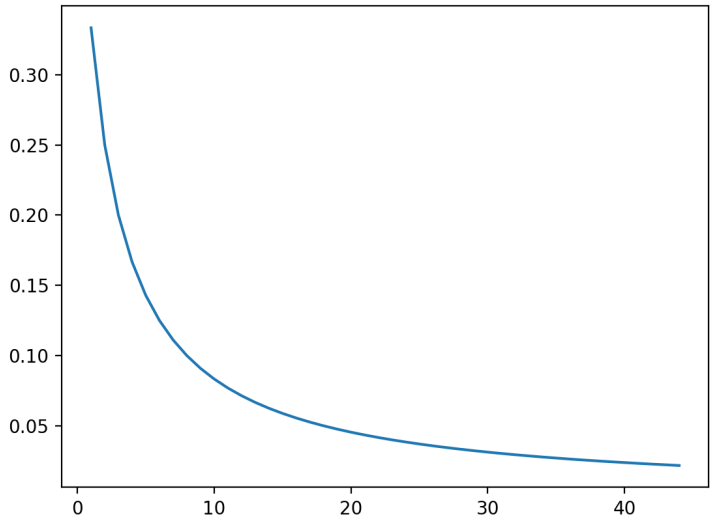
Q1~Q10

Q1

Round		1’s action	2’s action	1’s belief	2’s belief	1’s payoff	2’s payoff		
Round	1	2	2	[1. 2.]	[1. 2.]	[1. 6.]	[1. 6.]	(0.33, 0.67)	(0.33, 0.67)
Round	2	2	2	[1. 3.]	[1. 3.]	[2. 9.]	[2. 9.]	(0.25, 0.75)	(0.25, 0.75)
Round	3	2	2	[1. 4.]	[1. 4.]	[3. 12.]	[3. 12.]	(0.20, 0.80)	(0.20, 0.80)
Round	4	2	2	[1. 5.]	[1. 5.]	[4. 15.]	[4. 15.]	(0.17, 0.83)	(0.17, 0.83)
Round	5	2	2	[1. 6.]	[1. 6.]	[5. 18.]	[5. 18.]	(0.14, 0.86)	(0.14, 0.86)
Round	6	2	2	[1. 7.]	[1. 7.]	[6. 21.]	[6. 21.]	(0.12, 0.88)	(0.12, 0.88)
Round	7	2	2	[1. 8.]	[1. 8.]	[7. 24.]	[7. 24.]	(0.11, 0.89)	(0.11, 0.89)
Round	8	2	2	[1. 9.]	[1. 9.]	[8. 27.]	[8. 27.]	(0.10, 0.90)	(0.10, 0.90)
Round	9	2	2	[1. 10.]	[1. 10.]	[9. 30.]	[9. 30.]	(0.09, 0.91)	(0.09, 0.91)
Round	10	2	2	[1. 11.]	[1. 11.]	[10. 33.]	[10. 33.]	(0.08, 0.92)	(0.08, 0.92)
Round	11	2	2	[1. 12.]	[1. 12.]	[11. 36.]	[11. 36.]	(0.08, 0.92)	(0.08, 0.92)
Round	38	2	2	[1. 39.]	[1. 39.]	[38. 117.]	[38. 117.]	(0.03, 0.97)	(0.03, 0.97)
Round	39	2	2	[1. 40.]	[1. 40.]	[39. 120.]	[39. 120.]	(0.02, 0.98)	(0.02, 0.98)
Round	40	2	2	[1. 41.]	[1. 41.]	[40. 123.]	[40. 123.]	(0.02, 0.98)	(0.02, 0.98)
Round	41	2	2	[1. 42.]	[1. 42.]	[41. 126.]	[41. 126.]	(0.02, 0.98)	(0.02, 0.98)
Round	42	2	2	[1. 43.]	[1. 43.]	[42. 129.]	[42. 129.]	(0.02, 0.98)	(0.02, 0.98)
Round	43	2	2	[1. 44.]	[1. 44.]	[43. 132.]	[43. 132.]	(0.02, 0.98)	(0.02, 0.98)
Round	44	2	2	[1. 45.]	[1. 45.]	[44. 135.]	[44. 135.]	(0.02, 0.98)	(0.02, 0.98)

The game converges in round 44

Yes, it converges to (2, 2). The players both choose strategy 2 at the last several rounds.



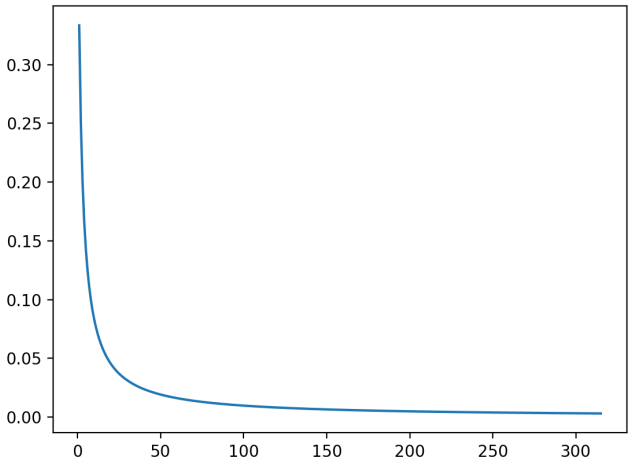
By this figure, it is also clear that it converges to the pure-strategy Nash equilibrium. Note that the x-axis means i-th round and the y-axis represents the first player’s belief history for the first strategy.

Q2

case1: converges to (2, 2)

Round		1’s action	2’s action	1’s belief	2’s belief	1’s payoff	2’s payoff		
Round	1	2	2	[1. 2.]	[1. 2.]	[4. 6.]	[4. 6.]	(0.33, 0.67)	(0.33, 0.67)
Round	2	2	2	[1. 3.]	[1. 3.]	[5. 9.]	[5. 9.]	(0.25, 0.75)	(0.25, 0.75)
Round	3	2	2	[1. 4.]	[1. 4.]	[6. 12.]	[6. 12.]	(0.20, 0.80)	(0.20, 0.80)
Round	4	2	2	[1. 5.]	[1. 5.]	[7. 15.]	[7. 15.]	(0.17, 0.83)	(0.17, 0.83)
Round	5	2	2	[1. 6.]	[1. 6.]	[8. 18.]	[8. 18.]	(0.14, 0.86)	(0.14, 0.86)
Round	6	2	2	[1. 7.]	[1. 7.]	[9. 21.]	[9. 21.]	(0.12, 0.88)	(0.12, 0.88)
Round	7	2	2	[1. 8.]	[1. 8.]	[10. 24.]	[10. 24.]	(0.11, 0.89)	(0.11, 0.89)
Round	8	2	2	[1. 9.]	[1. 9.]	[11. 27.]	[11. 27.]	(0.10, 0.90)	(0.10, 0.90)

