Response to Minor Comments: Boundary Multiple Measurement Vectors for Multi-Coset Sampler

Dong Xiao, Jian Wang and Yun Lin

We sincerely appreciate the reviewers' comments on our work. As the major comments of Reviewer 1F03 have been responded, here we only focus on the minor comments of reviewers.

Comment 1. *Reviewer 3B93 concerned that the algorithms used for comparison appear to be somewhat outdated.*

Response: We provide a new experiment that compares the recent algorithm DTMP in the literature [16] with SI-SSP. The high sparsity case which has been studied in many compressed sensing literature absolutely can be still valid for SI-SSP and details can be seen in Fig. 1.

In our experiments, we consider a multi-band signal with frequency range $[-5, 5]$GHz and $N_{\text{sig}} = 3$ PU's signals, whose bandwidths are $B = 100$ and 500MHz corresponding to $\frac{L}{M} = 2$ and 6, respectively. We divide the frequency range into $L = 100$ cosets and use $p \in [6, 20]$ channels or $p \in [36, 50]$ channels to get the MCS signals, with delay coefficients randomly generated. We set conditions where $\frac{L}{M} = 6$, $N_{\text{sig}} = 3$ and the $|\text{supp}(\mathbf{X})| = 18$. The sparsity of $\mathbf{X}$ is a very high value for the signal length $L = 100$.

Moreover, we add Gaussian white noise to the signal with signal-to-noise ratio (SNR) of $\{-10, 10, 30\}$dB. In bMMV, we consider $r = 4$ partitioning sub-matrices and set $\omega = 0.8$ to the weighting matrix $\mathbf{W}$. In SI-SSP, we select triple noise variance as the tolerance. In DTMP, we select the hyper-parameter constant $C = \{3, 0.001, 0.001\}$ for SNR $= \{-10, 10, 30\}$. To compare the recovery accuracy of SI-SSP with DTMP, we use the detection probability P_d and false alarm probability P_f as metrics.

From Fig. 1, SI-SSP has a better recovery performance than DTMP algorithm in both the low and high signal sparsity cases.

Comment 2. *Reviewer 1F31 concerned that there is lack of the computational analysis in the experimental results.*

We give the computational analysis of SI-SSP. We consider two main steps i) the subsequent selection and ii) pruning operations of SI-SSP for computational analysis. First, in each iteration, the proxy step requires $O(pNL/r)$ computations for a sub-matrix in general. Second, the identification need $O(L)$ computing cost. Then, the support merger step will cost $O(K)$ computations, where $K = |\text{supp}(\mathbf{X})|$ is the signal sparsity. Finally, we consider that SI-SSP repeats $O(K)$ iterations and there are r sub-matrices, the total time complexity of the subsequent selection is upper-bounded by $O(pNL + rL + rK)$.

In each iteration, the cost of pruning operations contain $O(pK^2N/r)$ computations for computing the projections and $O(K)$ for pruning with a sub-matrix. Then, we could sum the two-part costs to get $O(pK^2N/r + K)$ computations. Furthermore, we consider that SI-SSP repeats $O(K)$ iterations and there are r sub-matrices, the total time complexity of SI-SSP is upper-bounded by $O(pN(L + K^2))$. When the signal is very sparse, in particular, when $K^2 < O(L)$, the total complexity of SI-SSP is upper-bounded by $O(pNL)$.

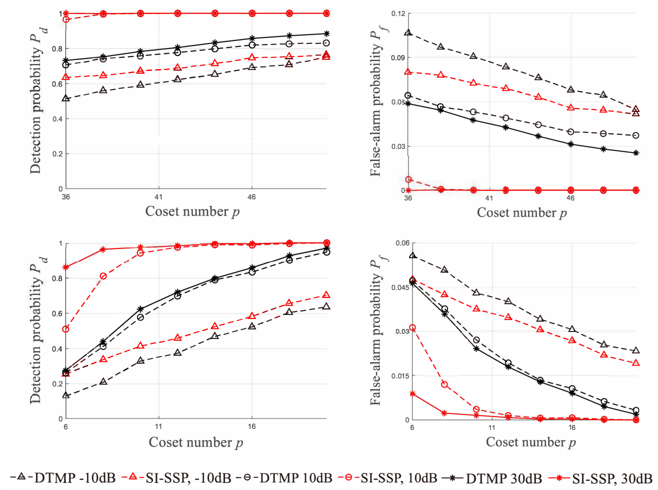

Fig. 1. P_d and P_f of DTMP and SI-SSP against p from $6 \sim 20$ and $36 \sim 50$ with SNR = $\{-10, 10, 30\}$ dB. From above to below: the signal with $\frac{L}{M} = 6$ is decomposed into $r = 4$ sub-matrices and the signal with $\frac{L}{M} = 2$ is decomposed into $r = 4$ sub-matrices.

Comment 3. *Reviewer 1F03 concerned that there may exist mismatch between the RIP definition and analysis.*

We have already explained this concern in the rebuttals. Now, we provide the complex form of the restricted isometry property (RIP) framework:

Definition 1. *A sensing matrix $\mathbf{A} \in \mathbb{C}^{m \times n}$ is said to satisfy the s-order RIP if for any s-sparse ($\|\mathbf{x}\|_0 \le s$) signal $\mathbf{x} \in \mathbb{C}^n$*

$$(1 - \delta) \|\mathbf{x}\|_2^2 \le \|\mathbf{A}\mathbf{x}\|_2^2 \le (1 + \delta) \|\mathbf{x}\|_2^2, \qquad \text{(S.1)}$$

where $0 \le \delta < 1$. The infimum of δ, denoted by δ_s, is called the restricted isometry constant (RIC) of $\mathbf{A}$.

Obviously, the RIP condition can be used to analyze both complex and real matrices ($\mathbf{A}$ and $\mathbf{X}$).

Besides, we have carefully checked typos throughout the paper.