mathbf{x}(\mathbf{D}(\mathbf{x}) \rightarrow \mathbf{P}(\mathbf{Sam}, \mathbf{x}))\forall \mathbf{y}(((\mathbf{x}>\mathbf{0}) \wedge (\mathbf{y}>\mathbf{0})) \rightarrow \mathbf{x}:\mathbf{y}>\mathbf{0})$
   (e)
   English Statement: The reciprocal of every positive number less than one is greater than one.
   Logical Expression: $\forall \mathbf{x}((\mathbf{0}<\mathbf{x}<\mathbf{1}) \rightarrow (\frac{\mathbf{1}}{\mathbf{x}}>\mathbf{1})$
   (f)
   English Statement: There is no smallest number.
   Logical Expression: $\neg\exists \mathbf{x} \forall \mathbf{y}(\mathbf{x} \leq \mathbf{y})$
   (g)
   English Statement: Every number other than 0 has a multiplicative inverse.
   Logical Expression: $\forall \mathbf{x} \exists \mathbf{y}((\mathbf{x} \neq \mathbf{0}) \rightarrow (\mathbf{x}\mathbf{y}=\mathbf{1}))$
2. Exercise 1.10.7
   (c)
   English Statement: There is at least one new employer who missed the deadline.
   Logical Expression: $\exists \mathbf{x}(\mathbf{N}(\mathbf{x}) \wedge \mathbf{D}(\mathbf{x}))$
   (d)
   English Statement: Sam knows the phone number of everyone who missed the deadline.
   Logical Expression: $\forall \mathbf{x} \rightarrow \mathbf{P}(\mathbf{Sam}, \mathbf{x}))$
   (e)
   English Statement: There is a new employee who knows everyone's phone number.
   Logical Expression: $\exists \mathbf{x} \forall \mathbf{y}(\mathbf{N}(\mathbf{x}) \wedge \mathbf{P}(\mathbf{x}, \mathbf{y}))$
   (f)
   English Statement: Exactly one new employee missed the deadline.
   Logical Expression: $\exists \mathbf{x} \forall \mathbf{y}(\mathbf{N}(\mathbf{x}) \wedge \mathbf{D}(\mathbf{x}) \wedge ((\mathbf{x} \neq \mathbf{y}) \wedge \mathbf{N}(\mathbf{y})) \rightarrow \neg \mathbf{D}(\mathbf{y})))$
3. Exercise 1.10.10
   (c)
   English Statement: Every student has taken at least one class other than Math 101.
   Logical Expression: $\forall \mathbf{x} \exists \mathbf{y}((\mathbf{y} \neq \mathbf{Math101}) \wedge \mathbf{T}(\mathbf{x}, \mathbf{y}))$
   (d)
   English Statement: There is a student who has taken every math class other than Math 101.

Logical Expression: $\exists \mathbf{x} \forall \mathbf{y}((\mathbf{y} \neq \mathbf{Math101}) \rightarrow \mathbf{T}(\mathbf{x}, \mathbf{y}))$
(e)
English Statement: Everyone other than Sam has taken at least two different math classes.
Logical Expression: $\forall \mathbf{x} \exists \mathbf{y} \exists \mathbf{z}((\mathbf{x} \neq \mathbf{Sam}) \rightarrow ((\mathbf{y} \neq \mathbf{z}) \wedge \mathbf{T}(\mathbf{x}, \mathbf{y}) \wedge \mathbf{T}(\mathbf{x}, \mathbf{z})))$
(f)
English Statement: Sam has taken exactly two math classes.
Logical Expression: $\exists \mathbf{x} \exists \mathbf{y} \forall \mathbf{z}((\mathbf{y} \neq \mathbf{y}) \wedge \mathbf{T}(\mathbf{Sam}, \mathbf{x}) \wedge \mathbf{T}(\mathbf{Sam}, \mathbf{y}) \wedge ((\mathbf{z} \neq \mathbf{x} \wedge \mathbf{z} \neq \mathbf{y}) \rightarrow \neg \mathbf{T}(\mathbf{Sam}, \mathbf{z}))$

Question 12

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.8.2

(b)

$$Logical\,Expression : \forall x(P(x) \lor D(x) \lor (P(x) \land D(x)))$$
$$Negation : \neg\forall x(P(x) \lor D(x) \lor (P(x) \land D(x)))$$
$$Applying\,De\,Morgan's\,Law : \exists x\neg(P(x) \lor D(x) \lor (P(x) \land D(x)))$$
$$\exists x(\neg P(x) \land \neg D(x) \land \neg(P(x) \land D(x)))$$
$$\exists x(\neg P(x) \land \neg D(x) \land (\neg P(x) \lor \neg D(x)))$$

English: There is a patient who was not given the placebo or not given the medication (or both).