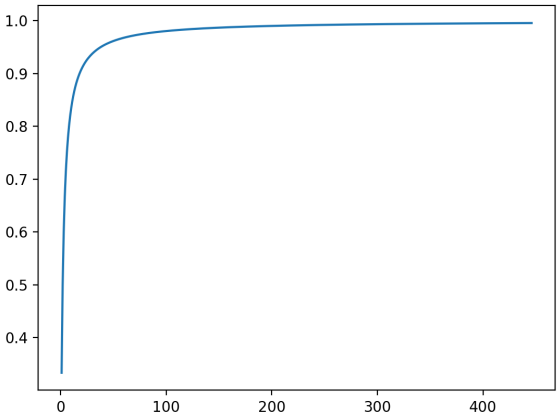
```
Round   307         2          2        [  1. 308.]     [  1. 308.]     [310. 924.]     [310. 924.]   (0.00, 1.00)  (0.00, 1.00)
Round   308         2          2        [  1. 309.]     [  1. 309.]     [311. 927.]     [311. 927.]   (0.00, 1.00)  (0.00, 1.00)
Round   309         2          2        [  1. 310.]     [  1. 310.]     [312. 930.]     [312. 930.]   (0.00, 1.00)  (0.00, 1.00)
Round   310         2          2        [  1. 311.]     [  1. 311.]     [313. 933.]     [313. 933.]   (0.00, 1.00)  (0.00, 1.00)
Round   311         2          2        [  1. 312.]     [  1. 312.]     [314. 936.]     [314. 936.]   (0.00, 1.00)  (0.00, 1.00)
Round   312         2          2        [  1. 313.]     [  1. 313.]     [315. 939.]     [315. 939.]   (0.00, 1.00)  (0.00, 1.00)
Round   313         2          2        [  1. 314.]     [  1. 314.]     [316. 942.]     [316. 942.]   (0.00, 1.00)  (0.00, 1.00)
Round   314         2          2        [  1. 315.]     [  1. 315.]     [317. 945.]     [317. 945.]   (0.00, 1.00)  (0.00, 1.00)
Round   315         2          2        [  1. 316.]     [  1. 316.]     [318. 948.]     [318. 948.]   (0.00, 1.00)  (0.00, 1.00)
The game converges in round 315
```



case1: converges to (1, 1)

```
Round          1's action  2's action  1's belief      2's belief      1's payoff      2's payoff
Round     1         1           2        [1. 2.]         [2. 1.]         [4. 6.]         [5. 3.]      (0.33, 0.67)  (0.67, 0.33)
Round     2         2           1        [2. 2.]         [2. 2.]         [6. 6.]         [6. 6.]      (0.50, 0.50)  (0.50, 0.50)
Round     3         1           1        [3. 2.]         [3. 2.]         [8. 6.]         [8. 6.]      (0.60, 0.40)  (0.60, 0.40)
Round     4         1           1        [4. 2.]         [4. 2.]         [10.  6.]       [10.  6.]    (0.67, 0.33)  (0.67, 0.33)
Round     5         1           1        [5. 2.]         [5. 2.]         [12.  6.]       [12.  6.]    (0.71, 0.29)  (0.71, 0.29)
Round     6         1           1        [6. 2.]         [6. 2.]         [14.  6.]       [14.  6.]    (0.75, 0.25)  (0.75, 0.25)

Round   440         1          1        [440.   2.]     [440.   2.]     [882.   6.]     [882.   6.]   (1.00, 0.00)  (1.00, 0.00)
Round   441         1          1        [441.   2.]     [441.   2.]     [884.   6.]     [884.   6.]   (1.00, 0.00)  (1.00, 0.00)
Round   442         1          1        [442.   2.]     [442.   2.]     [886.   6.]     [886.   6.]   (1.00, 0.00)  (1.00, 0.00)
Round   443         1          1        [443.   2.]     [443.   2.]     [888.   6.]     [888.   6.]   (1.00, 0.00)  (1.00, 0.00)
Round   444         1          1        [444.   2.]     [444.   2.]     [890.   6.]     [890.   6.]   (1.00, 0.00)  (1.00, 0.00)
Round   445         1          1        [445.   2.]     [445.   2.]     [892.   6.]     [892.   6.]   (1.00, 0.00)  (1.00, 0.00)
Round   446         1          1        [446.   2.]     [446.   2.]     [894.   6.]     [894.   6.]   (1.00, 0.00)  (1.00, 0.00)
The game converges in round 446
```

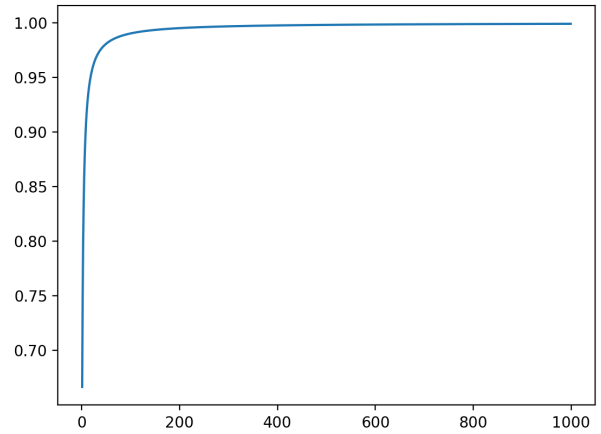


Yes, it can converge to the both of the pure-strategy Nash equilibria. In both cases, the players both choose (2, 2) and (1, 1) at the last several rounds. The figures also show it.

The only thing we have to do is run this code a few times, and because of the randomness, it converges to one of these two results.

Q3

```
Round           1's action  2's action  1's belief      2's belief      1's payoff      2's payoff
Round     1         1           1        [2. 1.]         [2. 1.]         [2. 0.]         [2. 0.]      (0.67, 0.33)  (0.67, 0.33)
Round     2         1           1        [3. 1.]         [3. 1.]         [3. 0.]         [3. 0.]      (0.75, 0.25)  (0.75, 0.25)
Round     3         1           1        [4. 1.]         [4. 1.]         [4. 0.]         [4. 0.]      (0.80, 0.20)  (0.80, 0.20)
Round     4         1           1        [5. 1.]         [5. 1.]         [5. 0.]         [5. 0.]      (0.83, 0.17)  (0.83, 0.17)
Round     5         1           1        [6. 1.]         [6. 1.]         [6. 0.]         [6. 0.]      (0.86, 0.14)  (0.86, 0.14)
Round     6         1           1        [7. 1.]         [7. 1.]         [7. 0.]         [7. 0.]      (0.88, 0.12)  (0.88, 0.12)
Round     7         1           1        [8. 1.]         [8. 1.]         [8. 0.]         [8. 0.]      (0.89, 0.11)  (0.89, 0.11)
Round   990         1           1      [991.   1.]     [991.   1.]     [991.   0.]     [991.   0.]   (1.00, 0.00)  (1.00, 0.00)
Round   991         1           1      [992.   1.]     [992.   1.]     [992.   0.]     [992.   0.]   (1.00, 0.00)  (1.00, 0.00)
Round   992         1           1      [993.   1.]     [993.   1.]     [993.   0.]     [993.   0.]   (1.00, 0.00)  (1.00, 0.00)
Round   993         1           1      [994.   1.]     [994.   1.]     [994.   0.]     [994.   0.]   (1.00, 0.00)  (1.00, 0.00)
Round   994         1           1      [995.   1.]     [995.   1.]     [995.   0.]     [995.   0.]   (1.00, 0.00)  (1.00, 0.00)
Round   995         1           1      [996.   1.]     [996.   1.]     [996.   0.]     [996.   0.]   (1.00, 0.00)  (1.00, 0.00)
Round   996         1           1      [997.   1.]     [997.   1.]     [997.   0.]     [997.   0.]   (1.00, 0.00)  (1.00, 0.00)
Round   997         1           1      [998.   1.]     [998.   1.]     [998.   0.]     [998.   0.]   (1.00, 0.00)  (1.00, 0.00)
Round   998         1           1      [999.   1.]     [999.   1.]     [999.   0.]     [999.   0.]   (1.00, 0.00)  (1.00, 0.00)
Round   999         1           1      [1000.   1.]   [1000.   1.]   [1000.   0.]   [1000.   0.]   (1.00, 0.00)  (1.00, 0.00)
The game converges in round 999
```

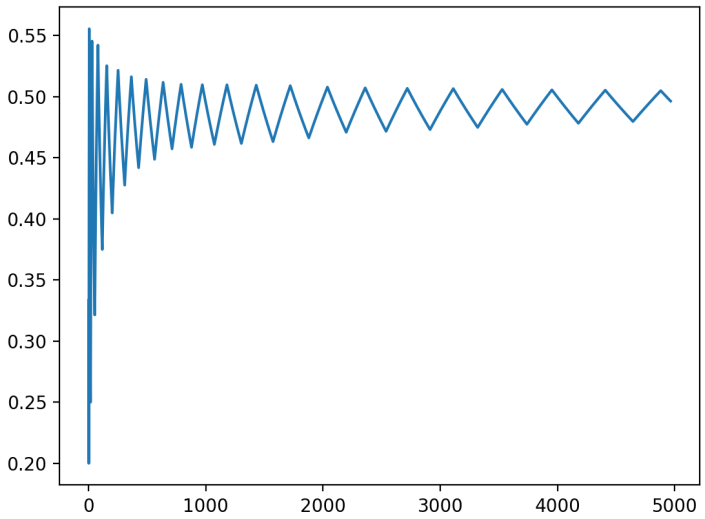


It can only converge to (1, 1). Although (2, 2) is also a pure-strategy Nash equilibria, it will never be chosen by the players. The reason is that no player will choose strategy 2 when strategy 1 is chosen by another player, or the payoff will be 0.

