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Extended Bridge to CS
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Homework 2
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Question 5

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.12.2

(b)
$$\begin{array}{l} p \to (q \land r) \\ \neg q \\ \hline \therefore \neg q \end{array}$$

1.	$p \to (q \land r)$	Hypothesis
2.	$\neg p \lor (q \land q)$	Conditional Identity, 1
3.	$(\neg p \lor q) \land (\neg p \lor r)$	Distributive Law, 3
4.	$(q \lor \neg p) \land (\neg p \lor r)$	Commutative Law, 3
5.	$\neg q$	Hypothesis
6.	$\neg p \land (\neg p \lor r)$	Disjunctive Syllogism, 4,5
7.	$\neg p$	Absorption Law, 6

(e)
$$\begin{array}{l} p \lor q \\ \neg p \lor r \\ \neg q \\ \hline \therefore r \end{array}$$

1.	$p \lor q$	Hypothesis
2.	$\neg p \lor r$	Hypothesis
3.	$q \lor r$	Resolution, 1,2
4.	$\neg q$	Hypothesis
5.	r	Disjunctive Syllogism, 4,5

2. Exercise 1.12.3

(c)
$$\begin{array}{l} p \lor q \\ \neg p \\ \hline \therefore q \end{array}$$

1.	$p \lor q$	Hypothesis
2.	$\neg\neg p \lor q$	Double Negation Law, 1
3.	$\neg p \to q$	Conditional Identity, 2
4.	$\neg p$	Hypothesis
5.	q	Modus Ponens, 4

3. Exercise 1.12.5

(c) p = buy a new car; q = buy a new house; r = get a job

$$\begin{array}{l} (p \land q) \to r \\ \neg r \\ \hline \therefore \neg p \end{array}$$

1.	$\neg r$	Hypothesis
2.	$(p \land q) \to r$	Hypothesis
3.	$\neg(p \land q)$	Modus Tollens, 1,2
4.	$\neg p \lor \neg q$	De Morgan's Law, 3

Since $\neg p \lor \neg q \neq \neg p$, this argument is invalid.

(d) p = buy a new car; q = buy a new house; r = get a job

$$\begin{array}{l} (p \land q) \to r \\ \neg r \\ q \\ \hline \therefore \neg p \end{array}$$

1.	$\neg r$	Hypothesis
2.	$(p \land q) \to r$	Hypothesis
3.	$\neg(p \land q)$	Modus Tollens, 1,2
4.	$\neg p \lor \neg q$	De Morgan's Law, 3
5.	$\neg q \lor \neg p$	Commutative Law, 4
6.	q	Hypothesis
7.	$\neg p$	Disjunctive Syllogism, 5,6

This argument is valid.

4. Exercise 1.13.3

‘ (b)
$$\begin{array}{l} \exists x(P(x) \lor Q(x)) \\ \exists x \neg Q(x) \\ \hline \therefore \exists x P(x) \end{array}$$

	P(x)	Q(x)
a	F	T
b	F	F

$\exists x(P(x) \lor Q(x))$ will be true if P(a) is false and Q(a) is true, and $\exists x \neg Q(x)$ would be true if Q(b) is false. However, if P(a) and P(b) are both false, $\exists x P(x)$ will be false, making this argument invalid.

5. Exercise 1.13.5

(d) C(x): x missed class; D(x): x had detention; x is a student in class

$$\begin{array}{l} \forall x(C(x) \to D(x)) \\ \text{Penelope is a student in the class.} \\ \neg C(Penelope) \\ \hline \therefore \neg D(Penelope) \end{array}$$

1.	$\forall x(C(x) \to D(x))$	Hypothesis
2.	Penelope is a student in the class.	Hypothesis
3.	$C(Penelope) \to C(Penelope)$	Universal Instantiation, 1,2
4.	$\neg C(Penelope)$	Hypothesis

Since $\neg C(Penelope) \neq \neg D(Penelope)$, this argument is invalid.

