## Rebuttals to Reviews: Boundary Multiple Measurement Vectors for Multi-Coset Sampler

Dong Xiao, Jian Wang and Yun Lin

We provide a new experiment that compares the recent algorithm DTMP in the literature [16] with SI-SSP. The high sparsity case which has been studied in many compressed sensing literature absolutely can be still valid for SI-SSP and details can be seen in Fig. 1. In our experiments, we consider a multi-band signal with frequency range [-5,5]GHz and  $N_{\rm sig}=3$  PU's signals, whose bandwidths are B=100 and 500MHz corresponding to  $\frac{L}{M}=2$  and 6, respectively. We divide the frequency range into L=100 cosets and use  $p\in[6,20]$  channels or  $p\in[36,50]$  channels to get the MCS signals, with delay coefficients randomly generated. We set conditions where  $\frac{L}{M}=6$ ,  $N_{\rm sig}=3$  and the  $|{\rm supp}({\bf X})|=18$ . The sparsity of  ${\bf X}$  is a very high value for the signal length L=100.

Moreover, we add Gaussian white noise to the signal with signal-to-noise ratio (SNR) of  $\{-10,10,30\}$ dB. In bMMV, we consider r=4 partitioning sub-matrices and set  $\omega=0.8$  to the weighting matrix W. In SI-SSP, we select triple noise variance as the tolerance. In DTMP, we select the hyper-parameter constant  $C=\{3,0.001,0.001\}$  for SNR  $=\{-10,10,30\}$ . To compare the recovery accuracy of SI-SSP with DTMP, we use the detection probability  $P_d$  and false alarm probability  $P_f$  as metrics.

From Fig. 1, SI-SSP has a better recovery performance than DTMP algorithm in both the low and high signal sparsity cases.

We give the computational analysis of SI-SSP. First, in each iteration, the subsequent selection requires O(pNL/r) computations for a sub-matrix in general. Second, in each iteration, the cost of computing the projections is of  $O(pK^2N/r)$  the order for a sub-matrix where  $K = |\mathrm{supp}(\mathbf{X})|$ . Then, we could sum the two-part costs to get  $O(pK^2N/r + pNL/r)$  computations. Finally, we consider that SI-SSP repeats O(K) iterations and there r sub-matrices, the total time complexity of SI-SSP is upper-bounded by  $O(pN(L+K^2))$ . When the signal is very sparse, in particular, when  $K^2 < O(L)$ , the total complexity of SI-SSP is upper-bounded by O(pNL).

The restricted isometry property (RIP) framework can be defined on the complex field. We have:

**Definition 1.** A sensing matrix  $\mathbf{A} \in \mathbb{C}^{m \times n}$  is said to satisfy the s-order RIP if for any s-sparse  $(\|\mathbf{x}\|_0 \leq s)$  signal  $\mathbf{x} \in \mathbb{C}^n$ 

$$\left(1-\delta\right)\left\|\mathbf{x}\right\|_{2}^{2}\leq\left\|\mathbf{A}\mathbf{x}\right\|_{2}^{2}\leq\left(1+\delta\right)\left\|\mathbf{x}\right\|_{2}^{2},\tag{S.1}$$

where  $0 \le \delta < 1$ . The infimum of  $\delta$ , denoted by  $\delta_s$ , is called the restricted isometry constant (RIC) of A.

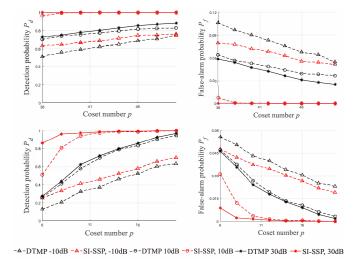


Fig. 1.  $P_d$  and  $P_f$  of DTMP and SI-SSP against p from  $6 \sim 20$  and  $36 \sim 50$  with SNR =  $\{-10, 10, 30\}$  dB. From above to below: the signal with  $\frac{L}{M} = 6$  is decomposed into r = 4 sub-matrices and the signal with  $\frac{L}{M} = 2$  is decomposed into r = 4 sub-matrices.

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