Q4

```
Round  4954         1           2       [2465. 2491.]   [3930. 1026.]   [4982. 4930.]   [3930. 4104.]   (0.50, 0.50)  (0.79, 0.21)
Round  4955         1           2       [2465. 2492.]   [3931. 1026.]   [4984. 4930.]   [3931. 4104.]   (0.50, 0.50)  (0.79, 0.21)
Round  4956         1           2       [2465. 2493.]   [3932. 1026.]   [4986. 4930.]   [3932. 4104.]   (0.50, 0.50)  (0.79, 0.21)
Round  4957         1           2       [2465. 2494.]   [3933. 1026.]   [4988. 4930.]   [3933. 4104.]   (0.50, 0.50)  (0.79, 0.21)
Round  4958         1           2       [2465. 2495.]   [3934. 1026.]   [4990. 4930.]   [3934. 4104.]   (0.50, 0.50)  (0.79, 0.21)
Round  4959         1           2       [2465. 2496.]   [3935. 1026.]   [4992. 4930.]   [3935. 4104.]   (0.50, 0.50)  (0.79, 0.21)
Round  4960         1           2       [2465. 2497.]   [3936. 1026.]   [4994. 4930.]   [3936. 4104.]   (0.50, 0.50)  (0.79, 0.21)
Round  4961         1           2       [2465. 2498.]   [3937. 1026.]   [4996. 4930.]   [3937. 4104.]   (0.50, 0.50)  (0.79, 0.21)
Round  4962         1           2       [2465. 2499.]   [3938. 1026.]   [4998. 4930.]   [3938. 4104.]   (0.50, 0.50)  (0.79, 0.21)
Round  4963         1           2       [2465. 2500.]   [3939. 1026.]   [5000. 4930.]   [3939. 4104.]   (0.50, 0.50)  (0.79, 0.21)
Round  4964         1           2       [2465. 2501.]   [3940. 1026.]   [5002. 4930.]   [3940. 4104.]   (0.50, 0.50)  (0.79, 0.21)
The game converges in round 4964
```

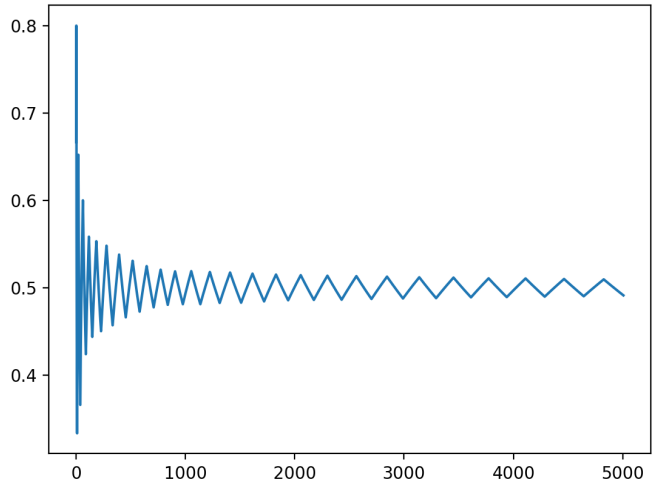
Yes. It can converge to the mixed-strategy in some cases though the probability is not exactly the same as (0.8, 0.2) and (0.5, 0.5). However, if we keep playing for more rounds, it will surely converge to (0.8, 0.2) and (0.5, 0.5).



In the above figure, it has a zigzag like feature, which is not that similar with previous pure strategy ones. That's because the players alternate back and forth between playing strategy 1 and 2.

Q5

```
Round  4991        1         2        [2460. 2533.]   [2486. 2507.]   [2533. 2460.]   [2486. 2507.]   (0.49, 0.51)  (0.50, 0.50)
Round  4992        1         2        [2460. 2534.]   [2487. 2507.]   [2534. 2460.]   [2487. 2507.]   (0.49, 0.51)  (0.50, 0.50)
Round  4993        1         2        [2460. 2535.]   [2488. 2507.]   [2535. 2460.]   [2488. 2507.]   (0.49, 0.51)  (0.50, 0.50)
Round  4994        1         2        [2460. 2536.]   [2489. 2507.]   [2536. 2460.]   [2489. 2507.]   (0.49, 0.51)  (0.50, 0.50)
Round  4995        1         2        [2460. 2537.]   [2490. 2507.]   [2537. 2460.]   [2490. 2507.]   (0.49, 0.51)  (0.50, 0.50)
Round  4996        1         2        [2460. 2538.]   [2491. 2507.]   [2538. 2460.]   [2491. 2507.]   (0.49, 0.51)  (0.50, 0.50)
Round  4997        1         2        [2460. 2539.]   [2492. 2507.]   [2539. 2460.]   [2492. 2507.]   (0.49, 0.51)  (0.50, 0.50)
Round  4998        1         2        [2460. 2540.]   [2493. 2507.]   [2540. 2460.]   [2493. 2507.]   (0.49, 0.51)  (0.50, 0.50)
Round  4999        1         2        [2460. 2541.]   [2494. 2507.]   [2541. 2460.]   [2494. 2507.]   (0.49, 0.51)  (0.50, 0.50)
Round  5000        1         2        [2460. 2542.]   [2495. 2507.]   [2542. 2460.]   [2495. 2507.]   (0.49, 0.51)  (0.50, 0.50)
Round  5001        1         2        [2460. 2543.]   [2496. 2507.]   [2543. 2460.]   [2496. 2507.]   (0.49, 0.51)  (0.50, 0.50)
Round  5002        1         2        [2460. 2544.]   [2497. 2507.]   [2544. 2460.]   [2497. 2507.]   (0.49, 0.51)  (0.50, 0.50)
Round  5003        1         2        [2460. 2545.]   [2498. 2507.]   [2545. 2460.]   [2498. 2507.]   (0.49, 0.51)  (0.50, 0.50)
Round  5004        1         2        [2460. 2546.]   [2499. 2507.]   [2546. 2460.]   [2499. 2507.]   (0.49, 0.51)  (0.50, 0.50)
Round  5005        1         2        [2460. 2547.]   [2500. 2507.]   [2547. 2460.]   [2500. 2507.]   (0.49, 0.51)  (0.50, 0.50)
Round  5006        1         2        [2460. 2548.]   [2501. 2507.]   [2548. 2460.]   [2501. 2507.]   (0.49, 0.51)  (0.50, 0.50)
The game converges in round 5006
```



Yes. It can converge to the mixed-strategy Nash equilibrium. For both of the players, the probability to choose each strategy is approximately 0.5 according to the result. That's because the players alternate back and forth between playing strategy 1 and 2.