(e) C(x): x missed class; D(x): x had detention; A(x): x got an A; x is a student in class

$$\begin{array}{l} \forall x((C(x) \lor D(x)) \to A(x)) \\ \text{Penelope is a student in the class.} \\ A(Penelope) \\ \hline \therefore \neg D(Penelope) \end{array}$$

1.	$\forall x((C(x) \lor D(x)) \to A(x))$	Hypothesis
2.	Penelope is a student in the class.	Hypothesis
3.	$(C(Penelope) \lor D(Penelope)) \to A(Penelope)$	Universal Instantiation, 1,2
4.	$\neg A(Penelope)$	Hypothesis
5.	$\neg(C(Penelope) \lor D(Penelope))$	Modus Tollens, 3,4
6.	$\neg C(Penelope) \land \neg D(Penelope)$	De Morgan's Law, 5
7.	$\neg D(Penelope)$	Simplification, 6

This argument is valid.

Question 6

Solve the following questions from the Discrete Math zyBook:

1. Exercise 2.3.1
   (d)
   Theorem: The product of two odd integers is an odd integer.
   Proof: Let x and y be odd integers. We shall prove that (x)(y) is odd.
   1.) Since x is odd, there is an integer k such that $x = 2k + 1$. Since y is odd, there is an integer j such that $y = 2j + 1$.
   2.) We can plug $2k + 1$ in for x and $2j + 1$ in for y into the equation (x)(y) to get $(x)(y) = (2k + 1)(2j + 1)$.
   3.) $4kj + 2k + 2j + 1 = 2(2kj + k + j) + 1$
   4.) Since k and j are integers, then $2kj + k + j$ is also an integer.
   5.) Since $(x)(y) = 2m + 1$, where $m = 2kj + k + j$ is an integer, then (x)(y) is odd.
   ■

2. Exercise 2.4.3
   (b)
   Theorem: If x is a real number and $x \leq 3$, then $12 - 7x + x^2 \geq 0$.
   Proof: Assume that x is a real number and $x \leq 3$. We shall prove that $12 - 7x + x^2 \geq 0$.
   1.) Factor out the quadratic equation $12 - 7x + x^2 \geq 0$ to get $(x - 3)(x - 4) \geq 0$.
   2.) Subtract 3 from both sides of the equation $x \leq 3$ to get $x - 3 \leq 0$.
   3.) Since $x - 3 \leq 0$, then $x - 4 \leq -1$, which is also less than 0.
   4.) Since $x - 3 \leq 0$ and $x - 4 \leq 0$, then $(x - 3)(x - 4) \geq 0$. Therefore, $12 - 7x + x^2 \geq 0$.
   ■

Question 7

Solve the following questions from the Discrete Math zyBook:

1. Exercise 2.5.1

   (d)

   Theorem: For every integer, n, if $n^2 - 2n + 7$ is even, then n is odd.

   Proof: Assume that n is even. We shall prove that $n^2 - 2n + 7$ is odd.

   1.) Since n is even, there is an integer k such that n=2k.

   2.) Plug 2k in for n in the equation $n^2 - 2n + 7$ to get $(2k)^2 - 2(2k) + 7$.

   3.) $(2k)^2 - 2(2k) + 7 = 4k^2 - 4k + 7 = 2(2k^2 - 2k + 3) + 1$

   4.) Since k is an integer, then $k^2 - 2k$ is also an integer.

   5.) Since $n^2 - 2n + 7 = 2m + 1$, where m=$k^2 - 2k$ is an integer, then $n^2 - 2n + 7$ is odd.

   ■

2. Exercise 2.5.4

   (a)

   Theorem: For every pair of real numbers x and y, if $x^3 + xy^2 \leq x^2y + y^3$, then $x \leq y$.

   Proof: Assume x and y are real numbers, where $x > y$. We shall prove that $x^3 + xy^2 > x^2y + y^3$.

   1.) Factor out the equation $x^3 + xy^2 > x^2y + y^3$ to get $x(x^2 + y^2) > y(x^2 + y^2)$.

   2.) Divide both sides by $x^2 + y^2$ to get $x > y$.

   3.) Since $x^3 + xy^2 > x^2y + y^3$ can be simplified to $x > y$, then $x^3 + xy^2 > x^2y + y^3$.

