# Response to the Comments: Boundary Multiple Measurement Vectors for Multi-Coset Sampler

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## I. New Experiment

We provide a new experiment that compares the recent algorithm DTMP in the literature [16] with SI-SSP. The high sparsity case which has been studied in many compressed sensing literature absolutely can be still valid for SI-SSP and details can be seen in Fig. 1. In our experiments, we consider a multi-band signal with frequency range $[-5, 5]$GHz and $N_{\text{sig}} = 3$ PU's signals, whose bandwidths are $B = 100$ and 500MHz corresponding to $\frac{L}{M} = 2$ and 6, respectively. We divide the frequency range into $L = 100$ cosets and use $p \in [6, 20]$ channels or $p \in [36, 50]$ channels to get the MCS signals, with delay coefficients randomly generated. We set conditions where $\frac{L}{M} = 6$, $N_{\text{sig}} = 3$ and the $|\text{supp}(\mathbf{X})| = 18$. The sparsity of $\mathbf{X}$ is a very high value for the signal length $L = 100$.

Moreover, we add Gaussian white noise to the signal with signal-to-noise ratio (SNR) of $\{-10, 10, 30\}$dB. In bMMV, we consider $r = 4$ partitioning sub-matrices and set $\omega = 0.8$ to the weighting matrix $\mathbf{W}$. In SI-SSP, we select triple noise variance as the tolerance. In DTMP, we select the hyper-parameter constant $C = \{3, 0.001, 0.001\}$ for SNR $= \{-10, 10, 30\}$. To compare the recovery accuracy of SI-SSP with DTMP, we use the detection probability $P_d$ and false alarm probability $P_f$ as metrics.

From Fig. 1, SI-SSP has a better recovery performance than DTMP algorithm in both the low and high signal sparsity cases.

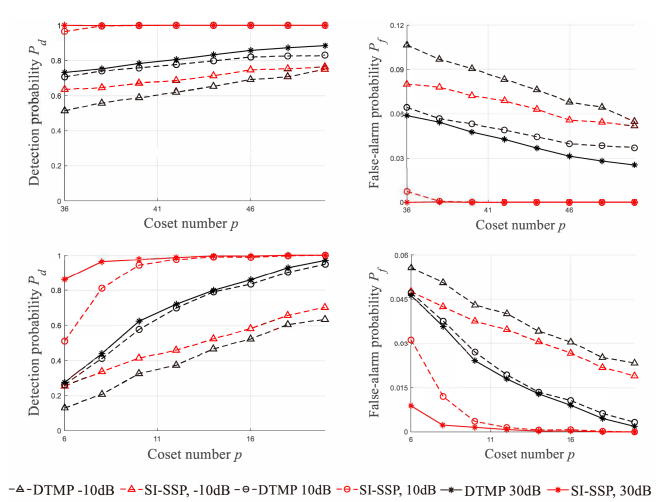


Fig. 1. $P_d$ and $P_f$ of DTMP and SI-SSP against $p$ from $6 \sim 20$ and $36 \sim 50$ with SNR = $\{-10, 10, 30\}$ dB. From above to below: the signal with $\frac{L}{M} = 6$ is decomposed into $r = 4$ sub-matrices and the signal with $\frac{L}{M} = 2$ is decomposed into $r = 4$ sub-matrices.

## II. Computational Analysis

We give the computational analysis of SI-SSP. We divide SI-SSP into two main steps i) the subsequent selection and ii) pruning operations for computational analysis. First, in each iteration, the proxy step requires $O(pNL/r)$ computations for a sub-matrix in general. Second, the identification need $O(L)$ computing cost. Then, the support merger step will cost $O(K)$ computations, where $K = |\text{supp}(\mathbf{X})|$ is the signal sparsity. Finally, we consider that SI-SSP repeats $O(K)$ iterations and there $r$ sub-matrices, the total time complexity of the subsequent selection is upper-bounded by $O(pNL + rL + rK)$.

In each iteration, the cost of computing the projections and the pruning operations are of $O(pK^2N/r)$ and $O(K)$ the order for a sub-matrix. Then, we could sum the two-part costs to get $O(pK^2N/r + K)$ computations. Furthermore, we consider that SI-SSP repeats $O(K)$ iterations and there $r$ sub-matrices, the total time complexity of SI-SSP is upper-bounded by $O(pN(L + K^2))$. When the signal is very sparse, in particular, when $K^2 < O(L)$, the total complexity of SI-SSP is upper-bounded by $O(pNL)$.

## III. Complex Form of RIP

The restricted isometry property (RIP) framework can be defined on the complex field. We have:

**Definition 1.** *A sensing matrix $\mathbf{A} \in \mathbb{C}^{m \times n}$ is said to satisfy the $s$-order RIP if for any $s$-sparse ($\|\mathbf{x}\|_0 \leq s$) signal $\mathbf{x} \in \mathbb{C}^n$*

$$(1 - \delta) \|\mathbf{x}\|_2^2 \leq \|\mathbf{A}\mathbf{x}\|_2^2 \leq (1 + \delta) \|\mathbf{x}\|_2^2, \tag{S.1}$$

*where $0 \leq \delta < 1$. The infimum of $\delta$, denoted by $\delta_s$, is called the restricted isometry constant (RIC) of $\mathbf{A}$.*

Obviously, the RIP condition can be used to analyze both complex and real matrices ($\mathbf{A}$ and $\mathbf{X}$).