Q6

For pure-strategy Nash equilibria

case1: (2, 2)

Round	1's action	2's action	1's belief	2's belief	1's payoff	2's payoff		
Round 1	2	2	[1. 2.]	[1. 2.]	[10. 20.]	[10. 20.]	(0.33, 0.67)	(0.33, 0.67)
Round 2	2	2	[1. 3.]	[1. 3.]	[10. 30.]	[10. 30.]	(0.25, 0.75)	(0.25, 0.75)
Round 3	2	2	[1. 4.]	[1. 4.]	[10. 40.]	[10. 40.]	(0.20, 0.80)	(0.20, 0.80)
Round 4	2	2	[1. 5.]	[1. 5.]	[10. 50.]	[10. 50.]	(0.17, 0.83)	(0.17, 0.83)
Round 5	2	2	[1. 6.]	[1. 6.]	[10. 60.]	[10. 60.]	(0.14, 0.86)	(0.14, 0.86)
Round 6	2	2	[1. 7.]	[1. 7.]	[10. 70.]	[10. 70.]	(0.12, 0.88)	(0.12, 0.88)
Round 7	2	2	[1. 8.]	[1. 8.]	[10. 80.]	[10. 80.]	(0.11, 0.89)	(0.11, 0.89)
Round 92	2	2	[1. 93.]	[1. 93.]	[10. 930.]	[10. 930.]	(0.01, 0.99)	(0.01, 0.99)
Round 93	2	2	[1. 94.]	[1. 94.]	[10. 940.]	[10. 940.]	(0.01, 0.99)	(0.01, 0.99)
Round 94	2	2	[1. 95.]	[1. 95.]	[10. 950.]	[10. 950.]	(0.01, 0.99)	(0.01, 0.99)
Round 95	2	2	[1. 96.]	[1. 96.]	[10. 960.]	[10. 960.]	(0.01, 0.99)	(0.01, 0.99)
Round 96	2	2	[1. 97.]	[1. 97.]	[10. 970.]	[10. 970.]	(0.01, 0.99)	(0.01, 0.99)
Round 97	2	2	[1. 98.]	[1. 98.]	[10. 980.]	[10. 980.]	(0.01, 0.99)	(0.01, 0.99)
Round 98	2	2	[1. 99.]	[1. 99.]	[10. 990.]	[10. 990.]	(0.01, 0.99)	(0.01, 0.99)
Round 99	2	2	[1. 100.]	[1. 100.]	[10. 1000.]	[10. 1000.]	(0.01, 0.99)	(0.01, 0.99)

The game converges in round 99

case2: (1, 1)

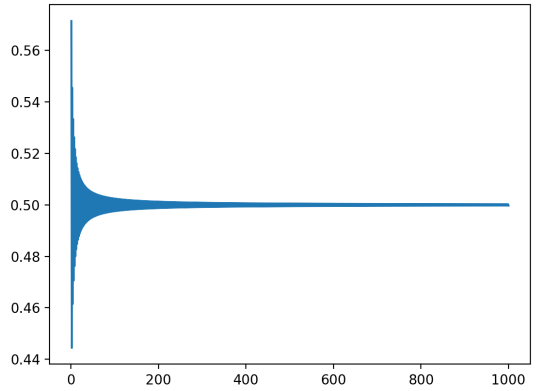
Round	1's action	2's action	1's belief	2's belief	1's payoff	2's payoff		
Round 1	1	2	[1. 2.]	[2. 1.]	[10. 20.]	[20. 10.]	(0.33, 0.67)	(0.67, 0.33)
Round 2	2	1	[2. 2.]	[2. 2.]	[20. 20.]	[20. 20.]	(0.50, 0.50)	(0.50, 0.50)
Round 3	1	1	[3. 2.]	[3. 2.]	[30. 20.]	[30. 20.]	(0.60, 0.40)	(0.60, 0.40)
Round 4	1	1	[4. 2.]	[4. 2.]	[40. 20.]	[40. 20.]	(0.67, 0.33)	(0.67, 0.33)
Round 5	1	1	[5. 2.]	[5. 2.]	[50. 20.]	[50. 20.]	(0.71, 0.29)	(0.71, 0.29)
Round 6	1	1	[6. 2.]	[6. 2.]	[60. 20.]	[60. 20.]	(0.75, 0.25)	(0.75, 0.25)
Round 7	1	1	[7. 2.]	[7. 2.]	[70. 20.]	[70. 20.]	(0.78, 0.22)	(0.78, 0.22)
Round 8	1	1	[8. 2.]	[8. 2.]	[80. 20.]	[80. 20.]	(0.80, 0.20)	(0.80, 0.20)
Round 9	1	1	[9. 2.]	[9. 2.]	[90. 20.]	[90. 20.]	(0.82, 0.18)	(0.82, 0.18)
Round 10	1	1	[10. 2.]	[10. 2.]	[100. 20.]	[100. 20.]	(0.83, 0.17)	(0.83, 0.17)
Round 11	1	1	[11. 2.]	[11. 2.]	[110. 20.]	[110. 20.]	(0.85, 0.15)	(0.85, 0.15)
Round 12	1	1	[12. 2.]	[12. 2.]	[120. 20.]	[120. 20.]	(0.86, 0.14)	(0.86, 0.14)
Round 13	1	1	[13. 2.]	[13. 2.]	[130. 20.]	[130. 20.]	(0.87, 0.13)	(0.87, 0.13)
Round 129	1	1	[129. 2.]	[129. 2.]	[1290. 20.]	[1290. 20.]	(0.98, 0.02)	(0.98, 0.02)
Round 130	1	1	[130. 2.]	[130. 2.]	[1300. 20.]	[1300. 20.]	(0.98, 0.02)	(0.98, 0.02)
Round 131	1	1	[131. 2.]	[131. 2.]	[1310. 20.]	[1310. 20.]	(0.98, 0.02)	(0.98, 0.02)
Round 132	1	1	[132. 2.]	[132. 2.]	[1320. 20.]	[1320. 20.]	(0.99, 0.01)	(0.99, 0.01)
Round 133	1	1	[133. 2.]	[133. 2.]	[1330. 20.]	[1330. 20.]	(0.99, 0.01)	(0.99, 0.01)
Round 134	1	1	[134. 2.]	[134. 2.]	[1340. 20.]	[1340. 20.]	(0.99, 0.01)	(0.99, 0.01)
Round 135	1	1	[135. 2.]	[135. 2.]	[1350. 20.]	[1350. 20.]	(0.99, 0.01)	(0.99, 0.01)
Round 136	1	1	[136. 2.]	[136. 2.]	[1360. 20.]	[1360. 20.]	(0.99, 0.01)	(0.99, 0.01)
Round 137	1	1	[137. 2.]	[137. 2.]	[1370. 20.]	[1370. 20.]	(0.99, 0.01)	(0.99, 0.01)
Round 138	1	1	[138. 2.]	[138. 2.]	[1380. 20.]	[1380. 20.]	(0.99, 0.01)	(0.99, 0.01)
Round 139	1	1	[139. 2.]	[139. 2.]	[1390. 20.]	[1390. 20.]	(0.99, 0.01)	(0.99, 0.01)
Round 140	1	1	[140. 2.]	[140. 2.]	[1400. 20.]	[1400. 20.]	(0.99, 0.01)	(0.99, 0.01)

The game converges in round 140

For mixed-strategy Nash equilibrium

(set initial belief to [1, 1.5], [1.5, 1])

```
Round   989         2         1       [496.  495.5]   [495.5 496. ]   [4960. 4955.]   [4955. 4960.]   (0.50, 0.50)  (0.50, 0.50)
Round   990         1         2       [496.  496.5]   [496.5 496. ]   [4960. 4965.]   [4965. 4960.]   (0.50, 0.50)  (0.50, 0.50)
Round   991         2         1       [497.  496.5]   [496.5 497. ]   [4970. 4965.]   [4965. 4970.]   (0.50, 0.50)  (0.50, 0.50)
Round   992         1         2       [497.  497.5]   [497.5 497. ]   [4970. 4975.]   [4975. 4970.]   (0.50, 0.50)  (0.50, 0.50)
Round   993         2         1       [498.  497.5]   [497.5 498. ]   [4980. 4975.]   [4975. 4980.]   (0.50, 0.50)  (0.50, 0.50)
Round   994         1         2       [498.  498.5]   [498.5 498. ]   [4980. 4985.]   [4985. 4980.]   (0.50, 0.50)  (0.50, 0.50)
Round   995         2         1       [499.  498.5]   [498.5 499. ]   [4990. 4985.]   [4985. 4990.]   (0.50, 0.50)  (0.50, 0.50)
Round   996         1         2       [499.  499.5]   [499.5 499. ]   [4990. 4995.]   [4995. 4990.]   (0.50, 0.50)  (0.50, 0.50)
Round   997         2         1       [500.  499.5]   [499.5 500. ]   [5000. 4995.]   [4995. 5000.]   (0.50, 0.50)  (0.50, 0.50)
Round   998         1         2       [500.  500.5]   [500.5 500. ]   [5000. 5005.]   [5005. 5000.]   (0.50, 0.50)  (0.50, 0.50)
Round   999         2         1       [501.  500.5]   [500.5 501. ]   [5010. 5005.]   [5005. 5010.]   (0.50, 0.50)  (0.50, 0.50)
Round  1000         1         2       [501.  501.5]   [501.5 501. ]   [5010. 5015.]   [5015. 5010.]   (0.50, 0.50)  (0.50, 0.50)
```



Yes. It can converge to three of the Nash equilibrium. It can be seen from the screenshots and figure above.