   ■

   (b)

   Theorem: For every pair of real numbers x and y, if $x + y > 20$, then $x > 10$ or $y > 10$.

   Proof: Assume it is not true that $x > 10$ or $y > 10$. We will prove that $x + y \leq 20$.

   1.) The assumption that it is not true that $x > 10$ or $y > 10$ is equivalent to the condition that $x \leq 10$ and $y \leq 10$ by De Morgan's Law.

   2.) Add the inequalities $x \leq 10$ and $y \leq 10$ to get $x + y \leq 20$.

   3.) Since $x + y \leq 20$ can be derived from $x \leq 10$ and $y \leq 10$, then $x + y \leq 20$.

   ■

3. Exercise 2.5.5

   (c)

   Theorem: For every non-zero real number x, if x is irrational, then $\frac{1}{x}$ is also irrational.

   Proof: Assume $\frac{1}{x}$ is rational, where x is a non-zero real number. We will prove that x is rational.

   1.) Since $\frac{1}{x}$ is rational, then $\frac{1}{x} = \frac{a}{b}$, where $b \neq 0$.

   2.) Solve for x in the equation $\frac{1}{x} = \frac{a}{b}$ to get $x = \frac{b}{a}$.

   3.) Since a and b are integers, $\frac{b}{a}$ is rational.

   4.) Since $x = \frac{b}{a}$, then x is rational. ■

Question 8

Solve the following questions from the Discrete Math zyBook:

1. Exercise 2.6.6

(c)

Theorem: The average of three real numbers is greater than or equal to at least one of the numbers.

Proof: Assume that there are three real numbers $x_1, x_2,$ and x_3. We will prove that these numbers are less than their average $\frac{x_1+x_2+x_3}{3}$.

1.) Therefore, $x_1 < \frac{x_1+x_2+x_3}{3}$, $x_2 < \frac{x_1+x_2+x_3}{3}$, and $x_3 < \frac{x_1+x_2+x_3}{3}$.

2.) Combine the inequalities: $x_1 + x_2 + x_3 < \frac{x_1+x_2+x_3}{3} + \frac{x_1+x_2+x_3}{3} + \frac{x_1+x_2+x_3}{3}$.

3.) This can be simplified as $x_1 + x_2 + x_3 < 3(\frac{x_1+x_2+x_3}{3}) = x_1 + x_2 + x_3 < x_1 + x_2 + x_3$.

This inequality cannot be true since a variable cannot be less than itself, thus forming a contradiction. Therefore, the assumption that $x_1, x_2,$ and x_3 are less than their average is incorrect, which means at least one of the three real numbers is greater than or equal to their average and the theorem is valid.

■

(d)

Theorem: There is no smallest number.

Proof: Assume that x is the smallest number. We will prove that there is a smallest number.

1.) Since there will always be an x-1, this is not true, and thus is a contradiction.

Therefore, the assumption that x is the smallest number is incorrect, which means there is no smallest number and the theorem is valid.

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Question 9

Solve the following questions from the Discrete Math zyBook:

1. Exercise 2.7.2

   (b)

   Theorem: If integers x and y have the same parity, then x+y is even.

   Proof:

   Case 1: x and y are even

   1.)Since x is even, there is an integer k such that x=2k. Since y is even, there is an integer j such that y=2k.

   2.) Plug 2k in for x and 2j in for y in the equation x+y to get x+y=2k+2j.

   3.) 2k+2j=2(k+j)

   4.) Since k and j are integers, then k+j is also an integer.

   5.) Since x+y=2m, where m=k+j is an integer, then x+y is even.

   Case 2: x and y are odd

   1.) Since x is odd, there is an integer k such that $x = 2k + 1$. Since y is odd, there is an integer j such that $y = 2j + 1$.

   2.) Plug 2k+1 in for x and 2j+1 in for y in the equation x+y to get x+y=(2k+1)+(2j+1).

   3.) x+y=(2k+1)+(2j+1)=2k+2j+2=2(k+j+1)

   4.) Since k and j are integers, then k+j+1 is also an integer.

   5.) Since x+y=2m, where m=k+j+1 is an integer, then x+y is even.

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