(c)

$$Logical\,Expression : \exists x(D(x) \land M(x))$$
$$Negation : \neg\exists x(D(x) \land M(x))$$
$$Applying\,De\,Morgan's\,Law : \forall x\neg(D(x) \land M(x))$$
$$\forall x(\neg D(x) \lor \neg M(x))$$

English: Every patient either was not given medication or did not have migraines.

(d)

$$Logical\,Expression : \forall x(P(x) \to M(X))$$
$$Negation : \neg\forall x(P(x) \to M(X))$$
$$Applying\,De\,Morgan's\,Law : \exists x\neg(P(x) \to M(X))$$
$$Conditional\,Identity : \exists x\neg(\neg P(x) \lor M(x))$$
$$Double\,Negative : \exists x(\neg\neg P(x) \land \neg M(x))$$
$$De\,Morgan's\,Law : \exists x(P(x) \land \neg M(x))$$

English: There is a patient who was given the placebo and did not have migraines.

(e)

$$Logical\,Expression : \exists x(M(x) \land P(x))$$
$$Negation : \neg\exists x(M(x) \land P(x))$$
$$Applying\,De\,Morgan's\,Law : \forall x\neg(M(x) \land P(x))$$
$$\forall x(\neg M(x) \lor \neg P(x))$$

English: Every patient either did not have migraines or was not given the placebo.

2. Exercise 1.9.4

(c)

$$
\begin{aligned}
Negation &: \neg\exists x\forall y(P(x,y) \to Q(x,y)) \\
De\,Morgan's\,Law &: \forall x\exists y\neg(P(x,y) \to Q(x,y)) \\
Conditional\,Identities &: \forall x\exists y\neg(\neg P(x,y) \lor Q(x,y)) \\
De\,Morgan's\,Law &: \forall x\exists y(\neg\neg P(x,y) \land \neg Q(x,y)) \\
Double\,Negative &: \forall \mathbf{x}\exists \mathbf{y}(\mathbf{P}(\mathbf{x},\mathbf{y}) \land \neg \mathbf{Q}(\mathbf{x},\mathbf{y}))
\end{aligned}
$$

(d)

$$
\begin{aligned}
Negation &: \neg\exists x\forall y(P(x,y) \leftrightarrow P(y,x)) \\
De\,Morgan's\,Law &: \forall x\exists y\neg(P(x,y) \leftrightarrow P(y,x)) \\
Conditional\,Identities &: \forall x\exists y\neg((P(x,y) \to P(y,x)) \land (P(y,x) \to P(x,y))) \\
Conditional\,Identities &: \forall x\exists y\neg((\neg P(x,y) \lor P(y,x)) \land (\neg P(y,x) \lor P(x,y))) \\
De\,Morgan's\,Law &: \forall x\exists y(\neg(\neg P(x,y) \lor P(y,x)) \lor \neg(\neg P(y,x) \lor P(x,y))) \\
De\,Morgan's\,Law &: \forall x\exists y((\neg\neg P(x,y) \land \neg P(y,x)) \lor (\neg\neg P(y,x) \land \neg P(x,y))) \\
Double\,Negative &: \forall \mathbf{x}\exists \mathbf{y}((\mathbf{P}(\mathbf{x},\mathbf{y}) \land \neg \mathbf{P}(\mathbf{y},\mathbf{x})) \lor (\mathbf{P}(\mathbf{y},\mathbf{x}) \land \neg \mathbf{P}(\mathbf{x},\mathbf{y})))
\end{aligned}
$$

(e)

$$
\begin{aligned}
Negation &: \neg(\exists x\exists y(P(x,y)) \land \forall x\forall y(Q(x,y))) \\
De\,Morgan's\,Law &: \neg\exists x\exists y(P(x,y)) \lor \neg\forall x\forall y(Q(x,y)) \\
De\,Morgan's\,Law &: \forall \mathbf{x}\forall \mathbf{y}(\neg \mathbf{P}(\mathbf{x},\mathbf{y})) \lor \exists \mathbf{x}\exists \mathbf{y}(\neg \mathbf{Q}(\mathbf{x},\mathbf{y}))
\end{aligned}
$$