Q7

For pure-strategy Nash equilibria

case1: (2, 1)

```
Round           1's action  2's action  1's belief      2's belief      1's payoff      2's payoff
Round     1         2           1       [2. 1.]         [1. 2.]         [1. 2.]         [2. 1.]      (0.67, 0.33)  (0.33, 0.67)
Round     2         2           1       [3. 1.]         [1. 3.]         [1. 3.]         [3. 1.]      (0.75, 0.25)  (0.25, 0.75)
Round     3         2           1       [4. 1.]         [1. 4.]         [1. 4.]         [4. 1.]      (0.80, 0.20)  (0.20, 0.80)
Round     4         2           1       [5. 1.]         [1. 5.]         [1. 5.]         [5. 1.]      (0.83, 0.17)  (0.17, 0.83)
Round     5         2           1       [6. 1.]         [1. 6.]         [1. 6.]         [6. 1.]      (0.86, 0.14)  (0.14, 0.86)
Round     6         2           1       [7. 1.]         [1. 7.]         [1. 7.]         [7. 1.]      (0.88, 0.12)  (0.12, 0.88)
Round     7         2           1       [8. 1.]         [1. 8.]         [1. 8.]         [8. 1.]      (0.89, 0.11)  (0.11, 0.89)
Round     8         2           1       [9. 1.]         [1. 9.]         [1. 9.]         [9. 1.]      (0.90, 0.10)  (0.10, 0.90)
Round    92         2           1       [93.  1.]       [ 1. 93.]       [ 1. 93.]       [93.  1.]    (0.99, 0.01)  (0.01, 0.99)
Round    93         2           1       [94.  1.]       [ 1. 94.]       [ 1. 94.]       [94.  1.]    (0.99, 0.01)  (0.01, 0.99)
Round    94         2           1       [95.  1.]       [ 1. 95.]       [ 1. 95.]       [95.  1.]    (0.99, 0.01)  (0.01, 0.99)
Round    95         2           1       [96.  1.]       [ 1. 96.]       [ 1. 96.]       [96.  1.]    (0.99, 0.01)  (0.01, 0.99)
Round    96         2           1       [97.  1.]       [ 1. 97.]       [ 1. 97.]       [97.  1.]    (0.99, 0.01)  (0.01, 0.99)
Round    97         2           1       [98.  1.]       [ 1. 98.]       [ 1. 98.]       [98.  1.]    (0.99, 0.01)  (0.01, 0.99)
Round    98         2           1       [99.  1.]       [ 1. 99.]       [ 1. 99.]       [99.  1.]    (0.99, 0.01)  (0.01, 0.99)
Round    99         2           1       [100.   1.]     [  1. 100.]     [  1. 100.]     [100.   1.]  (0.99, 0.01)  (0.01, 0.99)
The game converges in round 99
```

case2: (1, 2)

```
Round           1's action  2's action  1's belief      2's belief      1's payoff      2's payoff
Round     1         1           2       [1. 2.]         [2. 1.]         [2. 1.]         [1. 2.]      (0.33, 0.67)  (0.67, 0.33)
Round     2         1           2       [1. 3.]         [3. 1.]         [3. 1.]         [1. 3.]      (0.25, 0.75)  (0.75, 0.25)
Round     3         1           2       [1. 4.]         [4. 1.]         [4. 1.]         [1. 4.]      (0.20, 0.80)  (0.80, 0.20)
Round     4         1           2       [1. 5.]         [5. 1.]         [5. 1.]         [1. 5.]      (0.17, 0.83)  (0.83, 0.17)
Round     5         1           2       [1. 6.]         [6. 1.]         [6. 1.]         [1. 6.]      (0.14, 0.86)  (0.86, 0.14)
Round     6         1           2       [1. 7.]         [7. 1.]         [7. 1.]         [1. 7.]      (0.12, 0.88)  (0.88, 0.12)
Round     7         1           2       [1. 8.]         [8. 1.]         [8. 1.]         [1. 8.]      (0.11, 0.89)  (0.89, 0.11)
Round     8         1           2       [1. 9.]         [9. 1.]         [9. 1.]         [1. 9.]      (0.10, 0.90)  (0.90, 0.10)
```

```
Round    91        1         2       [ 1. 92.]      [92.  1.]      [92.  1.]      [ 1. 92.]    (0.01, 0.99)  (0.99, 0.01)
Round    92        1         2       [ 1. 93.]      [93.  1.]      [93.  1.]      [ 1. 93.]    (0.01, 0.99)  (0.99, 0.01)
Round    93        1         2       [ 1. 94.]      [94.  1.]      [94.  1.]      [ 1. 94.]    (0.01, 0.99)  (0.99, 0.01)
Round    94        1         2       [ 1. 95.]      [95.  1.]      [95.  1.]      [ 1. 95.]    (0.01, 0.99)  (0.99, 0.01)
Round    95        1         2       [ 1. 96.]      [96.  1.]      [96.  1.]      [ 1. 96.]    (0.01, 0.99)  (0.99, 0.01)
Round    96        1         2       [ 1. 97.]      [97.  1.]      [97.  1.]      [ 1. 97.]    (0.01, 0.99)  (0.99, 0.01)
Round    97        1         2       [ 1. 98.]      [98.  1.]      [98.  1.]      [ 1. 98.]    (0.01, 0.99)  (0.99, 0.01)
Round    98        1         2       [ 1. 99.]      [99.  1.]      [99.  1.]      [ 1. 99.]    (0.01, 0.99)  (0.99, 0.01)
Round    99        1         2       [  1. 100.]    [100.   1.]    [100.   1.]    [  1. 100.]   (0.01, 0.99)  (0.99, 0.01)
The game converges in round 99
```

For mixed-strategy Nash equilibrium

(set initial belief to [1, 1.5], [1, 1.5])

```
Round   989        1         1       [496.  495.5]  [496.  495.5]  [495.5 496. ]  [495.5 496. ]   (0.50, 0.50)  (0.50, 0.50)
Round   990        2         2       [496.  496.5]  [496.  496.5]  [496.5 496. ]  [496.5 496. ]   (0.50, 0.50)  (0.50, 0.50)
Round   991        1         1       [497.  496.5]  [497.  496.5]  [496.5 497. ]  [496.5 497. ]   (0.50, 0.50)  (0.50, 0.50)
Round   992        2         2       [497.  497.5]  [497.  497.5]  [497.5 497. ]  [497.5 497. ]   (0.50, 0.50)  (0.50, 0.50)
Round   993        1         1       [498.  497.5]  [498.  497.5]  [497.5 498. ]  [497.5 498. ]   (0.50, 0.50)  (0.50, 0.50)
Round   994        2         2       [498.  498.5]  [498.  498.5]  [498.5 498. ]  [498.5 498. ]   (0.50, 0.50)  (0.50, 0.50)
Round   995        1         1       [499.  498.5]  [499.  498.5]  [498.5 499. ]  [498.5 499. ]   (0.50, 0.50)  (0.50, 0.50)
Round   996        2         2       [499.  499.5]  [499.  499.5]  [499.5 499. ]  [499.5 499. ]   (0.50, 0.50)  (0.50, 0.50)
Round   997        1         1       [500.  499.5]  [500.  499.5]  [499.5 500. ]  [499.5 500. ]   (0.50, 0.50)  (0.50, 0.50)
Round   998        2         2       [500.  500.5]  [500.  500.5]  [500.5 500. ]  [500.5 500. ]   (0.50, 0.50)  (0.50, 0.50)
Round   999        1         1       [501.  500.5]  [501.  500.5]  [500.5 501. ]  [500.5 501. ]   (0.50, 0.50)  (0.50, 0.50)
Round  1000        2         2       [501.  501.5]  [501.  501.5]  [501.5 501. ]  [501.5 501. ]   (0.50, 0.50)  (0.50, 0.50)
```

With carefully picking the initial belief, it can converge to three of the Nash equilibrium. It can be seen from the screenshots above. In the pure-strategy case, it can correctly converge to (1, 2) and (2, 1). In the mixed-strategy case, it can converge to the same as the question statement.

Q8

For pure-strategy Nash equilibria

case1: (1, 1)

```
Round   535        1         1       [534.   3.]     [535.   2.]     [1602.    6.]   [1070.    6.]   (0.99, 0.01)  (1.00, 0.00)
Round   536        1         1       [535.   3.]     [536.   2.]     [1605.    6.]   [1072.    6.]   (0.99, 0.01)  (1.00, 0.00)
Round   537        1         1       [536.   3.]     [537.   2.]     [1608.    6.]   [1074.    6.]   (0.99, 0.01)  (1.00, 0.00)
Round   538        1         1       [537.   3.]     [538.   2.]     [1611.    6.]   [1076.    6.]   (0.99, 0.01)  (1.00, 0.00)
Round   539        1         1       [538.   3.]     [539.   2.]     [1614.    6.]   [1078.    6.]   (0.99, 0.01)  (1.00, 0.00)
Round   540        1         1       [539.   3.]     [540.   2.]     [1617.    6.]   [1080.    6.]   (0.99, 0.01)  (1.00, 0.00)
Round   541        1         1       [540.   3.]     [541.   2.]     [1620.    6.]   [1082.    6.]   (0.99, 0.01)  (1.00, 0.00)
Round   542        1         1       [541.   3.]     [542.   2.]     [1623.    6.]   [1084.    6.]   (0.99, 0.01)  (1.00, 0.00)
Round   543        1         1       [542.   3.]     [543.   2.]     [1626.    6.]   [1086.    6.]   (0.99, 0.01)  (1.00, 0.00)
Round   544        1         1       [543.   3.]     [544.   2.]     [1629.    6.]   [1088.    6.]   (0.99, 0.01)  (1.00, 0.00)
Round   545        1         1       [544.   3.]     [545.   2.]     [1632.    6.]   [1090.    6.]   (0.99, 0.01)  (1.00, 0.00)
Round   546        1         1       [545.   3.]     [546.   2.]     [1635.    6.]   [1092.    6.]   (0.99, 0.01)  (1.00, 0.00)
Round   547        1         1       [546.   3.]     [547.   2.]     [1638.    6.]   [1094.    6.]   (0.99, 0.01)  (1.00, 0.00)
The game converges in round 547
```

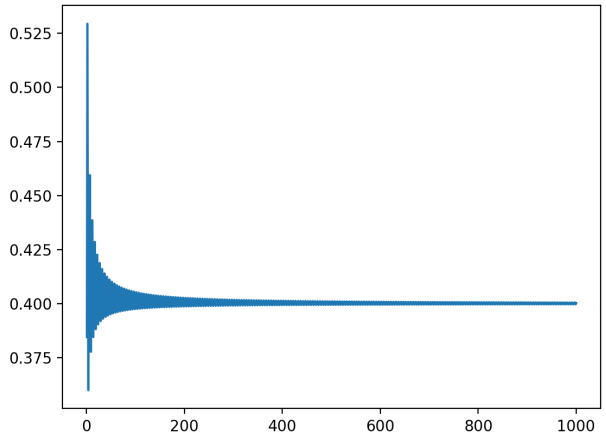
case2: (2, 2)

```
Round   536        2         2       [  2. 536.]    [  3. 535.]    [   6. 1072.]   [   6. 1605.]   (0.00, 1.00)  (0.01, 0.99)
Round   537        2         2       [  2. 537.]    [  3. 536.]    [   6. 1074.]   [   6. 1608.]   (0.00, 1.00)  (0.01, 0.99)
Round   538        2         2       [  2. 538.]    [  3. 537.]    [   6. 1076.]   [   6. 1611.]   (0.00, 1.00)  (0.01, 0.99)
Round   539        2         2       [  2. 539.]    [  3. 538.]    [   6. 1078.]   [   6. 1614.]   (0.00, 1.00)  (0.01, 0.99)
Round   540        2         2       [  2. 540.]    [  3. 539.]    [   6. 1080.]   [   6. 1617.]   (0.00, 1.00)  (0.01, 0.99)
Round   541        2         2       [  2. 541.]    [  3. 540.]    [   6. 1082.]   [   6. 1620.]   (0.00, 1.00)  (0.01, 0.99)
Round   542        2         2       [  2. 542.]    [  3. 541.]    [   6. 1084.]   [   6. 1623.]   (0.00, 1.00)  (0.01, 0.99)
Round   543        2         2       [  2. 543.]    [  3. 542.]    [   6. 1086.]   [   6. 1626.]   (0.00, 1.00)  (0.01, 0.99)
Round   544        2         2       [  2. 544.]    [  3. 543.]    [   6. 1088.]   [   6. 1629.]   (0.00, 1.00)  (0.01, 0.99)
Round   545        2         2       [  2. 545.]    [  3. 544.]    [   6. 1090.]   [   6. 1632.]   (0.00, 1.00)  (0.01, 0.99)
Round   546        2         2       [  2. 546.]    [  3. 545.]    [   6. 1092.]   [   6. 1635.]   (0.00, 1.00)  (0.01, 0.99)
Round   547        2         2       [  2. 547.]    [  3. 546.]    [   6. 1094.]   [   6. 1638.]   (0.00, 1.00)  (0.01, 0.99)
The game converges in round 547
```

For mixed-strategy Nash equilibrium

(set initial belief to [1.25, 1], [1, 1.25])

```
Round   993        1          2        [398.25 597.  ] [597.   398.25] [1194.75 1194.  ]      [1194.   1194.75]   (0.40, 0.60)  (0.60, 0.40)
Round   994        1          2        [398.25 598.  ] [598.   398.25] [1194.75 1196.  ]      [1196.   1194.75]   (0.40, 0.60)  (0.60, 0.40)
Round   995        2          1        [399.25 598.  ] [598.   399.25] [1197.75 1196.  ]      [1196.   1197.75]   (0.40, 0.60)  (0.60, 0.40)
Round   996        1          2        [399.25 599.  ] [599.   399.25] [1197.75 1198.  ]      [1198.   1197.75]   (0.40, 0.60)  (0.60, 0.40)
Round   997        2          1        [400.25 599.  ] [599.   400.25] [1200.75 1198.  ]      [1198.   1200.75]   (0.40, 0.60)  (0.60, 0.40)
Round   998        1          2        [400.25 600.  ] [600.   400.25] [1200.75 1200.  ]      [1200.   1200.75]   (0.40, 0.60)  (0.60, 0.40)
Round   999        1          2        [400.25 601.  ] [601.   400.25] [1200.75 1202.  ]      [1202.   1200.75]   (0.40, 0.60)  (0.60, 0.40)
Round  1000        2          1        [401.25 601.  ] [601.   401.25] [1203.75 1202.  ]      [1202.   1203.75]   (0.40, 0.60)  (0.60, 0.40)
```



With carefully picking the initial belief, it can converge to three of the Nash equilibrium. It can be seen from the screenshots and figure above. In the pure-strategy case, it converges to (1, 1) and (2, 2). In the mixed-strategy case, it can converge to (0.6, 0.4) and (0.4, 0.6).

Q9

For pure-strategy Nash equilibria

case1: (1, 1)

```
Round   126        1          1        [126.   2.]     [126.   2.]     [378. 254.]     [378. 254.]   (0.98, 0.02)  (0.98, 0.02)
Round   127        1          1        [127.   2.]     [127.   2.]     [381. 256.]     [381. 256.]   (0.98, 0.02)  (0.98, 0.02)
Round   128        1          1        [128.   2.]     [128.   2.]     [384. 258.]     [384. 258.]   (0.98, 0.02)  (0.98, 0.02)
Round   129        1          1        [129.   2.]     [129.   2.]     [387. 260.]     [387. 260.]   (0.98, 0.02)  (0.98, 0.02)
Round   130        1          1        [130.   2.]     [130.   2.]     [390. 262.]     [390. 262.]   (0.98, 0.02)  (0.98, 0.02)
Round   131        1          1        [131.   2.]     [131.   2.]     [393. 264.]     [393. 264.]   (0.98, 0.02)  (0.98, 0.02)
Round   132        1          1        [132.   2.]     [132.   2.]     [396. 266.]     [396. 266.]   (0.99, 0.01)  (0.99, 0.01)
Round   133        1          1        [133.   2.]     [133.   2.]     [399. 268.]     [399. 268.]   (0.99, 0.01)  (0.99, 0.01)
Round   134        1          1        [134.   2.]     [134.   2.]     [402. 270.]     [402. 270.]   (0.99, 0.01)  (0.99, 0.01)
Round   135        1          1        [135.   2.]     [135.   2.]     [405. 272.]     [405. 272.]   (0.99, 0.01)  (0.99, 0.01)
Round   136        1          1        [136.   2.]     [136.   2.]     [408. 274.]     [408. 274.]   (0.99, 0.01)  (0.99, 0.01)
Round   137        1          1        [137.   2.]     [137.   2.]     [411. 276.]     [411. 276.]   (0.99, 0.01)  (0.99, 0.01)
Round   138        1          1        [138.   2.]     [138.   2.]     [414. 278.]     [414. 278.]   (0.99, 0.01)  (0.99, 0.01)
Round   139        1          1        [139.   2.]     [139.   2.]     [417. 280.]     [417. 280.]   (0.99, 0.01)  (0.99, 0.01)
Round   140        1          1        [140.   2.]     [140.   2.]     [420. 282.]     [420. 282.]   (0.99, 0.01)  (0.99, 0.01)
```

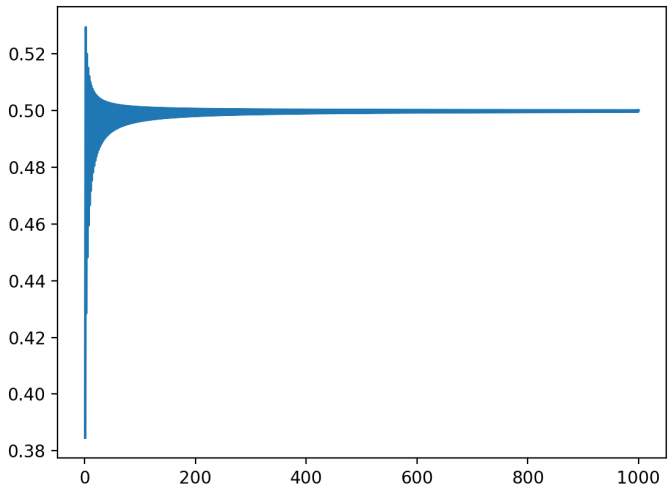
case2: (2, 2)

```
Round    85        2         2       [ 1. 86.]      [ 1. 86.]      [ 3. 88.]      [ 3. 88.]    (0.01, 0.99)  (0.01, 0.99)
Round    86        2         2       [ 1. 87.]      [ 1. 87.]      [ 3. 89.]      [ 3. 89.]    (0.01, 0.99)  (0.01, 0.99)
Round    87        2         2       [ 1. 88.]      [ 1. 88.]      [ 3. 90.]      [ 3. 90.]    (0.01, 0.99)  (0.01, 0.99)
Round    88        2         2       [ 1. 89.]      [ 1. 89.]      [ 3. 91.]      [ 3. 91.]    (0.01, 0.99)  (0.01, 0.99)
Round    89        2         2       [ 1. 90.]      [ 1. 90.]      [ 3. 92.]      [ 3. 92.]    (0.01, 0.99)  (0.01, 0.99)
Round    90        2         2       [ 1. 91.]      [ 1. 91.]      [ 3. 93.]      [ 3. 93.]    (0.01, 0.99)  (0.01, 0.99)
Round    91        2         2       [ 1. 92.]      [ 1. 92.]      [ 3. 94.]      [ 3. 94.]    (0.01, 0.99)  (0.01, 0.99)
Round    92        2         2       [ 1. 93.]      [ 1. 93.]      [ 3. 95.]      [ 3. 95.]    (0.01, 0.99)  (0.01, 0.99)
Round    93        2         2       [ 1. 94.]      [ 1. 94.]      [ 3. 96.]      [ 3. 96.]    (0.01, 0.99)  (0.01, 0.99)
Round    94        2         2       [ 1. 95.]      [ 1. 95.]      [ 3. 97.]      [ 3. 97.]    (0.01, 0.99)  (0.01, 0.99)
Round    95        2         2       [ 1. 96.]      [ 1. 96.]      [ 3. 98.]      [ 3. 98.]    (0.01, 0.99)  (0.01, 0.99)
Round    96        2         2       [ 1. 97.]      [ 1. 97.]      [ 3. 99.]      [ 3. 99.]    (0.01, 0.99)  (0.01, 0.99)
Round    97        2         2       [ 1. 98.]      [ 1. 98.]      [  3. 100.]    [  3. 100.]   (0.01, 0.99)  (0.01, 0.99)
Round    98        2         2       [ 1. 99.]      [ 1. 99.]      [  3. 101.]    [  3. 101.]   (0.01, 0.99)  (0.01, 0.99)
Round    99        2         2       [  1. 100.]    [  1. 100.]    [  3. 102.]    [  3. 102.]   (0.01, 0.99)  (0.01, 0.99)
The game converges in round 99
```

For mixed-strategy Nash equilibrium

(set initial belief to [1.25, 1], [1, 1.25])

```
Round   984        2         1       [493.25 493.  ] [493.   493.25] [1479.75 1479.5 ]      [1479.   1479.25]   (0.50, 0.50)  (0.50, 0.50)
Round   985        1         2       [493.25 494.  ] [494.   493.25] [1479.75 1480.5 ]      [1482.   1481.25]   (0.50, 0.50)  (0.50, 0.50)
Round   986        2         1       [494.25 494.  ] [494.   494.25] [1482.75 1482.5 ]      [1482.   1482.25]   (0.50, 0.50)  (0.50, 0.50)
Round   987        1         2       [494.25 495.  ] [495.   494.25] [1482.75 1483.5 ]      [1485.   1484.25]   (0.50, 0.50)  (0.50, 0.50)
Round   988        2         1       [495.25 495.  ] [495.   495.25] [1485.75 1485.5 ]      [1485.   1485.25]   (0.50, 0.50)  (0.50, 0.50)
Round   989        1         2       [495.25 496.  ] [496.   495.25] [1485.75 1486.5 ]      [1488.   1487.25]   (0.50, 0.50)  (0.50, 0.50)
Round   990        2         1       [496.25 496.  ] [496.   496.25] [1488.75 1488.5 ]      [1488.   1488.25]   (0.50, 0.50)  (0.50, 0.50)
Round   991        1         2       [496.25 497.  ] [497.   496.25] [1488.75 1489.5 ]      [1491.   1490.25]   (0.50, 0.50)  (0.50, 0.50)
Round   992        2         1       [497.25 497.  ] [497.   497.25] [1491.75 1491.5 ]      [1491.   1491.25]   (0.50, 0.50)  (0.50, 0.50)
Round   993        1         2       [497.25 498.  ] [498.   497.25] [1491.75 1492.5 ]      [1494.   1493.25]   (0.50, 0.50)  (0.50, 0.50)
Round   994        2         1       [498.25 498.  ] [498.   498.25] [1494.75 1494.5 ]      [1494.   1494.25]   (0.50, 0.50)  (0.50, 0.50)
Round   995        1         2       [498.25 499.  ] [499.   498.25] [1494.75 1495.5 ]      [1497.   1496.25]   (0.50, 0.50)  (0.50, 0.50)
Round   996        2         1       [499.25 499.  ] [499.   499.25] [1497.75 1497.5 ]      [1497.   1497.25]   (0.50, 0.50)  (0.50, 0.50)
Round   997        1         2       [499.25 500.  ] [500.   499.25] [1497.75 1498.5 ]      [1500.   1499.25]   (0.50, 0.50)  (0.50, 0.50)
Round   998        2         1       [500.25 500.  ] [500.   500.25] [1500.75 1500.5 ]      [1500.   1500.25]   (0.50, 0.50)  (0.50, 0.50)
Round   999        1         2       [500.25 501.  ] [501.   500.25] [1500.75 1501.5 ]      [1503.   1502.25]   (0.50, 0.50)  (0.50, 0.50)
Round  1000        2         1       [501.25 501.  ] [501.   501.25] [1503.75 1503.5 ]      [1503.   1503.25]   (0.50, 0.50)  (0.50, 0.50)
```



With carefully picking the initial belief, it can converge to three of the Nash equilibrium. It can be seen from the screenshots and figure above. In the pure-strategy case, it can correctly converge to (1, 1) and (2, 2). In the mixed-strategy case, it can converge to the same as the question statement.

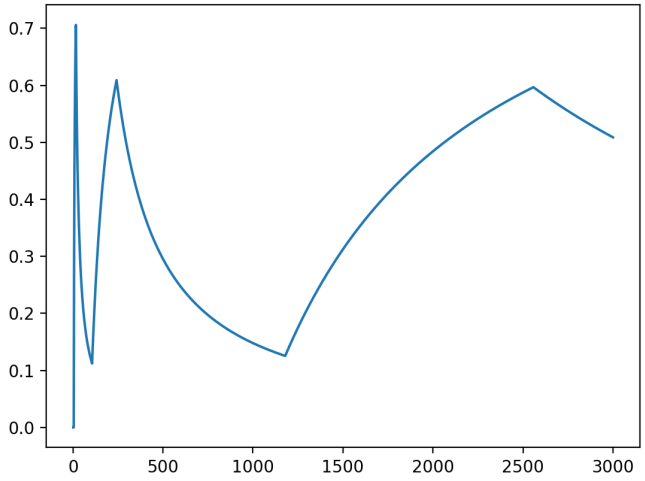
Q10

No.
Take the rock-paper-scissors game for example. Let's consider the game matrix below:

	R	S	P
R	0, 0	1, 0	0, 1
S	0, 1	0, 0	1, 0
P	1, 0	0, 1	0, 0

And setting the initial belief to [0, 1, 0], [1, 0, 0], maximum round to 3000, threshold to 0.0001.

We can see that even though the game is played for 3000 rounds, it still can't converge. The game will go into an infinite cycle. It can be observed by the figure below, which behaves pretty much differently to the previous games.



Note that from the screenshots in the previous ten questions, the probability for pure-strategy sometimes doesn't be exactly (0.00, 1.00) or (1.00, 0.00). However, it is obvious that if the games keep playing, the results will eventually be like that.

Code explanation

The class **FictiousPlay** contains information about each game, including number of the players, number of the actions (default to 2 in this case), current payoff, current belief, game matrix and current action.

```
class FictiousPlay:
    # Assume it always have only 2 players in this assignment
    # Then we can take advantages of it when implementing the code
    def __init__(self, matrix, player_num = 2, action_num = 2):
        self.matrix = matrix
        self.player_num = player_num
        self.action_num = action_num
        self.action_name = [i for i in range(action_num)]
        self.players_payoff = np.zeros(shape = (player_num, action_num))
        self.players_belief = np.ones(shape = (player_num, action_num))
        self.players_action = ["NA" for i in range(player_num)]
```

The function **calculate_payoff()** will calculate payoff depending on the current belief and game matrix. After the result is computed, it will also be updated to the game instance.

```
# calculate payoff depends on current belief
def calculate_payoff(self):
    for i in range(self.player_num): # for each player
        for j in range(self.action_num): # for each strategy
            payoff = 0
            for k in range(self.action_num): # consider the belief of player i
                payoff += self.players_belief[i][k] * self.matrix[i][j][k]
            self.players_payoff[i][j] = payoff
```

In the function **take_action(),** players will choose one strategy and the belief will be updated. If a player has more than one best response in each round, then the player will randomly pick one action.

```
# decide an action and update belief
def take_action(self):
    for i in range(self.player_num): # each player take best response based on payoff
        max_indices = np.where(self.players_payoff[i] == np.max(self.players_payoff[i]))[0]
        random_max_index = np.random.choice(max_indices)
        #print("Player {} take action {}".format(i, self.action_name[np.argmax(self.players_payoff[i])]))
        for j in range(self.player_num):
            if j != i:
                self.players_belief[j][random_max_index] += 1
        self.players_action[i] = self.action_name[random_max_index]
```

The **play()** function is one of the important parts in this assignment. For each round, two players will calculate the payoff and decide one strategy. After that, the row, which includes current round information, will be printed.

```
# play the game for n round or less when converge
def play(self, round = 1000, threshold = 0.005, plot = False):
    print("Round\t\t1's action  2's action\t1's belief\t2's belief\t1's payoff\t2's payoff")
    prev1 = 0
    prev2 = 0
    history = []
    y = []
    for i in range(round):
        self.calculate_payoff()
        self.take_action()
        self.calculate_payoff()

        player1_total = sum(self.players_belief[0])
        player2_total = sum(self.players_belief[1])
        prob1_1 = self.players_belief[0][0] / player1_total
        prob1_2 = self.players_belief[0][1] / player1_total
        prob2_1 = self.players_belief[1][0] / player2_total
        prob2_2 = self.players_belief[1][1] / player2_total
        history.append(prob1_1)
        y.append(i+1)
        print("Round {:>5d}\t{:>5d}\t{:>8d}\t{:10s}\t{:10s}\t{:10s}\t{:10s}   ({:.2f}, {:.2f})  ({:.2f}, {:.2f})".format(i+1,
                                                    self.players_action[0], self.players_action[1],
```

Note that for the last two columns in the output, it means the percentage of player1 and player2 belief.

```
    change1 = abs(prob1_1 - prev1)
    change2 = abs(prob2_1 - prev2)
    if change1 <= threshold and change2 <= threshold:
        print("The game converges in round {}".format(i+1))
        break
    prev1 = prob1_1
    prev2 = prob2_1
if plot:
    plt.plot(y, history)
    plt.show()
```

At the end, the function will compute the difference between current belief percentage and the previous one. If the difference is smaller than the threshold, the game stops.

This function also has another parameter called plot. When the parameter plot equals True, at the end of the function, the curve of player1's belief history will be printed. Therefore, we can observe if the game converges or not.

```
Q1 = FictiousPlay([
    [[-1, 1], [0, 3]],
    [[-1, 1], [0, 3]]
], [1, 2])
Q1.play()
```

For each game, we can easily initialize it with the game matrix. Then, call the play() function to start the game.

By default, the initial belief is an all one's array. We can also set beliefs using the set_belief() function to change the initial setting.